Resonance effects in distorted-wave Born approximation analyses of $(\vec{d}, p)$ analyzing powers at very low energies

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In very low energy (d, p) reactions, resonance effects are explicitly taken into account for zero-range distorted-wave Born approximation calculations by restricting the reaction amplitudes to those specified by the spin-parity of the resonance. The application of formulas of tensor analyzing powers to ${}^6\mathrm{Li}(\vec{d}, p){}^7\mathrm{Li}$(g.s.) reactions in resonance regions provides us a clear understanding of the characteristics of the measured analyzing powers and aides in the determination of the spin-parity of the resonance states concerned at $E_d = 90$, 600, and 960 keV.

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I. Introduction. Nuclear reactions at very low energies have been investigated with special interest in them as the source of the production of elements in the universe. In particular, the analyzing powers of polarized beams have attracted attention by providing detailed information of reaction mechanisms [1,2]. However, the method of theoretical analyses of analyzing power data has not always been established when resonances are concerned. For example, the compound nucleus (CN) theory has been useful in the analysis of low energy resonance reactions but is not suitable for application to the analyzing powers because the final channel is disconnected from the initial one because of the formation of the CN. On the other hand, direct reaction theories have been useful in the analysis of the analyzing power data but they have no definite program to include the resonance effect when the resonance mechanism is not clear. In the present report, we take up the distorted-wave Born approximation (DWBA) [3] as the direct reaction theory and examine the resonance effect in (d, p) reactions.

Recently, tensor analyzing powers T_{2q} $(q = 0, 1, 2)$ of polarized deuterons have been measured for the ${}^6\mathrm{Li}(\vec{d}, p){}^7\mathrm{Li}$(g.s., $3/2^-$) reaction at $E_d = 90$ keV [2], which is covered by a resonance region found in $\alpha + \alpha$ scattering. According to Ref. [4], the resonance pole is situated at 80 keV below the $d + {}^6\mathrm{Li}$ threshold with spin-parity 2^+ and width 800 keV. In Ref. [2], zero-range DWBA calculations that include distortions by central and spin-orbit potentials have been performed for the reaction but the calculations have not succeeded in reproducing the measured analyzing powers despite a wide search of the theoretical parameters. For example, a parameter set that reproduces the data of T_{20} and T_{22} gives T_{21} with the sign opposite to that of the measured and vice versa. Then the problem is to examine if the above difficulty of the DWBA is avoided when the resonance effects are considered.

A resonance state has its own spin-parity and in resonance reactions transition amplitudes designated by this spin-parity will be dominated over other amplitudes. We take account of the resonance nature of reactions by considering such superiority of the amplitude of the resonance spin-parity. Particularly, in an ideal case where contributions of nonresonance amplitudes are negligible, we restrict the reaction amplitudes to those that have the resonance spin-parity. Later our interest will focus on this ideal case. At present, the incident beam consists of mostly S waves due to the very low energy. Thus the total spin of the system is equal to the incident channel spin s_i defined by $\boldsymbol{s}_i = \boldsymbol{s}_d + \boldsymbol{s}_A$, where $\boldsymbol{s}_d$ and $\boldsymbol{s}_A$ are the spins of the deuteron and the target nucleus A. That is, the resonance spin should be one of $s_i = |s_d - s_A|, \ldots, s_d + s_A$. In the following sections, we expand the DWBA transition amplitude into terms of the channel spin s_i and see the contribution of each s_i term to observables, for instance, to the analyzing powers. When the resonance spin, I_R, is known, the choice $s_i = I_R$ provides the contribution of the resonance to the observables. When I_R is unknown, one determines the resonance spin by examining if the contribution of a specific s_i explains the experimental data.

