Spin-dependent Fermi liquid parameters and properties of polarized quark matter

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(Received 2 September 2008; published 20 January 2009)

We calculate the spin-dependent Fermi liquid parameters (FLPs), single-particle energies, and energy densities of various spin states of polarized quark matter. The expressions for the incompressibility K and sound velocity c_1 in terms of the spin-dependent FLPs and polarization parameter ξ are derived. Estimated values of K and c_1 reveal that the equation of state of the polarized matter is stiffer than the unpolarized one. Finally, we investigate the possibility of the spin polarization (ferromagnetism) phase transition.

DOI: 10.1103/PhysRevC.79.015205 PACS number(s): 21.65.Qr, 12.38.Bx, 12.39.−x, 26.60.−c

I. INTRODUCTION

One of the important research areas of contemporary high-energy physics has been the study of matter under extreme conditions. Such matter in the laboratory can be produced by colliding heavy ions at ultra-relativistic energies. Because of the asymptotic freedom of quantum chromodynamics (QCD), it is predicted that hadronic matter at high temperature and/or density can undergo a series of phase transitions such as confinement-deconfinement and/or chiral phase transitions [1,2]. In the high-density regime QCD predicts the existence of a color superconducting state [3–5]. The possibility of a spin-polarized quark liquid (i.e., the existence of a ferromagnetic phase in a dense quark system) has also been suggested recently [2,6]. The properties of a dense quark system are particularly relevant for the study of various astrophysical phenomena.

Part of the motivation for studying the ferromagnetic phase transition in dense quark matter (DQM), as mentioned in Ref. [6], is provided by the discovery of "magnetars" [7], where an extraordinarily high magnetic field ($\sim 10^{15}$ G) exists [6,8]. In Ref. [6], it is argued that the origin of such a high magnetic field can be attributed to the existence of spin-polarized quark matter [9]. To examine the possibility of ferromagnetism in DQM in Ref. [6] a variational calculation is performed in which it is observed that there exists a critical density below which spin-polarized quark matter is more energetically favorable than the unpolarized state. Subsequently, various other calculations were also performed to investigate this issue [2,4,5,7–10]. For example, in Ref. [5] it is shown that there is no contradiction between color superconductivity and ferromagnetism and both of these phases can coexist. In Ref. [10], the same problem was studied in the large-N_c and large-N_f limit while keeping N_c/N_f fixed, where it was shown that the spin-polarized state can exist; however, in the presence of magnetic screening, color superconductivity or dense chiral waves disappear. Such screening is now supported by the lattice calculation [10,11]. In Ref. [5] it is analytically shown that, if quarks are massless, ferromagnetism does not appear, which is consistent with the conclusion drawn in Ref. [10]. In Ref. [8] it is shown that ferromagnetism might appear in quark matter with a Goldstone boson current, where the magnetization is shown to be related to triangle anomalies.

In the present work, we apply relativistic Fermi liquid theory (RFLT) to study the possibility of a para-ferro phase transition in DQM. The RFLT was developed by Baym and Chin [12], who have shown how the various physical quantities (e.g., chemical potential μ, incompressibility K, and sound velocity c_1) can be expressed in terms of the Landau parameters (LPs) calculated relativistically. However, the formalism developed in Ref. [12] is valid for unpolarized matter and LPs calculated there are spin averaged.

In this paper we extend the formalism of RFLT and the required LPs are calculated by retaining their explicit spin dependencies. As a result, here various combination of parameters such as $f_{0,1}^{++}$, $f_{0,1}^{+-}$, $f_{0,1}^{-+}$, and $f_{0,1}^{--}$, corresponding to scattering involving up-up, up-down, down-up, or down-down spins, appear [12]. Once determined, these parameters are used to calculate quantities such as chemical potentials for the spin-up and spin-down quarks or the total energy density of the system as a function of $\xi = (n_q^+ - n_q^-)/n_q$ and n_q together with various other quantities, where n_q^+ and n_q^- correspond to densities of spin-up and spin-down quarks, respectively, and $n_q = n_q^+ + n_q^-$ denotes total quark density [6]. We also compare some of our results with those presented in Ref. [6], where a more direct approach was adopted to calculate the total energy density from the loop. In addition, the present work is extended further to estimate incompressibility and sound velocity in a dense quark system for a given fraction of spin-up or spin-down quarks.

Furthermore, in dealing with the massless gluons, we find that a naive series expansion fails and one has to use a hard density loop (HDL) corrected gluon propagator to get the finite result for the LPs involving scattering of like spins [13]. This however does not cause any problem for the calculation of various physical quantities such as chemical potential, exchange energy, and incompressibility. We shall see, even though f_0 and f_1 (suppressing spin indices) individually remain divergent, what appears in our case is the particular combination of these parameters where such divergences cancel.

The plan of the paper is as follows. In Sec. II, as mentioned before, we extend the formalism of RFLT to include explicit spin dependence. In Sec. III, we derive spin-dependent LPs resulting from one-gluon exchange (OGE) for polarized quark

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matter. Subsequently, we calculate the chemical potential and energy density. We find the density dependence of incompressibility K and first sound velocity c_1 with arbitrary spin polarization ξ. To compare with Ref. [6], we present ultra-relativistic and nonrelativistic results and study para-ferro phase transition of quark matter. Section IV is devoted to a summary and conclusion. In the Appendix, we calculate various LPs for unlike spin states of the scatterer.

