

Enhanced effect of quark mass variation in ^{229}Th and limits from Oklo dataV. V. Flambaum^{1,2,3} and R. B. Wiringa¹¹*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*²*School of Physics, University of New South Wales, Sydney 2052, Australia*³*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

(Received 30 July 2008; published 5 March 2009)

The effects of the variation of the dimensionless strong interaction parameter $X_q = m_q/\Lambda_{\text{QCD}}$ (m_q is the quark mass, Λ_{QCD} is the QCD scale) are enhanced about 1.5×10^5 times in the 7.6 eV “nuclear clock” transition between the ground and first excited states in the ^{229}Th nucleus and about 1×10^8 times in the relative shift of the 0.1 eV compound resonance in ^{150}Sm . The best terrestrial limit on the temporal variation of the fundamental constants, $|\delta X_q/X_q| < 4 \times 10^{-9}$ at 1.8 billion years ago ($|\dot{X}_q/X_q| < 2.2 \times 10^{-18}\text{y}^{-1}$), is obtained from the shift of this Sm resonance derived from the Oklo natural nuclear reactor data. The results for ^{229}Th and ^{150}Sm are obtained by extrapolation from light nuclei where the many-body calculations can be performed more accurately. The errors produced by such extrapolation may be smaller than the errors of direct calculations in heavy nuclei. The extrapolation results are compared with the “direct” estimates obtained using the Walecka model. A number of numerical relations needed for the calculations of the variation effects in nuclear physics and atomic spectroscopy have been obtained: for the nuclear binding energy $\delta E/E \approx -1.45 \delta m_q/m_q$, for the spin-orbit intervals $\delta E_{so}/E_{so} \approx -0.22 \delta m_q/m_q$, for the nuclear radius $\delta r/r \approx 0.3 \delta m_q/m_q$ (in units of Λ_{QCD}); for the shifts of nuclear resonances and weakly bound energy levels $\delta E_r \approx 10 \delta X_q/X_q$ MeV.

DOI: [10.1103/PhysRevC.79.034302](https://doi.org/10.1103/PhysRevC.79.034302)

PACS number(s): 06.20.Jr, 23.20.-g, 27.90.+b, 42.62.Fi

I. INTRODUCTION

Unification theories applied to cosmology suggest the possibility of variation of the fundamental constants in the expanding Universe (see, e.g., the review [1]). A review of recent results can be found, e.g., in Ref. [2]. In Ref. [3] it was suggested that there may be a five orders of magnitude enhancement of the variation effects in the low-energy transition between the ground and the first excited states in the ^{229}Th nucleus. This transition was suggested as a possible nuclear clock in Ref. [4]. Indeed, the transition is very narrow. The width of the excited state is estimated to be about 10^{-4} Hz [5]. The latest measurement of the transition energy [6] gives 7.6 ± 0.5 eV, compared to earlier values of 5.5 ± 1 eV [7] and 3.5 ± 1 eV [8]. Therefore, this transition may be investigated using laser spectroscopy where the relative accuracy has already reached 10^{-16} . Several experimental groups have already started working on this possibility [9]. However, a recent paper [10] claims that there is not any enhancement of the effects of the variation of the fundamental constants in this transition. The main aim of the present note is to demonstrate that the enhancement exists. We also estimate the relative shift of the 0.1 eV compound resonance in ^{150}Sm to obtain new limits on the variation of the fundamental constants from the Oklo natural nuclear reactor data [11–13].

We can measure only the variation of dimensionless parameters which do not depend on which units we use. In the standard model, the two most important dimensionless parameters are the fine structure constant $\alpha = e^2/\hbar c$ and the ratio of the electroweak unification scale determined by the Higgs vacuum expectation value (VEV) to the quantum chromodynamics (QCD) scale Λ_{QCD} [defined as the position of the Landau pole in the logarithm for the running strong coupling constant, $\alpha_s(r) \sim \text{constant}/\ln(\Lambda_{\text{QCD}}r/\hbar c)$]. The

variation of the Higgs VEV leads to the variation of the fundamental masses which are proportional to the Higgs VEV. The present work considers mainly effects produced by the variation of $X_q = m_q/\Lambda_{\text{QCD}}$ where $m_q = (m_u + m_d)/2$ is the average light quark mass. Within grand unification theories the relative variation of X_q may be 1–2 orders of magnitude larger than the variation of α [14]. Note that in the present work we do not consider effects of variation of the strange quark mass since they have larger uncertainty and should be treated separately. These effects were estimated in Refs. [3,15].

The results depend on the dimensionless parameter $X_q = m_q/\Lambda_{\text{QCD}}$. In all calculations it is convenient to assume that Λ_{QCD} is constant and calculate the dependence on the small parameter m_q . In other words, we measure all masses and energies in units of Λ_{QCD} and will simply restore Λ_{QCD} in the final results. Note that when a relative effect of the variation is enhanced it does not matter what units we use. The variation of the ratio of different units may be neglected anyway.

To study the time dependence of the fundamental constants in a laboratory one should measure the time dependence of the ratio of the ^{229}Th frequency ω to any other standard frequency ω_s (for example, to the frequency of the Cs primary standard or any optical standard) and extract the time dependence of the fundamental constants using the relation for the relative variations of dimensionless ratios:

$$\frac{d \ln(\omega/\omega_s)}{dt} = K_\alpha \frac{d \ln \alpha}{dt} + K_q \frac{d \ln X_q}{dt}. \quad (1)$$

The coefficients K_α and K_q must be calculated. If they are large, $K_{\alpha,q} \gg 1$, (as we find in Secs. II C, II D) it does not matter what standard ω_s we use since dependence of standards on the fundamental constants is not enhanced.

