Particle production in *p*-*p* collisions and predictions for $\sqrt{s} = 14$ TeV at the CERN Large Hadron Collider (LHC)

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We analyze recent data on particle production yields obtained in *p*-*p* collisions at SPS and RHIC energies within the statistical model. We apply the model formulated in the canonical ensemble and focus on strange particle production. We introduce different methods to account for strangeness suppression effects and discuss their phenomenological verification. We show that at RHIC the midrapidity data on strange and multistrange particle multiplicity can be successfully described by the canonical statistical model with and without an extra suppression effects. On the other hand, SPS data integrated over the full phase-space require an additional strangeness suppression factor that is beyond the conventional canonical model. This factor is quantified by the strangeness saturation parameter or strangeness correlation volume. Extrapolating all relevant thermal parameters from SPS and RHIC to LHC energy we present predictions of the statistical model for particle yields in *p*-*p* collisions at $\sqrt{s} = 14$ TeV. We discuss the role and the influence of a strangeness correlation volume on particle production in *p*-*p* collisions at LHC.

I. INTRODUCTION

Results from the CERN Large Hadron Collider (LHC) have a potential to usher in a new era of discoveries and insights into particle physics. In order to fully appreciate these it is important to have a clear understanding as to what we expect will happen at the LHC without new physics and new phenomena involved. This requires an extrapolation of trusted models to LHC energies. In this paper we use the statistical model for particle production which has been around since more than half a century [1–3] and discuss its predictions for LHC energy.

The statistical model has been very successful in describing hadron yields in central heavy-ion collisions [4–7]. It has provided a useful framework for describing centrality and system size dependence of particle production in heavy-ion collisions [8,9]. It has also led to new insights about its applicability in small systems like *p*-*p* and even e^+-e^- scattering [5,10–13].

In the following we concentrate on particle production in elementary collisions and discuss the predictions of the statistical model for particle yields in *p*-*p* collisions at the LHC at $\sqrt{s} = 14$ TeV center-of-mass energy.

Particle production calculated within the statistical model is quantified by a set of thermal parameters, these are: the temperature, the volume and the set of chemical potentials which are related to conserved charges. To make any predictions for particle production at LHC energies one needs methods to extrapolate the thermal parameters to higher energies. In this paper we first analyze the recent experimental data obtained in *p*-*p* collisions at Super Proton Synchrotron (SPS) and Relativistic Heavy Ion Collider (RHIC) and then we extrapolate these to LHC beam energies. This allows us to make predictions for different particle yield ratios at the

LHC as well as discuss their dependence on the values of extrapolated parameters. A very brief account of some of our results was given in [14]. In this paper we present a complete description of the extrapolation method which is based on our new analysis of the *p*-*p* data at SPS and RHIC energies.

The paper is organized as follows. In Sec. II we summarize the main features of the canonical statistical model. In Sec. III we present the analysis of SPS and RHIC data obtained in *p*-*p* collisions. In Sec. IV we introduce the extrapolation of the model to the LHC energy and discuss its predictions for particle production in *p*-*p* collisions. In the final section we present conclusions and summarize our results.

II. STATISTICAL MODEL

Systematic studies of particle yields extending over more than a decade, using experimental results at different beam energies, have revealed a clear underlying freeze-out pattern for particle yields in heavy-ion collisions [15]. A detailed comparison of different freeze-out criteria was made in [16] and we followed the one used previously [17] to extrapolate the temperature T and baryon chemical potential μ_B to the LHC energy. The expected hadron multiplicity ratios can be calculated directly from this extrapolation if the grand canonical (GC) description of charge conservation is adequate. This is because, in the GC ensemble, any particle multiplicity ratio is uniquely determined by the values of the temperature and the chemical potentials.

