Nuclear structure relevant to neutrinoless double β decay: The valence protons in ^{76}Ge and ^{76}Se

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The possibility of observing neutrinoless double β decay offers the opportunity of determining the effective neutrino mass *if* the nuclear matrix element were known. Theoretical calculations are uncertain, and the occupation of valence orbits by nucleons active in the decay is likely to be important. The occupation of valence proton orbits in the ground states of ^{76}Ge, a candidate for such decay, and ^{76}Se, the corresponding daughter nucleus, is determined by precisely measuring cross sections for proton-removing transfer reactions. As in previous work on neutron occupation, we find that the Fermi surface for protons is much more diffuse than previously thought, and the occupancies of at least three orbits change significantly between the two 0^+ ground states.

Major experimental efforts are under way to observe neutrinoless double β decay, an essential step in determining the nature of the neutrino [1]. If this process were to be observed, not only would it demonstrate that neutrinos are their own antiparticles, but the rate would also give a direct measure of the neutrino mass *if* the corresponding nuclear matrix element can be reliably calculated. For one of the likely candidates, ^{76}Ge, theoretical calculations have yielded answers that vary widely, and this uncertainty will be a major obstacle to deducing the neutrino mass [2]. It is important to explore experimental data that could help constrain theoretical models. The *difference* in the configuration of nucleons between the initial and final states (the 0^+ ground states of ^{76}Ge and ^{76}Se) is a major ingredient in the matrix element. We have undertaken several experiments to better define the knowledge of the two ground-state wave functions and the difference between them. In previous experiments, we focused on the difference in neutron configurations, determining the difference in neutron valence-orbit occupancies [3] and in correlations [4]. As was done for neutrons, here we utilize transfer reactions and the Macfarlane-French sum rules [5] to extract the proton occupancy of states from proton-removing reactions.

These reactions had been studied before [6], however, the two previous experiments dealt with germanium and selenium isotopes separately and were done at different times and apparently using slightly different techniques, so the relative cross sections may have additional uncertainties. In addition, the data in these publications are reported as spectroscopic factors and plots of angular distributions. From the plots, it is difficult to extract numbers with the accuracy required for the present purpose. Further, for a number of the $\ell = 3$ transitions, the spins were not known, leaving it ambiguous as to whether their strengths should be attributed to the $f_{5/2}$ or $f_{7/2}$ orbitals. To remove these ambiguities, we also carried out measurements for the $(d, ^3\text{He})$ reaction with vector polarized deuterons.

There are very few facilities remaining that are capable of carrying out precision reaction measurements of the type required here, with high-quality beams, the necessary beam energies, high-resolution spectrometers, and detector systems. We used the Research Center for Nuclear Physics (RCNP) facility at Osaka University to determine accurate cross sections for the $(d, ^3\text{He})$ reaction using the Grand Raiden (GR) spectrograph [7]. Evaporated targets of ^{76}Ge and ^{76}Se were used, as well as targets of ^{74}Ge and ^{78}Se as a check on systematic errors. The experiment was performed at a beam energy of 80 MeV, the lowest energy feasible, since the focal-plane detector of the GR requires passage through two foils between the spectrograph vacuum and the detectors. The angular distributions were found to be consistent with distorted wave Born approximation (DWBA) calculations, and examples are shown in Fig. 1(a).

To obtain accurate cross sections, the product of target thickness and spectrometer aperture was obtained from 10-MeV α particles at an angle of $30°$ on the assumption of Rutherford scattering. The AVF Cyclotron, usually the injector to the main Ring Cyclotron, was used to deliver a singly charged ^{4}He beam directly through a bypass beamline, to the target position. The spectrometer aperture, 60×40 mrad in the vertical and horizontal directions, was the same as for all the $(d, ^3\text{He})$ measurements, with the same current integrator to measure the total beam charge.

A problem remained regarding the measurements at 4.5°. It was suspected that the Faraday cup used for the beam measurement at 0° was partially obstructed by components of the spectrometer system. To appropriately normalize the data, a second spectrograph, the Large-Acceptance Spectrometer (LAS) placed at a fixed angle of 60°, was used at the same time as the GR. The ratio of the scattering yield in LAS to a beam current integrator was determined when the GR was at larger angles. This ratio was constant for a given target to the statistical accuracy of ~±3%.

High-statistics measurements of the unpolarized yield were carried out at the three angles, near the peaks of the angular distributions for $\ell = 1, 3,$ and 4, at 4.5, 8, and 12°, with the aperture subtending ±1°. The choice of the smallest angle was about half a degree larger than optimal because of limitations imposed by the counting rates from slit scattering at the very forward angles. The operation of the focal-plane detectors and the method of monitoring efficiencies and dead times have been described elsewhere [8].

