Systematic studies of properties of nuclei by parity violating electron scattering

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Two investigations by the parity violating electron scattering are performed theoretically: one is to investigate the properties of the surface for neutron-rich nuclei such as ¹²⁴Sn and ²⁰⁸Pb, and the other is to predict the parity violating asymmetries for some isotopic chains, such as Ba and Pb, which have been proposed for the atomic parity nonconservation experiments. For the first topic, the neutron and proton densities are taken to be the 2pF distributions. Results show that the parity violating asymmetries are very sensitive to the type of neutron density. It means that the parity violating electron scattering can be used to verify the type of neutron distribution in neutron-rich stable nuclei. For the second topic, the neutron and proton densities are obtained from the relativistic mean-field (RMF) theory. By combining the results for these two topics, we find that for various proton and neutron densities the amplitudes of the parity violating asymmetries correspond to the distances between the minima of the proton and neutron form factors. Our results can provide useful references for future experiments.

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parameter Fermi (2pF) model [24] $\rho_p(r) = \rho_0/\{1 + \exp[(r - c_p)/a_p]\}$, where c_p is the half-density radius, a_p is the

diffuseness parameter. It has been found that for medium and

heavy nuclei the diffuseness parameter is nearly a constant

I. INTRODUCTION

Very recently, we generalized the relativistic eikonal approximation [1] from the Coulomb electron scattering [2,3] to the parity violating electron scattering [4]. The properties of the parity violating asymmetries for three typical kinds of nuclei, Ca isotopes, N = 50 isotones, and N = Z doubly magic nuclei, have been analyzed [4]. It has been found that the parity violating asymmetry is very sensitive to the difference between the neutron and proton densities. So it is natural to expect that the parity violating electron scattering can be used to measure the neutron densities of nuclei with large difference between the neutron and proton densities. In this paper we will apply the parity violating electron scattering [5-17] to two interesting topics: one is to verify the type of the neutron density distribution for some neutron-rich stable nuclei, and the other is to predict the parity violating asymmetries for some isotopic chains, such as Ba and Pb, which have been proposed to perform the atomic parity nonconservation (PNC) experiments (see, e.g., Ref. [14]).

The size and shape of nuclei are important properties in nuclear physics. It is well known that finite nuclei have a misty surface. Many properties of nuclei are related to the surface. For example, in the nuclear shell model the spin-orbital coupling is a surface effect [18]. Its strength is proportional to the derivative of nuclear density. Therefore, the derivative of nuclear density at the surface needs detailed investigations. Up to now, the information about the size and shape of nuclei is mainly obtained from measurement of the charge (proton) densities of stable and long-lived nuclei by the high-energy electron scattering [19–34]. For many nuclei the proton densities can be fitted quite well by the two[24]. That is to say, at the surface the proton densities decay at nearly the same rate from nucleus to nucleus. It is natural to ask whether the surface thickness of the neutron density is the same as that of the proton density for a given nucleus, and whether the neutron densities have the same surface thickness for different nuclei. Due to the Coulomb interaction between protons, the β stability line departs from N = Z with the increase of the proton number. Therefore, the neutron and proton densities for a given neutron-rich stable nucleus should not be exactly equal. It has been assumed that the neutron excess will result in a neutron-skin at the nuclear surface. In the past, the thickness of the neutron-skin is generally defined as the difference between the rms radii of the neutron and proton densities: $\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$. However, as well known, the rms radius is only a gross property of the density distribution. The same rms radius can correspond to different densities. So how to measure the proton and neutron densities accurately is an important subject in nuclear physics. However, accurate neutron densities of nuclei are difficult to measure. Recently, the antiprotonic atom experiments [35,36] have shown that for many neutron-rich stable nuclei the half-density radius of neutron density is nearly equal to that of the proton density, and that the diffuseness parameter is larger than that of the proton density. This type of neutron density is called as the "neutron halo-type" distribution. Correspondingly, for the "neutron skin-type" distribution, the half-density radius of neutron density is larger than that of the proton density, and the surface thicknesses are same. From the points of the nuclear shell model, these two types of neutron densities may result in different asymptotic behavior of the nucleon wave functions. So it is necessary to find a more refined method to confirm the type of neutron density in neutron-rich

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stable nuclei. The parity violating electron scattering is a good candidate, which has been investigated extensively [5–17]. In early years, the parity violating electron scattering was investigated in the plane wave Born approximation (PWBA) (see, e.g., Ref. [8]). Then the Coulomb distortion effects were included by the phase shift method [10–14]. Very recently, we applied the relativistic eikonal approximation to the parity violating electron scattering [4].