II. FORMALISM

In Fermi liquid theory the total energy density E of an interacting system is the functional of occupation number n_p of the quasiparticle states of momentum p. The excitation of the system is equivalent to the change of occupation number by an amount δn_p. The corresponding energy density of the system is given by [12,14]

$$E = E^0 + \sum_s \int \frac{d^3p}{(2\pi)^3}\varepsilon^0_{ps}\delta n_{ps} + \frac{1}{2}\sum_{ss'}\int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} f_{ps,p's'}\delta n_{ps}\delta n_{p's'}, \quad (1)$$

where E^0 is the ground-state energy density and s is the spin index, and the quasiparticle energy can be written as

$$\varepsilon_{ps} = \varepsilon^0_{ps} + \sum_{s'}\int \frac{d^3p'}{(2\pi)^3} f_{ps,p's'}\delta n_{p's'}, \quad (2)$$

where ε^0_{ps} is the noninteracting single-particle energy. The interaction between quasiparticles is given by $f_{ps,p's'}$, which is defined to be the second derivative of the energy functional with respect to occupation functions,

$$f_{ps,p's'} = \frac{\delta^2 E}{\delta n_{ps}\delta n_{p's'}}. \quad (3)$$

Since the quasiparticles are well defined only near the Fermi surface, one assumes

$$\varepsilon_{ps} = \mu^s + v^s_f\left(p - p^s_f\right). \quad (4)$$

In Fermi liquid theory, the interaction parameter, $f_{ps,p's'}$, is expanded on the basis of Legendre polynomials, P_l [12,14]. The coefficients of this expansion are known as Fermi liquid parameters (FLPs), which are given by

$$f_l^{ss'} = (2l+1)\int \frac{d\Omega}{4\pi} P_l(\cos\theta) f_{ps,p's'}, \quad (5)$$

where θ is the angle between p and p', both taken to be on the Fermi surface, and the integration is over all directions of p [12]. Note that, unlike Refs. [12,14], here we retain explicit spin indices without performing spin summation. We restrict ourselves to $l \leqslant 1$ (i.e., f^s_0 and f^s_1), since any higher l contribution decreases rapidly as the scattering is dominated by small angles and the series converges (here, $f^s_l = \frac{1}{2}\sum_{s'} f_l^{ss'}$) [15].

The Landau Fermi liquid interaction $f_{ps,p's'}$ is related to the two-particle forward scattering amplitude via [12,14]

$$f_{ps,p's'} = \frac{m_q}{\varepsilon^0_p}\frac{m_q}{\varepsilon^0_{p'}}\mathcal{M}_{ps,p's'}, \quad (6)$$

where m_q is the mass of the quark and the Lorentz invariant matrix $\mathcal{M}_{ps,p's'}$ consists of the usual direct and exchange amplitude, which may, therefore, be evaluated by conventional Feynman rules. The dimensionless LPs are defined as $F^s_l = N^s(0) f^s_l$ [12], where $N^s(0)$, the density of states at the Fermi surface, is given by

$$N^s(0) = \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\delta(\varepsilon_{ps} - \mu^s) = \frac{g_{\rm deg}p_f^{s^2}}{2\pi^2}\left(\frac{\partial p}{\partial \varepsilon_{ps}}\right)_{p=p^s_f} \simeq \frac{g_{\rm deg}p^s_f\varepsilon^s_f}{2\pi^2}. \quad (7)$$

Here $g_{\rm deg}$ is the degeneracy factor. In our case $g_{\rm deg} = N_c N_f$, where N_c and N_f are the color and flavor indices for quark matter. For spin-up (+) and spin-down (−) quarks, the density of states will change accordingly. In this expression $(\partial p/\partial\varepsilon_{ps})_{p=p^s_f}$ is the inverse Fermi velocity $(1/v^s_f)$, which is related to the FLP F^s_1 by

$$\frac{1}{v^s_f} = (\partial p/\partial\varepsilon_{ps})_{p=p^s_f} = \left(\mu^s/p^s_f\right)\left(1 + F^s_1/3\right). \quad (8)$$

With Eq. (7) and Eq. (8) one reads the general relation as [16]

$$\varepsilon^s_f = \mu^s\left(1 + \tfrac{1}{3}F^s_1\right). \quad (9)$$

The compression modulus or incompressibility K of the system is defined by the second derivative of total energy density E with respect to the number density n_q, which is given by [16–20]

$$K = 9n_q\frac{\partial^2 E}{\partial n_q^2}. \quad (10)$$

Now we introduce a polarization parameter ξ by the equations $n^+_q = n_q(1+\xi)/2$ and $n^-_q = n_q(1-\xi)/2$ under the condition $0 \leqslant \xi \leqslant 1$ [6]. The Fermi momenta in the spin-polarized quark matter then are $p^+_f = p_f(1+\xi)^{1/3}$ and $p^-_f = p_f(1-\xi)^{1/3}$, where $p_f = (\pi^2 n_q)^{1/3}$ is the Fermi momentum of the unpolarized matter ($\xi = 0$). So, there are two Fermi surfaces corresponding to spin-up (+) and spin-down (−) states, such that $E \equiv E(n^+_q, n^-_q)$. We have

$$\frac{\partial E}{\partial n_q} = \frac{\partial E}{\partial n^+_q}\frac{\partial n^+_q}{\partial n_q} + \frac{\partial E}{\partial n^-_q}\frac{\partial n^-_q}{\partial n_q} = \frac{1}{2}[(1+\xi)\mu^+ + (1-\xi)\mu^-]. \quad (11)$$

Using Eq. (11), the incompressibility becomes [20]

$$K = \frac{9n_q}{4}\left[(1+\xi)^2\frac{\partial\mu^+}{\partial n^+_q} + (1-\xi)^2\frac{\partial\mu^-}{\partial n^-_q}\right] = \frac{9n_q}{4}\left[(1+\xi)^2\left(\frac{1+F^+_0}{N^+(0)}\right) + (1-\xi)^2\left(\frac{1+F^-_0}{N^-(0)}\right)\right], \quad (12)$$

where [12]

$$\frac{\partial \mu^s}{\partial n_q^s} = \frac{1+F_0^s}{N^s(0)}. \tag{13}$$

Similarly, the relativistic first sound velocity is given by the first derivative of pressure P with respect to energy density E. Since $P = \sum_s \mu^s n_q^s - E$ [16,20], we have

$$c_1^2 = \frac{\partial P}{\partial E} = \frac{\partial P}{\partial n_q}\frac{\partial n_q}{\partial E} = \left[\frac{(1+\xi)n_q^+ \frac{\partial \mu^+}{\partial n_q^+} + (1-\xi)n_q^- \frac{\partial \mu^-}{\partial n_q^-}}{(1+\xi)\mu^+ + (1-\xi)\mu^-}\right]$$
$$= \frac{n_q}{2[(1+\xi)\mu^+ + (1-\xi)\mu^-]}\left[(1+\xi)^2\left(\frac{1+F_0^+}{N^+(0)}\right) + (1-\xi)^2\left(\frac{1+F_0^-}{N^-(0)}\right)\right]. \tag{14}$$

