PHYSICAL REVIEW C **79**, 025804 (2009)

Influence of the σ-ω meson interaction on neutron star matter

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(Received 11 August 2008; revised manuscript received 18 November 2008; published 27 February 2009)

Relativistic mean field theory with nonlinear scalar self-interaction and isoscalar scalar-vector cross-interaction is used to study the properties of neutron star matter in β equilibrium with and without hyperons. The influence of σ-ω meson cross-interaction on the properties of neutron star matter and the mass-radius relation of neutron stars is examined with attractive and repulsive Σ potential, respectively. The calculated result indicates that the cross-interaction softens the equation of state (EOS) of nuclear (hadronic) matter and reduces the maximum mass of neutron stars. It also decreases the densities for hyperonization to occur and lowers the center density of neutron stars. The increase of the cross-interaction strength enhances the softening effect of hyperons on the EOS. Meanwhile the repulsive Σ potential stiffens slightly the EOS and influences obviously the composition of neutron star matter.

DOI: 10.1103/PhysRevC.79.025804 PACS number(s): 97.60.Jd, 13.75.Lb, 21.30.Fe, 26.60.Kp

I. INTRODUCTION

A quantized field theoretical description based on hadronic degrees of freedom, referred to as quantum hadrodynamics (QHD), was introduced by Walecka in 1974 [1]. It is a successful approach to describing the properties of nuclei and nuclear matter with the relativistic mean-field (RMF) approximation. For the original QHD, also known as the σ-ω model, a simple parameter set was used to model the nuclear force by exchanging the neutral (isoscalar) scalar sigma (σ) mesons and (isoscalar) vector omega (ω) mesons. These mesons have been found to be most important in describing the properties of nuclei and nuclear matter [2]. The prevalent models in the RMF framework also include the nonlinear self-coupling of σ mesons that is essential to reproduce the properties of nuclei quantitatively and give a reasonable value for the incompressibility of nuclear matter [3–6]. Moreover, the channel via ρ-meson exchange was introduced to describe the asymmetric nuclear matter [7]. In addition, the vector self-coupling ω^4 of ω mesons has been added to improve the description of finite nuclei [8], for instance, the shell effects in nuclei [9]. Applied to neutron star matter, it weakens the repulsive vector meson contribution, softens the equation of state (EOS) at high density and, in turn, induces neutron stars with relatively smaller maximum mass [10]. Compared with ω^4, the vector self-coupling ρ^4 of ρ mesons produces relatively minor changes in neutron star mass [11].

In recent years, great success has been achieved by considering the variations of meson coupling [12–15] or even density-dependent meson coupling [16–19]. There has also been progress in the effective field theoretical approach with higher order terms being included [20–22]. Moreover, meson-meson cross-interactions have also been studied to describe finite nuclei and nuclear matter [11,23–25]. In Ref. [25], the coupling between σ and ρ mesons was considered. It showed that such a coupling reduces the neutron skin to a more reasonable value relevant to neutron star mass. In Ref. [11], the vector ω-ρ cross-interaction was investigated to parametrize the nuclear matter property calculated by Dirac-Brueckner-Hartree-Fock theory, and it seemed to be a useful degree of freedom for describing asymmetric nuclear matter [23]. It was also shown that this cross-interaction softens the EOS of nuclear matter with a lowering of the hyperonization, enhances simultaneously the hyperon-induced decrease of the electron chemical potential, and, in turn, shifts the critical baryon density for the kaon condensation to take place to higher one.

Although the RMF theory has had great success in nuclear physics, nucleon-meson interactions or meson-meson interactions have not yet had a unified form in different models. It is not known which interaction terms the nature favors, therefore new interactions deserve to be studied with the development of nuclear experiment. Recently, an effective model with the coupling of σ and ω mesons in RMF theory, dubbed SIG-OM [26], has been proposed. The properties of finite nuclei and nuclear matter were investigated in this model, showing that an excellent description of binding energies and charge radii of nuclei over a large range of isospin could be achieved. However the composition of baryons considered in Ref. [26] is only protons and neutrons. To discuss the property of neutron stars, one usually should take into account not only nucleons but also hyperons. The main aim of this article is then to extend this model to include all baryon octets and study the influence of the cross-interaction on the properties of nuclear matter and neutron stars.

With the progress of astronomical observation and nuclear experiment, astrophysics phenomena and nuclear physics are combined more and more tightly. Because the density in the core of a neutron star is several times the nuclear saturation density, the classical view of neutron star matter composed of protons, neutrons, and electrons is insufficient and more realistic compositions are needed. At high density, hyperons, kaon condensation, and quarks may appear and much attention has been paid to these issues (see, for example, Refs. [27–51]).

