PHYSICAL REVIEW C **79**, 014603 (2009)

Feasibility of extracting a Σ^- admixture probability in the neutron-rich $^{10}_{\Lambda}$Li hypernucleus

T. Harada,[1,*] A. Umeya,[1] and Y. Hirabayashi[2]

[1]*Research Center for Physics and Mathematics, Osaka Electro-Communication University, Neyagawa, Osaka 572-8530, Japan*
[2]*Information Initiative Center, Hokkaido University, Sapporo 060-0811, Japan*

(Received 30 October 2008; published 9 January 2009)

We theoretically examine production of the neutron-rich $^{10}_{\Lambda}$Li hypernucleus by a double-charge exchange (π^-, K^+) reaction on a ^{10}B target with distorted-wave impulse approximation calculations. We calculate the inclusive spectrum at the incident momentum 1.20 GeV/c by a one-step mechanism $\pi^- p \to K^+\Sigma^-$ via Σ^- doorways caused by a $\Sigma^- p \leftrightarrow \Lambda n$ coupling. The resultant spectrum can explain the magnitude of the recent experimental data, so that the Σ^- admixture probability in $^{10}_{\Lambda}$Li is found to be of the order of 10^{-1}%. The (π^-, K^+) reaction provides the ability to extract properties of wave functions with Λ-Σ coupling effects in neutron-rich nuclei, as well as the reaction mechanism.

DOI: 10.1103/PhysRevC.79.014603 PACS number(s): 21.80.+a, 24.10.Eq, 25.80.Hp, 27.20.+n

It has been discussed that a study of strangeness in nuclei would provide new information on nuclear physics and astrophysics [1]. The presence of hyperons in high-density nuclear medium significantly affects the maximal mass of neutron stars and compact stars, because it makes the Equation of State (EoS) soften [2]. The negatively charged Σ^- hyperon was expected to play an essential role in the description of neutron stars, whereas the baryon fraction is found to depend on properties of hypernuclear potentials in neutron stars. On the other hand, several theoretical studies [3] have suggested that a repulsive component in Σ^--nucleus potentials is needed to reproduce the observed spectra of (π^-, K^+) reactions on nuclear targets [4], and also the strong level-shifts and widths of the Σ^- atomic x-ray data. This repulsion originates from the ΣN $T = 3/2$, $^3S_1$ channel that corresponds to a quark Pauli-forbidden state in the baryon-baryon system [5]. However, because a strong $\Sigma^- p \to \Lambda n$ conversion occurs at the nuclear surface, it is difficult to extract the nature of the Σ^- hyperon in nuclear medium from such experimental data on nuclear targets.

One of the most promising subjects for the examination of the hypernuclear potential in a neutron-excess environment is a study of neutron-rich Λ hypernuclei [6]. The Λ hyperon in nuclei is known to act as a nuclear "glue" and can often make the system bound even if a core-nucleus is unbound, e.g., $^{6}_{\Lambda}$He. In addition, it is suggested that in s-shell Λ hypernuclei an attractive mechanism appears because of the Λ-Σ coupling that is related to a three-body ΛNN force [7], and their Σ-mixing probabilities are 1–2%, as discussed in few-body calculations [8]. This situation is found to be more coherently enhanced in the neutron-excess environment [9]. Therefore, we believe that there are a lot of exotic neutron-rich Λ hypernuclei beyond the neutron drip line.

The experimental attempts to produce neutron-rich Λ hypernuclei were carried out by reactions based on a double-charge exchange (DCX) mechanism, $(K^-_{\text{Stopped}}, \pi^+)$ [10,11] and (π^-, K^+) [12]. Further experiments in the nuclear (π^-, K^+) reactions are also planned at J-PARC [13]. The production of the neutron-rich Λ hypernuclei by the DCX reaction (π^-, K^+) would conventionally proceed by a two-step mechanism of the meson charge-exchange, $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+\Lambda$, as shown in Fig. 1(a), or $\pi^- p \to K^0\Lambda$ followed by $K^0 p \to K^+ n$. Another exotic mechanism is a one-step process, $\pi^- p \to K^+\Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling in Λ hypernuclei, as shown in Fig. 1(b). Tretyakova and Lanskoy [14] theoretically found that the two-step mechanism in the ^{10}B(π^-, K^+) reaction is more dominant compared to the one-step one, where the Σ^- admixture probability is as small as on the order of 10^{-2}%. Thus they claimed that the magnitude of the cross section of the $^{10}_{\Lambda}$Li bound state in the (π^-, K^+) reaction is as large as 38–67 nb/sr at the incident momentum $p_\pi = 1.05$ GeV/c (0°), where the cross section of the conventional (π^+, K^+) reaction on nuclear targets is at its maximum [15].