In Eqs. (12) and (14), $N^{\pm}(0)$ and $F_0^{\pm}$ correspond to the density of states at the Fermi surface and the dimensionless LP for spin-up (+) and spin-down (−) quarks, respectively. For unpolarized matter, $\xi = 0$, implying $\mu^+ = \mu^-$, $F_0^+ = F_0^-$, and $N^+(0) = N^-(0)$. From Eqs. (12) and (14) we have the well-known result that $K = 9n_q \frac{\partial \mu}{\partial n_q}$ [16] and $c_1^2 = \frac{n_q}{\mu}\frac{\partial \mu}{\partial n_q}$ [12].

III. LANDAU PARAMETERS FOR POLARIZED QUARK MATTER

In this section we calculate LPs for quark matter with explicit spin dependencies. We choose spin s along the z axis [i.e., $s \equiv (0, 0, \pm 1)$] and represent spin-up and spin-down states by their signs. For a four-dimensional description of the polarization state, it is convenient to define a four-vector a^μ, which, in the rest frame of each quark, is the same as the three-dimensional vector s; since s is an axial vector, a^μ is a four-pseudovector. This four-vector is orthogonal to the four-momentum in the rest frame [in which $a^\mu = (0, s)$, $P^\mu = (m_q, 0)$]; in any frame we therefore have $a^\mu P_\mu = 0$ [6,21,22].

The components of the four-vector a^μ in a frame in which the particle is moving with momentum p are found by a Lorentz transformation from the rest frame [22],

$$a = s + \frac{p(s\cdot p)}{m_q(\varepsilon_p + m_q)}; \quad a^0 = \frac{p\cdot s}{m_q}, \tag{15}$$

with $\varepsilon_p = \sqrt{p^2 + m_q^2}$. We can define a projection operator $P(a)$ on each spin polarization as $P(a) = \frac{1}{2}(1 + \gamma^5 \not{a})$. Accordingly, the polarization density matrix ρ is given by the expression

$$\rho(P, s) = \frac{1}{2m_q}(\not{P} + m_q)P(a), \tag{16}$$

which is normalized by the condition $\mathrm{Tr}\,\rho(P, s) = 1$. The mean value of the spin is then given by the quantity [22]

$$s_{\mathrm{av}} = \frac{1}{2}\frac{m_q}{\varepsilon_p}\mathrm{Tr}\,(\rho\gamma_0\Sigma) = \frac{1}{2}\frac{m_q}{\varepsilon_p}\mathrm{Tr}\,(\rho\gamma_5\not{\gamma})$$
$$= \frac{1}{2}\frac{m_q}{\varepsilon_p}\left(s + \frac{p(s\cdot p)}{m_q(\varepsilon_p + m_q)}\right), \tag{17}$$

which is reduced to $s_{\mathrm{av}} = \frac{1}{2}s$ in the nonrelativistic limit.

We consider the color-symmetric forward scattering amplitude of the two quarks around the Fermi surface by the OGE interaction. The direct term does not contribute as it involves the trace of single color matrices such as $\mathrm{Tr}\,\lambda_a$, which vanish. Thus the leading contribution comes from the exchange (Fock) term [6]:

$$\mathcal{M}^{\mathrm{ex}}_{ps,p's'} = -\frac{1}{3}\sum_i \frac{1}{3}\sum_j [\bar{U}_\beta(P')g(t^a)_{ji}\gamma^\mu U_\alpha(P)] \times \left(\frac{-g_{\mu\nu}}{(P-P')^2}\right)[\bar{U}_\alpha(P)g(t^a)_{ij}\gamma^\nu U_\beta(P')]$$
$$= \frac{4}{9}\frac{g^2}{(P-P')^2}\mathrm{Tr}\,[\gamma_\mu \rho(P,s)\gamma^\mu \rho(P',s')], \tag{18}$$

where α, β is the flavor level, i, j is the quark color index, $t^a (= \lambda_a/2)$ is the color matrix, and g is the coupling constant. Since the gluon is flavor blind, the u-channel diagrams contribute only when $\alpha = \beta$ (i.e., the scattering of quarks with the same flavor [23]). This means that the Fermi sphere of each flavor makes an independent contribution. Thus the potential energy receives a factor N_f. However, the quarks with different colors can take part in the exchange process, giving rise to a factor N_c^2. Eventually, the potential energy density is proportional to $N_f N_c^2 g^2$. For the kinetic energy density, there arises an overall factor $N_c N_f$. Thus, the factor $N_c N_f$ factors out of the total energy density and the competition between the kinetic and potential energies is not influenced by the number of flavors. The number of flavors neither encourages nor discourages ferromagnetism [10].