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As for hyperons, experimental effort has resulted in some significant data. The recent Nagara event [52] has provided an identification of ${}_{\Lambda\Lambda}^{\;6}$He production with precise $\Lambda\Lambda$ binding energy value $B_{\Lambda\Lambda} = 7.25 \pm 1.19^{+0.18}_{-0.11}$ MeV. This suggested that the effective $\Lambda\Lambda$ interaction should be considerably weaker ($\Delta B_{\Lambda\Lambda} \sim 1$ MeV) than that deduced from earlier measurement ($\Delta B_{\Lambda\Lambda} \sim 5$ MeV). However, we still lack accurate knowledge about Σ-N interaction, even though there have been some hints of a high-density repulsion of Σ-N interaction as indicated by the extrapolated atomic data [53], the absence of bound Σ^- hypernuclear states [54], and the experimental data of (π^-, K^+) on a series of targets [55]. As for hyperon-meson coupling constants, they are usually derived from the SU(6) quark model or constrained by reasonable hyperon potentials. The meson-nucleon coupling constants are generally determined by fitting the properties of nuclear matter at saturation density or ground-state properties of finite nuclei. On the other hand, the astronomical observation, for example, the masses and radii of neutron stars, can also provide constraints on the EOS determined by nuclear data. In this article, we then take both the attractive and the repulsive forms of Σ potential in our calculation and examine the effect of the Σ potential on the properties of astronuclear matter and neutron stars.

This article is organized as follows. In Sec. II, we describe the effective Lagrangian with the additional cross-interaction term $\sigma^2\omega^2$ and give the relevant formulas in the RMF theory. In Sec. III, the calculated results of some properties of nuclear (hadronic) matter and neutron stars are given and compared with those without the cross-interaction term. Finally, a summary is given in Sec. IV.

II. THE MODEL

To describe the properties of hadronic matter, the RFM theory is usually implemented, in which baryons interact via the exchange of mesons. The baryons considered here include nucleons (p and n) and hyperons (Λ, Σ, and Ξ) investigated for the first time by Glendenning [56]. The exchanged mesons include the isoscalar scalar meson (σ), the isoscalar vector meson (ω), the isovector vector meson (ρ), and the cross-interaction term $\sigma^2\omega^2$ introduced in Ref. [26]. For neutron star matter in β equilibrium, the effective Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = {} & \sum_B \bar{\Psi}_B[i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\ & - g_{\rho B}\gamma_\mu\vec{t}\cdot\vec{\rho}^{\,\mu}]\Psi_B + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) \\ & - \frac{1}{3}bm(g_\sigma\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu} \\ & + \sum_\ell \bar{\Psi}_\ell(i\gamma_\mu\partial^\mu - m_\ell)\Psi_\ell - \frac{1}{2}g_{\sigma\omega}\sigma^2\omega^2, \end{aligned} \tag{1}$$

where the symbol B includes the entire baryon octet ($p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$) and ℓ represents e^- and μ^-; the last term is the cross-interaction between σ and ω mesons. The antisymmetric tensors of vector mesons take the forms

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu, \quad \vec{G}_{\mu\nu} \equiv \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu.$$

Other parameters in the Lagrangian are m_B denoting the baryon free mass; m standing for the (mean) bare mass of proton and neutron; m_σ, m_ω, and m_ρ being masses assigned to the corresponding meson, respectively; and $g_{\sigma B}, g_{\omega B}, g_{\rho B}, b, c$, and $g_{\sigma\omega}$ referring to the meson-baryon or meson-meson coupling constants.

With the mean field approximation by which the operators of meson fields are replaced by their expectation values, we obtain the meson field equations as

$$g_\sigma\sigma = f_\sigma\left[\sum_B x_{\sigma B}\rho_B^S - bm(g_\sigma\sigma)^2 - c(g_\sigma\sigma)^3 - \frac{g_{\sigma\omega}}{g_\sigma}\sigma\omega^2\right], \tag{2}$$

$$g_\omega\omega = f_\omega\left[\sum_B x_{\omega B}\rho_B - \frac{g_{\sigma\omega}}{g_\omega}\sigma^2\omega\right], \tag{3}$$

$$g_\rho\rho = f_\rho\sum_B x_{\rho B}t_{3B}\rho_B, \tag{4}$$

where new forms of the coupling constants are adopted with the following definitions,

$$f_i = \left(\frac{g_i}{m_i}\right)^2, \quad x_{iB} = \frac{g_{iB}}{g_i}, \quad (i = \sigma, \omega, \rho),$$

and ρ_B and ρ_B^S are baryon density and scalar density, respectively, with

$$\rho_B = \frac{2}{(2\pi)^3}\int_0^{k_f^B} d^3k, \tag{5}$$

$$\rho_B^S = \frac{2}{(2\pi)^3}\int_0^{k_f^B} d^3k\frac{m_B^*}{\sqrt{k^2 + m_B^{*2}}}. \tag{6}$$

In the last two equations, k_f^B is the Fermi momentum and m_B^* is the effective mass of baryon B, which can be related to the scalar meson field as $m_B^* = m_B - g_{\sigma B}\sigma$. With the requirement of translational invariance and rotational symmetry of static, homogenous, infinite nuclear matter, only zero components—ω_0 and ρ_0—of the vector fields survive, and they are still denoted as ω and ρ in the above meson equations.