Recently, Saha *et al.* [12] have performed the first measurement of a significant yield for the $^{10}_{\Lambda}$Li hypernucleus in (π^-, K^+) reactions on a ^{10}B target, whereas no clear peak has been observed with the lack of the experimental statistics. The data show that the absolute cross section for $^{10}_{\Lambda}$Li at 1.20 GeV/c ($d\sigma/d\Omega \sim 11$ nb/sr) is twice larger than that at 1.05 GeV/c ($d\sigma/d\Omega \sim 6$ nb/sr). This incident-momentum dependence of $d\sigma/d\Omega$ exhibits a trend in the opposite direction for the theoretical prediction of Ref. [14]. This might mean that the one-step mechanism is favored rather than the two-step mechanism, as pointed out in Ref. [12].

In this article, we theoretically investigate production of the neutron-rich $^{10}_{\Lambda}$Li hypernucleus by the DCX (π^-, K^+) reaction on a ^{10}B target at 1.20 GeV/c, within a distorted-wave impulse approximation (DWIA). To understand the mechanism of this reaction, we focus on the Λ spectrum populated by the one-step mechanism, $\pi^- p \to K^+\Sigma^-$ via Σ^- doorways due to the $\Sigma^- p \leftrightarrow \Lambda n$ coupling. This is the first attempt to extract the probability of the Σ^- admixture in a neutron-rich Λ hypernucleus from available experimental data phenomenologically. We also discuss a small contribution of the two-step processes in the (π^-, K^+) reactions within the eikonal approximation.

---

[*] FAX: +81-72-825-4689; Telephone: +81-72-824-1131 (ext. 4584); harada@isc.osakac.ac.jp

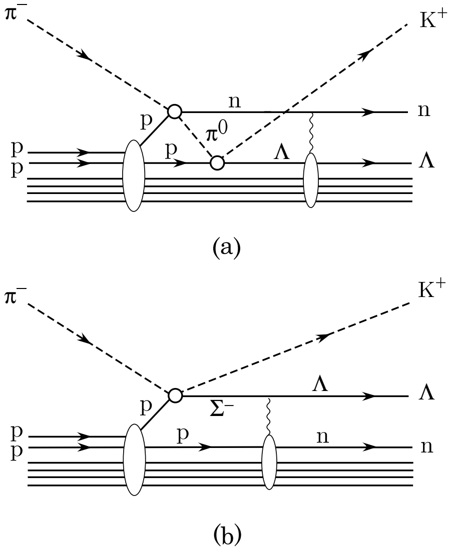

FIG. 1. Diagrams for DCX nuclear (π^-, K^+) reactions, leading to production of Λ hypernuclear states: (a) a two-step mechanism, $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+\Lambda$, and (b) a one-step mechanism, $\pi^- p \to K^+\Sigma^-$ via Σ^- doorways caused by the $\Sigma^- p \leftrightarrow \Lambda n$ coupling.