The shift of the resonance in ^{150}Sm has been found from the comparison of the abundances of ^{149}Sm and ^{150}Sm [11–13]. The isotope ^{149}Sm was transformed to ^{150}Sm by the neutron flux from the Oklo natural nuclear reactor. The cross section of this reaction is very sensitive to the position of the low-energy ($E_r \approx 0.1$ eV) compound resonance. The value of E_r extracted from the Oklo data is compared with the present value. To find the variation of the fundamental constants between the Oklo reactor time (1.8 billion years ago) and present time we use the formula for the shift of the resonance

$$\delta E_r = \left(k_q \frac{\delta X_q}{X_q} + k_\alpha \frac{\delta \alpha}{\alpha} \right) \text{ MeV}. \quad (2)$$

The coefficients k_q and k_α must be calculated (see Sec. III).

There is no doubt that the calculations in light nuclei may be performed with higher accuracy and more reliably than in heavy nuclei. The results are obtained starting from nucleon-nucleon interaction in vacuum, therefore, it is much easier to establish links with the fundamental QCD physics. On the other hand, such properties as the average nucleon binding energy and the spin-orbit interaction constant can be approximately predicted for all nuclei basing solely on the calculations in light nuclei (using well-known theoretical dependence on the number of neutrons and protons). This is the idea of our approach which is independent and complementary to the direct calculations in heavy nuclei, so it may be used as a general test of the accuracy of the theoretical predictions. Possibly, it is more accurate than the direct calculations in heavy nuclei based on the phenomenological forces (since it would be very hard to establish dependence of these phenomenological forces on quark mass).

II. THORIUM

To explain the origin of the enhancement we should present the small 7.6 eV interval between the ground and excited states in the ^{229}Th nucleus as a sum of a few components which nearly cancel each other and have different dependence on α we should separate the Coulomb energy from the remaining contributions to the energy. To study dependence on $X_q = m_q/\Lambda_{\text{QCD}}$ it is convenient to separate out the spin-orbit interaction energy:

$$\omega = E_b + E_{so} = 7.6 \text{ eV}. \quad (3)$$

Here E_b is the difference in bulk binding energies of the excited and ground states (including kinetic and potential energy but excluding the spin-orbit interaction) and E_{so} is the difference in the spin-orbit interaction energies $V_{ls}(\mathbf{l} \cdot \mathbf{s})$ in the excited and ground states. We make this separation because we expect E_b and E_{so} to have a very different dependence on $X_q = m_q/\Lambda_{\text{QCD}}$, as discussed below. In ^{229}Th the strength of the spin-orbit interaction is estimated to be $V_{ls} = -0.85$ MeV from Table 5-1 of Ref. [16]. The difference of $(\mathbf{l} \cdot \mathbf{s})$ between the excited and ground states can be easily calculated using the expansion of the wave functions over Nilsson orbitals presented in Table 4 of Ref. [17]: $E_{so} \approx 1.22 V_{ls} \approx -1.04$ MeV. (Note that without configuration

mixing, for the “pure” Nilsson excited state $[631]3/2^+$ and ground state $[633]5/2^+$, $E_{so} = 2 V_{ls}$.) Then Eq. (3) gives us $E_b \approx -E_{so} \approx 1$ MeV and

$$\frac{\delta \omega}{\omega} \approx \frac{E_{so}}{\omega} \left(\frac{\delta E_{so}}{E_{so}} - \frac{\delta E_b}{E_b} \right) = 1.3 \times 10^5 \left(\frac{\delta E_{so}}{E_{so}} - \frac{\delta E_b}{E_b} \right). \quad (4)$$

Qualitatively, we expect E_b and E_{so} to have a rather different dependence on X_q . In the Walecka model (which was used in Ref. [3] to estimate the enhancement factor) there is a significant cancellation between the σ and ω meson contributions to the mean-field potential and the total binding energy E , while the σ and ω mesons contribute with equal sign to the spin-orbit interaction constant V_{ls} [18]. A similar argument may be made from the variational Monte Carlo (VMC) calculations with realistic interactions used in Ref. [19] to evaluate binding energy dependence on X_q . These calculations use nucleon-nucleon potentials that fit NN scattering data together with three-nucleon potentials that reproduce the binding energies of light nuclei. The binding energy is the result of a significant cancellation between intermediate-range attraction due to two-pion exchange and short-range repulsion arising from heavy vector-meson exchange. However, spin-orbit splitting between nuclear levels has been found to be a coherent addition of short-range two-nucleon $\mathbf{l} \cdot \mathbf{s}$ interaction and multiple-pion exchange between three or more nucleons [20]. Thus if meson masses move in the same direction due to an underlying quark mass shift, contributions from pion exchange and heavy vector-meson exchange will cancel against each other in the binding energy, but reinforce each other in spin-orbit splittings.

A. Binding energies

The binding energy per nucleon and the spin-orbit interaction constant have a slow dependence on the nucleon number A . The total binding is dominated by the bulk terms, so we make the reasonable assumption that the relative variation of the bulk energy $\delta E/E$ with X_q is the same for the two levels in ^{229}Th and thus the relative variation of the bulk energy difference between this levels is $\delta E_b/E_b \approx \delta E/E$. Moreover, common factors in the theoretical dependence of the average nuclear parameters on the nucleon numbers cancel out in the relative variations. For example, the factor $A^{-1/3}$ in the dependence of the spin-orbit constant V_{ls} on A [16] cancels out in the relative variation $\delta E_{so}/E_{so}$ and the relative variation of V_{ls} is approximately the same in all nuclei. Therefore, it may be plausible to extract these relative variations from the type of calculations in light nuclei performed in Ref. [19]. The advantage of the light nuclei is that the calculations can be performed quite accurately, including different many-body effects. Their accuracy has been tested by comparison with the experimental data for the binding energies and by comparison of the results obtained using several sophisticated interactions (AV14, AV28, AV18 + UIX—see [19]). As the first step, the variations of the nuclear binding energies have been expressed in terms of the variations of nucleon, Δ , pion and vector-meson masses. The dependence of these masses on quark masses

TABLE I. Dimensionless derivatives $K = \frac{\delta E/E}{\delta X_q/X_q}$ of the binding energy over $X_q = m_q/\Lambda_{\text{QCD}}$.

