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Extended relativistic chiral mean field model for nuclear matter

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We study nuclear matter properties based on the chiral sigma model. We formulate the extended relativistic chiral mean-field model by using the tensor optimized shell-model prescription developed for the description of the tensor interaction in finite nuclei. We apply further the unitary correlation operator model prescription to the treatment of the short-range repulsion in the pion channel. The remaining effects are treated phenomenologically in terms of the ω-meson coupling term in the relativistic mean-field approximation. We discuss the equation of state and the chiral properties of nuclear matter.

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I. INTRODUCTION

Nuclear physics provides good opportunities for the study of the strong interaction. Chiral symmetry is the key concept for this study, where the pion appears as the Nambu-Goldstone boson of its spontaneous symmetry breaking and plays important roles for the description of nuclei. Therefore, for the discussion of nuclear properties as well as dynamical aspects of chiral symmetry, it is extremely important to have a theoretical framework for the pion nuclear dynamics in a manner consistent with the nuclear many-body problem. In particular, the proper treatment of pion exchange interaction in nuclear physics is very important for the discussion of chiral symmetry. Such attempts have been made in the relativistic framework for finite nuclei [1–3]. It was shown that the pion made a very important contribution in the formation of jj-closed-shell magic nuclei by providing a large spin-orbit splitting effect [3]. We point out that the large binding effect is caused by the strong tensor interaction, whose main origin is the one-pion exchange interaction, and that the Pauli blocking effect reduces the amount of the tensor contribution for spin-down single-particle state causing strong spin-orbit splitting effect [4,5].

As for the role of the pion for nuclear structure, important contribution has been made by the Argonne group for light nuclei in the variational Monte Carlo (VMC) method by using the realistic nucleon-nucleon interaction [6]. With introduction of three-body interaction providing a few MeV additional binding effect for ^{4}He, they have reproduced binding energies and several observables for light nuclei up to a mass number around 10 ($A \leqslant 10$). They have pointed out that matrix elements of the pion exchange interaction for ground-state wave functions are about 70∼80% of those of the entire two nucleon interaction. Furthermore, the tensor interaction provides almost 50% of the entire two body attraction, while the remaining 20∼30% is due to the central spin-spin interaction. The pion contribution increases with the mass number in the mass range they have calculated. These findings suggest that the pion contribution should be volume like and that even in nuclear matter the attraction due to the pion exchange interaction should be dominant.

For nuclear matter, Kaiser *et al.* have made an interesting work, where they have studied the role of the pion systematically using expansion in the Fermi momentum [7]. They have found that the pion exchange interaction plays the major role for providing the binding energy and the saturation property of nuclear matter. Particularly important in the pion contribution is the direct iterated one-pion exchange diagram, which is far dominant as compared with other terms such as the Fock term and the crossed 2p-2h excitation term. We would like to call the direct iterated pion exchange term as the two-particle-two-hole (2p-2h) excitation diagram in this article.

Another development was made recently by Ericson *et al.*, who have studied nuclear matter by using a chiral model Lagrangian [8]. However, their treatment of the one-pion exchange interaction differs from that of Kaiser *et al.* [7], and the resulting role of the pion exchange interaction on the property of nuclear matter is very different. The pion contribution is not as large as that of Kaiser *et al.*, whereas the role of the σ-meson exchange is largely attractive to provide sufficient binding energy. They have also studied chiral properties of nuclear matter such as the chiral condensate and the scalar susceptibility that are the first- and second-order derivatives of the grand potential with respect to the bare quark mass while keeping the chemical potential fixed.

One of the purposes of the present work is, therefore, to find out why completely different conclusions are derived from the work of Kaiser *et al.* [7] and the work of Ericson *et al.* [8]. After clarifying this, we would also like to study chiral properties of nuclear matter, because they are largely influenced by the treatment and the resulting roles of the pion contribution.

To work out nuclear matter properly by including pion contributions, we point out that there are important works in recent years on the treatment of the pion exchange potential,

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especially on the tensor interaction by means of the so-called tensor optimized shell model (TOSM) [9]. Myo *et al.* included all 2p-2h configurations with various angular momenta that mix in the ground state by the tensor interaction. The pion exchange interaction has a large tensor term, which should be treated in the TOSM prescription. Furthermore, the short-range repulsive interaction, which is caused by quark structure of the nucleon [10], is nicely treated in terms of the unitary correlation operator method (UCOM) by Feldmeier *et al.* [11, 12]. The central spin-spin interaction of the one-pion exchange interaction is subject to the short-range correlation, which are treated by the UCOM prescription. In our study we are able to use these two methods for the study of the effect of the pion in nuclear matter. Hence, the second purpose of the present study is to develop a method to treat the pion exchange interaction with the inclusion of the short range effect for the study of nuclear matter.

We shall study the role of the pion in nuclear matter using the Lagrangian of the extended chiral sigma model. We use the framework following the TOSM in the medium range region (0.5 fm $\lesssim r \lesssim$ 1.5 fm) and treat the short-range repulsion in terms of the UCOM ($r \lesssim$ 0.5 fm). We would like to use the Lagrangian of the extended chiral sigma model that contains ω-meson terms to provide a necessary repulsive effect for the construction of nuclear matter. We shall then describe the saturation property of nuclear matter and also the chiral condensate and nuclear susceptibility as functions of nuclear density.

This article is organized as follows. In Sec. II, we describe the Lagrangian of the extended chiral sigma model (ECSM) and introduce the TOSM prescription to treat the strong pion exchange interaction. We shall introduce also the form factor effect and the short-range correlation as treated in terms of the UCOM prescription. In Sec. III, we derive the explicit forms of the chiral condensate and the scalar susceptibility. In Sec. IV, we present numerical results for nuclear matter. We shall discuss the saturation property of nuclear matter and the change of the chiral condensate as well as the scalar susceptibility. Section V is devoted to the summary of the present work.

II. EXTENDED CHIRAL SIGMA MODEL

A. Extended sigma model Lagrangian and pion optimized energy minimization

We would like to start with the Lagrangian of the extended chiral sigma model introduced in our previous works [1–3,13],

$$\begin{aligned} L_{\sigma\omega} = {} & \bar{\psi}(i\gamma_\mu\partial^\mu - g_\sigma(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}) - g_\omega\gamma_\mu\omega^\mu)\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{\mu^2}{2}(\sigma^2+\vec{\pi}^2) \\ & - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\ & + \frac{1}{2}\tilde{g_\omega}^2(\sigma^2+\vec{\pi}^2)\omega_\mu\omega^\mu + c\sigma. \end{aligned} \tag{1}$$

The fields ψ, σ, and π are the nucleon, σ, and pion fields. The parameters μ and λ determine the masses of the π and σ fields and their three- and four-point interactions after the chiral symmetry breaking. The coupling constant g_σ and g_ω are for σNN and ωNN couplings. Another coupling constant $\tilde{g_\omega}$ is a characteristic part of the extended chiral sigma model that not only describes the interaction of the ω meson with the pion and σ meson but also provides the mass of the ω as we shall explain below [13]. In Eq. (1), we have introduced the explicit chiral symmetry breaking term, $c\sigma$. This term generates a finite mass of the pion and is related with the bare quark mass. Hence, this term plays the central role when we discuss the chiral properties of nuclear matter.

When chiral symmetry is spontaneously broken, the σ field takes a finite vacuum expectation value $\langle\sigma\rangle = f_\pi$, which is the pion decay constant. Rewriting the σ field as $\sigma \to f_\pi + \sigma$ and performing the Weinberg transformation [14], we get

$$\begin{aligned} L = {} & \bar{\psi}\left(i\gamma_\mu\partial^\mu - M_N - g_\sigma\sigma - \frac{g_A}{2f_\pi}\gamma_5\gamma_\mu\tau^a\partial^\mu\pi^a - g_\omega\gamma_\mu\omega^\mu\right)\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \lambda f_\pi\sigma^3 - \frac{\lambda}{4}\sigma^4 \\ & + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}m_\pi^2\pi^{a2} - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} \\ & + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \tilde{g_\omega}^2 f_\pi\sigma\omega_\mu\omega^\mu + \frac{1}{2}\tilde{g_\omega}^2\sigma^2\omega_\mu\omega^\mu. \end{aligned} \tag{2}$$

In general, the transformed Lagrangian contains higher-order terms in the pion field, but in Eq. (2) we have taken the lowest order term; we keep only the Yukawa interaction of the pseudovector type for the pion nucleon interaction. For the coupling constant of pion nucleon interaction, we have written $g_A/2f_\pi$ instead of $1/2f_\pi$ to meet with phenomenology. g_A is the nucleon axial vector coupling constant chosen as $g_A = 1.3$, which has been introduced by hand. In Eq. (2), the nucleon mass is generated by the σ condensate as $M_N = g_\sigma f_\pi$. The ω-meson mass is also generated in a similar way as the nucleon mass, $m_\omega = \tilde{g_\omega} f_\pi$. The two coupling constants, λ and μ, are expressed by the σ-meson mass and the pion mass as $\mu^2 = \frac{3m_\pi^2 - m_\sigma^2}{2}$ and $\lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}$. The ω-nucleon coupling constant, g_ω, and the σ-meson mass, m_σ, are kept as free parameters.

