Energies of ^{9}B(1/2$^+$) and ^{10}C(0_2^+)

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A potential-model calculation has given reasonably precise values for the energies of the $1/2^+$ first excited state of ^{9}B and the excited 0^+ state of ^{10}C. We here extend this calculation to allow for nonzero widths of the $1/2^+$ and $5/2^+$ levels of ^{9}Be and ^{9}B that are involved in the calculation. The ^{9}B $1/2^+$ peak energy (but not the resonance energy) appears to be well determined. The ^{10}C 0^+ energy is determined sufficiently well to make it unlikely that a state recently observed in ^{10}C at 4.20 MeV could be the 0^+ state, as has been suggested.

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In an inelastic scattering experiment, Curtis *et al.* [1] have observed a state of ^{10}C at an excitation energy of 4.20 MeV. On the basis of their observed angular distributions, they suggest that this may be the analog of the well-known 0^+ 6.179 MeV state in ^{10}Be. They discuss various descriptions of the states that might explain the almost 2 MeV energy difference. Fortune and Sherr [2] (hereafter FS), using a potential model, have, however, calculated the energy of the ^{10}C(0_2^+) state as 5.18 ± 0.11 MeV. We here consider modifications of the FS approach to see if an energy as low as 4.20 MeV is attainable.

FS give evidence that the 0_2^+, $T = 1$ state in ^{10}B is well described as two nucleons in the sd shell coupled to a ^{8}Be(g.s.) core, and they assume similar structures for the analog states in ^{10}Be and ^{10}C. FS use a potential model, with potential depths for ^{9}Be($1/2^+$) + $n(2s_{1/2})$ and ^{9}Be($5/2^+$) + $n(1d_{5/2})$ chosen to fit the ^{10}Be(0_2^+) energy of 6.179 MeV. The same depths are then used for ^{9}Be + p and ^{9}B + n to calculate the energy of the ^{10}B(0_2^+) state and for ^{9}B + p to calculate the energy of the ^{10}C(0_2^+) state. The ^{9}Be $1/2^+$ and $5/2^+$ states and the ^{9}B $5/2^+$ state have reasonably well-determined properties, but the energy of the ^{9}B($1/2^+$) state is quite uncertain [3]. The calculated energies of the ^{10}B(0_2^+) and ^{10}C(0_2^+) states depend on this energy and also on the d^2/s^2 ratio in the 0_2^+ states. By taking the d^2 fraction as $\beta^2 = 0.25 \pm 0.05$, FS fit the ^{10}B(0_2^+) energy, known to be 7.560 MeV, with a ^{9}B($1/2^+$) energy $E_x(^9\mathrm{B}_{1/2^+}) = 1.31 \pm 0.11$ MeV. Then the calculated energy of ^{10}C(0_2^+) is $E_x(^{10}\mathrm{C}_{0_2^+}) = 5.18 \pm 0.11$ MeV.

This FS value of $E_x(^9\mathrm{B}_{1/2^+})$ falls within the range of other calculated values, which extend from less than 1 MeV [4,5] to about 1.8 MeV [6]. Some of this variation is due to the use of different definitions for the energy of an unbound level, which can lead to very different values for broad levels such as ^{9}B($1/2^+$), for which calculated widths are of the order of 1 MeV. Thus Efros and Bang [5] find that the energy of ^{9}B($1/2^+$) defined by the complex-energy pole of the p-^{8}Be scattering amplitude is about 0.5 MeV less than the peak energy of the p-^{8}Be line shape. It is not clear how FS define the energy of an unbound level, when it is broad. In recent publications, Fortune [7] and Sherr [8] have given preferred definitions that are different. As FS point out, the ^{10}C(0_2^+) level is expected to be narrow, so that there is not much ambiguity in how its energy is defined.

The FS approach neglects the width of the ^{9}B($1/2^+$) state. To allow for a nonzero width, and to determine which definition of energy is appropriate, we repeat the calculations for the Coulomb displacement energies using R-matrix formulas [6] rather than the potential model. With the structure for the 0_2^+ states the same as that in FS, we obtain values of $E_x(^9\mathrm{B}_{1/2^+})$ and $E_x(^{10}\mathrm{C}_{0_2^+})$ that are close to the FS values. In this calculation, contributions from the point Coulomb interaction and from the different external wave functions are included [6],

$$\Delta E_C = \Delta H^c + \Delta L, \tag{1}$$

as these are the two contributions that are included (implicitly) in the FS potential approach. ΔH^c for the ^{10}B and ^{10}Be difference is assumed to be the same as that for the ^{9}B and ^{9}Be difference [6]. For the ^{10}C and ^{10}Be difference, ΔH^c is taken as twice that for the ^{9}B and ^{9}Be difference, plus the contribution from the Coulomb interaction between the two protons. The ΔL contributions depend on differences of shift factors, weighted by reduced widths [6]; e.g., for the ^{10}B and ^{10}Be difference,