A. Canonical suppression

The usual form of the statistical model in the grand canonical ensemble formalism cannot be used when either
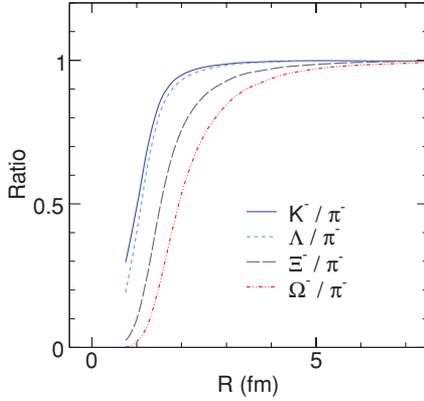
FIG. 1. (Color online) Particle ratios as a function of the volume radius R. The temperature $T = 170$ MeV and the baryon chemical potential $\mu_B = 1$ MeV were chosen according to the thermal conditions expected to be valid at LHC energy. All ratios are normalized to their grand canonical values.

the temperature or the volume or both are small, as a rule of thumb one needs $VT^3 > 1$ for a grand canonical description to hold [18,19]. Furthermore, even if this condition is met, if the abundance of a subset of particles carrying a conserved charge is small, the canonical suppression still appears even though the grand canonical description is valid for the bulk of the produced hadrons. There is by now a vast literature on the subject of canonical suppression and we refer to several articles (see, e.g., [5,20,21]).

The effects of canonical suppression are illustrated in Fig. 1 which shows particle ratios of strange and multistrange hadrons to pions normalized to the values in the grand canonical limit as a function of the radius of the system. The smaller the volume and the larger strangeness content of the particle the stronger the suppression of the yield of strange particles. This has been discussed in great detail in [13,22].

The analysis of the variations of particle ratios with the size of the system, e.g., via the number of participants, at SPS revealed [13] that the experimental data show stronger suppression of strange-particle yields than that expected in the canonical model [13,23,24]. Consequently, an additional suppression effect had to be included in order to quantify the observed yields of strange particles.

B. Strangeness correlation volume

One possible explanation for the failure of the canonical corrections is that strangeness can be conserved exactly in a small subvolume of the fireball, thus leading to a stronger canonical suppression, as seen in Fig. 1, even though the strange particles within such subvolumes are taken as being in chemical equilibrium. A modification of the statistical model was formulated in [13,23] that allows to quantify this extra suppression by the strangeness correlation volume (cluster size) within which the strangeness is conserved exactly. An alternative method was proposed in [10] to include in the canonical statistical model an additional factor γ_S that accounts
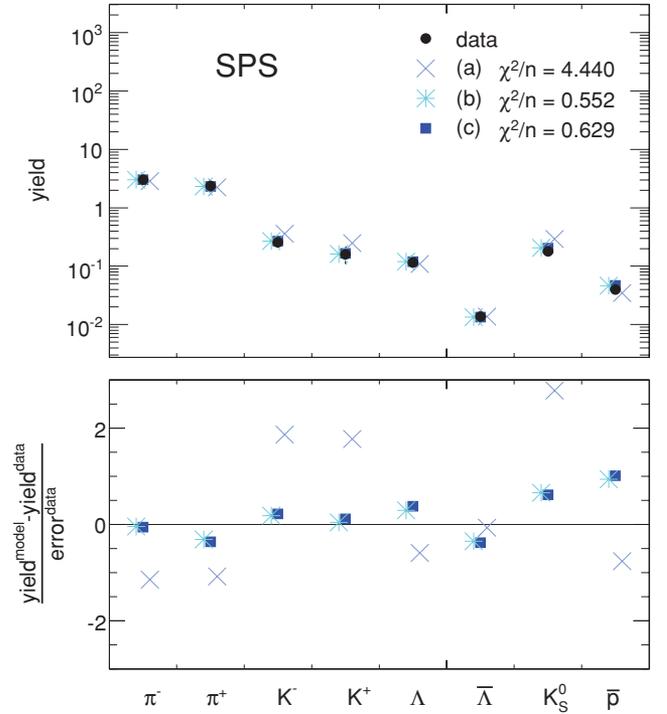

FIG. 2. (Color online) Integrated particle yields from p-p collisions at $\sqrt{s} = 17.3$ GeV [25–28] together with different model predictions: (a), (b), and (c) introduced in the text. The lower panel shows the deviation of the model fits to data.

for possible deviations of strange particle abundance from their chemical equilibrium distribution value.