We are able to treat the σ- and ω-meson fields in the relativistic mean-field approximation. However, the pion field does not develop a mean field in nuclear matter if the parity and charge are conserved for single-particle state ψ [1]. Hence we treat the pion contribution in terms of the pion exchange interaction between nucleons. For this purpose, we eliminate the pion field by using the equation of motion and rewrite the above Lagrangian in terms of the pion exchange interaction. We shall take here a static approximation and drop k^0 in the pion propagator. For later convenience, we write the Hamiltonian density, $\mathcal{H}$, instead of Lagrangian density as follows,

$$\begin{aligned} \mathcal{H} = {} & \psi^\dagger(-i\vec{\alpha}\cdot\vec{\nabla} + \beta M_N + \beta g_\sigma\sigma + g_\omega\omega)\psi + \frac{1}{2}m_\sigma^2\sigma^2 \\ & + \lambda f_\pi\sigma^3 + \frac{\lambda}{4}\sigma^4 - \frac{1}{2}m_\omega^2\omega^2 - \tilde{g_\omega}^2 f_\pi\sigma\omega^2 - \frac{1}{2}\tilde{g_\omega}^2\sigma^2\omega^2 \\ & - \frac{1}{2}\int d^3x' \int \frac{d^3k}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \bar{\psi}(x)\gamma_5\gamma_\mu\tau^a k^\mu\psi(x) \\ & \times \frac{e^{i\vec{k}(\vec{x}-\vec{x}')}}{\vec{k}^2 + m_\pi^2}\bar{\psi}(x')\gamma_5\gamma_\nu\tau^a k^\nu\psi(x'). \end{aligned} \tag{3}$$

We have written here explicitly only the time component of the ω-meson field to anticipate the mean-field treatment.

We introduce now a trial wave function for the ground state,

$$\Psi(\sigma, \omega, \psi) = \Psi(\sigma, \omega)\Psi_N(\{c_i\}). \quad (4)$$

Here, $\Psi(\sigma, \omega)$ is a coherent state defined by

$$\begin{aligned} \langle\Psi(\sigma, \omega)|\sigma|\Psi(\sigma, \omega)\rangle &= \sigma, \\ \langle\Psi(\sigma, \omega)|\omega_\mu|\Psi(\sigma, \omega)\rangle &= \delta_{\mu,0}\omega. \end{aligned} \quad (5)$$

We do not need an explicit form of the coherent state wave function for the σ and ω mesons. We need only the mean field values, σ and ω, which are hereafter classical fields. This procedure corresponds to the relativistic mean-field (RMF) approximation.

For the nucleon part $\Psi_N(\{c_i\})$, we write a variational wave function by including 2p-2h components for the optimization of pion contributions, following the TOSM prescription for finite nuclei,

$$\Psi_N(\{c_i\}) = c_0|0\rangle + \sum_i c_i|\text{2p-2h} : i\rangle, \quad (6)$$

where $|\text{2p-2h} : i\rangle$ is written explicitly as

$$|\text{2p-2h} : i\rangle = a^\dagger_{\vec{p}_1+\vec{k},s_1,t_1} a_{\vec{p}_1,s_2,t_2} a^\dagger_{\vec{p}_2-\vec{k},s_3,t_3} a_{\vec{p}_2,s_4,t_4}|0\rangle. \quad (7)$$

Here, $a^\dagger$ and a are creation and annihilation operators of nucleon state, and $|0\rangle$ denotes the ground state in the mean-field approximation. This equation shows creation of particle and hole states of nucleon carrying momenta as indicated by the subscripts of the nucleon creation and annihilation operators. The extra subscripts s and t denote the spin and isospin of the nucleon. For simplicity, these three indices $(\vec{p}, s, t)$ are expressed by a single label i in Eq. (7). The corresponding diagram is shown in Fig. 1.

The single-particle wave function is a product of a plane-wave solution, $u(\vec{p}, s)e^{-ipx}$ with a momentum $\vec{p}$ and a spin s, and an isospin wave function, $\chi(t)$, of isospin t,

$$\psi(x) = \frac{1}{\sqrt{V}} u(\vec{p}, s)e^{-ipx}\chi(t). \quad (8)$$

Here, V is the volume in which periodic boundary condition is imposed. We take the normalization condition for the spinor $u(\vec{p}, s)$ as $u^\dagger(\vec{p}, s)u(\vec{p}, s) = 1$.

We will now make variation of the total energy with respect to the parameters in the trial wave function; σ, ω and $\{c_i\}$. The total energy per unit volume E is written as

$$E(\sigma, \omega, \{c_i\}) = \langle\Psi| \int d^3x\mathcal{H}|\Psi\rangle/V = \langle\Psi|H|\Psi\rangle/V, \quad (9)$$

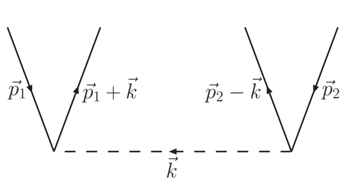

FIG. 1. Feynman diagram for the 2p-2h state.

where H is the Hamiltonian. Then, the minimization of E with respect to σ and ω provides

$$\begin{aligned} m_\sigma^2\sigma &= -g_\sigma\langle\Psi_N|\bar{\psi}\psi|\Psi_N\rangle + \tilde{g}_\omega{}^2 f_\pi\omega^2 \\ &\quad + \tilde{g}_\omega{}^2\sigma\omega^2 - 3\lambda f_\pi\sigma^2 - \lambda\sigma^3, \\ m_\omega^2\omega &= g_\omega\langle\Psi_N|\psi^\dagger\psi|\Psi_N\rangle - 2\tilde{g}_\omega{}^2 f_\pi\sigma\omega - \tilde{g}_\omega{}^2\sigma^2\omega. \end{aligned} \quad (10)$$

These two equations determine the mean-field values of σ and ω when the scalar density $\langle\Psi_N|\bar{\psi}\psi|\Psi_N\rangle$ and the vector density $\langle\Psi_N|\psi^\dagger\psi|\Psi_N\rangle$ of the ground state are given. For this we need an equation to determine Ψ_N.

We write the total energy as

$$\begin{aligned} \langle\Psi|H|\Psi\rangle &= c_0^* c_0\langle 0|H|0\rangle + \sum_j c_0^* c_j\langle 0|H|\text{2p-2h} : j\rangle \\ &\quad + \sum_i c_i^* c_0\langle\text{2p-2h} : i|H|0\rangle \\ &\quad + \sum_{i,j} c_i^* c_j\langle\text{2p-2h} : i|H|\text{2p-2h} : j\rangle. \end{aligned} \quad (11)$$

Here, the σ and ω meson fields take the mean-field values in the above matrix element of the Hamiltonian. The energy of the 0p-0h state is given by the expression of the RMF model,

$$\begin{aligned} \langle 0|H|0\rangle &= \frac{4}{V}\sum_{|\vec{p}|}^{p_F}\sqrt{\vec{p}^2 + M_N^{*2}} + g_\omega\omega\rho_V \\ &\quad + \frac{1}{2}m_\sigma^2\sigma^2 + \lambda f_\pi\sigma^3 + \frac{\lambda}{4}\sigma^4 - \frac{1}{2}m_\omega^2\omega^2 \\ &\quad - \tilde{g}_\omega{}^2 f_\pi\sigma\omega^2 - \frac{1}{2}\tilde{g}_\omega{}^2\sigma^2\omega^2, \end{aligned} \quad (12)$$

where the factor 4 of the first term includes spin and isospin degrees of freedom and ρ_V is

$$\rho_V = \frac{4}{V}\sum_{|\vec{p}|}^{p_F} 1 = 4\int^{p_F}\frac{d^3p}{(2\pi)^3} = \frac{2}{3\pi^2}p_F^3. \quad (13)$$

The nucleon effective mass is written as

$$M_N^* = M_N + g_\sigma\sigma. \quad (14)$$

In Eqs. (12) and (13), $\vec{p}$ is discretized by the quantization in a finite size box. However, we shall replace the summation by integration using the rule $\frac{1}{V}\sum_i \to \int\frac{d^3p}{(2\pi)^3}$ and write the momentum as a continuous variable.

For the matrix element between $|0\rangle$ and $|\text{2p-2h}\rangle$, we obtain

$$\begin{aligned} &\langle 0| - \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \int d^3x_1 d^3x_2 \int\frac{d^3k}{(2\pi)^3}\bar{\psi}(x_1)\gamma_5\gamma_\mu\tau^a k^\mu\psi(x_1) \\ &\quad\times\frac{e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2 + m_\pi^2}\bar{\psi}(x_2)\gamma_5\gamma_\nu\tau^a k^\nu\psi(x_2)|\text{2p-2h} : i\rangle \\ &= -\left(\frac{g_A}{2f_\pi}\right)^2\bar{u}(\vec{p}_1 + \vec{k}, s_1)\gamma_5\vec{\gamma}\cdot\vec{k}u(\vec{p}_1, s_2)\langle t_1|\tau^a|t_2\rangle \\ &\quad\times\frac{1}{\vec{k}^2 + m_\pi^2}\bar{u}(\vec{p}_2 - \vec{k}, s_3)\gamma_5\vec{\gamma}\cdot\vec{k}u(\vec{p}_2, s_4)\langle t_3|\tau^a|t_4\rangle. \end{aligned} \quad (15)$$

As for the 2p-2h matrix elements, we take an approximation of taking only the single-particle part and drop the contribution of the pion exchange interaction,

$$\langle \text{2p-2h} : i|H|\text{2p-2h} : j\rangle \sim \delta_{i,j}\frac{1}{V}\epsilon_{\text{2p-2h}} + \delta_{i,j}\langle 0|H|0\rangle, \quad (16)$$

where

$$\epsilon_{\text{2p-2h}} = \sqrt{(\vec{p}_1+\vec{k})^2 + M_N^{*2}} - \sqrt{\vec{p}_1^2 + M_N^{*2}} + \sqrt{(\vec{p}_2-\vec{k})^2 + M_N^{*2}} - \sqrt{\vec{p}_2^2 + M_N^{*2}}. \quad (17)$$