II. DWBA amplitude of (d,p) reactions in channel spin representation. The transition amplitude T_{fi} for $A(d, p)B$ reactions is described in the DWBA for stripping reaction models [3],

$$T_{fi} = \langle \Psi_f^{(-)} | v_{np} | \Psi_i^{(+)} \rangle, \tag{1}$$

where $\Psi_i^{(+)}(\Psi_f^{(-)})$ describes the incident (final) distorted wave with the outgoing (incoming) boundary condition and v_{np} is the neutron-proton interaction. These wave functions include internal wave functions of related nuclei, for example, in $\Psi_i^{(+)}$, those of the deuteron and the target A. We introduce the channel spin function $\chi_{s_i \nu_i}$, where ν's denote z components of spins, by composing the spin functions of the deuteron and the nucleus A, $\chi_{s_d \nu_d}$ and $\chi_{s_A \nu_A}$, as

$$\chi_{s_i \nu_i} = \sum_{\nu_d \nu_A} (s_d s_A \nu_d \nu_A | s_i \nu_i) \chi_{s_d \nu_d} \chi_{s_A \nu_A}. \tag{2}$$

Conversely,

$$\chi_{s_d \nu_d} \chi_{s_A \nu_A} = \sum_{s_i \nu_i} (s_d s_A \nu_d \nu_A | s_i \nu_i) \chi_{s_i \nu_i}. \tag{3}$$

Denote the plane wave of the d-A relative motion by ϕ_i. We introduce wave distortion on $\phi_i \chi_{s_i \nu_i}$ to produce the channel distorted wave $\psi_{s_i \nu_i}^{(+)}$, where the Coulomb and central interactions are considered as the source of the wave distortion, to see pure resonance effects. Such a choice of distortion has produced reasonable cross sections at low energies. The central interactions have some s_i dependencies reflecting the superiority of the resonance amplitude but their details depend on the resonance mechanism. At present, we utilize the s_i dependence to identify the transition amplitude concerned. Based on such considerations, from Eq. (3) we get

$$\Psi_i^{(+)} = \sum_{s_i\nu_i} (s_d s_A \nu_d \nu_A | s_i \nu_i) \psi_{s_i\nu_i}^{(+)}. \tag{4}$$

The final channel spin function $\chi_{s_f\nu_f}$ is given by

$$\chi_{s_f\nu_f} = \sum_{\nu_p\nu_B} (s_p s_B \nu_p \nu_B | s_f \nu_f) \chi_{s_p\nu_p} \chi_{s_B\nu_B}. \tag{5}$$

Denoting the final channel distorted wave by $\psi_{s_f\nu_f}^{(-)}$, where p-B central interactions are assumed in addition to the Coulomb interaction, the total distorted wave in the final state $\Psi_f^{(-)}$ is given by

$$\Psi_f^{(-)} = \sum_{s_f\nu_f} (s_p s_B \nu_p \nu_B | s_f \nu_f) \psi_{s_f\nu_f}^{(-)}. \tag{6}$$

Using Eqs. (4) and (6), we get the transition amplitude in the channel spin representation,

$$T_{fi} = \sum_{s_i\nu_i} \sum_{s_f\nu_f} (s_d s_A \nu_d \nu_A | s_i \nu_i)(s_p s_B \nu_p \nu_B | s_f \nu_f) \times \left\langle \psi_{s_f\nu_f}^{(-)} \middle| v_{np} \middle| \psi_{s_i\nu_i}^{(+)} \right\rangle. \tag{7}$$

To calculate $\langle \psi_{s_f\nu_f}^{(-)} | v_{np} | \psi_{s_i\nu_i}^{(+)} \rangle$, we introduce further simplifications. Describing the wave function of the deuteron internal motion by $\varphi_d(\boldsymbol{\xi})$ with the neutron-proton relative coordinate $\boldsymbol{\xi}$, the usual zero-range approximation, which is widely used as the established one, gives

$$v_{np}\varphi_d(\boldsymbol{\xi}) = D\delta(\boldsymbol{\xi}), \tag{8}$$

where D is a constant. The incident partial wave is restricted to the S wave, because of the very low incident energy. Then

$$v_{np}\psi_{s_i\nu_i}^{(+)} = \frac{D}{\sqrt{4\pi}} \delta(\boldsymbol{\xi}) R_{s_i}(k_i\rho) \chi_{s_i\nu_i}, \tag{9}$$

where the incident momentum is denoted by $\boldsymbol{k}_i$, the d-A relative coordinate by $\boldsymbol{\rho}$, and the radial part of the distorted wave by $R_{s_i}(k_i\rho)$.