In the next section we discuss how to choose the size of the cluster or γ_S in the canonical model to describe experimental data on strange particle production in p-p collisions. First, we use data from SPS and from RHIC energies and next we discuss the possible extrapolation of the model parameters to make predictions for particle production in p-p collisions at LHC energies.

III. DATA ANALYSIS

At the top SPS energy data are available for π, K, K_S^0, $\bar{p}$, and Λ yields integrated over the full phase-space [25–28] in p-p collisions. In Fig. 2 we show the comparison of the statistical model with these data for three different implementations of the canonical statistical model labeled (a), (b), and (c). In all cases the conservation of the electric charge and baryon number is formulated in the GC ensemble and is thus controlled by the corresponding chemical potentials. However, strangeness conservation is always treated canonically. We consider three different models:

(i) the strangeness suppression, at fixed T and μ_B, is controlled by the volume parameter which coincides with the system size,

(ii) an additional strangeness suppression is introduced through the γ_S-fugacity factor,

TABLE I. Statistical model parameters, extracted from the comparison of model (c) (upper table) and model (b) (lower table) with experimental data on *p*-*p* collisions. Full phase-space (4π integrated) data at $\sqrt{s} = 17.3$ GeV and midrapidity densities at $\sqrt{s} = 200$ GeV were used.

$\sqrt{s}$ (GeV)	T (MeV)	μ_B (MeV)	R (fm)	R_C (fm)
17.3	169 ± 16	225 ± 45	1.4 ± 0.3	1.0 ± 0.3
200	165 ± 4	14 ± 18	1.2 ± 0.1	1.3 ± 0.1

$\sqrt{s}$ (GeV)	T (MeV)	μ_B (MeV)	R (fm)	γ_S
17.3	164 ± 9	211 ± 27	1.5 ± 0.2	0.7 ± 0.1
200	165 ± 4	14 ± 18	1.2 ± 0.1	1.1 ± 0.1

(iii) the canonical suppression is controlled by the cluster volume which can be smaller than the system size.

From the comparisons shown in Fig. 2 it is clear the model (a) fails to describe the *p*-*p* data. Although, the canonical description of strangeness production is included in this model, the strange particle ratios exhibit large deviations from data. The discrepancies seen in Fig. 2 for model (a) could be even larger for multistrange particles. The reason of these discrepancies is due to the fact, that the volume of the system, fixed from the pion yields, is already too large to imply strangeness suppression. From the χ^2 fit to SPS data one gets, in model (a) the system size radius $R \simeq 1.3$. Consequently, as seen in Fig. 2, at this value of R the canonical suppression of single-strange particles is indeed negligible. Thus, for $S = \pm 1$ particles the model (a) is almost equivalent to the GC treatment of strangeness conservation.

From the above discussion it is clear, that at the SPS the strangeness suppression due to canonical effects alone is not sufficient to describe the 4π data. In models (b) and (c) we have included an additional suppression of strange particle phase-space by introducing either the γ_S factor or the strangeness correlation volume. From the comparisons of the model with data shown in Fig. 2 one sees that both these models describe the SPS data quite well with similar values for T and μ_B (see Table I). The particle yields calculated in these models are summarized in Table II.

In order to quantify the change of thermal parameters with collision energy we also analyze data on particle production

TABLE II. Particle yields (4π integrated) in minimum bias *p*-*p* collisions at $\sqrt{s} = 17.3$ GeV from Refs. [25–28] and fit results from models (b) and (c).