$$\Delta L = -\sum_{c=s,d} \gamma_c^2 \left[\frac{1}{2}\{S_c(^{10}\mathrm{B} \to {}^9\mathrm{B} + n) + S_c(^{10}\mathrm{B} \to {}^9\mathrm{Be} + p)\} - S_c(^{10}\mathrm{Be} \to {}^9\mathrm{Be} + n) \right], \tag{2}$$

with $\gamma_c^2 = \mathcal{S}_c\, \theta_{c,\mathrm{sp}}^2 \hbar^2/m_c\, a_c^2$, where a_c is the channel radius. The single-particle dimensionless reduced width is given by

$$\theta_{c,\mathrm{sp}}^2 = \frac{a_c}{2} \frac{u_c^2(a_c)}{\int_0^{a_c} \mathrm{d}r u_c^2(r)}, \tag{3}$$

where $u_c(r)$ is calculated in a central Woods-Saxon potential with conventional values of the radius and diffuseness parameters (1.25 and 0.65 fm), cut off at $r = a_c$. We use the average values of $\theta_{c,\mathrm{sp}}^2$ for the ^{9}Be + n and ^{9}Be + p channels, so that γ_c^2 may be taken the same for ^{10}Be as for ^{10}B. The spectroscopic factor $\mathcal{S}_c$ is given by $\mathcal{S}_s = 2(1 - \beta^2)$ and $\mathcal{S}_d = 2\beta^2$. With the conventional value $a_c = 1.45(A_1^{1/3} + A_2^{1/3})$ fm $= 4.47$ fm, and for $\beta^2 = 0.25$, we find $E_x(^9\mathrm{B}_{1/2^+}) = 1.29$ MeV and $E_x(^{10}\mathrm{C}_{0_2^+}) = 5.09$ MeV.

TABLE I. Values of $\bar{E}$ for $^{10}B(0_2^+) \to {}^9B(1/2^+) + n$, for values of E_r and γ^2 for $^9B(1/2^+)$ taken from Ref. [6].

Case	a (fm)	E_r [a] (MeV)	γ^2 (MeV)	E_m [a] (MeV)	$\bar{E}$ (MeV)
(a)	4	2.457	2.13	1.98	2.93
	5	2.124	1.36	1.79	2.69
	6	1.926	0.862	1.71	2.49
(b)	4	2.15	0.529	2.12	2.45
	5	1.99	0.423	1.96	2.30
	6	1.87	0.353	1.83	2.20

[a]Excitation energy.

The energy of the $^9B(1/2^+)$ state enters ΔE_C through the first shift factor in Eq. (2), for $c = s$. This term implies a sharp $1/2^+$ state. To allow for the width of the state, we replace S_s by $\bar{S}_s$, obtained by averaging the shift factor over the line shape of the state:

$$\bar{S}_s = \int_0^{E_{\max}} dE\ S_s(E_T - E)\rho(E) \bigg/ \int_0^{E_{\max}} dE\ \rho(E), \quad (4)$$

with

$$\rho(E) = \frac{P(E)}{[E_r - E - \gamma^2\{S(E) - S(E_r)\}]^2 + [\gamma^2 P(E)]^2}. \quad (5)$$

Here $P(E)$ and $S(E)$ are the penetration factor and shift factor for the $^8Be(g.s.) + p$ channel, with E being the channel energy. $E_T = -0.691$ MeV is the Q value for breakup of the $^{10}B(0_2^+)$ state into $^8Be(g.s.) + p + n$. Then the effective energy given by the FS approach, as an excitation energy in ^9B, is $\bar{E}$, where $S_s(E_T - Q_{gs} - \bar{E}) = \bar{S}_s$ (with $Q_{gs} = 0.185$ MeV).

In Table I, we give some values of $\bar{E}$ for given values of E_r and γ^2, taken from Table 3 of Ref. [6]. These are obtained using $E_{\max} = 7$ MeV, chosen because the one-level approximation assumed in Eq. (5) is not expected to be reasonable at high energies; a shell-model calculation [9] gives the second $1/2^+$ level about 5 MeV above the first, albeit with an appreciably smaller spectroscopic factor for the $^8Be(g.s.)$ channel. It is seen that $\bar{E}$ is somewhat greater than the resonance energy E_r and even greater than the peak energy E_m, which is the energy most likely to be determined if the $1/2^+$ level is identified in a reaction such as $^9Be(^3He,t)^9B$. If the FS value $E_x(^9B_{1/2^+}) = 1.31$ MeV is taken as a value of $\bar{E}$, this would suggest that the value of E_r is less than 1.31 MeV and that E_m is smaller still.

There are, however, other levels with appreciable widths involved in the FS procedure. In the compilation [3], the widths of the $1/2^+$ and $5/2^+$ levels in ^9Be and of the $5/2^+$ level in ^9B are given as 217, 282, and 550 keV, respectively. In the following, we use the FS potential-model approach, but now based on values of $\bar{E}$ for the ^9Be and ^9B levels rather than on the energy values listed in Ref. [3].