Now let us consider the DCX (π^-, K^+) reaction on the ^{10}B target within the DWIA. To fully describe the one-step process via Σ^- doorways, as shown in Fig. 1(b), we perform a Λ-Σ coupled-channel calculation [16], evaluating the production cross section of Λ hypernuclear states in $^{10}_{\Lambda}$Li. We assume a two-channel coupled wave function for simplicity, which is given by

$$|^{10}_{\Lambda}\mathrm{Li}\rangle = \varphi_\Lambda(\boldsymbol{r})|^9\mathrm{Li}\otimes\Lambda\rangle + \varphi_\Sigma(\boldsymbol{r})|^9\mathrm{Be}^*\otimes\Sigma^-\rangle, \tag{1}$$

where $\langle\varphi_\Lambda|\varphi_\Lambda\rangle + \langle\varphi_\Sigma|\varphi_\Sigma\rangle = 1$ and $\boldsymbol{r}$ denotes a relative coordinate between the core-nucleus and the hyperon. The probability of the Σ^- admixture in the Λ hypernucleus can be obtained by $P_{\Sigma^-} = \langle\varphi_\Sigma|\varphi_\Sigma\rangle$. It should be noticed that the core-excited state (^{9}Be*) in the Σ^- channel is assumed to be an effective state that can be fully coupled with the ^{9}Li core state via the Λ-Σ coupling, rather than the ^{9}Be$(\frac{3}{2}^-;\frac{1}{2})$ ground state. Thus, we assume $\Delta M = 80$ MeV effectively for the threshold-energy difference between ^{9}Li + Λ and ^{9}Be* + Σ^- channels. The mixed Σ^--hyperon in $^{10}_{\Lambda}$Li is regarded as a deeply bound particle in the nucleus, where the $\pi^- p \to K^+\Sigma^-$ transition takes place under an energy-off-shell condition.

To calculate the nuclear (π^-, K^+) spectrum, we employ the Green's function method [17], which is one of the most powerful treatments in a calculation of the spectrum that includes not only bound states but also continuum states with an absorptive potential for spreading components. The complete Green's function $\boldsymbol{G}$ describes all information concerning $(^9\mathrm{Li}\otimes\Lambda) + (^9\mathrm{Be}^*\otimes\Sigma^-)$ coupled-channel dynamics. We obtain it by solving the following equation with the hyperon-nucleus potential $\boldsymbol{U}$ numerically:

$$\boldsymbol{G} = \boldsymbol{G}^{(0)} + \boldsymbol{G}^{(0)}\boldsymbol{U}\boldsymbol{G}, \tag{2}$$

where

$$\boldsymbol{G} = \begin{pmatrix} G_{\Lambda\Lambda} & G_{\Lambda\Sigma} \\ G_{\Sigma\Lambda} & G_{\Sigma\Sigma} \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} U_{\Lambda\Lambda} & U_{\Lambda\Sigma} \\ U_{\Sigma\Lambda} & U_{\Sigma\Sigma} \end{pmatrix}, \tag{3}$$

and $\boldsymbol{G}^{(0)}$ is a free Green's function. By the complete Green's function, the inclusive K^+ double-differential laboratory cross section of Λ production on a nuclear target with a spin J_i (its z-component M_i) [16] by the one-step mechanism, $\pi^- p \to K^+\Sigma^-$ via Σ^- doorways, is given by

$$\frac{d^2\sigma}{d\Omega_K dE_K} = \beta\frac{1}{[J_i]}\sum_{M_i}(-)\frac{1}{\pi}\mathrm{Im}\sum_{\alpha\alpha'}\left\langle F_\Sigma^{\alpha\dagger}G_{\Sigma\Sigma}^{\alpha\alpha'}F_\Sigma^{\alpha'}\right\rangle, \tag{4}$$

where a production amplitude

$$F_\Sigma^\alpha = \bar{f}_{\pi^- p\to K^+\Sigma^-}\chi_{\boldsymbol{p}_K}^{(-)*}\chi_{\boldsymbol{p}_\pi}^{(+)}\langle\alpha|\hat{\psi}_p|i\rangle, \tag{5}$$

