Ground-state electric quadrupole moment of 31Al

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The ground-state electric quadrupole moment of ³¹Al ($I^{\pi} = 5/2^+$, $T_{1/2} = 644(25)$ ms) has been measured by means of *β*-ray-detected nuclear magnetic resonance spectroscopy using a spin-polarized ³¹Al beam produced in the projectile fragmentation reaction. The obtained *Q* moment, $|Q_{exp}(^{31}Al)| = 112(32) e$ mb, is in agreement with conventional shell model calculations within the *sd* valence space. Previous results on the magnetic moment also support the validity of the *sd* model in this isotope, and thus it is concluded that 31Al is located outside of the *island of inversion*.

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The ground states of Ne, Na, and Mg isotopes with neutron numbers around the magic number $N = 20$ have been known to show anomalously tight bindings since the 1970s [\[1,2\]](#page-3-0). Later spectroscopic studies have revealed that the first exited 2^+ levels are lowered [\[3,4\]](#page-3-0) and their *B*(*E*2) values are enhanced [\[5\]](#page-3-0) sizably in these isotopes, and the possibility of deformation has been proposed. Theoretical analyses [\[6\]](#page-3-0) have discussed the importance of 2p-2h excitations from the *sd* shell to the upper *pf* shell and concluded it is plausible that an inversion of amplitudes between the *sd* normal and the *pf* intruder configurations would lead to deformation of the ground states. The region of nuclei where such a phenomenon occurs is called the *island of inversion*. In elucidating the underlying mechanism for the inversion, the measurements of the electromagnetic moments have played an important role. For example in a series of neutron-rich Na isotopes, it has been found that, once entering the island of inversion, the ground-state magnetic dipole moment μ and electric quadrupole moment *Q* [\[7,8\]](#page-3-0) show clear deviations from the conventional shell model predictions [\[9\]](#page-3-0), indicating that μ and Ω are sensitive to changes in the nuclear configuration [\[10\]](#page-3-0). Also in the recent study of Mg isotopes, anomalous ground-state properties have been revealed through the μ -moment measurements [\[11,12\]](#page-3-0).

In the present work, the ground-state *Q* moment of ³¹Al ($I^{\pi} = 5/2^{+}$, $T_{1/2} = 644(25)$ ms) has been measured by means of *β*-ray-detected nuclear magnetic resonance $(\beta$ -NMR) spectroscopy [\[13\]](#page-3-0) applied on a projectile fragment $3¹$ Al implanted in an α -Al₂O₃ (corundum) single crystal in which a nonzero electric field gradient acts. A spin-polarized radioactive-isotope beam (RIB) of ³¹Al was obtained from the projectile fragmentation reaction [\[14\]](#page-3-0). Because the neutronrich aluminum isotopes are located in the neighborhood of the island of inversion, their electromagnetic moments would signify the possible onset of evolution in the nuclear structure that ultimately leads to the *inversion* phenomenon. So far, the μ moments of ^{31–34}Al [\[15,16\]](#page-3-0) and ^{30,32}Al [\[17\]](#page-3-0) have been reported. The obtained values of μ for ^{30–32}Al seem to stay within the conventional*sd* model predictions [\[9,18\]](#page-3-0), indicating that their structures are suitably described within the normal*sd* model space. Those of 33,34 Al having neutron numbers $N = 20$ and 21, on the other hand, seem to indicate deviations from the $0\hbar\omega$ shell model predictions [\[16,19\]](#page-3-0). Because the island of inversion is considered to involve the nuclear deformation, the *Q* moment would be a more suitable probe. It has been found in a recent measurement that 32Al has a very small *Q* moment, $|Q_{exp}(^{32}Al)| = 24(2)e$ mb [\[20\]](#page-3-0), characteristic of a simple $(\pi d_{5/2}^{-1} \otimes \nu d_{3/2}^{-1})^{J=1}$ configuration, indicating a spherical shape. The Q moment of 31 Al is important in elucidating how the nuclear shape evolves along the aluminum isotopes toward and beyond the 32 Al nuclide.