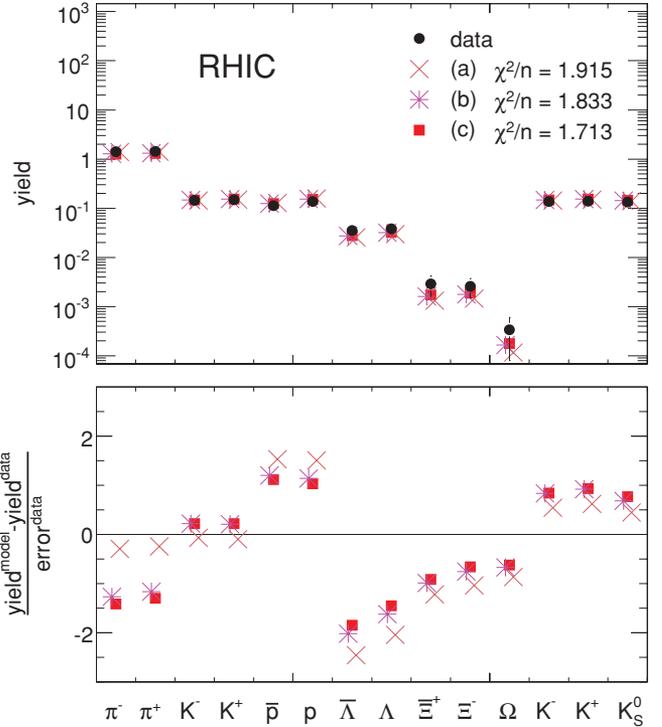| Particle	Data	Fit (b)	Fit (c)
π^+	3.02 ± 0.15	3.01	3.01
π^-	2.36 ± 0.11	2.32	2.32
K^+	0.258 ± 0.055	0.268	0.270
K^-	0.160 ± 0.050	0.162	0.166
Λ	0.116 ± 0.011	0.119	0.120
$\bar{\Lambda}$	0.0137 ± 0.0007	0.0135	0.0134
K_S^0	0.18 ± 0.04	0.21	0.20
$\bar{p}$	0.0400 ± 0.0068	0.0464	0.0469

FIG. 3. (Color online) Midrapidity particle densities from *p*-*p* collisions at $\sqrt{s} = 200$ GeV from STAR [29,30] together with the model fits as in Fig. 2. The lower panel shows the χ^2-deviations of the model fits to data.

in *p*-*p* collisions at RHIC. Here, particle yields are only available around midrapidity [29,30]. However, contrary to SPS the yields of multistrange particles are also available allowing for more complete verification of thermal models used in the analysis at SPS. In Fig. 3 we compare the predictions of models (a), (b), and (c) with RHIC data. The particle yields used in this study and the yields calculated in the model are summarized in Table III. The resulting thermal parameters for models (b) and (c) are indicated in Table I. The STAR Collaboration published the statistical model analysis of *p*-*p* data restricted to yields of pions, kaons, and (anti)protons [29], using the undersaturation factor γ_S. Our analysis agrees with Ref. [29] if one uses the same data set. With the hyperons included in the analysis there is a slight increase in γ_S whereas the temperature and chemical potential stay almost the same.

From Fig. 3 and Table III one sees that all models describe RHIC data within two standard deviations. At RHIC the pion yield at midrapidity is by more than a factor of two lower than at SPS in 4π, resulting in the corresponding decrease of the volume parameter and stronger strangeness suppression. In the context of the considered RHIC data it is rather difficult to definitely verify, that there is a need for an extra strangeness suppression effects going beyond the one already included in the standard canonical model. This is clear from Table III where in model (b) the $\gamma_S \sim 1$ and in model (c) the cluster and the system volume coincides within errors.

In the following, we concentrate on the statistical model predictions for particle yields at LHC energy. In view of the

TABLE III. Particle midrapidity densities in minimum bias p-p collisions at $\sqrt{s} = 200$ GeV from Refs. [29,30] and fit results from models (b) and (c). Charged kaons were analyzed with two different techniques and both measurements are included in the fits.