For the neutron star matter with baryons and charged leptons, the β equilibrium conditions are guaranteed with the following relations of chemical potentials for different particles,

$$\mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e, \tag{7}$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n, \tag{8}$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e, \tag{9}$$

$$\mu_\mu = \mu_e, \tag{10}$$

and the charge neutrality condition is fulfilled by

$$n_p + n_{\Sigma^+} = n_e + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-}, \tag{11}$$

where n_i is the number density of species i. The chemical potentials of baryons and leptons are expressed by

$$\mu_B = \sqrt{k_F^{B^2} + m_B^{*2}} + g_{\omega B}\omega + g_{\rho B} t_{3B}\rho, \tag{12}$$

$$\mu_\ell = \sqrt{k_F^{\ell^2} + m_\ell^{*2}}. \tag{13}$$

The total pressure and energy density of neutron star matter are given as

$$p = \sum_{i=B,\ell} \frac{1}{3}\frac{2}{(2\pi)^3}\int_0^{k_F^i} d^3k \frac{k^2}{\sqrt{k^2+m_i^{*2}}} - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{b}{3}m(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2}g_{\sigma\omega}\sigma^2\omega^2, \tag{14}$$

$$\varepsilon = \sum_{i=B,\ell} \frac{2}{(2\pi)^3}\int_0^{k_F^i} d^3k\sqrt{k^2+m_i^{*2}} + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{b}{3}m(g_\sigma\sigma)^3 + \frac{c}{4}(g_\sigma\sigma)^4 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 + \frac{1}{2}g_{\sigma\omega}\sigma^2\omega^2. \tag{15}$$

With the obtained EOS, the mass-radius relation and other relevant quantities of neutron star can be derived by solving the Tolman-Oppenheimer-Volkoff (TOV) equation [57,58],

$$\frac{dp}{dr} = \frac{[p(r)+\epsilon(r)][M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]}, \tag{16}$$

with

$$M(r) = 4\pi\int_0^R \epsilon(r) r^2 dr. \tag{17}$$

It is evident that the meson field equations involve six parameters, f_σ, f_ω, f_ρ, b, c, and $g_{\sigma\omega}$, which can be determined by fitting the bulk properties of nuclear matter at the saturation density. The properties of the nuclear matter at saturation density are usually taken as $\rho_0 = 0.153$ fm^{-3}, $E/A = -16.3$ MeV, $a_{\text{asym}} = 32.5$ MeV, $K = 240$ MeV, and $m^* = 0.78\ m$ (see, for example, Refs. [59,60]). We fixed the parameters with the same values except for $K = 265$ MeV (which has also been commonly used; see, for instance, Ref. [61]) being implemented here. As for the cross-coupling constant, $g_{\sigma\omega} = 35.7$ was obtained by fitting the properties of finite nuclei [26]. To investigate the effect of the cross-interaction between σ and ω mesons on the properties of nuclear matter and neutron stars, we deal with it more loosely in this article, taking several values around 35.7. The detail of the method of fitting the model parameters can be found in Ref. [60]. Because the cross-interaction term enters the meson equations, the contribution from it should be considered when one fits the bulk properties of nuclear matter. The fitted results of these parameters are listed in Table I.

TABLE I. Parameters used in our calculations for several values of $g_{\sigma\omega}$ by fitting the saturation properties of nuclear matter: $\rho_0 = 0.153$ fm^{-3}, $E/A = -16.3$ MeV, $a_{\text{asym}} = 32.5$ MeV, $K = 265$ MeV, and $m^* = 0.78\ m$.

	f_σ	f_ω	f_ρ	b	c
$g_{\sigma\omega} = 0$	9.60167	4.82864	4.79412	0.00654	0.00390
$g_{\sigma\omega} = 20$	8.54825	4.82864	4.79412	0.00036	0.01799
$g_{\sigma\omega} = 40$	7.49479	4.82864	4.79412	−0.00757	0.03596

For the meson-hyperon couplings, we take those in the SU(6) quark model for the vector coupling constants,

$$g_{\rho\Lambda} = 0,$$
$$g_{\rho\Sigma} = 2g_{\rho\Xi} = 2g_{\rho N},$$
$$g_{\omega\Lambda} = g_{\omega\Sigma} = 2g_{\omega\Xi} = \tfrac{2}{3}g_{\omega N}.$$