For the $1/2^+$ level of ^9Be, values of E_r and γ^2 obtained from fits to $^9Be(\gamma, n)^8Be$ data are given in Table I of Ref. [10]. We use the values from the fits B to the newer data [11] and to the Kuechler data [12], which fit the data well (see Figs. 1(b) and 2 of Ref. [10]), although they have very different values of E_r and of γ^2. These are for a channel radius $a = 4.35$ fm. For the $5/2^+$ levels, we take the compilation energies as values of E_r and choose γ^2 to fit the widths, taken as observed widths; this gives $\gamma^2 = 2.61$ MeV for the ^9Be level and 1.64 MeV for the ^9B level (we also do calculations using the mean value 2.12 MeV for each of ^9Be and ^9B). As a basic set of parameter values, we choose $E_{\max} = 5$ MeV for the ^9Be levels and 7 MeV for the ^9B levels, take the FS value $\beta^2 = 0.25$, and use the different γ^2 values for the $5/2^+$ levels; these lead to "basic" values (not necessarily best values). We then consider the changes in these values produced by small changes in the basic parameter values, taken one at a time. The uncertainties in the basic parameter values could be significantly greater than these assumed changes.

Basic values of $\bar{E}$ are given in Table II, while Table III gives the basic values of $E_x(^9B_{1/2^+})$ and $E_x(^{10}C_{0_2^+})$ expressed as resonance excitation energies, and their sensitivity to changes in the basic parameter values. It is seen that the value of $E_x(^9B_{1/2^+})$ is appreciably different for the two sets of values from Ref. [10]. These are, however, values of the resonance

TABLE II. Basic values of the effective energy $\bar{E}$, with E_r and γ^2 values for $^9Be(1/2^+)$ taken from Ref. [10] ($a = 4.35$ fm). All energies are in MeV.

J^π	$^9Be(J^\pi)$		$\bar{E}$			
	E_r [a]	γ^2	$^{10}Be \to {}^9Be + n$	$^{10}B \to {}^9B + n$	$^{10}B \to {}^9Be + p$	$^{10}C \to {}^9B + p$
$1/2^+$	0.0495	0.359	2.08	1.64	2.06	1.43
	0.272	2.40	3.05	2.53	3.01	2.16
$5/2^+$	1.384	2.61	3.40	3.15	3.40	3.14

[a]Channel energy.

TABLE III. Calculated excitation energies of ^{9}B($1/2^+$) and ^{10}C(0_2^+) for basic parameter values, and variations in these energies for changes in parameter values, with E_r and γ^2 values for ^{9}Be taken from Ref. [10]. All energies are in MeV.

^{9}Be($1/2^+$) E_r	γ^2	Basic values		Parameter changes	$E_{\rm max}$(^{9}Be) $5 \to 6$	$E_{\rm max}$(^{9}B) $7 \to 8$	β^2 $0.25 \to 0.30$	$\gamma^2(5/2^+)$ 2.61, 1.64 $\to$ 2.12
0.0495	0.359	$E_x(^9\mathrm{B}_{1/2^+})$	1.36	$\Delta E_x(^9\mathrm{B}_{1/2^+})$	+0.08	−0.06	−0.04	−0.03
		$E_x(^{10}\mathrm{C}_{0_2^+})$	5.01	$\Delta E_x(^{10}\mathrm{C}_{0_2^+})$	+0.02	−0.02	+0.00	−0.02
0.272	2.40	$E_x(^9\mathrm{B}_{1/2^+})$	1.74	$\Delta E_x(^9\mathrm{B}_{1/2^+})$	+0.37	−0.33	−0.07	−0.05
		$E_x(^{10}\mathrm{C}_{0_2^+})$	4.94	$\Delta E_x(^{10}\mathrm{C}_{0_2^+})$	+0.04	−0.05	+0.00	−0.02

energy; the corresponding values of the peak energy are 1.34 and 1.28 MeV, which are close to the value (1.29 MeV) obtained when the ^{9}Be and ^{9}B level widths are neglected and therefore close to the FS value (1.31 MeV). Small changes in the $E_{\rm max}$ values can produce large changes in the value of $E_x(^9\mathrm{B}_{1/2^+})$, although it does not change much if the difference $E_{\rm max}(^9\mathrm{B}) - E_{\rm max}(^9\mathrm{Be})$ is kept fixed. The value of $E_x(^{10}\mathrm{C}_{0_2^+})$ appears to be well determined at about 5.0 MeV, not far from the value given by FS. Changing the channel radius a from 4.35 to 5.0 fm, with parameter values taken from the fits E in Table I of Ref. [10], produces changes of less than 0.02 MeV in both $E_x(^9\mathrm{B}_{1/2^+})$ and $E_x(^{10}\mathrm{C}_{0_2^+})$.

If indeed the ^{10}C(0_2^+) level is at 4.20 MeV, as suggested by Curtis *et al.* [1], then some further modification of the FS approach seems necessary; e.g., by calculating and fitting the difference of the Coulomb displacement energies for the 0_2^+ and 0_1^+ $A = 10$ states, rather than by using the 0_2^+ energies only. Alternatively, the present result may be taken as an indication that the 4.20 MeV level is not the 0_2^+ state. It is of interest that Charity *et al.* [13], in an experiment similar to that of Curtis *et al.* [1], did not see a level at 4.20 MeV.

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