The experiment was carried out using the RIKEN projectile fragment separator RIPS [\[21\]](#page-3-0). The arrangement of RIPS for producing the spin-polarized RIB is essentially the same as that described in Ref. $[20]$. A beam of 31 Al was obtained from the fragmentation of ⁴⁰Ar projectiles at $E = 95$ *A* MeV on a 0*.*37 g*/*cm² thick 93Nb target. It has been revealed that a spin-polarized RIB is obtained in the projectile fragmentation reaction simply by selecting the angle and momentum of the outgoing fragments $[14]$. Thus, ³¹Al fragments emitted at angles $\theta_{\text{Lab}} = 1.3 - 5.7^\circ$ from the primary beam direction were accepted by RIPS using a beam swinger installed upstream of the target. Also, a range of momenta $p = 1.01 - 1.07 p_0$ was selected with a slit placed at the momentum-dispersive intermediate focal plane. Here $p_0 = 12.2 \text{ GeV}/c$ is the fragment momentum corresponding to the projectile velocity. The momentum distribution of ³¹Al, though not measured at this time, is presumed to have a peak at around p_0 with a width $\Delta p \sim 0.06$ *p*₀ (FWHM) that was obtained by considering an intrinsic width $\Delta p_{\text{Goldhaber}} \approx 0.55 \text{ GeV}/c$ from the Goldhaber

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FIG. 1. Schematic layout of the *β*-NMR apparatus.

model [\[22\]](#page-3-0) and the broadening effect $\Delta p_{\text{target}} \approx 0.40 \text{ GeV}/c$ due to a finite target thickness. The isotope separation was provided by combined analyses of the magnetic rigidity and momentum loss in the wedge-shaped degrader [\[21\]](#page-3-0). Then, the spin-polarized 31Al nucleus was transported to a *β*-NMR apparatus located at the final focus of RIPS and implanted in a stopper of α -Al₂O₃ single crystal of hexagonal structure. A static magnetic field $B_0 = 501.768(3)$ mT was applied to the stopper. The layout of the *β*-NMR apparatus is shown in Fig. 1. The α -Al₂O₃ crystal was cut into a $18 \times 32 \times$ 0*.*6 mm slab and was mounted in a stopper chamber so that the c axis was oriented parallel to the B_0 field. The stopper was kept in vacuum and cooled to a temperature $T = 70 \sim 100 K$ to suppress the spin-lattice relaxation of ³¹Al during the β decay. The spin-lattice relaxation time T_1 at $T = 100 K$ was evaluated to be $T_1 \approx 4.5$ *s*, based on the measured T_1 for ²⁷Al in α -Al₂O₃ at room temperature [\[23,24\]](#page-3-0) and the $T_1T^2Q^2 =$ const. law [\[25\]](#page-3-0).

The *Q* moment interacts with an electric field gradient *eq* acting at the site of the implanted nucleus in a single crystal stopper. The *eqQ* interaction causes the energy shift in the individual Zeeman magnetic sublevels. Thus, the *Q* moment is determined from the measurement of the frequencies for the resonance transition between the Zeeman $+$ quadrupole splitted sublevels, whose signal is detected as a change in the *β*-ray asymmetry (the *β*-NQR method). The *β* rays emitted from the implanted nucleus were detected with scintillator telescopes located above and below the stopper, each consisting of three 1-mm-thick plastic scintillators. The up/down ratio *R* of the β -ray counts is written as

$$
R = a \frac{1 + v/c \cdot A_{\beta} P}{1 - v/c \cdot A_{\beta} P} \simeq a(1 + 2A_{\beta} P),
$$
 (1)

where *a* is a constant factor representing asymmetries in the counter solid angles ($\Omega_{\beta} \approx 4\pi \times 0.26$ sr each) and efficiencies, v/c the velocity of the β particle, A_{β} the asymmetry parameter, and P the 31 Al nuclear spin polarization. The *β* rays lose energies of 960 keV on average upon penetrating the stopper material and a wall of the vacuum jacket and then