Atomic parity nonconservation (PNC) experiments are of great importance in testing the standard model at low energy [37–39]. However, the precision is limited by two reasons: (1) the low accuracy of the atomic theory, and (2) the low accuracy of the weak charge density which is mainly determined by the neutron density. The uncertainty caused by the first reason can be eliminated by taking the ratios of the atomic PNC observables for a given isotopic chain [37,38]. Cs, Ba, Yb, and Pb isotopic chains have been suggested for this purpose [14]. Therefore, accurate measurement of the neutron densities for these isotopic chains is a key point in interpreting the atomic PNC experiments. Accurate test of the standard model through the atomic PNC experiments needs the neutron radii with an accuracy of 1% which has not been available. In this paper we will predict the parity violating asymmetries for some isotopic chains in the relativistic eikonal approximation.

As for the first topic, we will take the 2pF model for the neutron and proton densities, and for the second one we will use the nucleon densities obtained from the relativistic mean-field (RMF) theory [40–42]. Our purposes are twofold: one is to see whether the parity violating electron scattering can identify the type of neutron density distribution for neutron-rich stable nuclei from the theoretical point of view, and the other is to see whether the behavior of the parity violating asymmetries can be interpreted consistently by the nucleon densities and/or form factors calculated from the 2pF model and the RMF model.

Another motivation of this paper is that the facilities for the parity violating electron scattering against stable nuclei have been established for many years [43–48]. For example, the highly stable electron beams at Jefferson Lab [48] make it possible to measure the electron scattering cross sections with high accuracy. In fact, the parity violating asymmetries for some light nuclei, ¹H, ²H, and ⁴He, have been measured (see for example Refs. [47,48]). Therefore, it appears feasible to measure the neutron densities of heavy stable nuclei, such as ¹²⁴Sn and ²⁰⁸Pb, in the future. In addition, in recent years a new generation of the electron-ion colliders at RIKEN [49–52] and GSI [53–55] develop rapidly. The main purpose of these facilities is to determine the size and shape of exotic nuclei. To provide useful references for future experiments, some theoretical studies about the electron scattering from exotic nuclei have been performed [56–61]. Of course, it will be more difficult to carry out the parity violating electron scattering from exotic nuclei. Even so, it is also interesting to see whether (at least in principle) the neutron densities of exotic nuclei can be measured by the parity violating electron scattering.

This paper is organized as follows. In Sec. II we give an outline of the formalism of the parity violating electron scattering in the relativistic eikonal approximation. In Sec. III we analyze the numerical results. Finally, a summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

In this section, we will outline the main formalism of the parity violating electron scattering in the relativistic eikonal approximation. More detailed descriptions can be found in Ref. [4]. The starting point of the relativistic eikonal approximation of electron scattering is the Dirac equation. In spirit, the relativistic eikonal approximation is a high-energy small angle approximation. In the high-energy limit, the rest mass of electron is negligible compared with its kinetic energy. Then the Dirac equations for the left- and right-handed electrons can be written as follows [10]:

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + V_{\pm}(r)]\Psi_{\pm} = E\Psi_{\pm},\tag{1}$$

where $V_+(r)$ and $V_-(r)$ are, respectively, the potentials felt by the right- and left-handed electrons. The corresponding wave functions for right- and left-handed electrons are Ψ_+ and Ψ_- , respectively. The potentials $V_{\pm}(r)$ include two components, the Coulomb potential $V_C(r)$ and the weak neutral axial potential A(r) [10]

$$V_{\pm}(r) = V_C(r) \pm A(r), \quad A(r) = \frac{G_F}{2\sqrt{2}} \rho_W(r),$$
 (2)

where the weak potential is determined by the weak charge density $\rho_W(r)$ [10],

$$\rho_W(r) = \int d^3 \mathbf{r}' G_E(|\mathbf{r} - \mathbf{r}'|) [(1 - 4\sin^2 \theta_W)\rho_p(\mathbf{r}') - \rho_n(\mathbf{r}')].$$
(3)