Having all the above ingredients, we can write the total energy as

$$\langle\Psi|H|\Psi\rangle = \langle 0|H|0\rangle\left(c_0^*c_0 + \sum_i c_i^*c_i\right) + \sum_i c_0^*c_i\langle 0|H|\text{2p-2h} : i\rangle + \sum_i c_i^*c_0\langle \text{2p-2h} : i|H|0\rangle + \sum_i c_i^*c_i\frac{1}{V}\epsilon_{\text{2p-2h}}. \quad (18)$$

Using the normalization condition

$$\langle\Psi|\Psi\rangle = c_0^*c_0 + \sum_i c_i^*c_i = 1, \quad (19)$$

we get the following equation by variation of $E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$ with respect to c_i^* as

$$c_0\langle \text{2p-2h} : i|H|0\rangle + c_i\frac{1}{V}\epsilon_{\text{2p-2h}} + \langle 0|H|0\rangle c_i - Ec_i = 0. \quad (20)$$

We can then solve for c_i as

$$c_i = \frac{-c_0\langle \text{2p-2h} : i|H|0\rangle}{\frac{1}{V}\epsilon_{\text{2p-2h}} + \langle 0|H|0\rangle - E}, \quad (21)$$

where E is the ground-state energy. This equation can be solved iteratively. We point out here that the expression for c_i in Eq. (21) becomes the one of perturbation if we drop $\langle 0|H|0\rangle - E$ in the denominator. In the actual calculation we make the self-consistent calculation with this term in Eq. (21). The self-consistency condition provides important effects in the structure of finite nuclei in the tensor optimized shell model [9].

We get the nucleon wave function, Ψ_N, by solving the above equations for c_i. Hence, we can calculate the vector and scalar densities for the mean-field equations,

$$\rho_V = \langle\Psi_N|\psi^\dagger\psi|\Psi_N\rangle = \langle 0|\psi^\dagger\psi|0\rangle + \sum_i c_i^*c_i\frac{1}{V}[u^\dagger(\vec{p}_1+\vec{k}, s_1)u(\vec{p}_1+\vec{k}, s_1) + u^\dagger(\vec{p}_2-\vec{k}, s_3)u(\vec{p}_2-\vec{k}, s_3) - u^\dagger(\vec{p}_1, s_2)u(\vec{p}_1, s_2) - u^\dagger(\vec{p}_2, s_4)u(\vec{p}_2, s_4)] = \frac{2}{3\pi^2}p_F^3,$$

$$\rho_S = \langle\Psi_N|\bar{\psi}\psi|\Psi_N\rangle = 4\int\frac{d^3p}{(2\pi)^3}\frac{M_N^*}{\sqrt{p^2+M_N^{*2}}} + \sum_i c_i^*c_i\frac{1}{V} \times\left(\frac{M_N^*}{\sqrt{(\vec{p}_1+\vec{k})^2+M_N^{*2}}} + \frac{M_N^*}{\sqrt{(\vec{p}_2-\vec{k})^2+M_N^{*2}}} - \frac{M_N^*}{\sqrt{p_1^2+M_N^{*2}}} - \frac{M_N^*}{\sqrt{p_2^2+M_N^{*2}}}\right), \quad (22)$$

where we have used the spinor normalization condition $u^\dagger(\vec{p}, s)u(\vec{p}, s) = 1$.

We work out now the summation of all the quantum states involving 2p-2h excitations; integration over $\vec{p}_1$, $\vec{p}_2$, and $\vec{k}$ for the case of perturbation. Here, we use the expression of Eq. (21) without $\langle 0|H|0\rangle - E$ in the denominator. After tedious calculation, we get the energy for the 2p-2h excitations,

$$\begin{aligned}E_{\text{2p-2h}} = &-\frac{3M_N^*g_A^4k_F^7c_0c_0^*}{\pi^2 f_\pi^4(2\pi)^4}\left\{\int_0^1 dx\frac{x^5}{(x^2+m_\pi^2/4k_F^2)^2}\right. \\ &\times\left[\frac{(58-80\ln 2)x^2}{15} - \frac{2x^4}{5} + \frac{8}{15}\ln(1-x^2) + \left(x - \frac{2x^3}{3} + \frac{x^5}{5}\right)\ln\left(\frac{1+x}{1-x}\right)\right] \\ &+\int_1^\infty dx\frac{x^5}{(x^2+m_\pi^2/4k_F^2)^2}\left[\frac{44x}{15} + \frac{8x^3}{15} + \left(-\frac{8x^3}{3} + \frac{8x^5}{15}\right)\ln\left(1-\frac{1}{x^2}\right)\right. \\ &\left.\left.+\left(\frac{8}{15} - \frac{8x^2}{3}\right)\ln\left(\frac{x+1}{x-1}\right)\right]\right\},\end{aligned} \quad (23)$$

where we have used $x = p/2k_F$ and $x^2/(x^2 + m_\pi^2/4k_F^2)$ term represents the one-pion exchange interaction. In actual calculation, we use the expression of Eq. (21) which include $\langle 0|H|0\rangle - E$ in the denominator for the equation of state and chiral condensate. It will become a self-consistent calculation with total energy E. But the expressions are very complicated and long, we do not write here explicitly. We notice that the second integration in Eq. (23) is divergent. We regularize it by introducing the dipole pion-nucleon form factor on the pion exchange interaction, which reflects the finite size effect of the nucleon. Furthermore, for the spin-spin interaction originating from the one-pion exchange interaction, we consider furthermore the effect of the short range correlation in terms of the UCOM.

B. Matrix elements with proper treatment of short range correlation

To compute two-body matrix elements properly, we need to treat the short-range correlation due to the strong short-range repulsion in the nucleon-nucleon interaction. Here, we follow the method of Neff-Feldmeier [11,12], which is named UCOM. We shall consider this short-range correlation in the central part of the pion interaction. We do not include this effect in the tensor part, because it requires a finite angular momentum between the two nucleon, and the short-range repulsive effect is suppressed due to the centrifugal force. This has been shown by Myo *et al.* in Ref. [9], where they compare the tensor matrix elements of the AV8′ and Bonn potentials. These tensor interactions are wildly different in the short range part, whereas the integrand matrix elements are similar due to the d-wave function that is pushed out by the strong centrifugal potential.

Now, the essence of the UCOM is to introduce the unitary transformation,

$$\psi = U\phi. \tag{24}$$

Here, ψ denotes a full wave function, whereas ϕ is a simple wave function without the short-range correlation. Hence, if we know U, we can express the full wave function ψ in terms of ϕ. The unitary correlation operator, U, is written as $U = e^{i\sum_{ij} g_{ij}}$ and $U^\dagger = U^{-1}$, where g_{ij} is a two-body Hermite operator [11]. Hence, we get the Schroedinger equation as

$$\begin{aligned} H\psi &= E\psi, \\ U^\dagger H U\phi &= E\phi. \end{aligned} \tag{25}$$

Because U is expressed in the form of exponential, the correlated Hamiltonian, $U^\dagger H U$, may have terms with more than three-body correlations. The approximation of UCOM is to take terms up to two-body correlation terms [12]. In this approximation, the short-range correlation is suitable because the correlator is effective only at very short distances between two nucleons. The two-body approximation is justified because the probability for three nucleons meet in their interaction range is small around the normal nuclear matter density.

The correlator U is to make the central interaction as

$$V(r) \to \tilde{V}(r) = U^\dagger V(r) U = V[R_+(r)], \tag{26}$$

where

$$R_+(r) = r + \alpha\left(\frac{r}{\beta}\right)^\eta \exp[-\exp(r/\beta)]. \tag{27}$$

The parameters α, β, and η are determined later. For the use of the central interaction after the UCOM prescription, we expand the modified central interaction in terms of the gaussian functions,

$$\tilde{V}(r) = V[R_+(r)] = \sum_i a_i \exp\left[-\left(\frac{r}{r_i}\right)^2\right]. \tag{28}$$

We can then easily perform the Fourier transform to obtain the momentum space expression as

$$\tilde{V}(k) = \sum_i a_i(\pi r_i^2)^{3/2} \exp\left(-\frac{\vec{k}^2 r_i^2}{4}\right). \tag{29}$$

Because we just consider the UCOM for the central part of the pion nucleon interaction, we work out now the part that contains spinors to separate the matrix element between $|0\rangle$ and $|\text{2p-2h}\rangle$ into the spin-spin part and the tensor part,

$$\begin{aligned} M &= \bar{u}(\vec{p}_1 + \vec{k}, s_1)\gamma_5\vec{\gamma}\cdot\vec{k}u(\vec{p}_1, s_2) \\ &\quad\times \frac{1}{\vec{k}^2 + m_\pi^2}\bar{u}(\vec{p}_2 - \vec{k}, s_3)\gamma_5\vec{\gamma}\cdot\vec{k}u(\vec{p}_2, s_4) \\ &= u^\dagger(\vec{p}_1 + \vec{k}, s_1)u^\dagger(\vec{p}_2 - \vec{k}, s_3)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}_1 \frac{\vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{k}}{\vec{k}^2 + m_\pi^2} \\ &\quad\times \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}_2 u(\vec{p}_1, s_2)u(\vec{p}_2, s_4), \end{aligned} \tag{30}$$

where the matrices with suffix 1 and 2 are those for particles 1 and 2, respectively. Now we can separate the pion exchange potential into the central and the tensor parts by using the relation,

$$\begin{aligned} &\frac{\vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{k}}{k^2 + m_\pi^2} \\ &\quad= \frac{1}{3}\left(\vec{\sigma}_1\cdot\vec{\sigma}_2\frac{k^2}{k^2 + m_\pi^2} + \frac{3\vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{k} - k^2\vec{\sigma}_1\cdot\vec{\sigma}_2}{k^2 + m_\pi^2}\right). \end{aligned} \tag{31}$$