deposit about 260 keV in each of the three scintillators. The threshold energy for the *β*-ray detection in triple coincidence among the three scintillators is evaluated [\[26\]](#page-3-0) to be 1740 keV, and the detection efficiency including the energy threshold and the detection solid angle is about 29% for the *β* rays from ³¹Al, for which the experimentally known major branches are to the $3/2^+$ ground state of ³¹Si (65%, Q_β = 7995 keV), to the 3*/*2+2317-keV state (26%), to the 5*/*2+1695-keV state (8%), and to the $1/2+752$ -keV state (1%) [\[27\]](#page-3-0). Because only the $E_8 \gtrsim$ 1500 keV (or $v/c \gtrsim 0.97$) portion of the spectrum was detected,
the ratio *R* in Eq. (1) is well approximated by the second exthe ratio R in Eq. (1) is well approximated by the second expression by simply setting $v/c \approx 1$. The adiabatic fast passage (AFP) technique [\[28\]](#page-3-0) was incorporated to pursue the reversal, but not the destruction, of the spin polarization. By taking a double ratio R/R_0 , where R_0 is the value for R measured without an oscillating magnetic field *B*1, the resonance frequency is derived from the position of a peak or dip deviating from unity. The measurements were carried out in the following sequence. The beam was pulsed with beam-on and bean-off periods of 1 and 1.126 s, respectively. In the beam-off period of a beam cycle, the B_1 field was applied for the first 63 ms duration. Then, the β rays were counted for 1 s, and in the last 63 ms of the the beam-off period the B_1 field was applied again to restore the spin direction so that the ratio R in the succeeding cycle might not be affected by the surviving activities. Such beam cycles were executed with *n* different sets of frequencies as described below and then with the B_1 field interrupted (the R_0) measurement). These $n + 1$ beam cycles were repeated until the sufficient measurement statistics were attained. By inserting the R_0 measurement in this way, the effect of long-term fluctuations in, e.g., the beam profile at the target should be removed. The B_1 field was applied in a direction perpendicular to the external field B_0 with a pair of coils located outside a vacuum jacket, in which the α -Al₂O₃ stopper was placed. In a first order perturbation theory, the resonance frequency *νm,m*+¹ between magnetic sublevels m and $m + 1$ of the spin I under the combined Zeeman and quadrupole interactions is given by

$$
\nu_{m,m+1} = \nu_{\rm L} - \nu_{\rm Q} \cdot (3\cos^2\theta_{\rm c\ axis} - 1) \frac{3(2m+1)}{8I(2I-1)},\tag{2}
$$

where v_L denotes the Larmor frequency, $v_O = \frac{eqQ}{h}$ the quadrupole coupling constant, *eq* the electric field gradient along the *c* axis (the additional term arising from a deviation *η* from the axial symmetry of the field gradient tensor is omitted, because η is reported to be small [\[29\]](#page-3-0)), and θ_{caxis} the angle between the *c* axis and the B_0 field. Q and h denote the Q moment and the Planck's constant, respectively. Inserting $I =$ $5/2$ and $\theta_{c \text{ axis}} = 0$ for the present ³¹Al experiment, Eq. (2) reads as

$$
v_{m,m+1}(v_Q)
$$

$$
= v_L - \frac{3}{40}(2m+1)v_Q
$$
\n(3)
\n
$$
= \begin{cases}\nv_L + (3/10)v_Q & \text{for } (m, m+1) = (-5/2, -3/2);\n\text{(frequency 'a'')}\n\end{cases}
$$
\n
$$
= \begin{cases}\nv_L + (3/20)v_Q & \text{for } (-3/2, -1/2), ('b'')\nv_L - (3/20)v_Q & \text{for } (-1/2, +1/2), ('c'')\nv_L - (3/10)v_Q & \text{for } (+3/2, +5/2), ('e'').\n\end{cases}
$$
\n(4)