$[J_i] = 2J_i + 1$, and the kinematical factor β expresses the translation from the two-body π^--p laboratory system to the π^--^{10}B laboratory system. $\bar{f}_{\pi^- p\to K^+\Sigma^-}$ is a Fermi-averaged amplitude for the $\pi^- p \to K^+\Sigma^-$ reaction in nuclear medium, and $\chi_{\boldsymbol{p}_K}^{(-)}$ and $\chi_{\boldsymbol{p}_\pi}^{(+)}$ are the distorted waves for outgoing K^+ and incoming π^- mesons, respectively, taking into account the recoil effects [18]. $\langle\alpha|\hat{\psi}_p|i\rangle$ is a hole-state wave function for a struck proton in the target, where α denotes the complete set of eigenstates for the system. The inclusive Λ spectrum in Eq. (4) can be decomposed into different physical processes [16,17] by using the identity

$$\begin{aligned}\mathrm{Im}(F_\Sigma^\dagger G_{\Sigma\Sigma}F_\Sigma) = {} & F_\Sigma^\dagger\Omega^{(-)\dagger}\big(\mathrm{Im}G_\Lambda^{(0)}\big)\Omega^{(-)}F_\Sigma \\ & + F_\Sigma^\dagger G_{\Sigma\Lambda}^\dagger(\mathrm{Im}U_\Lambda)G_{\Lambda\Sigma}F_\Sigma \\ & + F_\Sigma^\dagger G_{\Sigma\Sigma}^\dagger(\mathrm{Im}U_\Sigma)G_{\Sigma\Sigma}F_\Sigma,\end{aligned} \tag{6}$$

where $\Omega^{(-)}$ is the Möller wave operator.

The diagonal (optical) potentials for $\boldsymbol{U}$ in Eq. (3) are given by the Woods-Saxon (WS) form:

$$U_Y(r) = (V_Y + iW_Y g(E_\Lambda))f(r) \tag{7}$$

for $Y = \Lambda$ or Σ^-, where $f(r) = [1 + \exp((r-R)/a)]^{-1}$ with $a = 0.6$ fm, $r_0 = 1.088 + 0.395A^{-2/3}$ fm, and $R = r_0(A-1)^{1/3} = 2.42$ fm for the mass number $A = 10$ [19]. Here we used $V_\Lambda = -30$ MeV for the ^{9}Li $\otimes$ Λ channel and assumed $V_\Sigma = 0$ MeV to describe the effective Σ state of the ^{9}Be* $\otimes$ Σ^- channel. The spreading imaginary potential, ImU_Y, can describe complicated excited states for $^{10}_{\Lambda}$Li; $g(E_\Lambda)$ is an energy-dependent function that linearly increases from 0 at $E_\Lambda = 0$ MeV to 1 at $E_\Lambda = 60$ MeV, as often used in nuclear optical models. The strength parameter W_Y should be adjusted appropriately to reproduce the data. The coupling Λ-Σ potential $U_{\Sigma\Lambda}$ in off-diagonal parts for $\boldsymbol{U}$ is written by

$$U_{\Sigma\Lambda}(r) = \langle^9\mathrm{Be}^*\otimes\Sigma^-|\frac{1}{\sqrt{3}}\sum_j v_{\Sigma\Lambda}(\boldsymbol{r}_j,\boldsymbol{r})\boldsymbol{\tau}_j\cdot\boldsymbol{\phi}|^9\mathrm{Li}\otimes\Lambda\rangle, \tag{8}$$

where $v_{\Sigma\Lambda}(\boldsymbol{r}_j,\boldsymbol{r})$ is a two-body ΛN-ΣN potential including the spin-spin interaction, $\boldsymbol{\tau}_j$ denotes the jth nucleon isospin operator, and $\boldsymbol{\phi}$ is defined as $|\Sigma\rangle = \boldsymbol{\phi}|\Lambda\rangle$ in isospin space [20]. Here we assumed $U_{\Sigma\Lambda}(r) = V_{\Sigma\Lambda}f(r)$ in a real potential for simplicity, where $V_{\Sigma\Lambda}$ is an effective strength parameter.

We will attempt to determine the values of W_Σ and $V_{\Sigma\Lambda}$ phenomenologically by fitting to a spectral shape of the experimental data.