On the right hand side, the first term is the central part and the second term is the tensor part. By including the short-range correlation with the UCOM for the central part, we can express the interaction as

$$\frac{1}{3}\left\{\vec{\sigma}_1\cdot\vec{\sigma}_2[\tilde{V}(k) - V(k)] + \frac{3\vec{\sigma}_1\cdot\vec{k}\vec{\sigma}_2\cdot\vec{k}}{k^2 + m_\pi^2}\right\}, \tag{32}$$

where $V(k) = \vec{k}^2/(\vec{k}^2 + m_\pi^2)$ is the central part of the original interaction and $\tilde{V}(k)$ is the modified one by the UCOM. For simplicity, we shall take the nonrelativistic approximation for the evaluation of the spin matrix elements,

$$\begin{aligned} M = &\ \frac{1}{3}[\tilde{V}(k) - V(k)]\langle s_1|\sigma_1^a|s_2\rangle\langle s_3|\sigma_2^a|s_4\rangle \\ &+ \frac{\langle s_1|\vec{\sigma}_1\cdot\vec{k}|s_2\rangle\langle s_3|\vec{\sigma}_2\cdot\vec{k}|s_4\rangle}{k^2 + m_\pi^2}. \end{aligned} \tag{33}$$

Here, we have dropped the exchange term contribution, although it is straightforward to work out.

Finally, we have to make a self-consistent calculation to get the solution of the whole system by energy minimization. We make the nonrelativistic approximation for the nucleon energy $\sqrt{p_1^2 + m^2}$ and also for the spin matrix elements. Because we solve the above equations step by step, we shall provide the squared quantity for the matrix element between $|0\rangle$ and $|\text{2p-2h}\rangle$. The squared matrix elements are written as

$$\begin{aligned} &|\langle 0|H|\text{2p-2h}: i\rangle|^2 \\ &\quad= \left(\frac{g_A}{2f_\pi}\right)^4 \frac{1}{V^2}\theta(|\vec{p}_1 + \vec{k}| - k_F)\theta(|\vec{p}_2 - \vec{k}| - k_F) \\ &\quad\quad\times\theta(k_F - |\vec{p}_1|)\theta(k_F - |\vec{p}_2|)(\mathrm{Tr}\{\tau^a\tau^b\})^2 \\ &\quad\quad\times\left(\frac{1}{9}[\tilde{V}(k) - V(k)]^2\,\mathrm{Tr}\left\{\sigma_1^a\sigma_1^b\right\}\mathrm{Tr}\left\{\sigma_2^a\sigma_2^b\right\}\right. \\ &\quad\quad+ \frac{\mathrm{Tr}\{(\vec{\sigma}_1\cdot\vec{k})^2\}\,\mathrm{Tr}\{(\vec{\sigma}_2\cdot\vec{k})^2\}}{\left(k^2 + m_\pi^2\right)^2} + \frac{1}{3}\frac{\tilde{V}(k) - V(k)}{k^2 + m_\pi^2} \end{aligned}$$

$$\times\left[\mathrm{Tr}\left\{\sigma_1^a\vec{\sigma}_1\cdot\vec{k}\right\}\mathrm{Tr}\left\{\sigma_2^a\vec{\sigma}_2\cdot\vec{k}\right\}+\mathrm{Tr}\left\{\vec{\sigma}_1\cdot\vec{k}\sigma_1^b\right\}\mathrm{Tr}\left\{\vec{\sigma}_2\cdot\vec{k}\sigma_2^b\right\}\right]\Big)$$
$$=\left(\frac{g_A}{2f_\pi}\right)^4\frac{1}{V^2}\theta(|\vec{p}_1+\vec{k}|-k_F)\theta(|\vec{p}_2-\vec{k}|-k_F)\times\theta(k_F-|\vec{p}_1|)\theta(k_F-|\vec{p}_2|)\times 12\left\{\frac{12}{9}[\tilde{V}(k)-V(k)]^2+\frac{4k^4}{(k^2+m_\pi^2)^2}+\frac{8}{3}\frac{k^2[\tilde{V}(k)-V(k)]}{k^2+m_\pi^2}\right\}, \tag{34}$$

where we have used $\mathrm{Tr}\{\sigma^a\sigma^b\}=2\delta^{ab}$, $\mathrm{Tr}\{\vec{\sigma}\cdot\vec{a}\vec{\sigma}\cdot\vec{b}\}=2\vec{a}\cdot\vec{b}$, and $\mathrm{Tr}\{\sigma^a\vec{\sigma}\cdot\vec{k}\}=2k^a$, where k^a is the a-th component of $\vec{k}$. Because $V(k)=\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}$, we get

$$|\langle 0|H|2\text{p-}2\text{h}:i\rangle|^2=\left(\frac{g_A}{2f_\pi}\right)^4\frac{12}{V^2}\theta(|\vec{p}_1+\vec{k}|-k_F)\theta(|\vec{p}_2-\vec{k}|-k_F)\times\theta(k_F-|\vec{p}_1|)\theta(k_F-|\vec{p}_2|)\times\left\{4V^2(k)+\frac{4}{3}[\tilde{V}^2(k)-V^2(k)]\right\}. \tag{35}$$

Therefore, when we consider the UCOM effect in our program, we just have to use the modified matrix elements of Eq. (35) instead of the corresponding part appearing in Eq. (23), which contains $V^2(k)$.

Finally, we consider the nucleon form factor that is associated with the finite size of the nucleon [15]. Here, we employ the dipole form factor in which we replace the one-pion exchange interaction $\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}$ in the momentum space by

$$\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}\to\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}\left(\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\vec{k}^2}\right)^2. \tag{36}$$

We can then manipulate this as

$$\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}\left(\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\vec{k}^2}\right)^2=\left(\frac{\vec{k}^2}{\vec{k}^2+m_\pi^2}-\frac{\vec{k}^2}{\vec{k}^2+\Lambda^2}\right)\left(\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\vec{k}^2}\right)=\frac{\Lambda^2}{\vec{k}^2+\Lambda^2}-\frac{m_\pi^2}{\vec{k}^2+m_\pi^2}-\frac{\Lambda^2-m_\pi^2}{2\Lambda}\frac{\partial}{\partial\Lambda}\frac{\Lambda^2}{\Lambda^2+\vec{k}^2}. \tag{37}$$

This form indicates that calculations can be performed with the Yukawa interaction. This is true even for the case $\vec{k}^2$ is included in the numerator.

III. CHIRAL CONDENSATE AND SCALAR SUSCEPTIBILITY OF NUCLEAR MATTER

Ericson *et al.* [8] have studied various chiral quantities associated with partial restoration of chiral symmetry and scalar susceptibility in nuclear matter. They are related with the first and second order derivatives of the grand potential with respect to the bare quark mass m_q at a fixed chemical potential μ. If we are able to work out nuclear matter with quark degrees of freedom explicitly, we can use this definition for the chiral condensate and the scalar susceptibility. In most of chiral model studies of nuclear matter, including the one of ours and Ericson *et al.* [8], quarks are confined in hadrons and the dynamical variables at low energies are baryons and mesons with chiral symmetry. In the present chiral model, the explicit breaking of chiral symmetry is expressed by the term $c\sigma$ in the Lagrangian (1), which is written as $c(f_\pi+\sigma)$ in the present notation. The parameter c is related to the pion mass by $c=f_\pi m_\pi^2$ and the pion mass and the quark mass are related through the Gell-Mann-Oakes-Renner (GOR) relation, $f_\pi^2 m_\pi^2=-2m_q\langle\bar{q}q\rangle_{\text{vac}}$. We use the suffix "vac" to distinguish it from the one of nuclear matter, $\langle\bar{q}q\rangle_\rho$ at a density ρ.

We start with the energy density of nuclear matter written as

$$E=\frac{4}{(2\pi)^3}\int d^3p\,\theta(p_F-p)E_p^*+V(\sigma)+\frac{g_\omega^2\rho^2}{2m_\omega^{*2}}+E_{2\text{p-}2\text{h}}, \tag{38}$$

with $V(\sigma)=\frac{\mu^2}{2}(f_\pi+\sigma)^2+\frac{\lambda}{4}(f_\pi+\sigma)^4-c(f_\pi+\sigma)$. We note again that σ here is the mean-field value in nuclear matter. In Eq. (38) m_ω^* is the effective mass of the ω meson

$$m_\omega^*=\tilde{g}_\omega(f_\pi+\sigma)=\tilde{g}_\omega\bar{S}, \tag{39}$$

where we have introduced $\bar{S}$ for the mean-field value of the σ field, including the pion decay constant. We shall use this notation frequently later. Furthermore, the nucleon single-particle energy is given by $E_p^*=\sqrt{\vec{p}^2+M_N^{*2}}$, where the effective nucleon mass M_N^* is given in Eq. (14). The pion contribution is also included by the 2p-2h energy as given in Eq. (23).