^2H	^3H	^3He	^4He	^5He	^6He	^6Li	^7He	^7Li	^7Be	^8Be	^9Be
-1.39	-1.44	-1.55	-1.08	-1.24	-1.50	-1.36	-1.93	-1.50	-1.57	-1.35	-1.59

have been taken from Refs. [21,22]. The results for the relative variations of the total binding energies are presented in Table I (in the present work we add ^6He , ^7He , and ^9Be to this table). We see that all the results are close to the average value $\delta E/E \approx -1.45 \delta X_q/X_q$. The maximal deviations are for ^4He , which is especially tightly bound, and for ^7He , which is a resonant state.

B. Spin-orbit intervals

To find the dependence of the spin-orbit constant V_{ls} on m_q/Λ_{QCD} we calculate the spin-orbit splitting between the $p_{1/2}$ and $p_{3/2}$ levels in ^5He , ^7He , ^7Li , and ^9Be in the present work. We use the Argonne v_{18} two-nucleon and Urbana IX three-nucleon (AV18 + UIX) interaction which provides our best results for small nuclei (see Ref. [19] for details and references). In all calculations it is convenient to keep $\Lambda_{\text{QCD}} = \text{constant}$, i.e., measure the quark mass m_q in units of Λ_{QCD} . We restore Λ_{QCD} in the final answers. As the first step we calculate the binding energies of the ground and excited states shown in Table II and their dependence on the nucleon, Δ , pion, and vector-meson masses, $\Delta\mathcal{E}(m_H) = \frac{\delta E/E}{\delta m_H/m_H}$, shown in Table III. To find the dependence of these energies on the quark mass, we utilize the results of a Dyson-Schwinger equation (DSE) study of sigma terms in light-quark hadrons [21]. Equations (85–86) of that work give the rate of hadron mass variation as a function of the average light current-quark mass $m_q = (m_u + m_d)/2$ as

$$\frac{\delta m_H}{m_H} = \frac{\sigma_H}{m_H} \frac{\delta m_q}{m_q}, \quad (5)$$

with σ_H/m_H values of 0.498 for the pion, 0.030 for the ρ -meson, 0.043 for the ω -meson, 0.064 for the nucleon, and 0.041 for the Δ . The values for the ρ and ω -mesons were reduced to 0.021 and 0.034 in subsequent work [22]. We use an average of the ρ and ω terms of 0.030 for our short-range mass parameter m_V .

It is convenient to present the result for the variation of the spin-orbit splitting in the following form:

$$\delta E_{so} = \delta E_{1/2} - \delta E_{3/2} = E_{1/2} \frac{\delta E_{1/2}}{E_{1/2}} - E_{3/2} \frac{\delta E_{3/2}}{E_{3/2}}. \quad (6)$$

Accidentally, the calculated spin-orbit constant in ^5He is the same as in ^{229}Th , $V_{ls} = -0.83$ MeV (the $p_{1/2}$ - $p_{3/2}$ splitting in ^5He is $1.5V_{ls}$). The spin-orbit constant in ^9Be is larger than in ^{229}Th , in accord with the expected dependence $A^{-1/3}$ (see, e.g., Ref. [16]). The spin-orbit interval sensitivity coefficients K_{so} defined from

$$\frac{\delta E_{so}}{E_{so}} = K_{so} \frac{\delta m_q}{m_q} \quad (7)$$

for the quark mass variation in ^5He , ^7He , ^7Li , and ^9Be are -0.27 , -0.16 , -2.58 , and -0.22 , respectively. The ^5He , ^7He , and ^9Be values are all very similar, as all these nuclei are essentially one nucleon outside a 0^+ core. The ^7Li value is anomalously large because its ground and first excited states are primarily a triton outside an alpha core, so although δE_{so} is comparable to ^9Be , E_{so} is very small and not typical of the single-particle spin-orbit interaction we seek. Excluding the ^7Li result gives us an average value of $K_{so} = -0.22$ to use in ^{229}Th . Note that the estimate based on the Walecka model, outlined in Sec. V below, gives a very similar value $K_{so} = -0.2$.

C. Frequency shift

Substituting $\delta E_{so}/E_{so} = -0.22 \delta X_q/X_q$ and $\delta E_b/E_b = -1.45 \delta X_q/X_q$ into Eq. (4) we obtain the following energy shift for the 7.6 eV transition in ^{229}Th :

$$\delta\omega = 1.2 \frac{\delta X_q}{X_q} \text{ MeV}. \quad (8)$$

This corresponds to the frequency shift $\delta\nu = 3 \times 10^{20} \delta X_q/X_q$ Hz. The width of this transition is 10^{-4} Hz so one may hope to get the sensitivity to the variation of X_q about 10^{-25} per year. This is 10^{11} times better than the current atomic clock limit on the variation of X_q , $\sim 10^{-14}$ per year (see, e.g., Ref. [2]).