For the ^{10}B($3^+;0$) target nucleus, we use single-particle wave functions for a proton, which are calculated by a WS potential [21] with $V_0^N = -61.36$ MeV fitting to the charge radius of 2.45 fm [22]. Because of the large momentum transfer $q \simeq$ 270–370 MeV/c in the (π^-, K^+) reaction, we simplify the computational procedure for the distorted waves, $\chi_{\boldsymbol{p}_K}^{(-)}$ and $\chi_{\boldsymbol{p}_\pi}^{(+)}$, with the help of the eikonal approximation. To reduce ambiguities in the distorted waves, we adopt the same parameters used in calculations for the Λ and Σ^- quasifree spectra in nuclear ($\pi^\mp$, K^+) reactions [18]. Here we used total cross sections of $\sigma_\pi = 32$ mb for $\pi^- N$ scattering and $\sigma_K = 12$ mb for K^+N, and we used $\alpha_\pi = \alpha_K = 0$, as the distortion parameters [18]. The Fermi-averaged amplitude $\bar{f}_{\pi^- p \to K^+\Sigma^-}$ is obtained by the optimal Fermi-averaging for the $\pi^- p \to K^+\Sigma^-$ t-matrix [18]; we used 20μb/sr as the laboratory cross section of $d\sigma/d\Omega = |\bar{f}_{\pi^- p \to K^+\Sigma^-}|^2$.

Now let us examine the dependence of the spectral shape on two important parameters, W_Σ and $V_{\Sigma\Lambda}$, by comparing the calculated inclusive Λ spectrum for $^{10}_{\Lambda}$Li with the data of ^{10}B(π^-, K^+) experiments at KEK [4]. The cross sections of the data are three orders of magnitude less than those for $^{10}_{\Lambda}$B in ^{10}B(π^+, K^+) reactions. In Fig. 2, we show the calculated spectra by the one-step mechanism at $p_\pi = 1.20$ GeV/c (6°) for the several values of $-W_\Sigma$ when we use $V_{\Sigma\Lambda} = 11$ MeV, which leads to the Σ^--mixing probability of $P_\Sigma = 0.57\%$ in the $^{10}_{\Lambda}$Li bound state. We have the peak of the bound state with a $[0p_{\frac{3}{2}}^{-1}s_{\frac{1}{2}}^{\Lambda}]_{2^-}$ configuration at $E_\Lambda \simeq -10.0$ MeV, and the peaks of the excited states with $[0p_{\frac{3}{2}}^{-1}p_{\frac{3}{2},\frac{1}{2}}^{\Lambda}]_{3^+,1^+}$ configurations at $E_\Lambda =$ 1–3 MeV. Because the non-spin-flip processes with the large momentum transfer q are known to dominate in the $\pi^- p \to K^+\Sigma^-$ reaction at 1.20 GeV/c, these spin-stretched states are mainly populated. We find that the value of $-W_\Sigma$ significantly affects a shape of the Λ spectrum for the continuum states that can be populated via $\Sigma^- p \to \Lambda n$ processes in ^{9}Be* together with the core-nucleus breakup; this Λ strength mainly arises from a term of $G_{\Sigma\Sigma}^{\dagger}(\mathrm{Im}U_\Sigma)G_{\Sigma\Sigma}$ in Eq. (6). We recognize that the calculated spectrum with $-W_\Sigma =$ 20–30 MeV can reproduce the shape of the data in the continuum region [12], and these values of $-W_\Sigma$ are consistent with the analysis of Σ^- production by the (π^-, K^+) reactions [18]. Obviously, the parameter W_Σ does not contribute to the spectrum of the bound state. It should be noticed that the contribution of the two-step processes in the continuum spectrum is rather small (see the dashed curve in Fig. 2).