The scalar couplings are usually fixed by fitting hyperon potentials with $U_Y^N = x_{\omega Y}V - x_{\sigma Y}S$, where $S = g_\sigma\sigma$ and $V = g_\omega\omega$ are the values of the scalar and vector field strengths at saturation density [59,62]. The Λ-N interaction has been well studied and $U_\Lambda^N = -28$ MeV was obtained with bound Λ hypernuclear states (see, for instance, Ref. [63]). One of the unsettled issues in hypernuclear physics is the Σ-N interaction in nuclear matter. An attractive potential was generally used in the past for Σ to be bounded in nuclear matter [31,64]. However, a detailed scan for Σ hypernuclear states turned out to give negative results [54,55]. The study of Σ^- atoms also showed strong evidence for a sizable repulsive potential in the nuclear core at $\rho = \rho_0$ [65–67]. A recent review again confirmed the repulsive nature of the Σ^- potential with a new geometric analysis of the Σ^- atom data [68]. Therefore, for the Σ-N interaction, we consider two cases: $U_\Sigma^N = -30$ MeV, as used in Refs. [31,62,64], and $U_\Sigma^N = 30$ MeV, as used in Refs. [63,69]. Besides, the $\Xi - N$ interaction in nuclear matter is attractive with the potential $U_\Xi^N = -18$ MeV [63,69]. We take then such a value in our calculation.

III. NUMERICAL RESULT AND DISCUSSION

A. The effects on the most simple neutron star

Before taking into account the neutron star matter with hyperons, we first investigate the most simple neutron star whose baryon composition includes only neutrons and protons. For the description of such simple neutron stars, the discussion in the last section still works with the exclusion of hyperons Λ, Σ, and Ξ from the Lagrangian and the meson field equations. The relevant parameters by fitting the saturation nuclear matter properties are listed in Table I for different values of $g_{\sigma\omega}$. With these parameters, we calculate the EOS of the nuclear matter and the mass-radius relations of neutron stars for the cases with and without the cross-interaction. The obtained results of the EOS and the mass-radius relation are illustrated in the curve set marked Part A of Figs. 1 and 2, respectively. The curve set marked Part A in Fig. 1 indicates that the σ-ω interaction term softens the equation of state of the matter whose baryon composition consists of only nucleons.

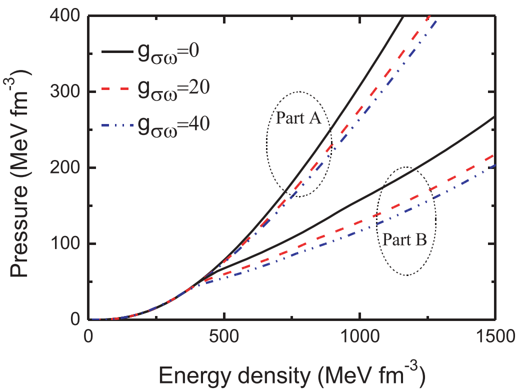

FIG. 1. (Color online) The equations of state of the neutron star matter in the cases without and with the σ-ω meson cross-interaction and Σ-N interaction $U_{\Sigma}^{N} = 30$ MeV. The curve set marked Part A are for the matter whose-baryon composition consists of only nucleons; the curve set marked Part B are for the one including the whole baryon octet (p, n, Λ, Σ^-, Σ^0, Σ^+, Ξ^0, Ξ^-).

Figure 2 shows evidently that, for $g_{\sigma\omega} = 0$, $g_{\sigma\omega} = 20$, and $g_{\sigma\omega} = 40$, the calculated maximum masses of neutron stars without hyperons are 2.05 $M_{\odot}$, 1.97 $M_{\odot}$, and 1.94 $M_{\odot}$, respectively. These results are consistent with Fig. 1, because the stiffer EOS produces the larger maximum neutron star mass. We also calculated the maximum masses of neutron stars for the parameter set with $m^* = 0.7\,m$, and $M_{\max} =$ 2.35 $M_{\odot}$, 2.21 $M_{\odot}$, and 2.18 $M_{\odot}$ were obtained for $g_{\sigma\omega} =$ 0, $g_{\sigma\omega} = 20$, and $g_{\sigma\omega} = 40$, respectively. It is apparent that the maximum masses of the neutron stars without the cross-interaction are larger than those with the cross-interaction, and the larger the $g_{\sigma\omega}$ is the smaller the maximum mass is. This manifests in the σ-ω cross-interaction playing the role of softening the EOS of the neutron star matter. Moreover, Fig. 2 also shows that the smaller effective mass of the nucleon gives

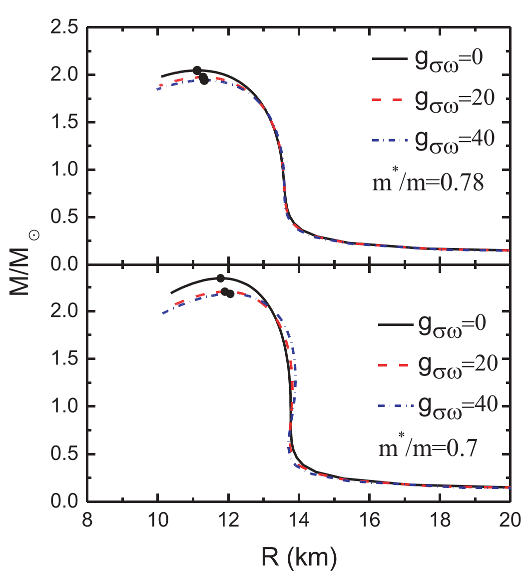

FIG. 2. (Color online) Calculated mass-radius relations of neutron stars not including hyperons in the cases without and with the σ-ω meson cross-interaction.