The grand potential is written as $\Omega=E-\mu\rho$ with μ being the chemical potential, which is obtained by taking derivative of the energy density with respect to density ρ,

$$\mu=\frac{\partial E}{\partial\rho}=E_F^*+\frac{g_\omega^2\rho}{m_\omega^{*2}}+\frac{\partial E_{2\text{p-}2\text{h}}}{\partial\rho}. \tag{40}$$

If we take the derivative of the grand potential with respect to the quark mass, we can obtain the quark condensate in nuclear matter as

$$\langle\bar{q}q\rangle_\rho=\frac{1}{2}\left(\frac{\partial\Omega}{\partial m_q}\right)_\mu=\frac{1}{2}\frac{\partial c}{\partial m_q}\left(\frac{\partial\Omega}{\partial c}\right)\simeq\frac{1}{2}\frac{\partial c}{\partial m_q}\left(-f_\pi-\sigma+\frac{\partial E_{2\text{p-}2\text{h}}}{\partial c}\right)=\langle\bar{q}q\rangle_{\text{vac}}\left(1+\frac{\sigma}{f_\pi}-\frac{1}{f_\pi^2}\frac{\partial E_{2\text{p-}2\text{h}}}{\partial m_\pi^2}\right). \tag{41}$$

We note that there is a contribution to the quark condensate from the 2p-2h energy that contains pion mass. In the last step we have used the GOR relation and take the derivative of c with respect to m_q, $\frac{\partial c}{\partial m_q}=-\frac{2}{f_\pi}\langle\bar{q}q\rangle_{\text{vac}}$.

The nuclear many-body effects have been worked out for the chiral condensate in Eq. (41). The modification of the nucleon mass due to hadron interaction could be treated through the sigma condensate σ. However, we have to add the pion cloud contribution to the nucleon in the free space for the quark condensate. This pion cloud contribution is a nucleon property, which is unchanged in nuclear matter. This point was discussed by various authors [8,16–18]. We shall follow the discussion of Ericson *et al.* [8,16] and add the pion cloud contribution to the above expression,

$$\langle \bar{q}q\rangle_\rho = \langle \bar{q}q\rangle_{\rm vac}\left[1-\frac{\sigma_N^{(\pi)}\rho}{f_\pi^2 m_\pi^2}+\frac{\sigma}{f_\pi}-\frac{1}{f_\pi^2}\frac{\partial E_{\text{2p-2h}}}{\partial m_\pi^2}\right]. \qquad (42)$$

We take the pion cloud contribution to the nucleon σ term $\sigma_N^{(\pi)} = 20$ MeV for our presentation [8,16]. This value of the pion cloud contribution provides the entire σ term of 50 MeV.

The scalar susceptibility, χ_S, is obtained by taking further a derivative of the chiral condensate with respect to the quark mass [19,20]. We separate the scalar susceptibility into two terms,

$$\chi_S = \chi_S^{\rm nucl} + \chi_S^{\rm pion}. \qquad (43)$$

The pion term, $\chi_S^{\rm pion}$, has purely pionic nature, which comes from explicit dependence of the energy coming from pion exchange interaction on the mass of pion,

$$\chi_S^{\rm pion} = 2\frac{\langle \bar{q}q\rangle_{\rm vac}^2}{f_\pi^4}\left[\frac{\partial^2 E_{\text{2p-2h}}}{\partial\left(m_\pi^2\right)^2}\right]. \qquad (44)$$

The nuclear term, $\chi_S^{\rm nucl}$, consists of two parts. One is a term explicitly dependent on c and the other a term implicitly dependent on c, which is a derivative of the density, ρ, with respect to c at fixed μ,

$$\chi_S^{\rm nucl} = -2\frac{\langle \bar{q}q\rangle_{\rm vac}^2}{f_\pi^2}\left[\left(\frac{\partial \bar{S}}{\partial c}\right)_\mu - \frac{1}{f_\pi}\left(\frac{\partial^2 E_{\text{2p-2h}}}{\partial\rho\,\partial m_\pi^2}\right)_\mu\left(\frac{\partial\rho}{\partial c}\right)_\mu\right]. \qquad (45)$$

To evaluate the derivative $(\partial\bar{S}/\partial c)_\mu$, we have to use the energy minimization condition for σ,

$$\frac{\partial E}{\partial\sigma} = g_\sigma\rho_s + V'(\sigma) - \frac{g_\omega^2\rho^2}{\tilde{g_\omega}^2\bar{S}^3} = 0, \qquad (46)$$

where ρ_s is the scalar density. Here, we have ignored the contribution from the term of $E_{\text{2p-2h}}$, because the σ dependence of $E_{\text{2p-2h}}$ is very small, which we have verified explicitly. We take derivative of $V'(\sigma)$ with respect to c,

$$\frac{\partial V'(\sigma)}{\partial c} = -g_\sigma\frac{\partial\rho_s}{\partial c} + \frac{\partial}{\partial c}\left(\frac{g_\omega^2\rho^2}{\tilde{g_\omega}^2\bar{S}^3}\right). \qquad (47)$$

For the left-hand side of Eq. (47), we can use the definition of effective σ mass, $m_\sigma^{*2} = \frac{\partial^2 E}{\partial\sigma^2}$. Thus,

$$\frac{\partial V'(\sigma)}{\partial c} = m_\sigma^{*2}\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu. \qquad (48)$$

To evaluate these partial differentiations on the right-hand side of Eq. (47), we must consider the dependence of the density ρ and $\bar{S}$ on c at fixed μ,

$$\text{r.h.s.} = \left(\frac{\partial\bar{S}}{\partial c}\right)_\mu\frac{\partial}{\partial\bar{S}}\left(-g_\sigma\rho_s + \frac{g_\omega^2\rho^2}{\tilde{g_\omega}^2\bar{S}^3}\right) + \left(\frac{\partial\rho}{\partial c}\right)_\mu\frac{\partial}{\partial\rho}\left(-g_\sigma\rho_s + \frac{g_\omega^2\rho^2}{\tilde{g_\omega}^2\bar{S}^3}\right). \qquad (49)$$

In Eq. (49), two derivatives $(\partial\bar{S}/\partial c)_\mu$ and $(\partial\rho/\partial c)_\mu$ appear. For the last term, we can evaluate it by using the density at fixed μ,

$$\rho = \frac{2}{3\pi^2}\left[\left(\mu - \frac{g_\omega^2\rho}{\tilde{g_\omega}^2\bar{S}^2} - \frac{\partial E_{\text{2p-2h}}}{\partial\rho}\right)^2 - M_N^{*2}\right]^{\frac{3}{2}}. \qquad (50)$$

Knowing the expression about $(\partial\rho/\partial c)_\mu$, we can get $(\partial\bar{S}/\partial c)_\mu$,

$$\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu = \frac{1}{m_\sigma^{*2}} - \frac{\left(g_\sigma^{*2}\frac{M_N^*}{E_F^*} + g_\sigma^* m_\sigma^{*2}X^\pi\right)\Pi_0(0)}{m_\sigma^{*4}[1 - V_{\rm res}\Pi_0(0)]}, \qquad (51)$$

with $\Pi_0(0) = -2M_N^* k_F/\pi^2$, which is the nonrelativistic free Fermi gas particle-hole polarization propagator. Here, g_σ^* can be read as the effective σ coupling constant

$$g_\sigma^* = g_\sigma - \frac{2g_\omega^2\rho}{\tilde{g_\omega}^2\bar{S}^3}\frac{E_F^*}{M_N^*}, \qquad (52)$$

and the variables X^π and $V_{\rm res}$ are defined as

$$X^\pi = \frac{1}{f_\pi}\frac{\partial^2 E^\pi}{\partial\rho\,\partial m_\pi^2} + \frac{M_N^*}{E_F^*}\frac{\sigma_N^{(\pi)}}{f_\pi m_\pi^2}$$
$$V_{\rm res} = \frac{E_F^*}{M_N^*}\left(\frac{g_\omega^2}{m_\omega^{*2}} + \frac{\partial^2 E^\pi}{\partial\rho^2}\right) - \frac{M_N^*}{E_F^*}\frac{g_\sigma^{*2}}{m_\sigma^{*2}}. \qquad (53)$$

Here, $V_{\rm res}$ has two contributions due to the σ-meson attraction and the ω-meson repulsion in addition to the 2p-2h pion contribution. The nuclear part of scalar susceptibility, $\chi_S^{\rm nucl}$, contains the coupling of the scalar quark density fluctuations to the nuclear particle-hole excitations,

$$\chi_S^{\rm nucl} = -2\frac{\langle \bar{q}q\rangle_{\rm vac}^2}{f_\pi^2}\left[\frac{1}{m_\sigma^{*2}} - \frac{1}{m_\sigma^{*2}} \times\left(1 + \frac{m_\sigma^{*2}}{g_\sigma^*}\frac{E_F^*}{M_N^*}X^\pi\right)^2\Pi_{ss}(0)\frac{1}{m_\sigma^{*2}}\right], \qquad (54)$$

where $\Pi_{ss}(0)$ is the full scalar polarization propagator,

$$\Pi_{ss}(0) = g_\sigma^{*2}\frac{M_N^*}{E_F^*}\Pi_0(0)\,[1 - V_{\rm res}\Pi_0(0)]^{-1}. \qquad (55)$$

The full scalar polarization propagator diverges at some density smaller than the nuclear matter density. This instability is caused by the density fluctuation of uniform nuclear matter into nonuniform nuclear matter [19].