On the other hand, the Λ-Σ coupling potential plays an essential role in the formation of the Λ bound state. In Fig. 3, we show the dependence of the cross section for the bound state in $^{10}_{\Lambda}$Li on the values of $V_{\Sigma\Lambda}$ when $-W_\Sigma =$ 20 MeV. We find that the calculated spectrum for the bound state is quite sensitive to $V_{\Sigma\Lambda}$; when we use $V_{\Sigma\Lambda} = 4$, 8, 10, 11, and 12 MeV, the probabilities of the Σ^- admixture in the 2^- bound state are $P_{\Sigma^-} = 0.075$, 0.30, 0.47, 0.57, and 0.68%, respectively. The positions of the peaks for $^{10}_{\Lambda}$Li are slightly shifted downward by $\Delta E_\Lambda \simeq -\langle U_{\Sigma\Lambda}\rangle^2/\Delta M \simeq -\Delta M \cdot P_{\Sigma^-}$, e.g., -456 keV for $V_{\Sigma\Lambda} = 11$ MeV, which is about 4–5 times larger than that of $^{7}_{\Lambda}$Li [23]. For the order of $V_{\Sigma\Lambda} =$ 10–12 MeV ($P_{\Sigma^-} =$ 0.47–0.68%), the calculated spectra can fairly reproduce the data, whereas it is not appropriate for a detailed study of the structure of $^{10}_{\Lambda}$Li because of the simple single-particle picture we adopted here. Such a Σ^- admixture seems to be consistent with recent microscopic calculations [7,8,24]. This consistency of $V_{\Sigma\Lambda}$ considerably enhances the reliability of our calculations. Consequently, the calculated spectrum by the one-step mechanism explains the ^{10}B(π^-, K^+) data. This fact implies that the one-step mechanism dominates in the

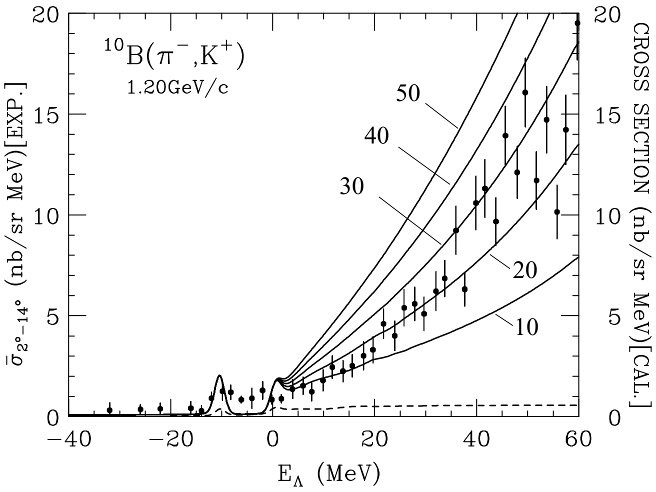

FIG. 2. Calculated inclusive Λ spectra obtained by the one-step mechanism in the ^{10}B(π^-, K^+) reaction at $p_\pi = 1.20$ GeV/c (6°), together with the experimental data [12]. The solid curves denote the K^+ spectra by $-W_\Sigma = 10, 20, 30, 40$, and 50 MeV when $V_{\Sigma\Lambda} = 11$ MeV ($P_\Sigma = 0.57\%$), with a detector resolution of 2.5 MeV FWHM. The dashed curve denotes the inclusive Λ spectrum obtained by the two-step mechanism.

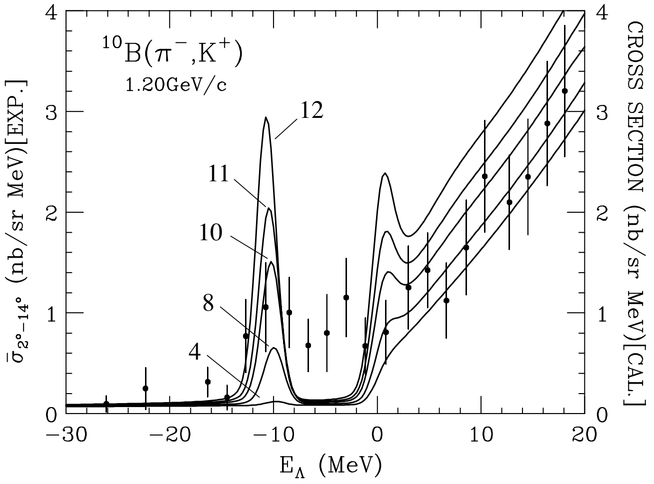

FIG. 3. Calculated inclusive Λ spectra obtained by the one-step mechanism near the Λ threshold in the ^{10}B(π^-, K^+) reaction at 1.20 GeV/c (6°), by changing $V_{\Sigma\Lambda}$ for the Λ-Σ coupling potential. The experimental data are taken from Ref. [12]. The solid curves denote $V_{\Sigma\Lambda} = 4$, 8, 10, 11, and 12 MeV when $-W_\Sigma = 20$ MeV, with a detector resolution of 2.5 MeV FWHM.