Particle	Data	Fit (b)	Fit (c)
π^-	1.42 ± 0.11	1.28	1.26
π^+	1.44 ± 0.11	1.31	1.30
K^-	0.145 ± 0.013	0.148	0.148
K^+	0.150 ± 0.014	0.153	0.153
$\bar{p}$	0.113 ± 0.010	0.125	0.124
p	0.138 ± 0.013	0.153	0.151
$\bar{\Lambda}$	0.0351 ± 0.0039	0.0272	0.0279
Λ	0.0385 ± 0.0042	0.0317	0.0324
$\bar{\Xi}^+$	0.0029 ± 0.0013	0.0016	0.0017
Ξ^-	0.0026 ± 0.0011	0.0018	0.0019
$\Omega^- + \bar{\Omega}^+$	0.00034 ± 0.00026	0.00017	0.00018
K^- (kink)	0.137 ± 0.013	0.148	0.148
K^+ (kink)	0.140 ± 0.014	0.153	0.153
K_S^0	0.134 ± 0.014	0.144	0.145

results obtained at SPS and at RHIC, we limit our attention only to model (c) and its extrapolation to LHC energies.

IV. PARTICLE RATIOS IN p-p COLLISIONS AT LHC

The extrapolation of particle ratios to LHC energy requires estimates of the temperature, the chemical potential, and the cluster volume. From our analysis made at SPS and RHIC energies (summarized in Tables II and III) it is clear that no variation in the temperature is expected between SPS, RHIC, and LHC. A strong decrease of μ_B from SPS to RHIC seen in our results, together with the previous systematics on the beam energy dependence of μ_B obtained from freeze-out conditions in heavy-ion collisions [16], indicate that all chemical potentials should be very small at LHC. In the following, we use $T \simeq 170$ MeV and $\mu_B \simeq 1$ MeV from Ref. [17] as appropriate thermal parameters at LHC.
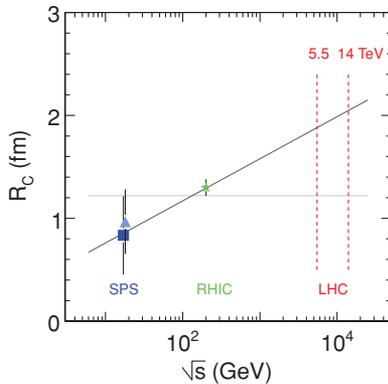

FIG. 4. (Color online) Cluster radius R_C as a function of energy, extracted from the present analysis of SPS data (triangle) and RHIC data (star). Also shown is the previous analysis [13] of a smaller set of SPS data (square). The lines illustrate possible evolutions toward LHC energies as discussed in the text.

TABLE IV. Particle ratios for thermal conditions expected at LHC. The extreme values of the cluster size R_C span the band of expected numerical values. The grand canonical values in the last column are included for comparison, for a detailed discussion see Ref. [17].

Ratio	$R_C = 1$ fm	$R_C = 2$ fm	Grand canon.
p/π^-	0.0970	0.0920	0.0914
K^+/π^+	0.0871	0.169	0.180
K^-/π^-	0.0870	0.169	0.179
Λ/p	0.179	0.436	0.473
Ξ^-/Λ	0.0397	0.130	0.160
Ω^-/Ξ^-	0.0358	0.131	0.186
Ω^-/K^-	2.83×10^{-4}	4.06×10^{-3}	7.19×10^{-3}
Ω^-/π^-	2.46×10^{-5}	6.85×10^{-4}	1.29×10^{-3}

The only parameter that remains largely uncertain in extrapolating from SPS and RHIC to LHC is the size of the cluster described by the radius R_C quantifying the strangeness suppression. Two limiting cases for the extrapolation are used in the model and shown in Fig. 4:

(i) Saturation of the correlation radius at $R_C \simeq 1$ fm,
(ii) An increase of R_C from SPS and RHIC to LHC.

In the first case, namely of an energy independent R_C and at fixed temperature, the strangeness suppression at LHC will be the same as at RHIC. Consequently, different ratios of strange to nonstrange particle yields will be modified only through the variation in μ_B which can be quantified by the $\exp(\pm\mu_B/T)$ Boltzmann factor.