Once the true value for v_Q is inserted, sweeping the B_1 field across $v_{m,m+1}(v_Q)$ leads to a reversal of population between sublevels m and $m + 1$ (the adiabatic fast passage technique). Note that the full reversal of spin, $+5/2 \leftrightarrow$ −5*/*2*,* +3*/*2 ↔ −3*/*2*,* +1*/*2 ↔ −1*/*2, requires a sequence of stepwise reversals between two contiguous sublevels [\[30\]](#page-3-0). The B_1 field was applied in $I(2I + 1) = 15$ steps in a sequence *abcdeabcdabcaba* within the *B*¹ application period of 63 ms duration. In each step the frequency *ν* was swept from $v_{m,m+1}(v_Q^{\text{lower}})$ to $v_{m,m+1}(v_Q^{\text{upper}})$ with v_Q^{lower} and v_Q^{upper} denoting the lower and upper bounds of the searched v_O region. Prior to the present *Q*-moment measurement, a *β*-NMR experiment was carried out on 31 Al in a Si crystal using the same apparatus and B_0 setting. The obtained magnetic moment $|\mu(^{31}Al)| = 3.824(8)$ μ_N is in good agreement with the previously reported values [\[15,16\]](#page-3-0). For the Larmor frequency *ν*_L in Eq. [\(3\)](#page-1-0), we adopted a value $v_L = 5850(12)$ kHz obtained from this measurement.

Thus, the β -ray count ratio R/R_0 is expected to differ from unity when the B_1 field sequence is executed for the v_O region that includes the true v_O value. Figure 2 shows the measured R/R_0 ratio for ³¹Al in α -Al₂O₃ as a function of ν _O (the β -NQR spectrum). The horizontal bar attached to the data point (solid circle) indicates the v_Q region over which the $B₁$ field frequency was swept. The vertical bar represents the error in R/R_0 arising from the *β*-ray counting statistics. The R/R_0 value at the dip bottom shows a displacement of 5.3 standard deviation from unity, clearly indicating the occurrence of the AFP spin reversal. The width of the dip, however, appears to be substantially broader than a calculated width 653 kHz (FWHM) for the present AFP sweep width (see horizontal bars attached to the data points in Fig. 2). Although an externally implanted Al ion would be most likely to stop at the site of the same element in the host crystal, there may be a possibility that some portion of the implanted ions stop at other, metastable sites. Also, there might be another case where the implanted ions stop at the site of ²⁷Al in α -Al₂O₃ but some of them

FIG. 2. A NQR spectrum obtained in an α -Al₂O₃ crystal for the ground state of ³¹Al. The R/R_0 ratio is plotted as a function of v_0 . The vertical bar attached to the data point represents the statistical error due to *β*-counting statistics, while the horizontal bar indicates the width of *ν*_O frequency sweep. The result of the least- $χ²$ fitting analysis is shown by a dotted curve.

are accompanied by a near-by lattice defect produced by the implantation, thus leading to a shifted NQR resonance. In these cases the NQR spectrum would show a broadened dip.

The obtained NQR spectrum was fitted with a function,

$$
F(v_Q) = a \int G_{\sigma}(\xi) \cdot \mathcal{F}_{\text{AFP}}(v_Q - v_Q^{(0)} - \xi) d\xi + b,\qquad(5)
$$

with four free parameters $v_Q⁽⁰⁾$, σ , a , and b to be determined through the fitting. The $F(v_0)$ function is a Gaussian convolution of a theoretical shape function $\mathcal{F}_{\text{A}\text{FP}}(x)$ of a detuning x (i.e., $f(x)$ of Eq. (4) in Ref. [\[31\]](#page-3-0)). The parameter $v_O⁽⁰⁾$ represents the position of the dip, from which the quadrupole coupling constant *eqQ/h* will be deduced. The parameter σ is the width of the Gaussian function $G_{\sigma}(\xi)$ $\equiv (\sqrt{2\pi}\sigma)^{-1} \exp(-\xi^2/2\sigma^2)$, representing extrabroadening effects that are not included in the function $\mathcal{F}_{\text{AFP}}(x)$. Extrabroadening may arise from distribution of the *eq* value due to lattice defects or impurities in the stopper crystal and misalignments in the *c*-axis orientation and *νL* setting. In the present analysis such effects are expressed as a finite value of *σ*. The extrabroadening was not included in the preliminary reports [\[30,32\]](#page-3-0).