Without loss of generality, for the calculation of energy density and other related quantities, we consider one-flavor quark matter. With the help of polarization density matrices given in Eq. (16), we have from Eq. (18) the interaction amplitude as [6]

$$\mathcal{M}^{\mathrm{ex}}_{ps,p's'} = \frac{2g^2}{9m_q^2}\frac{1}{(P-P')^2}\Big\{2m_q^2 - P.P' - (p\cdot s)(p'\cdot s') + m_q^2(s\cdot s') + \frac{1}{(\varepsilon_p + m_q)(\varepsilon_{p'} + m_q)} \times [m_q(\varepsilon_p + m_q)(p'\cdot s)(p'\cdot s') + m_q(\varepsilon_{p'} + m_q) \times (p\cdot s)(p\cdot s') + (p\cdot p')(p\cdot s)(p'\cdot s')]\Big\}. \tag{19}$$

From Eq. (6) the quasiparticle interaction parameter is given by

$$f^{\mathrm{ex}}_{ps,p's'} = \frac{m_q}{\varepsilon_p}\frac{m_q}{\varepsilon_{p'}}\mathcal{M}^{\mathrm{ex}}_{ps,p's'}. \tag{20}$$

Here the spin may be either parallel ($s = s'$) or antiparallel ($s = -s'$). Thus scattering possibilities are denoted by (+, +), (+, −), (−, −), etc. Motivated by Ref. [15], in analogy with isospin we define the spin-dependent interaction parameters as $f^+_{pp'} = \frac{1}{2}(f^{++}_{pp'} + f^{+-}_{pp'})$ and $f^-_{pp'} = \frac{1}{2}(f^{--}_{pp'} + f^{-+}_{pp'})$. Note that $f^{+-}_{pp'} = f^{-+}_{pp'}$.

For $(+,+)$ scattering the interaction parameter is given by

$$f^{++}_{pp'|p=p'=p_f^+} = -\frac{g^2}{9\varepsilon_f^{+2}}\frac{1}{p_f^{+2}(1-\cos\theta)}\Big\{2m_q^2 - p_f^{+2}(1-\cos\theta) - p_f^{+2}\cos\theta_1\cos\theta_2 + \frac{1}{(\varepsilon_f^+ + m_q)^2} \times [m_q(\varepsilon_f^+ + m_q)p_f^{+2}(\cos^2\theta_1 + \cos^2\theta_2) + p_f^{+4}\cos\theta\cos\theta_1\cos\theta_2]\Big\}, \quad (21)$$

where $\hat{p}\cdot\hat{s} = \cos\theta_1$, $\hat{p}'\cdot\hat{s} = \cos\theta_2$, and the Fermi energy is $\varepsilon_f^+ = (p_f^{+2} + m_q^2)^{1/2}$. Since spin and momentum have no preferred direction, we have done an angular average of the spin-dependent parameter [24]:

$$\overline{f^{++}}_{pp'|p=p'=p_f^+} = \int\frac{d\Omega_1}{4\pi}\int\frac{d\Omega_2}{4\pi}f^{++}_{pp'|p=p'=p_f^+} = -\frac{g^2}{9\varepsilon_f^{+2}p_f^{+2}(1-\cos\theta)} \times\left[2m_q^2 - p_f^{+2}(1-\cos\theta) + \frac{2m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)}\right]. \quad (22)$$

With the help of Eq. (5) along with Eq. (22) one can find the LPs, but it is to be noted that $f_{0,1}^{++}$ and $f_{0,1}^{--}$ are individually divergent because of the term $(1-\cos\theta)$ in the denominator of the interaction parameter.[1] This divergence disappears if one uses a Debye screening mass for gluons or equivalently an HDL corrected gluon propagator while evaluating the scattering amplitudes [13,24]. Note that the combination $(f_0^{++(--)} - \frac{1}{3}f_1^{++(--)})$ is, however, finite as in this case the divergences cancel and we do not calculate the LPs separately. It would, however, be interesting to see how the results are modified if HDL calculations are performed to evaluate $f_{0,1}^{++(--)}$, $f_{0,1}^{+-}$ and the corresponding physical quantities. The numerical estimates suggest that, for the results that we present here, the effect of HDL corrections are expected to be small.

From Eq. (5),

$$f_0^{++} - \frac{1}{3}f_1^{++} = -\frac{g^2}{18\varepsilon_f^{+2}p_f^{+2}}\int_{-1}^{+1}\left[2m_q^2 - p_f^{+2}(1-\cos\theta) + \frac{2m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)}\right]d(\cos\theta) = -\frac{g^2}{9\varepsilon_f^{+2}p_f^{+2}}\left[2m_q^2 - p_f^{+2} + \frac{2m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)}\right]. \quad (23)$$

This combination will appear in the calculation of the chemical potential and other relevant quantities. For $(+,-)$ scattering, the angular averaged interaction parameter yields

$$f^{+-}\Big|_{p=p_f^+,p'=p_f^-} = \frac{g^2}{9\varepsilon_f^+\varepsilon_f^-}\left\{1 - \left[\frac{m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)} + \frac{m_q p_f^{-2}}{3(\varepsilon_f^- + m_q)}\right] \times \frac{1}{\left(m_q^2 - \varepsilon_f^+\varepsilon_f^- + p_f^+ p_f^-\cos\theta\right)}\right\}. \quad (24)$$

It is to be noted that individual LPs for scattering of unlike spin states are finite (i.e., free of divergences), in contrast to the case involving scattering of like spin states. (For details see the Appendix.)