The corresponding relative energy shift is

$$\frac{\delta\omega}{\omega} = 1.5 \times 10^5 \frac{\delta X_q}{X_q}. \quad (9)$$

This enhancement coefficient may be compared with the coefficient 0.4×10^5 from Ref. [3] and 0.7×10^5 from

 TABLE II. Experimental and calculated energies for the ground ($p_{3/2}$) and first excited ($p_{1/2}$) states of $A = 5, 7, 9$ nuclei in MeV.

	$^5\text{He}(\frac{3}{2}^-)$	$^5\text{He}^*(\frac{1}{2}^-)$	$^7\text{He}(\frac{3}{2}^-)$	$^7\text{He}^*(\frac{1}{2}^-)$	$^7\text{Li}(\frac{3}{2}^-)$	$^7\text{Li}^*(\frac{1}{2}^-)$	$^9\text{Be}(\frac{3}{2}^-)$	$^9\text{Be}^*(\frac{1}{2}^-)$
AV18 + UIX	-25.26	-24.02	-21.77	-19.56	-33.33	-33.02	-45.39	-42.01
Expt.	-27.41	-26.23	-28.83	-26.23	-39.24	-38.77	-58.16	-55.38

TABLE III. Dimensionless derivatives $\Delta\mathcal{E}(m_H) = \frac{\delta E/E}{\delta m_H/m_H}$ of the binding energy to the different hadron masses and the sensitivity K after folding in the DSE values of $\delta m_H/m_H$.

	${}^5\text{He}(\frac{3}{2}^-)$	${}^5\text{He}^*(\frac{1}{2}^-)$	${}^7\text{He}(\frac{3}{2}^-)$	${}^7\text{He}^*(\frac{1}{2}^-)$	${}^7\text{Li}(\frac{3}{2}^-)$	${}^7\text{Li}^*(\frac{1}{2}^-)$	${}^9\text{Be}(\frac{3}{2}^-)$	${}^9\text{Be}^*(\frac{1}{2}^-)$
$m_N + \delta_N$	13.31	13.83	19.34	21.34	15.53	15.48	16.09	17.12
δ_Δ	-10.24	-10.72	-14.92	-16.63	-11.96	-11.88	-12.39	-13.27
m_π (+TNI)	-5.82	-6.07	-8.78	-9.73	-6.91	-6.88	-7.27	-7.76
m_V	40.87	42.84	60.46	67.54	48.11	47.81	50.21	53.85
$K = \frac{\delta E/E}{\delta m_q/m_q}$	-1.24	-1.29	-1.93	-2.13	-1.50	-1.49	-1.59	-1.70

Ref. [23]. The calculations in Ref. [23] have been done using the relativistic mean field theory (extended Walecka model) and some basic ideas from Ref. [3]. Thus, in this work we obtain an even larger enhancement. Here we present the relative variations from Refs. [3,23] for the new measured value 7.6 eV of the frequency ω (the old value was 3.5 eV, and we multiplied the numbers from [3,23] by $(3.5 \text{ eV})/\omega$). The difference between the results of different approaches looks pretty large. However, this is only a reflection of the current accuracy of all three calculations. The present aim is to show that the enhancement does exist.

Note that because of the huge enhancement it does not matter what units one will use to measure the frequency ω . In the calculations above we assumed that ω is measured in units of Λ_{QCD} . However, the variation of the ratio of any popular frequency standard to Λ_{QCD} does not have such enhancement and may be neglected.

D. Coulomb energy and effect of α variation

We also would like to comment about the possible enhancement of α variation. Reference [10] claims that this enhancement is impossible since the ground and excited states differ in the neutron state only and the neutron is neutral. Therefore, the ground and excited states have the same Coulomb energy and the interval does not change when α varies. We do not agree with this conclusion. Indeed, the total Coulomb energy of the ${}^{229}\text{Th}$ nucleus is 900 MeV (see, e.g., [16]) which is 10^8 times larger than the energy difference $\omega = 7.6 \text{ eV}$. Therefore, to have an enhancement it is enough to change the proton density distribution (deformation parameter) by more than 10^{-8} . Any change in the neutron state influences the nuclear mean field and proton distribution. For example, neutron removal changes the Coulomb energy of ${}^{229}\text{Th}$ by 1.3 MeV [16]. This gives us an upper estimate (and a natural scale) for the change of the Coulomb energy in the 7.6 eV ${}^{229}\text{Th}$ transition. One should expect a fraction of MeV change in any neutron transition in heavy nuclei. According to [17] the weight of admixed octupole vibrations to the 7.6 eV state exceeds 20%. Octupole vibrations involve both protons and neutrons. Therefore, the proton density distribution in the excited state is different from the ground state and this difference is only an order of magnitude smaller than the difference in neutron distribution.

The existence of the enhancement was confirmed by the direct calculation in Ref. [23]. The authors performed the

calculation of the change of the nuclear mean field acting on neutrons induced by the change of the proton density due to the variation of α . They obtained the enhancement coefficient 4×10^3 . This corresponds to the Coulomb energy difference 0.03 MeV. As it was mentioned above, an additional enhancement may come from the change of the nuclear deformation. Anyway, there is no doubt that the enhancement of the sensitivity to α variation in ${}^{229}\text{Th}$ does exist.