The wave distortion in the final state is applied to the p-B relative motion ϕ_f and the distorted wave $\phi_f^{(-)}$ is given by the partial-wave expansion,

$$\phi_f^{(-)} = 4\pi \sum_{\ell_p m_p} (i)^{\ell p} Y_{\ell_p m_p}^*(\hat{\boldsymbol{k}}_f) Y_{\ell_p m_p}(\hat{\boldsymbol{\rho}}) R_{\ell_p}\left(\frac{A}{A+1}k_f\rho\right), \tag{10}$$

where the final momentum is denoted by $\boldsymbol{k}_f$ and the radial part of the distorted wave by $R_{\ell_p}(\frac{A}{A+1}k_f\rho)$. The final channel distorted wave is given by

$$\psi_{s_f\nu_f}^{(-)} = \phi_f^{(-)} \chi_{s_f\nu_f}, \tag{11}$$

where $\chi_{s_f\nu_f}$ is obtained by Eq. (5) with $\chi_{s_B\nu_B}$, which is given by the stripping model. The neutron is captured into an (ℓ_n, j_n) orbit around the target nucleus A. Then

$$\chi_{s_B\nu_B} = \sum_{\mu_n} (j_n s_A \mu_n \nu_A | s_B \nu_B) \phi_{j_n\mu_n} \chi_{s_A\nu_A}, \tag{12}$$

with

$$\phi_{j_n\mu_n} = \sum_{m_n} (s_n \ell_n \nu_n m_n | j_n \mu_n) \chi_{s_n\nu_n} Y_{\ell_n m_n}(\hat{\boldsymbol{\rho}}) R_n(\beta\rho), \tag{13}$$

where $R_n(\beta\rho)$ is the radial wave function of the captured neutron and β is calculated from the binding energy of the neutron as usual. By the use of Eqs.(9)–(13), we get

$$\begin{aligned}
&\left\langle \psi_{s_f\nu_f}^{(-)} \middle| v_{np} \middle| \psi_{s_i\nu_i}^{(+)} \right\rangle \\
&\quad = i^{\ell_n} \sqrt{4\pi(2s_d+1)(2j_n+1)} \sum_J (2J+1) W(s_p s_n s_i s_A; s_d J) \\
&\quad \times W(\ell_n j_n J s_A; s_n s_B) \sum_M (s_p J \nu_p M | s_i \nu_i) \\
&\quad \times \sum_{m_n} (J \ell_n M m_n | s_B \nu_B)(s_p s_B \nu_p \nu_B | s_f \nu_f) Y_{\ell_n m_n}^*(\hat{\boldsymbol{k}}_f) I(s_i),
\end{aligned} \tag{14}$$

with

$$I(s_i) = D \int R_{\ell_p=\ell_n}^*\left(\frac{A}{A+1}k_f\rho\right) R_n(\beta\rho) R_{s_i}(k_i\rho) \rho^2 d\rho. \tag{15}$$

Here the Racah coefficient $W(s_p s_n s_i s_A; s_d J)$ describes the change of the order of the sum of three vectors from $(\boldsymbol{s}_p + \boldsymbol{s}_n = \boldsymbol{s}_d, \boldsymbol{s}_d + \boldsymbol{s}_A = \boldsymbol{s}_i)$ to $(\boldsymbol{s}_n + \boldsymbol{s}_A = \boldsymbol{J}, \boldsymbol{s}_p + \boldsymbol{J} = \boldsymbol{s}_i)$. A similar change is applied to the final state.

III. Cross section and tensor analyzing powers and $^6\mathrm{Li}(\vec{d},p)^7\mathrm{Li}$ *reaction.* Denote T matrix by $\boldsymbol{M}$. Differential cross sections $d\sigma/d\Omega$ and the tensor analyzing powers T_{2q} [5] are

$$\frac{d\sigma}{d\Omega} = \frac{1}{3(2s_A+1)} \mathrm{Tr}(\boldsymbol{M}\boldsymbol{M}^\dagger) \tag{16}$$

and

$$T_{2q} = \frac{1}{\mathrm{Tr}(\boldsymbol{M}\boldsymbol{M}^\dagger)} \mathrm{Tr}\left(\boldsymbol{M}\tau_q^2\boldsymbol{M}^\dagger\right), \tag{17}$$