From the fitting analysis of the NQR spectrum, we obtained $v_O⁽⁰⁾ = 1824(119)$ kHz for the dip position. The resulting curve $F(v_0)$ is shown in Fig. 2 by a dotted line. Although the parameter $v_Q^{(0)}$ is determined with a rather small uncertainty ($\delta^{fit}v_Q^{(0)} = 119$ kHz) from the fitting procedure, the actual spectrum is dominated by a much larger broadening, $\sigma = 506$ kHz, whose origins are not well pinpointed, as discussed above. We therefore take into account the extra width σ as an independent error and assign an experimental error $\delta v_0^{(0)} = 520$ kHz, as shown in Table I. As a result, we obtain a quadrupole coupling constant $|v_0|$ = $|eqQ/h| = 1824(520)$ kHz. The Q moment of ³¹ Al is deduced from the relation $|Q^{(3)}| = |Q^{(27)}| \cdot v_Q^{(31)}| \cdot v_Q^{(27)}|$, where $Q(^{27}Al)$ and $v_Q(^{27}Al)$ denote the *Q* moment of ²⁷Al and the quadrupole coupling constant of ²⁷Al in α -Al₂O₃, respectively. By inserting the recently reported *Q* moment $Q(^{27}Al) = 146.6(10) e$ mb [\[33\]](#page-3-0) and quadrupole coupling constant $v_Q(^{27}$ Al) = 2389(2) kHz [\[34\]](#page-3-0), the ground-state *Q* moment ³¹Al is determined as $|Q_{exp}(^{31}Al)| = 112(32) e$ mb.

In Fig. [3,](#page-3-0) the experimentally known *Q* moments for the neutron-rich aluminum isotopes [\[20,33,35\]](#page-3-0) including the

TABLE I. Uncertainties taken into account for the determination of the $|Q_{\text{exp}}(^{31}\text{Al})|$ moment. Those uncertainties were converted into the corresponding v_Q frequencies.

Resonance vO	1824 kHz
Statistical error	
Fitting error	119 kHz
Systematic errors	
Ambiguity from the resonance width	506 kHz
Uncertainty of the electric field gradient	13 kHz
Ambiguity of the θ_{caxis} -angle setting	0.9 kHz
Total	520 kHz
\rightarrow $ Q_{exp}(^{31}Al) $	$112 \pm 32 e$ mb

FIG. 3. Experimental (solid circle) and theoretical (solid and dotted lines) *Q* moments of neutron-rich aluminum isotopes as a function of mass number; nuclear spins are also shown. Theoretical values are obtained from shell model calculations within the *sd* shell with the USD interaction, using the constant effective charges $e_p = 1.3$ and $e_n = 0.5$ (solid line) and isospin-dependent effective charges [31] (dotted line).

present data are plotted as a function of the mass number *A*. Also, the results of shell model calculations within the *sd* shell [9,18] are shown by the solid line. The calculations reproduce the observed trend of the *Q* moments in the ²⁷−32Al region fairly well: |*Q*exp| stays almost constant at |*Q*exp| ∼ 150 *e* mb, but suddenly decreases to a very small value at $A = 32$ [20]. These calculations have employed effective charges $e_p = 1.3$ for protons and $e_n = 0.5$ for neutrons. One could include the effect of isospin dependence of the effective

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charges, which had been pointed out in Ref. [36] and was observed experimentally for the first time in the *Q* moments of boron isotopes [31]. The dotted line shows the calculated *Q* with the effective charges varied with isospin according to the expression given in Ref. [31]. The use of the isospin-dependent *e*_p and *e_n* reduces the calculated *Q* by 10 ∼ 15% in the ^{27–32}Al isotopes and improves the agreement particularly in the 31Al Q moment, although the experimental error in $Q(^{31}Al)$ is not small. Finally, in contrast to the approximate accordance of the *sd* shell calculations with experiment, an anticipation that the deformation might set in somewhere along the chain of Al isotopes proves to be not the case at least until $A = 32$, because sizes of the experimental *Q* presented in Fig. 3 are much smaller than those expected for deformations of $\beta \sim 0.5$ occurring in the neighboring nucleus $\frac{30}{Mg}$ [37].

In summary, the ground-state Q moment of 31 Al has been determined by the *β*-NQR method, using the fragmentationinduced spin polarization. The obtained Q for ³¹Al as well as known |*Q*| values for other neutron-rich aluminum isotopes were found to be well explained by shell model calculations within the *sd* shell. Because the magnetic moment of 31 Al recently determined [15,16] is explained with the same calculations, it is concluded that $31\text{ }\text{Al}$ is located outside the island of inversion.

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