III. SHIFT OF THE RESONANCE IN SAMARIUM AND LIMITS FROM THE OKLO NATURAL NUCLEAR REACTOR DATA

In Refs. [24,25] we derived a simple formula to estimate the shift of the resonance or weakly bound energy level due to the variation of the fundamental constants. Let us assume a Fermi gas model in a square well nuclear potential of the radius R and depth V_0 . The energy of a single-particle energy level or resonance is determined as

$$E_r \approx \left\langle \frac{p^2}{2m} \right\rangle - V_0. \quad (10)$$

The momentum p in the square well is quantized, $p \approx \text{constant}/R$. Therefore,

$$E_r = \frac{K}{2mR^2} - V_0. \quad (11)$$

For a resonance or a weakly bound level $E_r \approx 0$ ($E_r \ll V_0$) and the constant $K \approx 2mR^2V_0$. Then we have

$$\begin{aligned} \delta E_r &= -\frac{K}{2mR^2} \left(\frac{\delta m}{m} + \frac{2\delta R}{R} \right) - \delta V_0 \\ &\approx -V_0 \left(\frac{\delta m}{m} + \frac{2\delta R}{R} + \frac{\delta V_0}{V_0} \right). \end{aligned} \quad (12)$$

This equation is also valid for a compound state with several excited particles. Indeed, the position of the compound state or resonance relative to the bottom of the potential well is determined mainly by the kinetic energy which scales as $1/R^2$ (both the Fermi energy and sum of the single-particle excitation energies scale this way). The shift of the resonance due to the residual interaction between excited particles ($\sim 0.1 \text{ MeV}$) is small in comparison with the depth of the potential well ($V_0 \approx 50 \text{ MeV}$) and may be neglected. Note that the depth of the potential V_0 is approximately the same in light and heavy nuclei. The radius of the well $R \approx 1.2A^{1/3}r_0$, therefore, the relative variation $\delta R/R = \delta r_0/r_0$ is the same too. Thus, the

TABLE IV. Binding energies S of the valence nucleon and the shift of the energy level $-\frac{\delta S}{\delta m_q/m_q}$ due to quark mass variation in MeV.

	^5He	^6He	^6Li	^7He	^7Li	^7Be	^8Be	^9Be
S_{expt}	-0.89	1.86	4.59	-0.43	7.25	5.61	18.90	1.67
S_{calc}	-2.24	-0.30	2.96	-3.19	5.11	3.52	16.89	-3.24
$-\frac{\delta S_{\text{expt}}}{\delta m_q/m_q}$	3.42	9.92	9.52	11.73	15.35	15.53	17.24	16.19
$-\frac{\delta S_{\text{calc}}}{\delta m_q/m_q}$	1.62	6.12	7.06	4.58	11.62	11.45	15.82	6.52

resulting shift of the resonance both in light and heavy nuclei is given by Eq. (12) and we may extrapolate the accurate result for light nuclei to the resonance in ^{150}Sm .

In Table IV we present binding energies of the valence nucleon, $S = -E$ (in MeV), and shift of the energy level (resonance), $\frac{\delta E}{\delta m_q/m_q} = -\frac{\delta S}{\delta m_q/m_q}$, due to the variation of the quark mass (in units MeV $\delta X_q/X_q$) in light nuclei with $A = 5, 6, 7, 8, 9$. In the derivation of Eq. (12) it was assumed that the valence nucleon is localized inside the potential well. This is not the case for ^5He where the valence nucleon is localized mainly outside the narrow potential well produced by the ^4He core. As a result the potential $\langle V \rangle$ averaged over the valence neutron wave function $1p_{3/2}$ is significantly smaller than the depth of the potential V_0 . This explains why the shift in ^5He (proportional to $\langle V \rangle$)—see Eq. (12) and Ref. [25] is much smaller than the shift in other nuclei. Another extreme case is ^8Be where $|E_r|$ is too large and the condition $E_r \ll V$ is not fulfilled. The results for other nuclei are reasonably close to the average value

$$\delta E_r \approx 10 \frac{\delta X_q}{X_q} \text{ MeV}. \quad (13)$$

We assume this shift for the 0.1 eV resonance in ^{150}Sm . This value does not contradict the order-of-magnitude estimates in Refs. [2,24,25]. Finally, we can add to this shift the contribution of α variation from Ref. [26] ($\delta E_r = -1.1 \pm 0.1 \text{ MeV } \delta\alpha/\alpha$). The total shift of the resonance in ^{150}Sm is

$$\delta E_r = 10 \left(\frac{\delta X_q}{X_q} - 0.1 \frac{\delta\alpha}{\alpha} \right) \text{ MeV}. \quad (14)$$

Now we can extract limits on the variation of X_q from the measurements of δE_r . Pioneering work in this area was done in Ref. [26]. We will use recent measurements [11–13] where the accuracy is higher. Reference [13] has given $|\delta E_r| < 20 \text{ MeV}$. Then Eq. (14) gives

$$\left| \frac{\delta X_q}{X_q} - 0.1 \frac{\delta\alpha}{\alpha} \right| < 2 \times 10^{-9}. \quad (15)$$

Reference [12] has given $-73 < \delta E_r < 62 \text{ MeV}$. This gives

$$\left| \frac{\delta X_q}{X_q} - 0.1 \frac{\delta\alpha}{\alpha} \right| < 7 \times 10^{-9}. \quad (16)$$

Reference [11] has given $-11.6 < \delta E_r < 26.0 \text{ MeV}$. This gives

$$\left| \frac{\delta X_q}{X_q} - 0.1 \frac{\delta\alpha}{\alpha} \right| < 2.6 \times 10^{-9}. \quad (17)$$

The limits on δE_r have been presented with 2σ range. Note that Ref. [11] has presented also the second, nonzero solution (it exists since the resonance has two tails): $-101.9 < \delta E_r < -79.6 \text{ MeV}$. However, Ref. [13] tentatively ruled out this solution based on the data for the shift of a similar resonance in the Gd nucleus.