where τ_q^2 is the spin tensor that describes the tensor polarization of the incident deuteron. Using Eq. (14) for matrix elements of $\boldsymbol{M}$, we obtain by some Racah algebra manipulations, for $y \parallel \vec{k}_i \times \vec{k}_f$ and $z \parallel \vec{k}_i$,

$$\begin{aligned}
\mathrm{Tr}(\boldsymbol{M}\boldsymbol{M}^\dagger) &= \sum_{s_i} 3(2s_B+1)(2j_n+1)(2s_i+1) \\
&\times \sum_J (2J+1) W^2\left(\frac{1}{2}\frac{1}{2}s_i s_A; 1J\right) \\
&\times W^2\left(\ell_n j_n J s_A; \frac{1}{2}s_B\right) |I(s_i)|^2
\end{aligned} \tag{18}$$

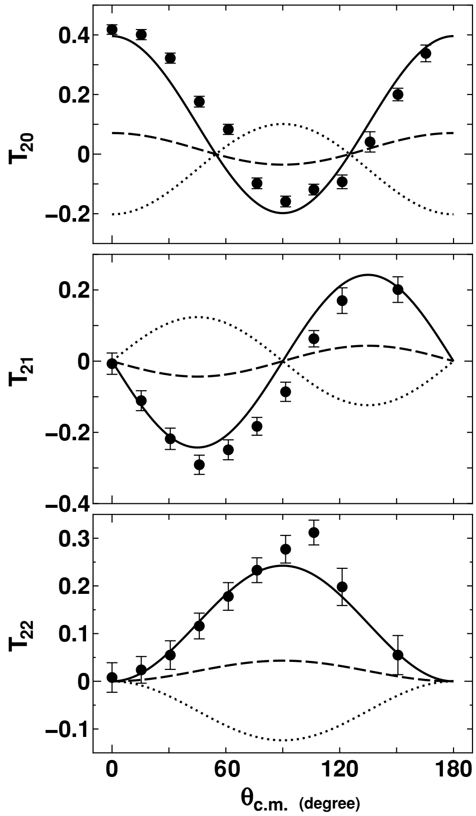

FIG. 1. Comparison of T_{2q} $(q = 0, 1, 2)$ between experimental data and calculations at $E_d = 90$ keV in $^6\mathrm{Li}(\vec{d}, p)^7\mathrm{Li}$(g.s.). The data are taken from Ref. [2]. The lines describe the calculations by Eqs. (21)–(23); the dotted ones are for $I_R = 1$ with $j_n = \frac{1}{2}$, the dashed ones are for $I_R = 1$ with $j_n = \frac{3}{2}$, and the solid ones for $I_R = 2$ with $j_n = \frac{1}{2}$ or $\frac{3}{2}$.

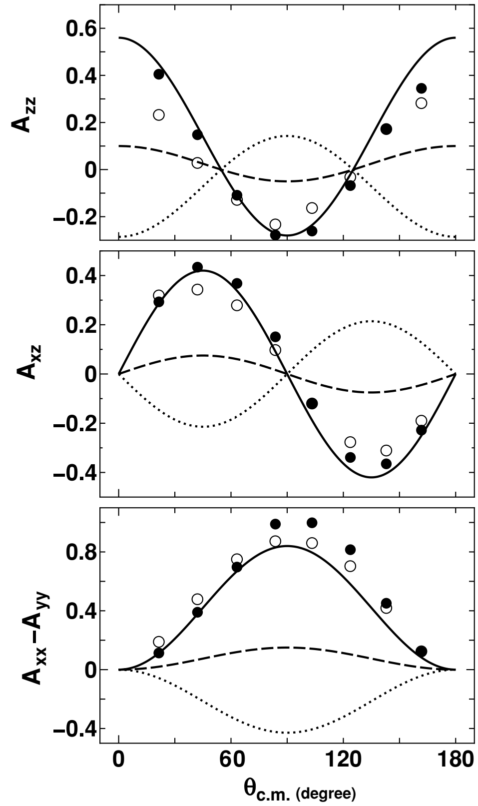

FIG. 2. Comparison of T_{2q} $(q = 0, 1, 2)$ between experimental data and calculations at $E_d = 600$ keV and 960 keV in $^6\mathrm{Li}(\vec{d}, p)^7\mathrm{Li}$(g.s.). The data are taken from Ref. [6]; the closed circles (open circles) are for 600 keV (960 keV). When the data of both energies overlap, the 600 keV data are plotted. For a description of the lines, see the caption of Fig. 1.