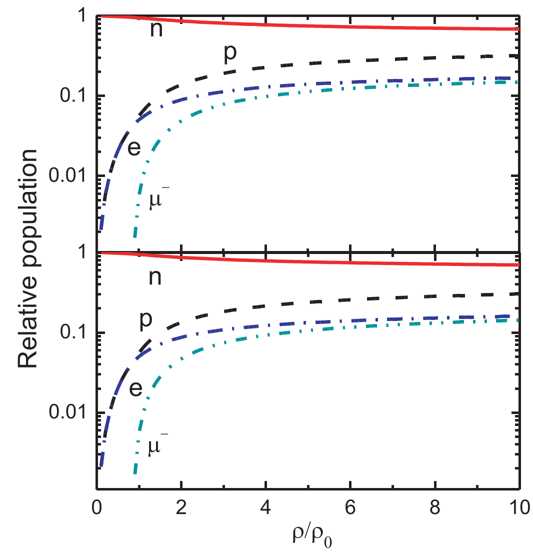

FIG. 3. (Color online) Calculated results of the relative populations of neutron stars not including hyperons as functions of the total nucleon density in the cases without and with the σ-ω cross-interaction (The upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$).

a bigger difference in the mass-radius relations of neutron stars for different cross-interaction constants.

To show the distribution of the compositions in the interiors of the neutron stars, we display the variation behavior of the relative populations of different species with respect to the baryon density and the radius of the neutron star in Figs. 3 and 4, respectively. These figures show that the σ-ω meson cross-interaction hardly influences the relative populations at the low density region of the neutron star without hyperons

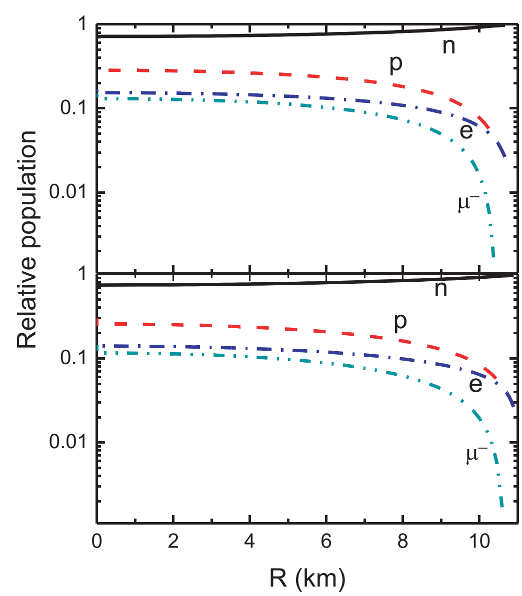

FIG. 4. (Color online) Calculated results of the relative populations of the neutron stars not including hyperons as functions of the radius of the neutron star in the cases without and with the σ-ω cross-interaction (The upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$).

but slightly decreases that of the charged particles (protons, electrons, and muons) in the high density region (close to the center of the neutron star).

B. The effects on neutron stars including hyperons

In this section, we study the influence of the σ-ω cross-interaction on neutron stars with the inclusion of hyperons. It is known that the appearance of hyperon degrees of freedom has a significant effect on the global properties of hadron matter and neutron stars, lowering the total pressure of the system and softening the equation of state, because it suppresses the overall Fermi energy and momentum of baryons and leptons. We take $U_\Lambda^N = -28$ MeV and $U_\Xi^N = -18$ MeV to determine the scalar coupling constants $g_{\sigma\Lambda}$, $g_{\sigma\Xi}$. As for the Σ hyperon, we consider the attractive potential $U_\Sigma^N = -30$ MeV and the repulsive potential $U_\Sigma^N = 30$ MeV, respectively. To supplement Table I, the relevant scalar coupling constants $x_{\sigma\Lambda} = 0.60367$, $x_{\sigma\Xi} = 0.32314$, and $x_{\sigma\Sigma} = 0.61432$ (0.29483) are derived for the attractive (repulsive) Σ potential. The calculated results of the EOS of the neutron star matter including hyperons for the repulsive Σ potential $U_\Sigma^N = 30$ MeV are displayed in the curve set marked Part B of Fig. 1 (The results for $U_\Sigma^N = -30$ MeV in the case of including hyperons are not plotted here, because they are only slightly softer than that of $U_\Sigma^N = 30$ MeV). And the obtained mass-radius relations of neutron stars including hyperons with different Σ potentials are illustrated in Fig. 5. The curve set marked Part B of Fig. 1 shows that the appearance of hyperons softens the EOS definitely and the σ-ω cross-interaction softens the EOS further. Figure 5 demonstrates clearly that, no matter whether the Σ potential is attractive or repulsive, the σ-ω meson cross-interaction suppresses the maximum masses of neutron stars, which is similar to the result of neutron stars not including hyperons. However, for the two different Σ potentials, it is hard to distinguish the difference of mass-radius relations with the same $g_{\sigma\omega}$ from the figure. For clarity, we list the calculated results of the maximum masses of the neutron stars with hyperons in Table II for different parameter sets. Table II shows that the maximum masses of neutron stars with repulsive Σ potential are slightly larger than those with attractive Σ potential for the same $g_{\sigma\omega}$. In addition, comparing the obtained maximum masses of neutron stars including hyperons with the maximum masses of those not including hyperons, one can learn that the emergence of hyperons suppresses the maximum masses by about 22, 23, and 25% for the three values of the σ-ω meson cross-interaction, respectively. It indicates that the σ-ω meson cross-interaction decreases the maximum masses of neutron stars generally, and the increase of the cross-interaction strength further enhances the softening effect of hyperons on the EOS. For the consideration of the sensitivity of the parameter set, we also calculated the maximum masses of neutron stars with $m^* = 0.7\ m$, and obtained the maximum masses of the neutron stars as 1.71 $M_\odot$, 1.56 $M_\odot$, and 1.50 $M_\odot$ for $U_\Sigma^N = -30$ MeV and 1.74 $M_\odot$, 1.57 $M_\odot$, and 1.51 $M_\odot$ for $U_\Sigma^N = 30$ MeV.