IV. NUMERICAL RESULTS

We would like to start with modification of the pion exchange interaction due to the nucleon form factor and the short-range correlation effect. We show first the effect of the form factor in Fig. 2, where the momentum dependent part

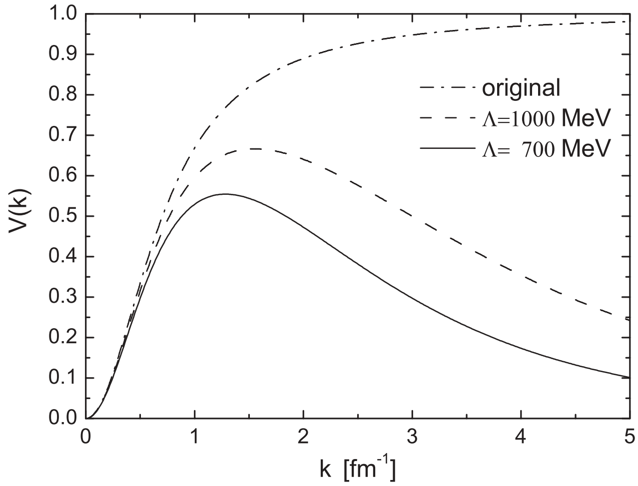

FIG. 2. The momentum-dependent part of the pion exchange interaction with and without the form factor as function of momentum. The dot-dashed curve corresponds to the interaction without the form factor and the dashed curve denotes the one with the form factor of $\Lambda = 1000$ MeV and the solid curve the one with $\Lambda = 700$ MeV.

of the one-pion exchange interaction, $V(\vec{k}) = \vec{k}^2/(\vec{k}^2 + m_\pi^2)$, is shown. The pion exchange interaction without the form factor increases with momentum, whereas that with the form factor decreases in the high-momentum region. As the cutoff momentum, Λ, is decreased, the pion exchange interaction decreases particularly in the high-momentum region. Due to this behavior, the energy gain associated with the pion is smaller for a smaller Λ by cutting down the contribution from the high-momentum region.

Next, the effect of the short-range correlation is shown in Fig. 3. The pion exchange interaction can be separated into the tensor interaction and the spin-spin central interaction. The tensor interaction requires the change of the quantum number of angular momentum by two, $\Delta l = 2$, because the tensor operator is written as $S_{12}(\hat{r}) = \sqrt{24\pi}[[\sigma_1 \cdot \sigma_2]^{(2)} \times Y_2(\hat{r})]^{(0)}$. Hence, the lowest angular momentum involved in the matrix element of the tensor interaction is between s state ($l = 0$) and d state ($l = 2$). The d state is pushed away from the central region due to the centrifugal potential, $V_c(r = 0.5\ \text{fm}) \sim 1000$ MeV, and hence the effect of the short-range correlation is negligible for the tensor interaction. Therefore, in the following discussion, we shall neglect the short-range correlation effect for the tensor part of the pion exchange interaction [9].

However, the effect of the short-range correlation is large and essential for the spin-spin central interaction, because it works between s states. In this case, there is no centrifugal barrier to protect two nucleons to approach into the short-range region. We shall take care of the short-range correlation effect for the spin-spin central interaction by the UCOM. We choose two sets of parameters. One is the set of Feldmeier and Neff, $\alpha = 0.94$ fm, $\beta = 1$ fm, and $\eta = 0.37$ [11], and the other set is $\alpha = 0.8$ fm, $\beta = 0.6$ fm, and $\eta = 0.37$. These two sets of UCOM parameters provide the function $R_+(r)$ as function of the relative coordinate of two nucleons, r, as shown in Fig. 3. They also provide modification of the wave function at $r \lesssim 0.5$ fm as shown in the right panel of Fig. 3 [$\psi = U\phi$ of Eq. (24)]. It is important to express the behavior of the short-range part of modified wave functions as closely as possible to the rigorous relative wave function [12].

The spin-spin central part with the UCOM effect, $\tilde{V}(r)$ and $\tilde{V}(k)$ in Eqs. (28) and (29), are shown in Fig. 4, where we show the case with $\Lambda = 700$ MeV. We compare the spin-spin central part of the pion exchange interaction in the coordinate and momentum space with and without the UCOM effect. We see that the short-range part of the central interaction is largely reduced by the UCOM effect. When the parameter β is large, the effect of the short-range correlation is large. As we see in the figure, the UCOM effect cuts down the central part of the pion exchange interaction by about 20∼40%.

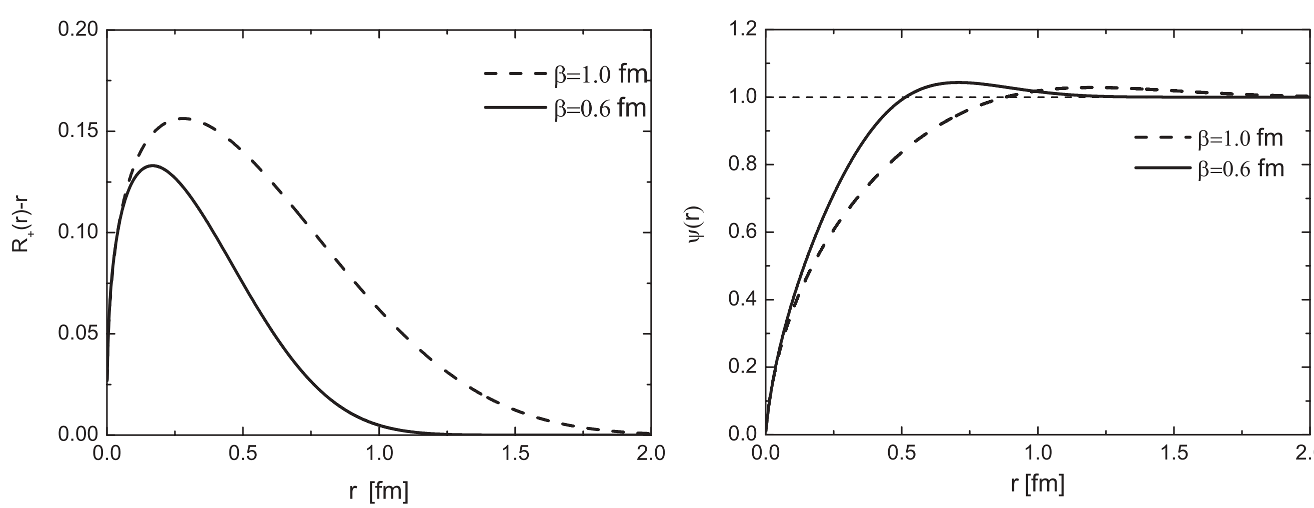

FIG. 3. $R_+(r)$ of the UCOM transformation with a shorter-range correlation function ($\beta = 0.6$ fm) shown by the solid curve and the one with longer-range correlation function ($\beta = 1$ fm) shown by the dashed curve in the left panel. These transformation functions modify the wave function near the origin as shown in the right panel, respectively. Here, the uncorrelated wave function used for presentation is a constant $\psi = 1$ as shown by the thin dashed line.

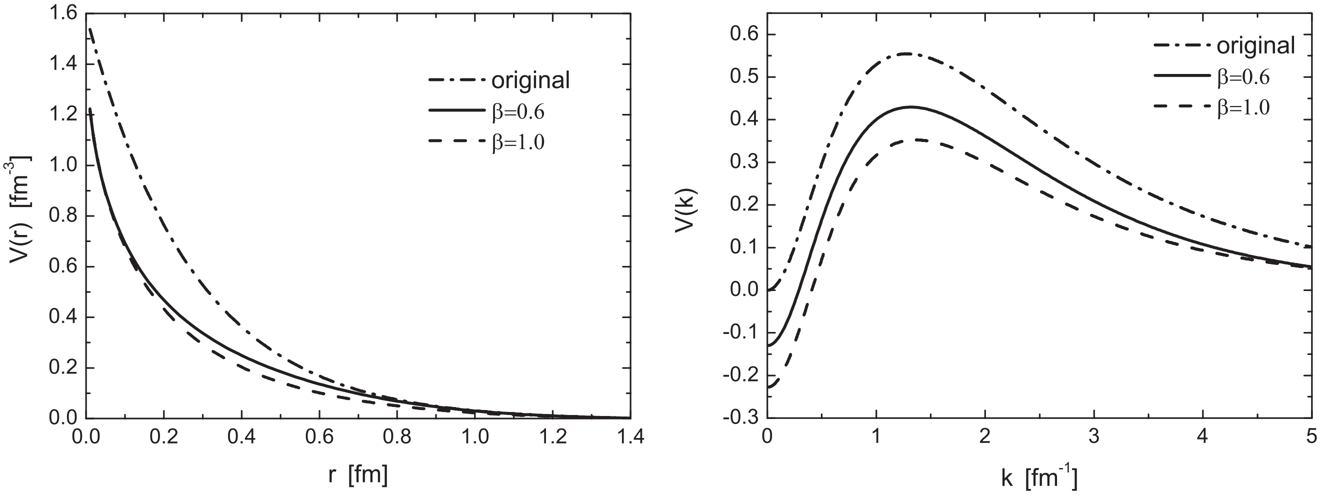

FIG. 4. The spin-spin central part of the pion exchange interaction with and without UCOM for the case $\Lambda = 700$ MeV in coordinate space in the left panel and one in momentum space in the right panel. The dot-dashed curve corresponds to the interaction without UCOM, the solid curve to UCOM with $\beta = 0.6$ fm, and the dashed curve to $\beta = 1$ fm.