The polarization measurements were carried out separately. A vector-polarized deuteron beam ($A_y = -0.520 \pm 0.010$) was used, and the measurement was made with the GR spectrograph at 10°. DWBA calculations indicate that the optimal angle for detecting the difference in asymmetry between $j^\pi = 5/2^-$ and $j^\pi = 7/2^-$ states is near 10°. The mean observed values of asymmetries for states with known spins we determined to be -0.064 ± 0.011 for $j^\pi = 5/2^-$ and $+0.032 \pm 0.013$ for $j^\pi = 7/2^-$. Spin assignments were made for $\ell = 3$ transitions whose j values were ambiguous, and four definite $5/2^-$ assignments were made, including spins of the 198-keV level in ^{73}Ga and the 229-keV level in ^{75}Ga, both with dominant spectroscopic factors, as well as the 1167-keV level in ^{75}Ga and the 1165-keV one in ^{75}As. All other $\ell = 3$ transitions with significant strengths were found to have asymmetries consistent with $j^\pi = 7/2^-$. The results for one of the targets, ^{76}Ge, are shown in Fig. 1(b). It so happens that for $\ell = 1$ transitions, the signs of asymmetries for $\ell + 1/2$ and $\ell - 1/2$ states at this angle are opposite to those for $\ell = 3$, which is qualitatively consistent with DWBA predictions.

To extract spectroscopic factors, the finite-range DWBA program PTOLEMY [9] was used with several sets of optical-model parameters from the literature [10]. The observed cross sections for $\ell = 1, 3,$ and 4 were divided by the DWBA cross sections. A single overall normalization was established for the $(d,^3\text{He})$ data by requiring that the total occupancy of the valence proton orbits add up to four for Ge and six for Se.

The DWBA normalizations obtained in this fashion had a mean value of 0.63 with an rms variation of 23% between the different parameter sets. However, the summed spectroscopic factors were very nearly the same and varied by less than 0.1 units. The values given in Table I are the average of results from analyses with the various potential sets.

We note that "absolute" spectroscopic factors for "good" single-particle states in doubly-magic nuclei are usually around ~0.6 because of short-range correlations [11]. Such correlations are a uniform property of nuclei, formally they move a given fraction of spectroscopic strength to very high excitation energy, and this fraction does not change appreciably between nearby nuclei or configurations. Thus
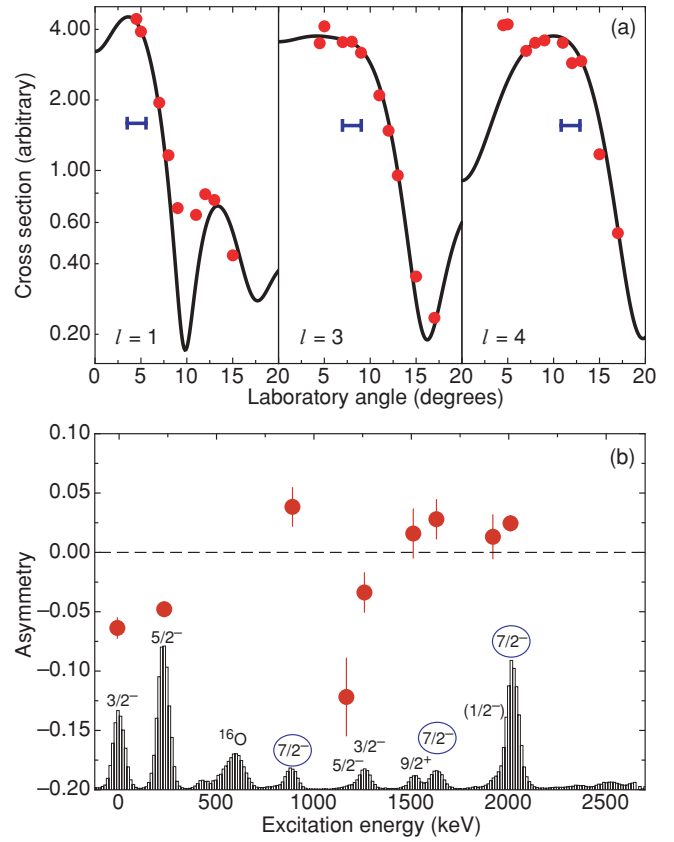

FIG. 1. (Color online) (a) Measured angular distributions for $\ell = 1, 3,$ and 4 on ^{76}Ge (points) compared with DWBA calculations (lines). The intervals shown (horizontal bars) indicate the angular range covered in the high-statistics measurements. (b) Solid points are asymmetries with the vector-polarized deuteron beam, between spin up and spin down for ^{76}Ge, with the data summed over the peaks. A histogram representing the spectrum on the focal plane (in arbitrary units) is inserted at the bottom, with spin assignments shown. The circled spins were determined in this experiment.

the normalization obtained by our procedure is meant as a way of estimating the relative populations of valence orbits—and not as a measurement of absolute spectroscopic factors.