The calculated results of the variation behavior of the relative populations of all compositions with respect to the total baryon density are demonstrated in Figs. 6 and 7 for

TABLE II. Calculated results of the maximum masses of neutron stars including hyperons with different parameter sets (The upper row is the result with $U_\Sigma^N = -30$ MeV, and the lower row is that with $U_\Sigma^N = 30$ MeV).

	$g_{\sigma\omega} = 0$	$g_{\sigma\omega} = 20$	$g_{\sigma\omega} = 40$
$M_{\max}(M_\odot)$	1.60	1.51	1.45
$M_{\max}(M_\odot)$	1.61	1.52	1.46

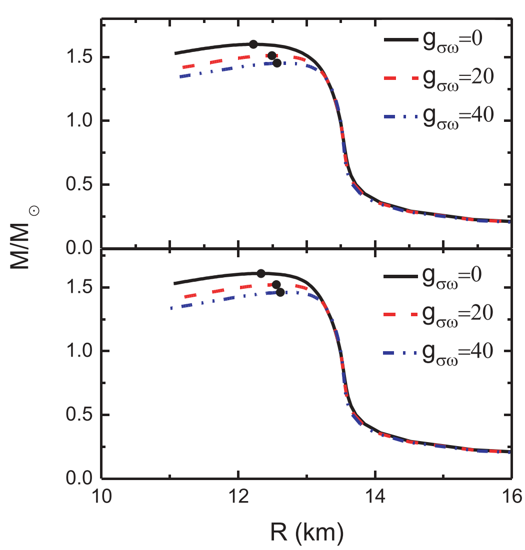

FIG. 5. (Color online) Calculated mass-radius relation of neutron stars in the cases without or with the σ-ω meson cross-interaction for $U_\Sigma^N = -30$ MeV (upper panel) and $U_\Sigma^N = 30$ MeV (lower panel), respectively.

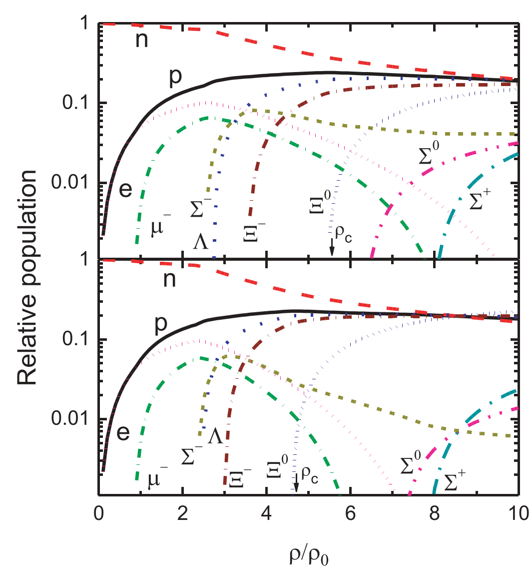

FIG. 6. (Color online) Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of attractive Σ potential with respect to the total baryon density (the upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$; ρ_c denotes the baryon density at the center of the neutron star).