In the second case, of increasing R_C with $\sqrt{s}$ and at fixed temperature, the strangeness suppression at LHC will be weaker than at RHIC leading possibly to an equilibrated, canonical system without any additional suppression. Scenario (ii) is naturally expected if R_C coincides with the size of the system. In this case, the R_C should scale with the number of pions in the final state. This scenario could be verified experimentally at LHC by comparing the strange/nonstrange particle ratios in p-p collisions for events with different pion multiplicities. If valid, then for sufficiently high pion multiplicity at large $\sqrt{s}$ the strangeness production normalized to pion multiplicities could converge to the results expected in heavy-ion collisions. However, in view of the known data in elementary and heavy-ion collisions this is a very unlikely scenario. An increase of R_C with $\sqrt{s}$ shown in Fig. 4 is expected to saturate at higher energies, or to be much weaker than linear. Due to a lack of data the actual dependence of $R_C = R_C(\sqrt{s})$ is not known. The LHC data are essential to understand this behavior of strangeness suppression and its energy dependence.

In Fig. 5 and Table IV we summarize predictions of the statistical model for different particle ratios at LHC energy. We compare the results obtained under the GC description of strangeness conservation with the canonical model (c) with the parameter R_C describing the size where strangeness is conserved exactly in a system. Changing the value of R_C from 1 to 2 fermi implies a dramatic change in the ratios involving multistrange particles (see Fig. 5). It is interesting to note,
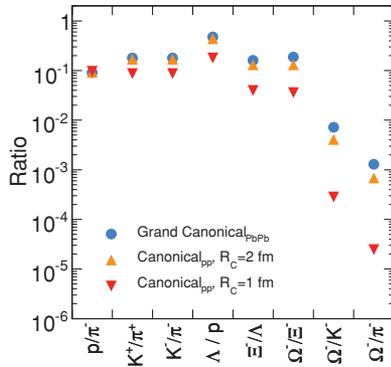
FIG. 5. (Color online) Predictions for various particle ratios using different values for R_C.
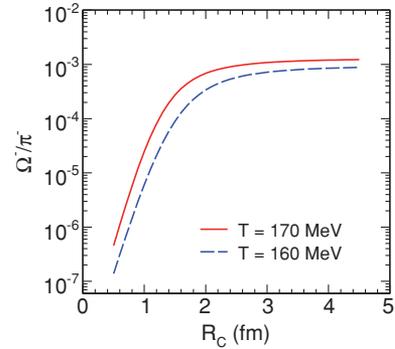

FIG. 6. (Color online) Particle ratio Ω/π as a function of R_C for two assumed temperatures, $T = 160$ MeV (dashed) and $T = 170$ MeV (full line).

that even for $R_C = 2$ fm the Ξ/π and Ω/π ratios differ from their GC values, whereas K/π and Λ/π are already well consistent with GC results. The Ω/π is particularly sensitive to changes in R_C while its temperature dependence is rather moderate as seen in Figs. 5 and 6. Consequently, the Ω/π ratio is an excellent observable to probe strangeness suppression and correlated strangeness production in *p-p* collisions at LHC energy.

V. SUMMARY

We have analyzed, within the statistical model, recent data on particle production in *p-p* collisions at SPS and RHIC energies. The models were formulated in the canonical ensemble with respect to strangeness conservation and extended to implement extra strangeness suppression either by the off-equilibrium γ_S or by a strangeness correlation volume. We have shown, that at RHIC the midrapidity data can be successfully described by the canonical statistical model with and without any extra suppression effects. On the other hand, the full phase-space SPS data require additional strangeness

suppression that is beyond the conventional canonical model. Extrapolating all relevant thermal parameters from SPS and RHIC up to LHC energy we have made predictions for particle yields in *p-p* collisions at $\sqrt{s} = 14$ TeV. We have discussed the role and the influence of strangeness correlation volume on particle production in *p-p* collisions. We have argued that the Ω/π ratio is an excellent probe of strangeness correlations and/or strangeness suppression mechanism in *p-p* collisions. We have indicated that comparing strangeness production at the LHC in events with different pion multiplicities can provide a deep insights into our understanding of strangeness suppression from *A-A* to *p-p* collisions.

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