(π^-, K^+) reaction, and our calculations provide the capability to extract a production mechanism from the data of this reaction. Some discrepancy between the results and the data in the bound-state region surrounding $E_\Lambda \simeq -5$ MeV might be improved by a sophisticated shell-model calculation with configuration mixing [24].

The early theoretical prediction by Tretyakova and Lanskoy [14] differs from the present result. They have shown that the Σ^--mixing probabilities on p-shell nuclei are on the order of 10^{-3}–10^{-2}%, which is smaller than ours by one or more orders of magnitude, within Hartree-Fock single-particle calculations based on two-body ΛN-ΣN effective interactions [7]. The Λ-Σ coupling in the Hartree-Fock states seems to be hindered by the lack of active configurations. For the two-step mechanism, $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+\Lambda$ or $\pi^- p \to K^0\Lambda$ followed by $K^0 p \to K^+ n$, in the DCX ${}^{10}\mathrm{B}(\pi^-, K^+)$ reaction, we roughly estimate the integrated laboratory cross sections of $d\sigma/d\Omega$ for the ${}^{10}_{\Lambda}\mathrm{Li}$ bound state with a harmonic oscillator model in the eikonal approximation [25]. In Table I, we show that the calculated value of $d\sigma/d\Omega$ at 1.20 GeV/c (6°) by the two-step mechanism is rather small (1–2 nb/sr) compared to that obtained by the one-step mechanism. (See also Fig. 2.) The incident-momentum dependence of $d\sigma/d\Omega$ in the data is similar to that in the one-step mechanism. Therefore, we believe that the one-step mechanism is dominant in the (π^-, K^+) reaction at 1.20 GeV/c. The ${}^{10}\mathrm{B}(\pi^-, K^+)$ experiment at KEK [12] might be interpreted as a measurement of the Σ^- admixture in the Λ hypernucleus. The Σ^- admixture gives a key for understanding the EoS and neutron stars [9].

In conclusion, the calculated spectrum of the ${}^{10}_{\Lambda}\mathrm{Li}$ hypernucleus by the one-step mechanism via Σ^- doorways fully explains the data of the DCX ${}^{10}\mathrm{B}(\pi^-, K^+)$ reaction at 1.20 GeV/c, rather than the two-step mechanism. The result shows that the Σ^- admixture probability in the ${}^{10}_{\Lambda}\mathrm{Li}$ bound state is on the order of 10^{-1}%. The sensitivity to the potential parameters implies that the nuclear (π^-, K^+) reactions with much less background experimentally provide a high ability for the theoretical analysis of precise wave functions in the neutron-rich Λ hypernuclei. The detailed analysis based on microscopic nuclear calculations is required for forthcoming J-PARC experiments [13]. This investigation is in progress.

TABLE I. Calculated results of the integrated lab cross sections of $d\sigma/d\Omega$ for the ${}^{10}_{\Lambda}\mathrm{Li}$ 2^- bound state with two-step and one-step processes in ${}^{10}\mathrm{B}(\pi^-, K^+)$ reactions at 6° compared with the data [12]. The value in the bracket is a lower limit one with Λ quasifree corrections.

p_π (GeV/c)	Two-step[a] (nb/sr)	One-step[b] (nb/sr)	Exp. [12] (nb/sr)
1.05	∼1.6	2.4	5.8 ± 2.2[c]
1.20	∼1.2	5.4	11.3 ± 1.9[c] (9.6 ± 2.0)

[a] Sum of the cross sections via $\pi^- p \to \pi^0 n$ followed by $\pi^0 p \to K^+\Lambda$ and $\pi^- p \to K^0\Lambda$ followed by $K^0 p \to K^+ n$, by a simple harmonic oscillator model.