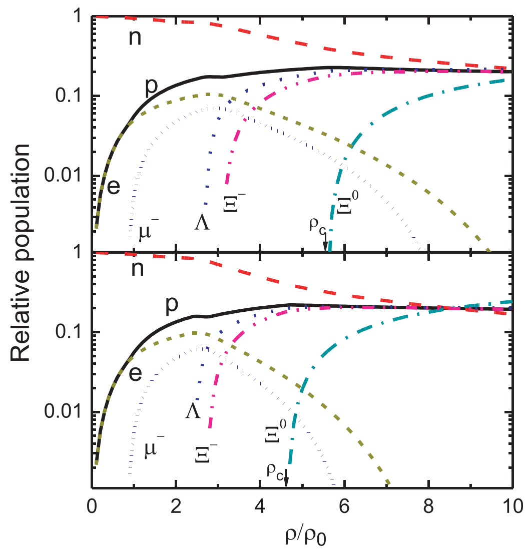

FIG. 7. (Color online) Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of repulsive Σ potential with respect to the total baryon density (the upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$; ρ_c stands for the baryon density at the center of the neutron star).

the attractive and repulsive Σ potentials, respectively. Figure 6 shows that, for the attractive Σ potential $U_\Sigma^N = -30$ MeV, hyperons Σ^- and Λ appear at $2 \sim 3\rho_0$. In the case with the σ-ω meson cross-interaction, the relative population of Σ^- hyperon is suppressed when the baryon density is larger than $3\rho_0$. The figure also indicates that the Ξ^- hyperons with negative charge, too, appear at a lower baryon density. Besides, Ξ^0 hyperons emerge also at a density lower than that without the cross-interaction, but the appearance of Σ^0 hyperons is delayed to a higher density. As for leptons, their relative populations increase with the ascent of the density in the low density region. Then they decrease with the increase of the baryon density and disappear at some critical densities. The critical densities for the leptons to disappear in the case with the cross-interaction are lower than those without the cross-interaction. Figure 7 presents the relative populations with the repulsive Σ potential. Compared with Fig. 6 for the attractive Σ potential, the main difference is that Σ hyperons do not appear up to the maximum density considered here, $10\rho_0$, beyond the central density of neutron star. Besides attributing to the repulsive Σ potential, the absence of Σ hyperons in the dense neutron star matter can also be elucidated by investigating finite systems of strange hadronic matter [62,69]. As pointed out in Ref. [49], one can construct a system with arbitrary nucleons and hyperons forming a big multi-hypernucleus. The system is stable with the forbiddance by Pauli principle of such reactions as $\Lambda + \Lambda \leftrightarrow \Xi + N$ and $\Sigma + N \rightarrow \Lambda + N$. The first reaction releases an energy of $Q \approx 25$ MeV and the second reaction an energy of $Q \approx 80$ MeV, so that Σ hyperons can be hardly stabilized in hypernuclear systems. Because Σ hyperons do not appear in the reasonable range of baryon density for neutron star matter, Ξ^- hyperons emerge at the relatively lower baryon density region to keep the charge neutrality of the whole system.

Furthermore, comparing the role of the $\sigma^2\omega^2$ term with that of the $\omega^2\rho^2$ interaction given in Ref. [23], we notice that, although they both soften the equation of state, the contribution of the $\sigma^2\omega^2$ cross-interaction to the composition distribution of neutron star matter is different from that of the $\omega^2\rho^2$ interaction. For the cross-interaction term $\omega^2\rho^2$ with attractive Σ potential, the onset density of Σ^- hyperons is slightly lowered but the onset of Λ hyperons is shifted to higher density, resulting in a difference of their onset densities being more than $1.5\rho_0$. However, with the $\sigma^2\omega^2$ interaction, the onset densities of Σ^- and Λ are slightly affected and their difference is still small. Besides, for attractive Σ potential, the richness of the Λ hyperon is suppressed with the inclusion of the $\omega^2\rho^2$ interaction, but that of the Σ^- hyperon is suppressed with the $\sigma^2\omega^2$ interaction. For the $\sigma^2\rho^2$ interaction, previous calculation [25] showed that the neutron star has a smaller radius than that without such an interaction for the same neutron star mass $M = 1.4\ M_\odot$. This indicates that such a cross-coupling also plays the role of softening the equation of state. In addition, vector self-coupling ω^4 of ω mesons and ρ^4 of ρ mesons have been considered in Ref. [11]. Generally, the ω^4 term can soften the EOS at high density and give a major modification in the high-density behavior, but the ρ^4 term produces relatively minor changes in neutron star mass.

Finally, to describe the neutron star structure more completely, we demonstrate the variation behavior of the relative populations of all baryons and leptons against the radius of the neutron star in Fig. 8 for attractive Σ potential and in Fig. 9 for repulsive Σ potential, respectively. It is evident from the two figures that, although hyperons Σ^0 and Σ^+ can appear in very high density nuclear matter (as shown in Fig. 6) for

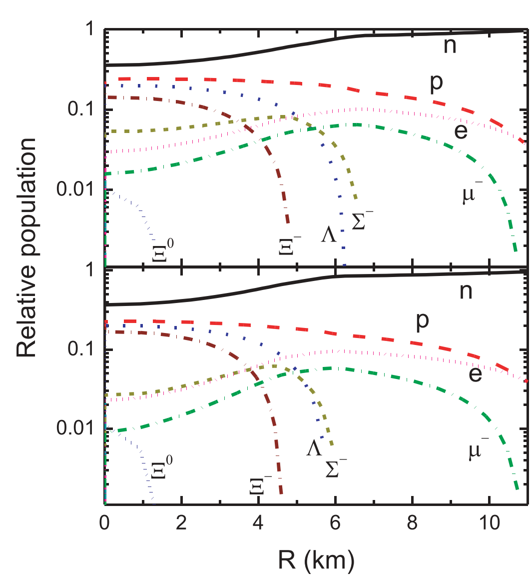

FIG. 8. (Color online) Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of attractive Σ potential with respect to the radius (the upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$).