A. Chemical potential

Now we proceed to calculate the chemical potential, which, in principle, will be different for spin-up and spin-down quarks, denoted by μ^s with s (or s') $= +,-$ for matter containing unequal densities of up and down quarks. To determine the chemical potential with arbitrary polarization ξ, we take the distribution function with explicit spin index (s or s'), so that variation of the distribution function gives [14,20,25]

$$\delta n_q^s = -N^s(0)\left[\sum_{s'} f_0^{ss'}\delta n_q^{s'} - \delta\mu^s\right], \quad (25)$$

where $N^s(0)$ is given by Eq. (7). Equation (25) yields

$$\frac{\partial\mu^s}{\partial n_q^s} = \frac{1}{N^s(0)} + \sum_{s'} f_0^{ss'}\frac{\partial n_q^{s'}}{\partial n_q^s}. \quad (26)$$

Separately for spin-up and spin-down states we have

$$\begin{pmatrix}\partial\mu^+\\ \partial\mu^-\end{pmatrix} = \begin{pmatrix}\frac{1}{N^+(0)} + f_0^{++} & f_0^{+-}\\ f_0^{-+} & \frac{1}{N^-(0)} + f_0^{--}\end{pmatrix}\begin{pmatrix}\partial n_q^+\\ \partial n_q^-\end{pmatrix}, \quad (27)$$

where the superscripts $++$ and $+-$ denote scattering of quasiparticle with up-up and up-down spin states. For unpolarized matter the upper and lower components become equal, which gives rise to the well-known result [12]

$$\mu d\mu = \left[p_f + \frac{g_{\rm deg}\mu p_f^2}{2\pi^2}\left(f_0 - \frac{1}{3}f_1\right)\right]dp_f. \quad (28)$$

In general the chemical potential (both for spin-up and spin-down) is the combination of like and unlike spin states. By adjusting the constant of integration [12], the chemical potential of spin-up quark turns out to be

$$\mu^+ = \varepsilon_f^+ - \frac{g^2}{6\pi^2\varepsilon_f^+}\left[\frac{11}{6}m_q^2\ln\left(\frac{p_f^+ + \varepsilon_f^+}{m_q}\right) + \frac{2}{3}p_f^+ m_q - \frac{p_f^+\varepsilon_f^+}{2}\right] + \frac{g^2}{72\pi^2\varepsilon_f^+}\left\{-\frac{2m_q^3}{p_f^+}\ln\left(\frac{p_f^+ + p_f^-}{p_f^+ - p_f^-}\right) + \frac{4m_q^2\varepsilon_f^+}{p_f^+}\left[\ln\left(\frac{p_f^+ + p_f^-}{p_f^+ - p_f^-}\right) + \ln\left(\frac{p_f^+\varepsilon_f^- + p_f^-\varepsilon_f^+}{p_f^+\varepsilon_f^- - p_f^-\varepsilon_f^+}\right)\right]\right.$$

[1] Denoted hereafter as $\overline{f_{pp'}} = f_{pp'}$.

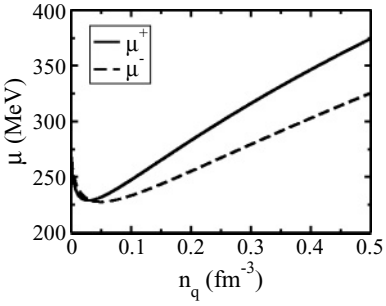

FIG. 1. Density dependence of chemical potential of spin-up and spin-down quarks denoted by solid and dashed curves, respectively.

$$
\begin{aligned}
&- 14m_q^2 \ln\left(\frac{p_f^- + \varepsilon_f^-}{m_q}\right) + 2m_q p_f^- \\
&- 3m_q p_f^- \ln\left(\frac{p_f^+ + p_f^-}{p_f^+ - p_f^-}\right) - \frac{m_q}{p_f^+}\left(2m_q^2 + 3p_f^{+2}\right) \\
&\times \ln\left(\frac{p_f^+\varepsilon_f^- + p_f^-\varepsilon_f^+}{p_f^+\varepsilon_f^- - p_f^-\varepsilon_f^+}\right) + 6m_q\varepsilon_f^+ \ln\left(\frac{p_f^- + \varepsilon_f^-}{m_q}\right) \\
&+ \frac{m_q}{p_f^+}\left[2\varepsilon_f^-\left(2m_q - \varepsilon_f^+\right) - p_f^{-2}\right] \\
&\times \ln\left(\frac{\varepsilon_f^-\varepsilon_f^+ - m_q^2 - p_f^- p_f^+}{\varepsilon_f^-\varepsilon_f^+ - m_q^2 + p_f^- p_f^+}\right) + 6p_f^-\varepsilon_f^-\Bigg\}. \qquad (29)
\end{aligned}
$$

In this equation the term in the first set of square brackets arises from the scattering of like spin states (++), and the term in curly braces comes from the scattering of unlike spin states (+−).

Similarly, for the spin-down quark, one may determine μ^- by replacing $p_f^\pm$ with $p_f^\mp$ and $\varepsilon_f^\pm$ with $\varepsilon_f^\mp$ in Eq. (29).

For the numerical estimation of these quantities, following Refs. [6,26], we take $\alpha_c = g^2/4\pi = 2.2$ as the fine structure constant of QCD and $m_q = 300$ MeV. In Fig. 1 we plot the chemical potential for spin-up and spin-down quarks as a function of density with order parameter $\xi = 0.5$. In real astrophysical calculations, the chemical potentials are determined by the β-equilibrium conditions, where the condition of charge neutrality is also imposed. In Fig. 1, however, we use density n_q and polarization parameter ξ as input parameters and Eq. (29) is used to determine μ for a system with one flavor.

B. Energy density

Once the value of μ is determined, one can readily calculate the exchange energy density by evaluating [12,17,25]

$$E_{\rm ex} = \int {\rm d}n_q(\mu - \varepsilon_f). \qquad (30)$$

After summing over the color degrees of freedom and evaluating over the Fermi surfaces, we have the exchange energy density. The latter, consisting of all types of scattering amplitudes, can be written as

$$E_{\rm ex} = E_{\rm ex}^{++} + E_{\rm ex}^{--} + E_{\rm ex}^{+-}, \qquad (31)$$

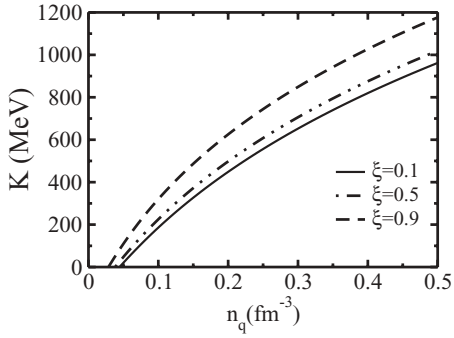

FIG. 2. Incompressibility K in quark matter as a function of density for different polarization parameters.