[b] $P_{\Sigma^-} = 0.57\%$ ($V_{\Sigma\Lambda} = 11$ MeV) is assumed.

[c] All the events for $-20\ \mathrm{MeV} \leqslant E_\Lambda \leqslant 0$ MeV.

ACKNOWLEDGMENTS

The authors are obliged to T. Fukuda, Y. Akaishi, and H. Nemura for many discussions and to A. Sakaguchi and H. Noumi for useful comments. They are pleased to acknowledge D. J. Millener for valuable discussions and comments. This work was supported by Grants-in-Aid for Scientific Research on Priority Areas (Nos. 17070002 and 20028010).

[1] For example, Special Issue on Recent Advances in Strangeness Nuclear Physics, edited by A. Gal and R. S. Hayano, Nucl. Phys. **A804**, 1 (2008).
[2] S. Balberg and A. Gal, Nucl. Phys. **A625**, 435 (1997); M. Baldo, G. F. Burgio, and H.-J. Schulze, Phys. Rev. C **58**, 3688 (1998), and references therein.
[3] E. Friedman and A. Gal, Phys. Rep. **452**, 89 (2007).
[4] P. K. Saha *et al.*, Phys. Rev. C **70**, 044613 (2004).
[5] Y. Fujiwara, Y. Suzuki, and C. Nakamoto, Prog. Part. Nucl. Phys. **58**, 439 (2007), and references therein.
[6] L. Maijling, Nucl. Phys. **A585**, 211c (1995).
[7] Y. Akaishi, T. Harada, S. Shinmura, and Khin Swe Myint, Phys. Rev. Lett. **84**, 3539 (2000).
[8] H. Nemura and Y. Akaishi, Nucl. Phys. **A738**, 254 (2004).
[9] Khin Swe Myint, T. Harada, S. Shinmura, and Y. Akaishi, Few-Body Syst. Suppl. **12**, 383 (2000); S. Shinmura, Khin Swe Myint, T. Harada, and Y. Akaishi, J. Phys. G **28**, 1 (2002).
[10] K. Kubota *et al.*, Nucl. Phys. **A602**, 327 (1996).
[11] M. Agnello *et al.*, Phys. Lett. **B640**, 145 (2006).
[12] P. K. Saha *et al.*, Phys. Rev. Lett. **94**, 052502 (2005).
[13] A. Sakaguchi *et al.*, *Production of Neutron-Rich Λ-Hypernuclei with the Double Charge-Exchange Reactions*, Proposal for Nuclear and Particle Physics Experiments at the J-PARC (2007), http://j-parc.jp/NuclPart/Proposal_e.html.
[14] T. Yu. Tretyakova and D. E. Lanskoy, Phys. At. Nucl. **66**, 1651 (2003); Nucl. Phys. **A691**, 51c (2001).
[15] H. Bandō, T. Motoba, and J. Žofka, Int. J. Mod. Phys. A **5**, 4021 (1990).
[16] T. Harada, Phys. Rev. Lett. **81**, 5287 (1998); Nucl. Phys. **A672**, 181 (2000).
[17] O. Morimatsu and K. Yazaki, Prog. Part. Nucl. Phys. **33**, 679 (1994), and references therein.
[18] T. Harada and Y. Hirabayashi, Nucl. Phys. **A744**, 323 (2004); **A759**, 143 (2005).
[19] D. J. Millener, C. B. Dover, and A. Gal, Phys. Rev. C **38**, 2700 (1988).
[20] N. Auerbach, Phys. Rev. C **35**, 1798 (1987).
[21] A. Bohr and M. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 238.
[22] H. de Vries, C. W. de Jager, and C. de Vries, At. Data Nucl. Data Tables **36**, 495 (1987).
[23] D. J. Millener, Lect. Notes Phys. **724**, 31 (2007); D. J. Millener, Nucl. Phys. **A804**, 84 (2008).
[24] A. Umeya and T. Harada, arXiv:0810.4591 [nucl-th].
[25] R. E. Chrien, C. B. Dover, and A. Gal, Czech. J. Phys. **42**, 1089 (1992).