The summed spectroscopic factors of Table I are the proton occupancies and are shown graphically in Fig. 2, for the $A = 76$ nuclei, along with the values of these occupancies from various theoretical calculations. The first QRPA calculations [12] were the only ones available at the time the neutron occupancies were published [3]. The more recent QRPA

TABLE I. Summed spectroscopic strengths.

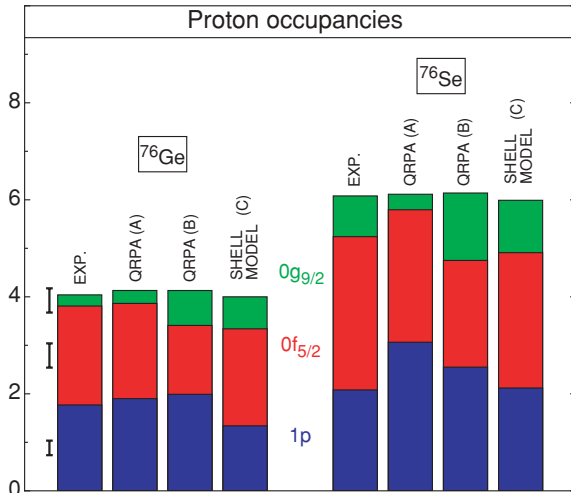| Target	Particles			
	$\ell = 1$	$\ell = 3$	$\ell = 4$	Sum
^{74}Ge	1.89	1.52	0.37	3.78
^{76}Ge	**1.75**	**2.04**	**0.23**	**4.02**
^{76}Se	**2.09**	**3.17**	**0.86**	**6.12**
^{78}Se	2.35	1.82	2.05	6.22
Uncertainty	±0.15	±0.25	±0.25	±0.38

FIG. 2. (Color online) Experimentally determined proton occupancies from Table I for the three valence orbits in ^{76}Ge and ^{76}Se. These are compared with QRPA calculations of Ref. [12] (a) and Ref. [13] (b) and the shell-model calculations of Ref. [14] (c). The experimental uncertainties are indicated by error bars on the left.

calculation [13] and a shell-model calculation [14] were published subsequently.

The uncertainties in the experimental values are difficult to estimate. Statistical errors in the summed strengths are less than 1%, and relative systematic errors between targets are believed to be less than 3%. The biggest uncertainties stem from possible missing states and, to a lesser extent, from the validity of the DWBA method in extracting spectroscopic factors. We estimate that the occupancy is determined to about 0.15 nucleons for the $1p$, and to 0.25 for the $0f_{5/2}$ and $0g_{9/2}$ orbits. These estimates of uncertainties are of necessity rather crude—we have some confidence in them because a single normalization gives the appropriate proton number for the four targets studied, with an rms deviation of ± 0.18 nucleons.

We also made measurements of the (^{3}He,d) reaction on the same targets and at the same scattering angles, at a corresponding ^{3}He beam energy of 72 MeV—determined by the average difference in Q values between the proton-adding and proton-removing reactions on these targets. We had intended to apply the sum rules to both the adding and removing reactions, however, when we apply the normalization obtained from the (d,^{3}He) data, the summed strengths for proton vacancies fall short of the expected values, indicating considerable missed strength. For $\ell = 1$, an average of 16% of the total strength (12, 14, 21, and 18%, respectively, for ^{74}Ge, ^{76}Ge, ^{76}Se, and ^{78}Se), for $\ell = 3$, 24% (25, 30, 4, and 35%), and for $\ell = 4$, 48% (56, 41, 57, 37%) are missing. A small correction, for the unobserved $T_{>}$ strength was made in the sums—these strengths are known from the measurements of the neutron vacancies.