and

$$\begin{aligned}
&\mathrm{Tr}\left(\boldsymbol{M}\tau_q^2\boldsymbol{M}^\dagger\right)\\
&\quad= \sum_{s_i s_i'} \frac{3\sqrt{2}}{5}(2s_B+1)(2\ell_n+1)(2j_n+1)\\
&\qquad\times\sqrt{(2s_i+1)(2s_i'+1)}(\ell_n\ell_n 00|20)\\
&\qquad\times\sum_{JJ'}(2J+1)(2J'+1)(-)^{s_B-J'}\\
&\qquad\times W\left(\frac{1}{2}\frac{1}{2}s_i s_A;1J\right)W\left(\frac{1}{2}\frac{1}{2}s_i' s_A;1J'\right)\\
&\qquad\times W\left(\ell_n j_n J s_A;\frac{1}{2}s_B\right)W\left(\ell_n j_n J' s_A;\frac{1}{2}s_B\right)\\
&\qquad\times W\left(\frac{1}{2}Js_i'2;s_i J'\right)W(J\ell_n J'\ell_n;s_B 2)(s_i'||\tau^2||s_i)\\
&\qquad\times \mathfrak{Re}\{I^*(s_i)I(s_i')\}P_{2q}(\cos\theta),
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
(s_i'||\tau^2||s_i) &= (-)^{1-s_A+s_i'}\sqrt{15(2s_i+1)(2s_i'+1)}\\
&\quad\times W(1s_i'1s_i;s_A 2).
\end{aligned} \tag{20}$$

The quantity $P_{2q}(\cos\theta)$ is the associated Legendre function and θ is the scattering angle. The results obtained above show the general features of the reaction observables; i.e., the differential cross section is isotropic and the tensor analyzing powers have a characteristic angular dependence, $T_{2q} \propto P_{2q}(\cos\theta)$, where the proportional factor depends on the choice of distortion potential.

In the case of the ideal resonance, one can set $s_i = s_i' = I_R$ in Eqs. (18) and (19). Then in T_{2q}, $\mathfrak{Re}\{I^*(s_i)I(s_i')\}$ of $\mathrm{Tr}(\boldsymbol{M}\tau_q^2\boldsymbol{M}^\dagger)$ becomes $|I(s_i)|^2_{s_i'=s_i}$ and is canceled by that of $\mathrm{Tr}(\boldsymbol{M}\boldsymbol{M}^\dagger)$. Thus we get T_{2q} independently from details of the distortion potential. To derive explicit expressions of T_{2q}, we consider the low energy $^6\mathrm{Li}(\vec{d}, p)^7\mathrm{Li}$(g.s.) reaction in the resonance region. At $E_d = 90$ keV, the measured cross section of the reaction is almost isotropic [2], as predicted above, ensuring the S-wave dominance in the incident channel. Then the resonance spin will be one of s_i and $s_i = 0, 1, 2$ due to $s_d = s_A = 1$. The neutron will be captured into a p orbit, i.e., $\ell_n = 1$, $j_n = \frac{1}{2}$ or $\frac{3}{2}$. Setting $s_B = \frac{3}{2}$, we get the analyzing powers for $I_R = 0$, $T_{2q} = 0$, and

$$\text{for } I_R = 1 \text{ with } j_n = \frac{3}{2}, \quad T_{2q} = \frac{1}{10\sqrt{5}}P_{2q}(\cos\theta), \tag{21}$$

$$\text{for } I_R = 1 \text{ with } j_n = \frac{1}{2}, \quad T_{2q} = -\frac{2}{7\sqrt{5}} P_{2q}(\cos\theta), \tag{22}$$

$$\text{for } I_R = 2 \text{ with } j_n = \frac{3}{2} \text{ or } \frac{1}{2}, \quad T_{2q} = \frac{14}{25\sqrt{5}} P_{2q}(\cos\theta). \tag{23}$$