which we evaluate numerically. The total kinetic energy density for spin-up and spin-down quarks is given by

$$E_{\rm kin} = \frac{3}{16\pi^2}\sum_{s=\pm}\left[p_f^s\varepsilon_f^s\left(\varepsilon_f^{s^2} + p_f^{s^2}\right) - m_q^4 \ln\left(\frac{\varepsilon_f^s + p_f^s}{m_q}\right)\right], \qquad (32)$$

where $\varepsilon_f^s = (p_f^{s^2} + m_q^2)^{1/2}$. The total energy is given by the sum of the kinetic energy and the interaction energy $E_{\rm ex}$, that is,

$$E_{\rm tot} = E_{\rm kin} + E_{\rm ex}. \qquad (33)$$

Now we calculate the incompressibility and the sound velocity by using Eqs. (12) and (14). In Figs. 2 and 3 we plot the density dependencies of the incompressibility and the sound velocity. This shows that, for higher values of the order parameter ξ, the incompressibility and the sound velocity become higher for the same value of density. Thus the equation of state for polarized quark matter is found to be stiffer than the unpolarized one.

C. Phase transition

Bloch first pointed out the possibility of ferromagnetism of an electron gas where the Fock exchange interaction induces spontaneous spin polarization [27]. Consider a spin-polarized electron gas interacting by the Coulomb interaction in a background of positively charged ions. Since the direct

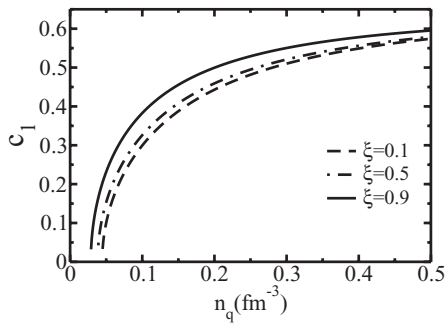

FIG. 3. First sound velocity c_1 in quark matter as a function of density for different polarization parameters.

interaction gives no contribution because of charge neutrality, the Fock exchange interaction gives the leading contribution as the interaction energy. For spontaneous ferromagnetism, the interaction energy dominates over the kinetic energy [6,21,28]. Therefore, if the exchange energy from the OGE interaction is negative and becomes greater than the kinetic energy at some density, the quark matter becomes polarized, giving rise to ferromagnetism [6].

To check whether our results for the total energy density are consistent with Ref. [6], we consider two limiting cases, corresponding to the ultra-relativistic (UR) and nonrelativistic (NR) regimes. In the UR limit, $p_f^s \gg m_q$, then using Eq. (29) we have

$$\mu^{+,\mathrm{ur}} = p_f^+ + \frac{\alpha_c}{3\pi}\left[p_f^+ + \frac{p_f^{-2}}{p_f^+}\right]. \tag{34}$$

Similarly one can find $\mu^{-,\mathrm{ur}}$ by replacing $p_f^{\pm}$ with $p_f^{\mp}$.

One can arrive at the same expression $\mu^{\pm,\mathrm{ur}}$ by taking the UR limit of the scattering amplitude. For $(+,+)$ scattering one gets the interaction parameter as

$$f_{pp'}^{++,\mathrm{ur}} = \frac{g^2}{9pp'}\left(1+\cos\theta_1\cos\theta_2\right). \tag{35}$$

After taking the angular average of the interaction parameter and with the help of Eq. (5), we find that f_1^{++} vanishes. Thus we have

$$f_{pp'}^{++,\mathrm{ur}}\Big|_{p=p'=p_f^+} = f_0^{++,\mathrm{ur}} = \frac{g^2}{9p_f^{+2}}. \tag{36}$$

Similarly, for $(+,-)$ scattering, the interaction parameter yields

$$f_{pp'}^{+-,\mathrm{ur}} = \frac{g^2}{9pp'}\left(1-\cos\theta_1\cos\theta_2\right). \tag{37}$$

The only existing LP is f_0^{+-} and other higher order LPs do not contribute. Hence we get

$$f_{pp'}^{+-,\mathrm{ur}}\Big|_{p=p_f^+,p'=p_f^-} = f_0^{+-,\mathrm{ur}} = \frac{g^2}{9p_f^+p_f^-}. \tag{38}$$

It is observed that, in the UR limit, all the LPs are finite. Now the chemical potential for the spin-up quark is found to be

$$\mu^{+,\mathrm{ur}} = p_f^+ + \frac{\alpha_c}{3\pi}\left[p_f^+ + \frac{p_f^{-2}}{p_f^+}\right]. \tag{39}$$

The chemical potential $\mu^{-,\mathrm{ur}}$ can be obtained by replacing $p_f^{\pm}$ with $p_f^{\mp}$ in Eq. (39).

Using Eqs. (30) and (31), the exchange energy densities are given by

$$\left.\begin{aligned} E_{\mathrm{ex}}^{++,\mathrm{ur}} &= \tfrac{\alpha_c}{8\pi^3}p_f^{+4},\\ E_{\mathrm{ex}}^{--,\mathrm{ur}} &= \tfrac{\alpha_c}{8\pi^3}p_f^{-4},\\ E_{\mathrm{ex}}^{+-,\mathrm{ur}} &= \tfrac{\alpha_c}{4\pi^3}p_f^{+2}p_f^{-2}. \end{aligned}\right\} \tag{40}$$

Thus the final expression for the exchange energy density in the UR limit is found to be

$$E_{\mathrm{ex}}^{\mathrm{ur}} = \frac{\alpha_c}{8\pi^3}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}+2(1-\xi^2)^{2/3}]. \tag{41}$$

This result is the same as in Ref. [6].