The shortfall may be a consequence of the fact that these nuclei are near the beginning of a major shell for protons, the centroids of the expected single-particle strength are at higher excitation energy, and the strength becomes appreciably fragmented in a high level-density region such that the many small individual components cannot be resolved. Indeed, the discrepancy is greatest for the $9/2^{+}$ orbit, which is at

the highest excitation energy. Previous work on the (^{3}He,d) reaction at lower bombarding energies [15], and some of it at higher energy resolution, has not identified significant components with the requisite ℓ values. We have also attempted to identify this missing strength by carrying out measurements at the Yale tandem accelerator of the (α,t) reaction, which is particularly selective for $\ell = 4$ transitions, where the problem is the most serious. No candidates for missing $\ell = 4$ strength were found with a strength greater than $\sim$4%. This is unlike the neutron-transfer case, where excellent consistency was obtained between the neutron-adding and -removing reactions with $\ell = 4$ using the (α,^{3}He) reaction and its inverse. We therefore conclude that we cannot utilize the data from proton-adding reactions in the present case and must rely on the consistency of the proton-removing reactions.

Indeed, the strength in the proton-removing reactions so close to the $Z = 28$ closed shell is restricted to relatively few states at low excitation energies and, as shown in Table I, gives self-consistent results among the four nuclei studied. Even though the summed strength was used for the normalization, the four determinations are independent, and the rms deviation from the expected value is 0.18 protons, as mentioned earlier, well within the estimated uncertainty. We believe that, together with the neutron occupation study [3], the internal consistency
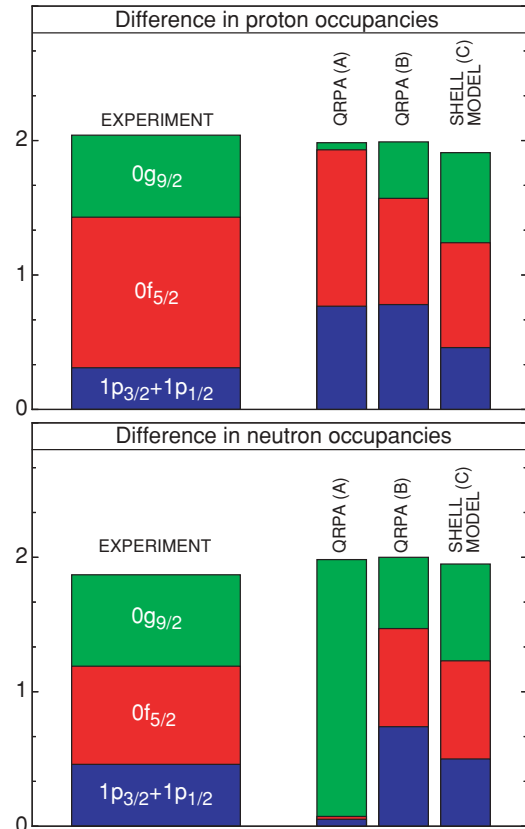

FIG. 3. (Color online) The difference between the proton and neutron occupancies [3] of the ground states of ^{76}Ge and ^{76}Se deduced from the present measurement and compared with theoretical calculations with the notation as in Fig. 2.

of these results perhaps constitutes the most quantitative test of the validity of the sum rules in one-nucleon transfer reactions.

Utilizing the information in Table I, we can state the differences in proton occupations between ^{76}Se and ^{76}Ge as 0.31 ± 0.15 in $\ell = 1$ (it appears that this difference is mostly in the $1p_{1/2}$ occupation), 1.12 ± 0.35 in $0f_{5/2}$, and 0.61 ± 0.35 in $0g_{9/2}$. The uncertainties were estimated taking possible correlations into account. This comparison is shown in Fig. 3, displaying both the neutron differences from Ref. [3] and the proton differences from the present work, along with the same set of calculations as in Fig. 2. As is evident from both Figs. 2 and 3, the $0g_{9/2}$ proton orbit is considerably more involved in the changes in the Fermi surface than the original QRPA [12] calculations suggested. The more recent calculations are consistent with the proton as well as the neutron data on occupancies. Some weak $\ell = 2$ transitions are also seen in the proton removal from the two Se isotopes. The spectroscopic factors indicate $1d_{5/2}$ strength of no more than 0.1 protons. Low-lying $7/2^-$ states seen in the (^{3}He,d) reaction indicate similarly low strengths. Though this evidence is not conclusive, we see no appreciable admixture from beyond $Z = 50$ or below $Z = 28$.

We have thus characterized the microscopic changes in the valence occupations for neutrinoless double β decay.

Measurements of neutron-pair transfer on these nuclei suggest that correlations between neutrons are very similar in these two ground states [4]. We hope to determine proton correlations in a future experiment. Although the most recent theoretical calculations in Refs. [13,14,16] appear to be closer to each other in their predictions for the ^{76}Ge matrix element, firm anchor points to the measured properties of these states are likely to be essential. At present, the theoretical calculations do not yet seem to be able to specify how the experimental observables that characterize the wave functions of these two ground states influence the matrix element for neutrinoless double β decay.

Our data are available online in the XUNDL database [17].

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