These are displayed in Fig. 1 and compared with the experimental data at $E_d = 90$ keV [2]. In Fig. 1, T_{2q} calculated for $I_R = 2$ agrees with the data with good quality for all q, indicating that the reaction takes place as the 2^+ resonance probably in high purity. This result is consistent with that of the previous analyses [2], where the resonance spin-parity is 2^+ and the fraction of the 2^+ configuration is about 90% of the reaction at $E_d = 90$ keV, showing the neglect of the nonresonant configurations to be a good approximation. The previous analyses treat the analyzing powers phenomenologically and the parameters involved are determined so as to fit the data, while at present the analyzing powers are derived theoretically without any adjustment once the resonance spin-parity is given. Such results provide a key for understanding the resonance mechanism by showing the stripping model to be favorable for the neutron transfer. In the case of no resonance, $\mathrm{Tr}(\boldsymbol{M}\tau_q^2\boldsymbol{M}^\dagger)$ is shown to be zero by calculating Eq. (19) explicitly, neglecting the s_i dependence of $I(s_i)$. Thus, in the present approximation, the observed analyzing powers are considered to be the product of the resonance.

Earlier the tensor analyzing powers of this reaction were measured at $E_d = 600$ and 960 keV [6]. These energies are in an off-resonance region for 2^+ in the conventional analyses [4]. However, an additional 2^+ resonance, which is situated at 500 keV above the $d + {}^6\mathrm{Li}$ threshold with the 1670 keV width, has been predicted by reanalyses of related cross sections [7]. Because this new resonance covers the above deuteron energies, it is interesting to compare the measured analyzing powers with our calculations. The measured analyzing powers are described in the cartesian representation. They are related to T_{2q} as follows,

$$A_{zz} = \sqrt{2}T_{20}, \quad A_{xz} = -\sqrt{3}T_{21}, \quad A_{xx} - A_{yy} = 2\sqrt{3}T_{22}. \tag{24}$$

Transforming the calculated T_{2q} to the cartesians, we compare in Fig. 2 the theoretical prediction with the data, where the cross section factor of the analyzing power data is assumed to be isotropic, as consistent with our theoretical cross section. In this figure, the measured analyzing powers at both 600 and 960 keV are described by the calculated ones for the 2^+ resonance assumption. This result supports the reanalyses in Ref. [7].

Here, contributions of spin-orbit and tensor interactions, higher partial waves in the incident channel, and transition amplitudes of nonresonance s_i are neglected. Despite such deficiencies, we have obtained finite contributions to the analyzing powers as the resonance effect by restricting the DWBA amplitudes to those which have the resonance spin-parity. This restriction brings about the definite spin dependence in the transition amplitudes and thus the analyzing powers are obtained without other spin-dependent interactions. The agreement with the data support the present theoretical development. Further, extensions of the theory are required for applications to the case where transitions by scalar interactions like v_{np} are forbidden due to the spin-parity conservation.

[1] H. Paetz gen. Schieck, Few-Body Syst. **5**, 171 (1988); H. Paetz gen. Schieck *et al.*, Phys. Lett. **B276**, 290 (1992); Y. Tagishi *et al.*, Phys. Rev. C **46**, R1155 (1992); R. M. Chasteler, H. R. Weller, D. R. Tilley, and R. M. Prior, Phys. Rev. Lett. **72**, 3949 (1994); W. H. Geist *et al.*, Phys. Rev. C **60**, 054003 (1999); B. Braizinha *et al.*, *ibid.* **69**, 024608 (2004); A. Sabourov *et al.*, *ibid.* **74**, 064611 (2006).
[2] M. Yamaguchi *et al.*, Phys. Rev. C **74**, 064606 (2006).
[3] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, MA, 1962).
[4] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
[5] G. G. Ohlsen, Rep. Prog. Phys. **35**, 717 (1972).
[6] M. Glor *et al.*, Nucl. Phys. **A286**, 31 (1977).
[7] P. R. Page, Phys. Rev. C **72**, 054312 (2005).