Similarly, from Eq. (32), the kinetic energy density in the UR limit takes the following form [6]:

$$E_{\mathrm{kin}}^{\mathrm{ur}} = \frac{3p_f^4}{8\pi^2}[(1+\xi)^{4/3}+(1-\xi)^{4/3}]. \tag{42}$$

In the NR limit, p (or p') $\ll m_q$, the interaction parameter reduces to a simple form

$$f_{ps,p's'}^{\mathrm{nr}} = -\frac{g^2}{9pp'}\left[\frac{1+s\cdot s'}{(1-\cos\theta)}\right]. \tag{43}$$

For a spin antiparallel interaction $s=-s'$, then $f_{ps,p's'}^{\mathrm{nr}}=0$. Thus the contribution from the scattering of quarks with unlike spin states vanishes and the dominant contribution to the energy density comes from the parallel spin states ($s=s'$). For (s,s) scattering, the interaction parameter yields

$$f_{pp'}^{\mathrm{nr},s}\Big|_{p=p'=p_f^s} = -\frac{2g^2}{9p_f^{s^2}(1-\cos\theta)}, \tag{44}$$

where $s=+$ or $-$ according to the scattering process. In the NR limit one gets

$$\left(f_0^s - \frac{1}{3}f_1^s\right) = -\frac{2g^2}{9p_f^{s^2}}. \tag{45}$$

The NR chemical potential μ^{nr} is given by

$$\mu^{\mathrm{nr},s} = m_q - \frac{g^2}{3\pi^2}p_f^s. \tag{46}$$

Using Eq. (30), the exchange energy density for $(+,+)$ scattering is given by

$$E_{\mathrm{ex}}^{++,\mathrm{nr}} = -\frac{g^2}{8\pi^4}p_f^4(1+\xi)^{4/3}. \tag{47}$$

Similarly, for $(-,-)$ scattering, we have

$$E_{\mathrm{ex}}^{--,\mathrm{nr}} = -\frac{g^2}{8\pi^4}p_f^4(1-\xi)^{4/3}. \tag{48}$$

As in the NR limit, $E_{\mathrm{ex}}^{+-} = E_{\mathrm{ex}}^{-+} = 0$ as mentioned before, so from Eq. (31) the exchange energy density yields

$$E_{\mathrm{ex}}^{\mathrm{nr}} = -\frac{\alpha_c}{2\pi^3}p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}]. \tag{49}$$

Thus the energy density, in this limit, becomes negative.

The kinetic energy density turns out to be [6]

$$E_{\mathrm{kin}}^{\mathrm{nr}} = \frac{3p_f^5}{20\pi^2 m_q}[(1+\xi)^{5/3}+(1-\xi)^{5/3}]. \tag{50}$$

In the NR limit, ferromagnetism can appear as a consequence of competition between the kinetic energy and the Coulomb potential energy [10]. The latter favors spin alignment owing to quantum effects. When the energy gain from the spin alignment dominates over the increase in the kinetic energy at some density, the unpolarized state suddenly turns into the completely polarized state [29]. In contrast, in the UR limit, the contribution to the energy density comes not only from the like spin states but also from the unlike spin states of the scatterer (see Ref. [6] for a detailed discussion).

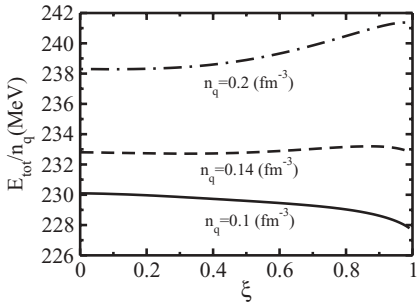

FIG. 4. The total energy of a quark liquid as a function of polarization parameter at $n_q = 0.1$ fm^{-3}, $n_q = 0.14$ fm^{-3}, and $n_q = 0.2$ fm^{-3}. The critical density is found to be $n_q^c = 0.14$ fm^{-3} in this case.

To check the consistency we compare our result derived in the RFLT approach with that of Ref. [6] derived from two loop ring diagrams. In Fig. 4 we plot $E_{\rm tot}/n_q$ as a function of polarization parameter ξ. The results clearly show that, for lower density ($<$0.14 fm^{-3}), total energy favors $\xi = 1$, which indicates a completely polarized state, whereas, at higher density, the system becomes unpolarized ($\xi = 0$). Thus the polarization parameter suddenly changes from $\xi = 1$ to $\xi = 0$ as one increases the number density of the system. So the phase transition is first order and the critical density n_q^c is around 0.14 fm^{-3}.

In Fig. 5 we show the total energy as a function of the polarization parameter for different densities. In every plot, there is a minima, which corresponds to a possible metastable state. We notice that when density increases metastable state arises for lower values of polarization parameter ξ. For example, at a density of $\sim$0.2 fm^{-3} a minima arises at $\xi = 0.1$ whereas at a density of $\sim$0.35 fm^{-3} a minima arises at $\xi = 0.03$. Thus the metastable state shows a tendency to disappear as the density increases.

IV. SUMMARY AND CONCLUSION

In this work we have applied RFLT to study the properties of dense quark matter. Accordingly, we calculate the FLPs by retaining their explicit spin dependencies. We also show how the physical quantities such as the chemical potential of spin-up and spin-down states, their energy densities, and incompressibility and sound velocity for polarized quark matter can be expressed in terms of these spin-dependent RFLPs. For scattering involving like spin states, the LPs $f_{0,1}^{++}$ and $f_{0,1}^{--}$ are found to diverge. However, we show that for the combination in which they appear in the calculation of the physical quantities such divergences cancel. For scattering involving unlike spin states no such divergence appears. The appearance of such divergences is related to the unscreened gluonic interaction between the quarks, and invoking a hard dense loop corrected gluon propagator may eliminate these divergences. We do not perform such a calculation here and postpone this for a future investigation. As far as the equation of state is concerned, in the present model we find that the equation of state for polarized quark matter is stiffer than that for unpolarized matter. In addition, we also show that there exists a metastable state that disappears at higher density, although it seems that the effect is tiny.