Based on the results above we conclude that $|\frac{\delta X_q}{X_q}| < 4 \times 10^{-9}$ (for simplicity, we omit the small contribution of α variation here). Assuming linear time dependence during the last 1.8 billion years we obtain the best terrestrial limit on the variation of the fundamental constants

$$\left| \frac{\dot{X}_q}{X_q} \right| < 2.2 \times 10^{-18} \text{ y}^{-1}. \quad (18)$$

IV. VARIATION OF NUCLEAR RADIUS

Variation of the nuclear radius is needed to calculate effects of the fundamental constant variation in microwave atomic clocks where the transition frequency depends on a probability of the electron to be inside the nucleus. Indeed, the hyperfine interaction constant in heavy atoms has some sensitivity to the nuclear radius (including the Cs hyperfine transition which defines the unit of time, the second, and is used as a reference in numerous atomic and molecular clock experiments). This dependence was also requested by Schiller who proposed new experiments with hydrogen-like ions to search for the variation of the fundamental constants [27].

In Table V we present a comparison of calculated and measured charge nuclear radii for the stable $A = 2, 7$ nuclei. Determination of the sensitivity of the nuclear radius to quark mass variation is a more involved calculation than for the energy. While the deuteron can be solved exactly, the VMC calculations for $A \geq 3$ nuclei of Ref. [19] have to be modified. This is because the variational bound for the energy is a quadratic function near its minimum in the space of variational parameters, but the radius is a linear function. In the previous VMC calculations, the variational parameters were fixed at the energy minimum for the nominal hadron masses corresponding to $\delta m_q = 0$, and then not allowed to vary as the energy was evaluated for different δm_H . Consequently the

TABLE V. Experimental and calculated point proton rms radii for stable $A = 2-9$ nuclei in fm.

	^2H	^3H	^3He	^4He	^6He	^6Li	^7Li	^7Be	^9Be
AV18 + UIX	1.967	1.58	1.77	1.45	1.92	2.46	2.34	2.45	2.40
Expt.	1.953	1.59	1.75	1.45	1.93	2.39	2.25		2.38

“size” of the trial wave function was essentially unchanged. For the radius determination, we must allow this size to vary. We do this by multiplying a set of variational parameters (those to which the radius is most sensitive) by a scale factor, and then carefully reoptimize this scale factor for each δm_H . This allows us to determine $\Delta r(m_H) = \frac{\delta r/r}{\delta m_H/m_H}$. The $\Delta \mathcal{E}(m_H) = \frac{\delta E/E}{\delta m_H/m_H}$ reported in Ref. [19] are unchanged in this new minimization.

This procedure works well for the $A = 3, 4$ nuclei, and the $\Delta r(m_H)$ is presented in Table VI, along with the total sensitivity to the quark mass, $K_r \equiv \frac{\delta r/r}{\delta m_q/m_q}$, obtained by folding in the DSE values for $\delta m_H/m_H$. However, because our trial functions for $A = 6-9$ nuclei are not inherently stable against breakup into subclusters, we need to make an additional constraint when calculating their sensitivity. For some of these nuclei, we have trial functions that asymptotically look like the appropriate subclusters bound in a Coulomb well with the experimental separation energy: ^6Li is asymptotically an alpha and a deuteron bound by 1.47 MeV, ^7Li is asymptotically an alpha and a triton bound by 2.47 MeV, and ^7Be is asymptotically an alpha and a ^3He bound by 1.59 MeV. (^6He is asymptotically a three-body $\alpha + n + n$ cluster and ^9Be is an $\alpha + \alpha + n$ cluster, so they cannot be treated this way.) For a quark mass shift $\delta m_q/m_q = \pm 0.01$, we know the total energy shift expected from our previous calculations. We subtract that portion attributable to the alpha and deuteron or trinucleon subclusters, and use the remaining energy shift to adjust the asymptotic separation energy of our trial function. This allows the size of both the subclusters and the well binding them to vary. For $A = 6, 7$, we have carried out this calculation for the total sensitivity K_r only, and not for the individual $\Delta r(m_H)$; these results are also given in Table VI.

The average value of K_r is about 0.3, which may serve as an estimate of the sensitivity for all nuclei. There are significant deviations from this value for the very weakly bound deuteron ^2H and very strongly bound ^4He ; the latter is probably a solid lower bound.

The dependence of the nuclear radius on fundamental constants manifests itself in microwave transitions in atomic clocks which are used to search for the variation of the fundamental constants (see, e.g., Refs. [2,27]). The dependence of the hyperfine transition frequency ω_h on nuclear radius r in atoms with an external s -wave electron is approximately given by the following expressions (in units of Λ_{QCD}):

$$\frac{\delta \omega_h}{\omega_h} = K_{hr} \frac{\delta r}{r} = K_{hr} K_r \frac{\delta m_q}{m_q} \approx 0.3 K_{hr} \frac{\delta m_q}{m_q}, \quad (19)$$

$$K_{hr} \approx -\frac{(2\gamma - 1)\delta_h}{1 - \delta_h}, \quad (20)$$

$$\delta_h \approx 2(3 \times 10^{-5} Z^{4/3})^{2\gamma-1}, \quad (21)$$

where $\gamma = (1 - Z^2\alpha^2)^{1/2}$. For the Cs atom microwave standard the nuclear charge $Z = 55$ and $K_{hr} = -0.03$; for the Hg^+ microwave clock $Z = 80$ and $K_{hr} = -0.09$.

We also calculated the dependence of the ^4He radius on α : $\frac{\delta r/r}{\delta \alpha/\alpha} = 0.0034$. For heavy nuclei the relative role of the Coulomb repulsion increases and the sensitivity to the α variation should be larger.