We present now numerical results of equation of state (EOS) for nuclear matter as function of nuclear matter density in Fig. 5. In the EOS, we choose the parameter set: $\alpha = 0.8$ fm, $\beta = 0.6$ fm, and $\eta = 0.37$ for the UCOM effect. We show the EOS for two cases of different Λ's, 635 and 700 MeV. Both cases can reproduce the saturation properties (saturation energy, $E/A = 16.8$ MeV, at density, $\rho_0 = 0.142$ fm^{-3}) by adjusting g_ω and m_σ as tabulated in Table I, set A and B, where other properties of the nuclear matter (incompressibility, K, and the nucleon effective mass, M^*) are also shown. The dashed curve corresponds to the result of chiral sigma model without the pionic effect, whereas the dot-dashed and solid curves correspond to the results with the pionic effect (ECSM) with sets A and B, respectively. They are compared with a phenomenological EOS, which is calculated by using the RMF with the TM1 parameters [21]. From Fig. 5, we observe that the nuclear matter of the chiral sigma model ($K \sim 650$ MeV) is stiffer than the phenomenological one of TM1 ($K \sim 280$ MeV). By including the pionic effect in the ECSM, we can obtain a softer nuclear matter, though the incompressibility ($K \sim 400$ MeV) is still larger than the phenomenological one. We can reduce further the incompressibility, K, by increasing Λ to have more contribution from the pion. The σ-meson mass, m_σ, however, has to increase for larger pionic contribution to reduce the σ-meson contribution. To keep the σ mass in a reasonable range, we chose $\Lambda = 700$ MeV, which is close to the one of Kaiser *et al.* [7].

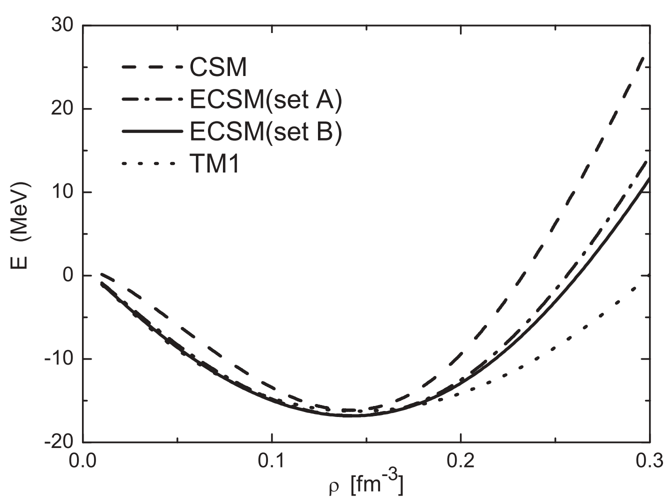

FIG. 5. The equation of state (EOS) of nuclear matter as function of nuclear density. The dashed curve corresponds to the result of chiral sigma model (CSM) without the pionic effect, the dot-dashed curve denotes the result of extended chiral sigma model (ECSM) with the inclusion of the pion exchange effect with the form factor of $\Lambda = 635$ MeV and the solid curve the ECSM result with $\Lambda = 700$ MeV. The dotted curve is the result of RMF with the parameter set of TM1 for comparison [21].

We show now various components for EOS in Fig. 6. For this calculation we take the parameter set B. In the left panel, the dotted curve shows the contribution of the central part of the pion exchange interaction. The tensor part shown by the dot-dashed curve provides a large contribution as it reproduces the binding energy of 15 MeV per nucleon at the saturation density. The contribution of the other interaction, due to σ and ω exchanges, is shown by the solid curve. At low density the $\sigma + \omega$ contribution provides large attraction as the tensor contribution, while it rapidly becomes repulsive at higher density due to the repulsion of ω. At the saturation density, the tensor part of the pion exchange interaction provides the attraction as large as the $\sigma + \omega$ interaction at the saturation

TABLE I. Two parameter sets used in our numerical calculations. Set A uses a smaller form factor, $\Lambda = 635$ MeV, and Set B uses a larger one, $\Lambda = 700$ MeV.

Set	Λ (MeV)	g_ω	m_σ (MeV)	K (MeV)	$M_N^*(\rho_0)/M_N$
A	635	6.894	867	435	0.837
B	700	6.89	896.5	401	0.850

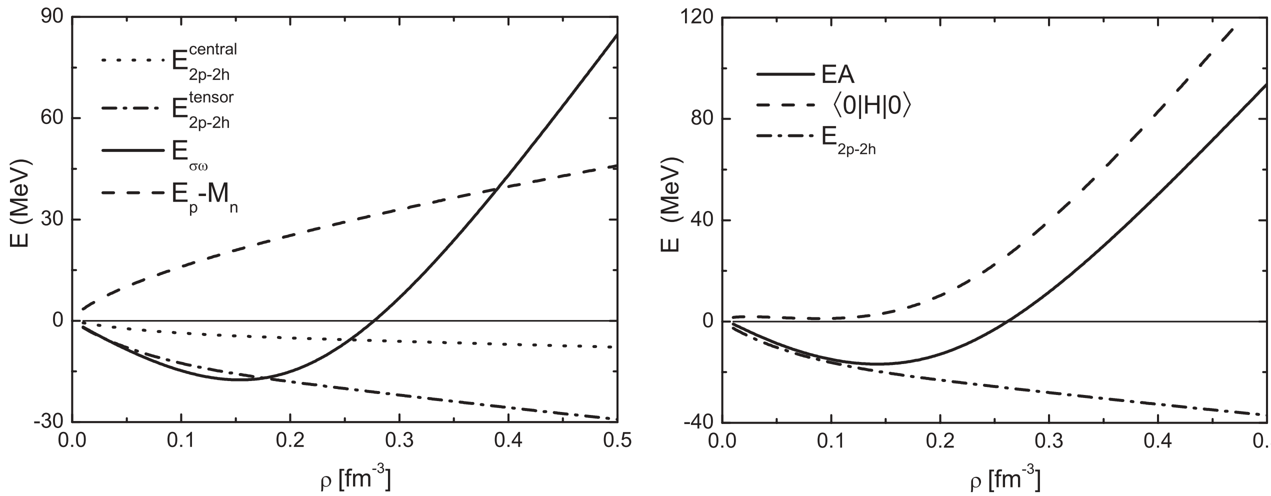

FIG. 6. Energy contributions from various terms for EOS as functions of nuclear density. In the left panel, the pion contribution due to the tensor part is shown by the dot-dashed curve, whereas that of the spin-spin part is shown by the dotted curve. The interaction energy due to the σ- and ω-meson contributions is shown by the solid curve. The kinetic energy is shown by the dashed curve. In the right panel, the dashed curve corresponds to the energy of the mean-field ground state. The dot-dashed curve corresponds to the pionic contribution, $E_{2p\text{-}2h}$. The whole energy is shown by the solid curve.

density. The kinetic energy is shown here by the dashed curve. We neglect here modification of the kinetic energy due to the UCOM effect. In the right panel, the dashed curve denotes the contribution of the mean-field part of the Hamiltonian, $\langle 0|H|0\rangle$. Unlike the standard RMF calculation, this 0p-0h part does not provide enough attraction for the binding of nuclear matter. The dot-dashed curve denotes the contribution coming from the pion exchange interaction, $E_{2p\text{-}2h}$. The total energy is shown by the solid curve, where the pion exchange contribution turns out to almost coincide with the whole energy until the saturation density.

We should compare our results with other studies. First, we compare with the results of the variational calculations for light nuclei by Argonne group [6]. They conclude that the contribution of the pion exchange interaction to the entire two-body interaction energy is about 70∼80%. If we see the contribution due to the tensor part for ^{4}He, it is about a half of the entire attraction. This feature is consistent with our present result as seen in Fig. 6. We find here that about 50% of attraction comes from the tensor part of the pion exchange interaction. Additionally we have a contribution from the spin-spin part, which adds the contribution from the tensor part to make the entire contribution of the pion exchange interaction about 60% of the entire two body attraction.

Our result is also qualitatively similar to the one of Kaiser *et al.* [7]. A large amount of attraction is caused by the pion exchange interaction. They calculated the pion 2p-2h contributions without consideration of the short-range correlation effect. The Pauli blocking effect, calculated separately in their study, is quite large to reduce the 2p-2h contribution by about one-third. In addition, the short-range correlation effect further cuts down the 2p-2h contribution. They have calculated the Fock term contribution of pion exchange interaction and the crossed diagram of 2p-2h excitations, which are much smaller than the dominant 2p-2h direct contribution. In our case, the Fock contribution from the pion exchange interaction is estimated to be about 7 MeV repulsive at the saturation density. However, in the present study, we have not included this contribution to be consistent with the RMF approximation used for σ and ω mesons. In fact, the inclusion of the Fock term contributions due to σ and ω exchanges provides slightly attractive energy, which overcomes the repulsive contribution from the pion exchange Fock term, resulting slightly a attractive result [8]. If we were to include the pion Fock contribution, we reduce somewhat the coupling strength of the ω-meson exchange interaction to get a similar energy of nuclear matter.

We also make comment on the study of Ericson *et al.* [8]. When we calculate the contribution of the pion, we have to include explicitly all the strength coming from the one-pion exchange interaction. This is achieved in our study in terms of the TOSM prescription [9], because the tensor component of the one-pion exchange interaction needs large 2p-2h excitations, which are taken care by Kaiser *et al.* [7]. However, Ericson and her collaborators treated this effect in terms of the G matrix by using the Landau-Migdal parameter, g'. Therefore, the tensor contribution is effectively included in the scalar part, $\sigma + \omega$, in their prescription. The binding energy can be calculated by choosing the coupling constants of the meson-nucleon couplings, and therefore the $\sigma + \omega$ contribution is overestimated.