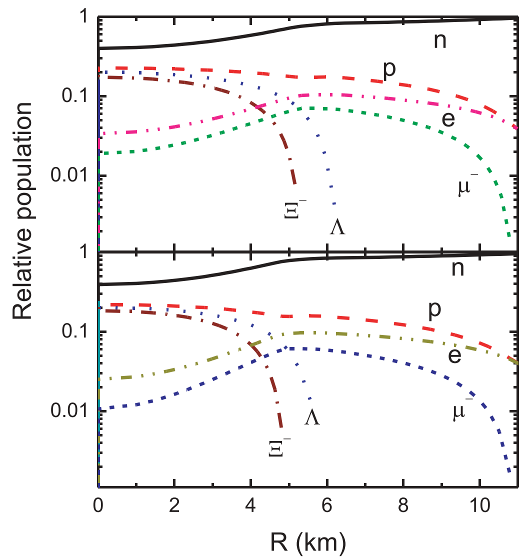

FIG. 9. (Color online) Calculated variation behavior of the relative populations of the compositions of neutron stars including hyperons in the case of repulsive Σ potential with respect to the radius (the upper panel is for $g_{\sigma\omega} = 0$ and the lower panel is for $g_{\sigma\omega} = 40$).

attractive Σ potential, they do not appear in the interior of neutron star because the central density is not high enough to reach the threshold of their production. It is the same for Ξ^0 hyperons in the case with repulsive Σ potential. Moreover, the hyperons Λ, $\Xi^-(\Lambda, \Sigma^-, \Xi^{0,-})$ exist in a smaller region (from the center of the star) with the σ-ω meson cross-interaction for repulsive (attractive) Σ potential than those without the cross-interaction.

IV. SUMMARY

The RMF theory has achieved great success in describing finite nuclei and (astro)nuclear matter, but baryon-meson and meson-meson interactions have not yet had a unified form in different models. Many forms of interactions have been constructed to fit nuclear data, and it is hard to determine which form of interaction is the important one before carrying out concrete calculations. On the other hand, more accurate observations will combine the astronomical physics with nuclear physics more and more tightly. The progress on one aspect will affect the other. Based on these considerations, in this article we investigate the properties of neutron stars with the σ-ω meson cross-interaction. The calculation shows that this cross-interaction softens the EOS of the nuclear (hadronic) matter and results in smaller maximum masses of neutron stars. Moreover, increasing the cross-interaction strength can enhance the softening effect of hyperons on the EOS. In addition to the cross-interaction, we emphasize that the hyperon potential has great influence on the properties, especially the compositions, of neutron stars. For the attractive potential $U_\Sigma^N = -30$ MeV, our calculation shows that almost all the hyperons can possibly appear in neutron star matter. With the cross-interaction, the Ξ^- and Ξ^0 hyperons appear at lower baryon densities than that of the case without the cross-interaction, but the appearance of Σ^0 hyperons is delayed to a higher density. At the same time the critical densities for the leptons to disappear are lower than those of the case without the cross-interaction. For the repulsive potential $U_\Sigma^N = 30$ MeV, a distinct feature is that Σ hyperons do not appear up to the maximal density considered here, $10\rho_0$, beyond the central density of neutron stars. Hence, hypernuclear physics serves as a key ingredient for the composition of dense neutron star matter.

The ever reported heaviest neutron star PSR J0751+1807 with mass about (2.1 ± 0.2) $M_\odot$ [70,71] has been revised down to (1.26 ± 0.14) $M_\odot$ [72]. Therefore the observed maximum mass of neutron stars is consistent with the current calculation with the inclusion of hyperons. However, for more complex consideration, the interactions by exchanging σ^*, ϕ, and δ mesons should be taken into account, and the boson condensates also play a very important role in describing the neutron star matter. All these factors have not yet been taken into account here for giving prominence to the cross-interaction in this article. With the consideration of kaon condensation and δ-meson channel interaction, it's possible to modify the maximum masses of neutron stars by adjusting kaon optical potential U_K or other parameters. Moreover, The three-body force [73–75] has not received sufficient attention but has been known for quite some time to be repulsive in nature. It may stiffen the equation of state of neutron star matter and gives a large modification to the structure of neutron stars. In addition, the hadron quark phase transition at high (energy) density is also important for further study of neutron star properties. Related investigations are under way.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Contracts 10425521, 10575005, and 10675007; the Major State Basic Research Development Program under Contract G2007CB815000; and the Key Grant Project of the Chinese Ministry of Education (CMOE) under Contact 305001.

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