V. ESTIMATES IN WALECKA MODEL

It is instructive to compare the results obtained by the extrapolation from light nuclei with some “direct” calculations. In this section we estimate the variations of the resonance positions and spin-orbit splittings in heavy nuclei using the Walecka model [28] where the strong nuclear potential is produced by scalar and vector meson exchanges:

$$V = -\frac{g_s^2}{4\pi} \frac{e^{-rm_s}}{r} + \frac{g_v^2}{4\pi} \frac{e^{-rm_v}}{r}. \quad (22)$$

Averaging Eq. (22) over the nuclear volume we can find the depth of the potential well [24]

$$V_0 = \frac{3}{4\pi r_0^3} \left(\frac{g_s^2}{m_s^2} - \frac{g_v^2}{m_v^2} \right). \quad (23)$$

TABLE VI. Dimensionless derivatives $\Delta r(m_H) = \frac{\delta r/r}{\delta m_H/m_H}$ of the point proton rms radii to the different hadron masses and the sensitivity K_r with respect to m_q after folding in the DSE values of $\delta m_H/m_H$.

	^2H	^3H	^3He	^4He	^6Li	^7Li	^7Be
$m_N + \delta_N$	-7.32	-4.81	-4.73	-3.04			
δ_Δ	4.07	3.32	3.28	2.18			
m_π (+TNI)	2.57	1.80	1.77	1.11			
m_V	-16.39	-12.97	-12.79	-8.50			
$K_r = \frac{\delta r/r}{\delta m_q/m_q}$	0.48	0.34	0.33	0.20	0.35	0.27	0.22

Here $2r_0 = 2.4$ fm is an internucleon distance. The result for the variation of the potential is

$$\frac{\delta V_0}{V_0} \approx -7.5 \frac{\delta m_S}{m_S} + 5.5 \frac{\delta m_V}{m_V} - 3 \frac{\delta r_0}{r_0}. \quad (24)$$

Here we have used $\frac{g_s^2}{m_S^2} / \frac{g_v^2}{m_V^2} = 266.9/195.7 = 1.364$ from Ref. [18]. There is an order of magnitude enhancement of the meson mass variation contributions due to the cancellation of the vector and scalar contributions in the denominator V_0 . Equation (12) for the variation of the resonance position becomes

$$\delta E_r \approx V_0 \left(7.5 \frac{\delta m_S}{m_S} - 5.5 \frac{\delta m_V}{m_V} - \frac{\delta m_N}{m_N} + \frac{\delta r_0}{r_0} \right). \quad (25)$$

We do not know the variation of r_0 in the Walecka model, therefore, to make a rough numerical estimate we neglect this term. As above we take dependence of the nucleon and meson masses on the current light quark mass $m_q = (m_u + m_d)/2$ from Refs. [21,22]: $\frac{\delta m_\omega}{m_\omega} = 0.034 \frac{\delta m_q}{m_q}$, $\frac{\delta m_N}{m_N} = 0.064 \frac{\delta m_q}{m_q}$, $\frac{\delta m_\sigma}{m_\sigma} = 0.013 \frac{\delta m_q}{m_q}$, $\frac{\delta m_\pi}{m_\pi} = 0.498 \frac{\delta m_q}{m_q}$. The vector meson in the Walecka model is usually identified with the ω -meson so $\frac{\delta m_V}{m_V} = 0.034 \frac{\delta m_q}{m_q}$. The scalar meson exchange, in fact, imitates both the σ meson exchange and two-pion exchange. Even if we neglect the two-pion exchange in zero approximation, there is virtual σ decay to two π . These virtual decays (loops on σ line in the NN -interaction diagrams with intermediate σ) very strongly modify the σ propagator and change its large distance asymptotics from $e^{-m_\sigma r}$ to $e^{-2m_\pi r}$ [29]. The mixing between σ and two π in m_S should increase the sensitivity coefficient for the variation of m_S . For an estimate we take an intermediate value between the neutron and vector meson mass sensitivity, $\frac{\delta m_S}{m_S} \sim 0.05 \frac{\delta m_q}{m_q}$. [Note that the positive contribution of $\frac{\delta r_0}{r_0}$ in Eq. (25) produces an effect similar to that of an increase of $\frac{\delta m_S}{m_S}$.] Then Eq. (25) gives

$$\delta E_r \sim 10 \frac{\delta X_q}{X_q} \text{ MeV}. \quad (26)$$

This rough estimate agrees with the result extrapolated from light nuclei. Note, however, that the accuracy of this estimate is very low due to the cancellations of different terms.

The scalar and vector mesons contribute with equal sign to the spin-orbit interaction constant V_{ls} [18]. Also, the spin-orbit interaction is inversely proportional to the nucleon mass m_N squared. Thus, we have

$$V_{ls} \propto \frac{1}{m_N^2} \left(\frac{g_s^2}{m_S^2} + \frac{g_v^2}{m_V^2} \right), \quad (27)$$

$$\frac{\delta E_{so}}{E_{so}} = -2 \left(\frac{\delta m_N}{m_N} + \frac{0.58 \delta m_S}{m_S} + \frac{0.42 \delta m_V}{m_V} \right) \approx -0.2 \frac{\delta m_q}{m_q}. \quad (28)$$

This estimate is close to the result ($-0.22 \frac{\delta m_q}{m_q}$) obtained by the extrapolation from light nuclei. Note, however, that here we neglected the effect of variation of r_0 which probably should increase the absolute value of the sensitivity coefficient.