Now we turn to the discussion of the chiral condensate as function of nuclear matter density, ρ. The chiral condensate in nuclear matter is an order parameter for spontaneous breaking of chiral symmetry. The chiral condensate in nuclear medium is defined as the first derivative of the grand potential with respect to the bare quark mass. The bare quark mass can be expressed by the pion mass by the Gell-Mann-Oakes-Renner (GOR) relation. The expression for chiral condensate in nuclear matter is given in Eq. (42). We show numerical result in Fig. 7 for chiral condensate as function of nuclear density. We find the quark condensate in ECSM reduces by about 30% at the saturation density from the value of the vacuum. As the density

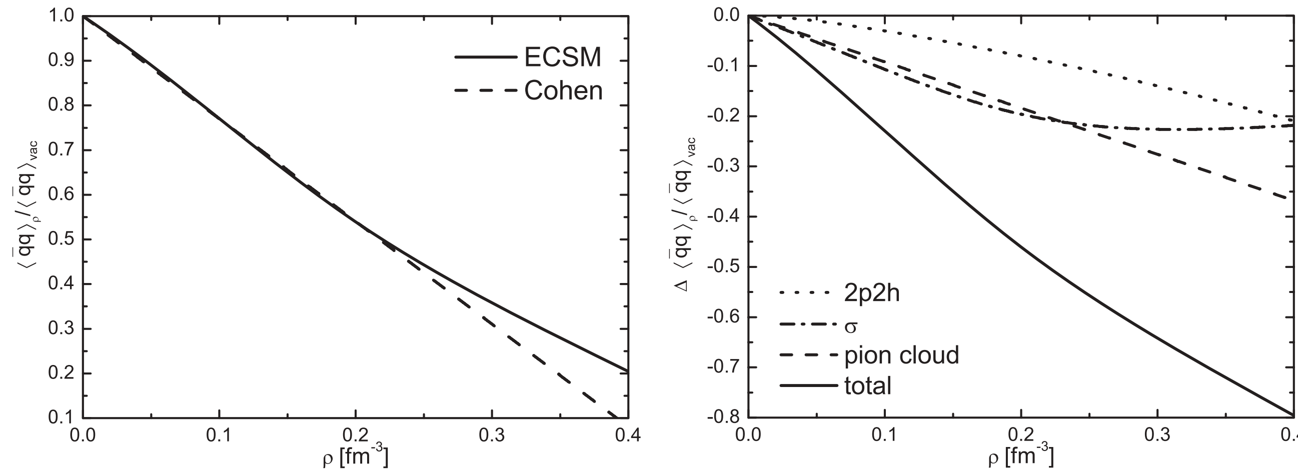

FIG. 7. The chiral condensate in nuclear matter as function of density is shown in the left figure. The ECSM result includes the pion cloud contribution $\sigma_N^{(\pi)} = 20$ MeV [Eq. (42)]. For comparison, the model-independent result is shown by the dot-dashed curve with $\sigma_N = 50$ MeV. In the right panel, various components of the chiral condensate are shown, where the total value is shown by the solid curve. The σ mean-field contribution is shown by the dot-dashed curve, the 2p-2h contribution by the dotted curve, and the pion cloud contribution by the dashed curve.

increases, the pion contribution becomes larger. We include here the contribution of the pion cloud effect to the chiral condensate with $\sigma_N^{(\pi)} = 20$ MeV to be consistent with the σ term value of 50 MeV in the free space. The final result is not very much different from that of the model independent calculation [22], where the chiral condensate, $\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle$, is proportional to the first order of nuclear matter density and becomes smaller linearly as ρ is increased. However, our curve becomes gradually flat in the high-density region, $\rho > 0.25$ fm^{-3}. This is caused by the σ-meson field, as shown in the right panel of Fig. 7.

Nuclear scalar susceptibility is also very interesting. The result of the scalar susceptibility is shown in Fig. 8. The nuclear contribution, $\chi_S^{\rm nucl}$, has a singular property at around the density $\rho = 0.75\rho_0$, which is caused by zero in the denominator of the full scalar polarization propagator. The pion contribution, $\chi_s^{\rm pion}$, which is related with the behavior of pion mass, increases slowly with nuclear density. At the density about $\rho = 2.0\rho_0$, the pion contribution, $\chi_s^{\rm pion}$, becomes larger than the nuclear part, $\chi_s^{\rm nucl}$, and controls the total scalar susceptibility. We find the susceptibility has the similar property as that of Ericson *et al.* [8]. We also included $g_\sigma \frac{\partial \rho_S}{\partial \bar{s}}$ term, which corresponds to the nuclear response associated with the $N\bar{N}$ excitation in the effective σ mass. The effective σ mass is larger by 40% than that in Ref. [8] at the density $\rho = 3\rho_0$, and therefore the nuclear contribution, $\chi_S^{\rm nucl}$, is obviously smaller than the pion contribution.

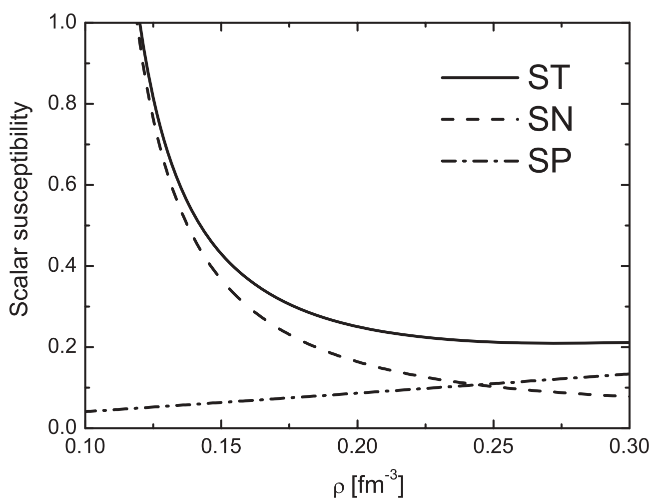

FIG. 8. Scalar susceptibility as function of nuclear density. The dashed curve corresponds to the nuclear contribution, the dot-dashed curve corresponds to the pion contribution, and the solid curve corresponds to total susceptibility. The singular behavior is caused by the instability of uniform nuclear matter to nonuniform nuclear matter.

V. SUMMARY

We have developed the extended chiral sigma model to study the chiral properties of nuclear matter. We have taken the chiral sigma model Lagrangian, including the ω-meson term, whose mass is generated by the chiral condensate as the case of the nucleon mass. We made Weinberg transformation to obtain the nonlinear realization of the extended chiral sigma model Lagrangian. Because the pion field cannot be treated in the mean-field approximation, we have included 2p-2h excitations to take care of the pion contributions. We have then expressed the variational wave function as a product of a coherent state for the mean fields of σ and ω, and the nucleon wave function written as a linear combination of the mean-field nuclear matter wave function (0p-0h) and 2p-2h wave function. We have kept the phenomenological concept of the relativistic mean-field approximation and take only the direct diagrams. All the phenomenological aspect was considered to be included in ω-meson couplings. We also kept the σ-meson mass as a parameter to obtain the saturation property.

We have considered further two effects to study the role of pion in nuclear matter. One is the effect of the nucleon size, which is expressed by the form factor in the coupling of the pion with the nucleon. The other is the effect of the short-range repulsion caused by the quark content of the nucleon, which is treated in this article by the UCOM. Our framework of taking the 2p-2h configurations is based on the success of the treatment of the tensor interaction in finite nuclei in terms of the TOSM [9]. With these methods (TOSM+UCOM), we are able to take the nucleon-nucleon interaction directly for the study of nuclear structure.

We have calculated the EOS of nuclear matter and other important properties related with the role of pion in nuclear matter. We have obtained the saturation property within ECSM, where the pion exchange interaction has very important contribution to provide binding of nucleons. This result is in accordance with the VMS calculation for light nuclei by the Argonne group [6]. However, the incompressibility of our present calculation is slightly larger, $K \sim 400$ MeV, than the phenomenological value.

We have also calculated the chiral properties of nuclear matter. We have obtained the chiral condensate by taking variation of the grand potential with respect to the bare quark mass, which is related with the pion mass by the GOR relation. The chiral condensate decreases monotonically as the nuclear matter density is increased; at around the saturation density the chiral condensate is reduced by about 30% of that of the vacuum. Hence, in our present model chiral symmetry is recovered by 30% at the saturation density. However, the σ condensate associated with the change of the nucleon property is reduced by about 15%. We have studied also the scalar susceptibility of the nuclear matter. The susceptibility diverges at around $0.75\rho_0$ due to the instability of uniform nuclear matter to nonuniform nuclear matter. The susceptibility decreases gradually with density, whereas the pionic effect increases with density. The net result is similar to the one of the Ericson *et al*. [8].

In this article, we focused on the study of the role of pion in nuclear matter. We developed a new framework with the use of the TOSM and UCOM. However, in the present work, we did not take into account the entire feature of the nucleon-nucleon interaction such as the strong short-range repulsion and the ρ-meson effects. All these effects are at this moment treated phenomenologically in terms of the ω-nucleon and the σ-meson mass. We need to work out all these effects in a systematic manner, which is, however, difficult to perform at this moment. We shall leave them for future works.

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