We reconfirm that DQM can exhibit ferromagnetism at low density, as was originally suggested in Ref. [6]. However, the density at which the spin-polarized ferromagnetic state in the present model might appear depends strongly on the quark mass. The critical density increases with increasing mass. In Fig. 4, we observe that states with ξ appear only below

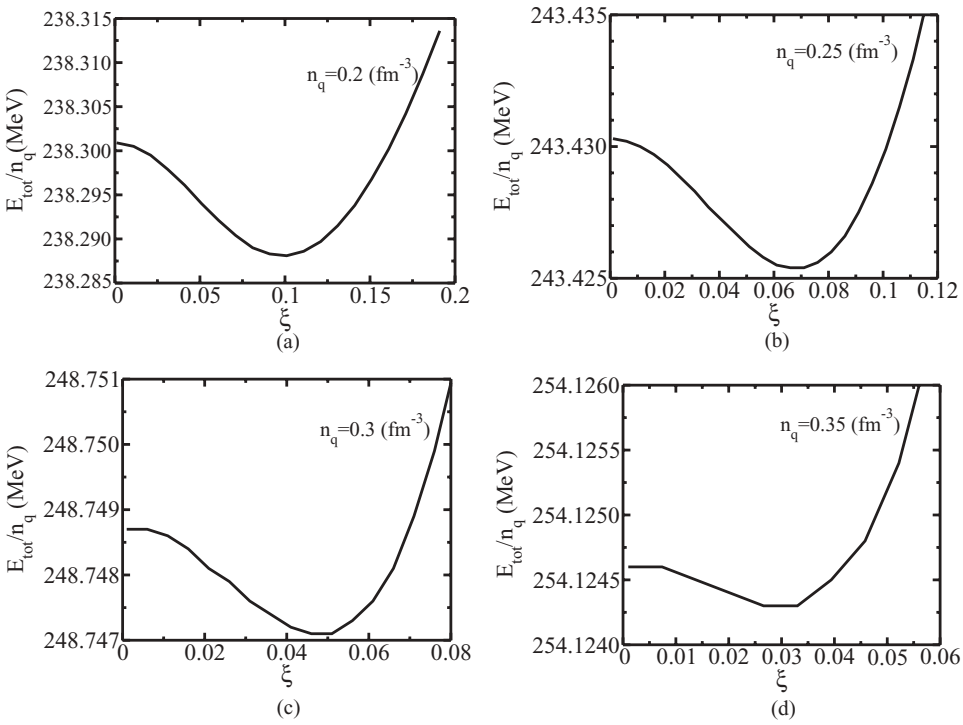

FIG. 5. The metastable ferromagnetic state as a function of polarization parameter for different densities.

or around normal nuclear density where deconfined quark matter is not likely to exist. We cannot, however, ascertain the critical density from the present analysis in which we restrict ourselves only to OGE diagrams and a one-flavor system. In this regime, multigluon exchange processes [10] might play an important role. Furthermore, the correlations as given by the ring diagrams can also change the conclusion. Further work therefore is necessary to understand the existence of ferromagnetic quark matter in real multiflavor systems that might appear in astrophysics.

ACKNOWLEDGMENTS

The authors would like to thank P. Roy for his critical reading of the manuscript.

APPENDIX

In the text the interaction parameter $f_{pp'}^{+-}$ for unlike spin states was calculated. Here we give a detailed expression of the Landau parameters. With the help of Eq. (5), the LPs are given by

$$f_0^{+-} = \frac{g^2}{18\varepsilon_f^+\varepsilon_f^-} \times \left[2 + \frac{m_q\left[\varepsilon_f^- p_f^{+2} + m_q\left(p_f^{+2} + p_f^{-2}\right) + \varepsilon_f^+ p_f^{-2}\right]}{3p_f^+ p_f^-(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)} \times \ln\left(\frac{m_q^2 - p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}{m_q^2 + p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}\right)\right] \tag{A1}$$

and

$$f_1^{+-} = \frac{g^2}{18\varepsilon_f^+\varepsilon_f^-}\left[6 - \frac{2m_q\left[\varepsilon_f^- p_f^{+2} + m_q\left(p_f^{+2} + p_f^{-2}\right) + \varepsilon_f^+ p_f^{-2}\right]}{p_f^+ p_f^-(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)} + \left(\frac{m_q\left(m_q^2 - \varepsilon_f^+\varepsilon_f^-\right)\left[\varepsilon_f^- p_f^{+2} + m_q\left(p_f^{+2} + p_f^{-2}\right) + \varepsilon_f^+ p_f^{-2}\right]}{p_f^{+2} p_f^{-2}(m_q + \varepsilon_f^+)(m_q + \varepsilon_f^-)}\right)\ln\left(\frac{m_q^2 + p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}{m_q^2 - p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}\right)\right]. \tag{A2}$$

Using Eqs. (A1) and (A2) we have

$$f_0^{+-} - \frac{1}{3}f_1^{+-} = \frac{g^2}{18\varepsilon_f^+\varepsilon_f^-}\left\{2 - \left[\frac{m_q p_f^{+2}}{3(\varepsilon_f^+ + m_q)} + \frac{m_q p_f^{-2}}{3(\varepsilon_f^- + m_q)}\right] \times \left[-\frac{2}{p_f^+ p_f^-} + \frac{\left(p_f^+ p_f^- + m_q^2 - \varepsilon_f^+\varepsilon_f^-\right)}{\left(p_f^{+2} p_f^{-2}\right)} \times \ln\left(\frac{m_q^2 + p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}{m_q^2 - p_f^+ p_f^- - \varepsilon_f^+\varepsilon_f^-}\right)\right]\right\}. \tag{A3}$$

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