VI. CONCLUSION

At the moment one can hardly calculate the sensitivity coefficient for the dependence of the strong interaction on the quark mass m_q with an accuracy better than a factor of 2. Moreover, it is hard to identify this dependence in phenomenological interactions which are used for the calculations in heavy nuclei. For example, it is not obvious that the scalar and vector mesons in the Walecka model are actually equivalent to free σ and ω mesons in particle physics. Therefore, to test conclusions obtained using the Walecka model, we explored a complementary approach. We performed the calculations in light nuclei where the interactions are well-known and the accuracy of the calculations is high. The binding energy per nucleon E_b , the spin-orbit interaction constant V_{ls} and the nuclear radius r have a slow dependence as a function of the nucleon number A . Moreover, the common factors (like $A^{-1/3}$ in the spin-orbit constant V_{ls} and $A^{1/3}$ in the nuclear radius) cancel out in the relative variations $\delta r/r$, $\delta V_{ls}/V_{ls}$, and $\delta E_b/E_b$. Therefore, we can extract these relative variations from the calculations in light nuclei and use them in heavy nuclei. The errors produced by such extrapolation may be smaller than the errors of direct calculations in heavy nuclei. So far, this extrapolation and direct calculations using Walecka model give comparable values of the enhancement factors in ^{229}Th and ^{150}Sm .

ACKNOWLEDGMENTS

V.V.F. is grateful to H. Feldmeier for useful discussions. This work is supported by the US Department of Energy, Office of Nuclear Physics, under contract DE-AC02-06CH11357, and by the Australian Research Council. Calculations were made at Argonne's Laboratory Computing Resource Center.

- [1] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).
- [2] V. V. Flambaum, Int. J. Mod. Phys. A **22**, 4937 (2007).
- [3] V. V. Flambaum, Phys. Rev. Lett. **97**, 092502 (2006).
- [4] E. Peik and Chr. Tamm, Europhys. Lett. **61**, 181 (2003).
- [5] E. V. Tkalya, A. N. Zherikhin, and V. I. Zhudov, Phys. Rev. C **61**, 064308 (2000); A. M. Dykhne and E. V. Tkalya, Pis'ma Zh. Eksp. Teor. Fiz. **67**, 233 (1998) [JETP Lett. **67**, 251 (1998)].

- [6] B. R. Beck, J. A. Becker, P. Beiersdorfer, G. V. Brown, K. J. Moody, J. B. Wilhelmy, F. S. Porter, C. A. Kilbourne, and R. L. Kelley, Phys. Rev. Lett. **98**, 142501 (2007).
- [7] Z. O. Guimarães-Filho and O. Helene, Phys. Rev. C **71**, 044303 (2005).
- [8] R. G. Helmer and C. W. Reich, Phys. Rev. C **49**, 1845 (1994).

- [9] E. Peik, Talk at workshop “In search for variation of fundamental constants and mass scales”, Perimeter Institute, 14–18 July 2008; E. Hudson, Talk at workshop “In search for variation of fundamental constants and mass scales”, Perimeter Institute, 14–18 July 2008; D. Habbs (private communication); D. DeMille (private communication); J. Torgerson (private communication).
- [10] A. C. Hayes and J. L. Friar, Phys. Lett. **B650**, 229 (2007).
- [11] C. R. Gould, E. I. Sharapov, and S. K. Lamoreaux, Phys. Rev. C **74**, 024607 (2006).
- [12] Yu. V. Petrov, A. I. Nazarov, M. S. Onegin, V. Yu. Petrov, and E. G. Sakhnovsky, Phys. Rev. C **74**, 064610 (2006).
- [13] Y. Fujii, A. Iwamoto, T. Fukahori, T. Ohnuki, M. Nakagawa, H. Hidaka, Y. Oura, and P. Möller, Nucl. Phys. **B573**, 377 (2000).
- [14] W. J. Marciano, Phys. Rev. Lett. **52**, 489 (1984); X. Calmet and H. Fritzsch, Eur. Phys. J. C **24**, 639 (2002); P. Langacker, G. Segré, and M. J. Strassler, Phys. Lett. **B528**, 121 (2002); T. Dent and M. Fairbairn, Nucl. Phys. **B653**, 256 (2003); C. Wetterich, J. Cosmol. Astropart. Phys. 10 (2003) 002; Phys. Lett. **B561**, 10 (2003).
- [15] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **67**, 083507 (2003).
- [16] A. Bohr and B. R. Mottelson, *Nuclear Structure* (W. A. Benjamin, New York, 1974), Vol. II.
- [17] K. Gulda *et al.*, Nucl. Phys. **A703**, 45 (2002).
- [18] R. Brockmann and W. Weise, Phys. Rev. C **16**, 1282 (1977).
- [19] V. V. Flambaum and R. B. Wiringa, Phys. Rev. C **76**, 054002 (2007).
- [20] S. C. Pieper and V. R. Pandharipande, Phys. Rev. Lett. **70**, 2541 (1993).
- [21] V. V. Flambaum, A. Höll, P. Jaikumar, C. D. Roberts, and S. V. Wright, Few-Body Syst. **38**, 31 (2006).
- [22] A. Höll, P. Maris, C. D. Roberts, and S. V. Wright, Nucl. Phys. Proc. Suppl. **161**, 87 (2006).
- [23] Xiao-tao He and Zhong-zhou Ren, J. Phys. G: Nucl. Part. Phys. **34**, 1611 (2007).
- [24] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **65**, 103503 (2002).
- [25] V. F. Dmitriev and V. V. Flambaum, Phys. Rev. D **67**, 063513 (2003).
- [26] A. I. Shlyakhter, Nature (London) **264**, 340 (1976); Yu. V. Petrov, Sov. Phys. Usp. **20**, 937 (1977); T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996).
- [27] S. Schiller, Phys. Rev. Lett. **98**, 180801 (2007).
- [28] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
- [29] V. V. Flambaum and E. V. Shuryak, Phys. Rev. C **76**, 065206 (2007).