Here, $\rho_n(r)$ and $\rho_p(r)$ are point neutron and proton densities, respectively. They are normalized to the neutron and proton numbers, respectively, $\int d^3 \mathbf{r} \rho_n(\mathbf{r}) = N$, $\int d^3 \mathbf{r} \rho_p(\mathbf{r}) = Z$. $G_E(r) = \frac{\Lambda^3}{8\pi} e^{-\Lambda r}$ with $\Lambda = 4.27$ fm⁻¹ is the electric form factor of the proton. $\sin^2 \theta_W \approx 0.23$ is the Weinberg angle, and $G_F = 1.16639 \times 10^{-5}$ GeV⁻² is the Fermi constant. Since the potentials felt by the electron are dependent on the helicity states, the cross sections at the same scattering angle for different helicity states will be not exactly equal. The parity violating asymmetry A_{LR} is defined as the difference between the cross sections of different helicity states [10]

$$A_{LR} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-},\tag{4}$$

where σ^+ and σ^- are the cross sections of the right- and left-handed electrons scattered by the potential $V_+(r)$ and $V_-(r)$, respectively. Following the standard procedure of the relativistic eikonal approximation of electron scattering [1–3], we obtain the parity violating asymmetry A_{LR} as follows [4]:

$$A_{LR}(q) = \frac{2\text{Re}[(I_1(q) + I_2(q))^* I_W(q)]}{|I_1(q) + I_2(q)|^2},$$
(5)

where the amplitudes $I_1(q)$ and $I_W(q)$ can be calculated numerically, and $I_2(q)$ can be integrated out in terms of the Lommel's function [1–4].

III. NUMERICAL RESULTS AND ANALYSES

Now let us start to investigate the first problem, that is, whether the parity violating electron scattering can verify the type of neutron density for neutron-rich stable nuclei. At first, we will take ¹²⁴Sn as an example to investigate this problem. We take the two-parameter Fermi (2pF) proton (charge) distribution for ¹²⁴Sn: $\rho_p(r) = \rho_0/\{1 + \exp[(r - \rho_0)/(1 + \exp[$ $(c_p)/a_p$], where $c_p = 5.490$ and $a_p = 0.534$ fm are taken from Ref. [24]. Though it is difficult to measure the neutron densities of nuclei, the difference between the rms radii of neutron and proton densities can be measured easily [62-65]. So in calculations the values of Δr_{np} are taken to be the experimental data. The radii difference $\Delta r_{np} = 0.23$ fm of ¹²⁴Sn is taken from Ref. [35]. In order to see to what extent the parity violating asymmetry is sensitive to the difference between proton and neutron densities at the nuclear surface, we take two extreme cases for the neutron density: (a) neutron halo-type distribution: $c_n = c_p$, $a_n > a_p$; (b) neutron skin-type distribution: $c_n > c_p$, $a_n = a_p$. The half-density radius c_n and the diffuseness parameter a_n can be obtained by the relation [35] $\langle r_n^2 \rangle \approx \frac{3}{5}c_n^2 + \frac{7}{5}\pi^2 a_n^2$ assuming either $a_n = a_p$ or $c_n = c_p$. For the halo-type distribution $a_n = 0.668$ fm, and for the skin-type one $c_n = 5.819$ fm. ρ_0 can be obtained by the condition $\int \rho_n(\mathbf{r}) d^3 \mathbf{r} = N$.

In order to see clearly the difference between the halo- and skin-type neutron distributions, we introduce a quantity S(r). The value of S(r) is taken as $\rho_n/\rho_p(\rho_p/\rho_n)$ if the neutron density is larger (smaller) than the proton density at large radius. The difference in shape between the neutron and proton densities can be described by the derivative of S(r). As for ¹²⁴Sn, the neutron density is larger than the proton density at the surface due to the large neutron excess (N - Z = 24). Therefore, S(r) is taken as $S(r) = \rho_n/\rho_p$. The values of S(r) for the skin- and halo-type of neutron distributions are shown in Fig. 1. From this figure one can see that S(r) for the skin-type distribution changes more sharply than that of the halo-type one near the half-density radius of the proton density. In Ref. [4], we have shown that the value of S(r) in the RMF



FIG. 1. The values of S(r) for ¹²⁴Sn with the skin-type and halotype neutron distributions. Solid line denotes the value of S(r) for the halo-type neutron distribution ($c_n = c_p, a_n > a_p$); dashed line denotes the value of S(r) for the skin-type neutron distribution ($c_n > c_p, a_n = a_p$).



FIG. 2. Upper part: the parity violating asymmetries of ¹²⁴Sn for the skin-type and halo-type neutron distributions in the eikonal approximation at incident energy E = 850 MeV; middle part: the proton and neutron form factors for the halo-type neutron distribution $(c_n = c_p, a_n > a_p)$; lower part: the proton and neutron form factors for the skin-type neutron distribution $(c_n > c_p, a_n = a_p)$. $|F(q)|^2$ denotes $|F_n(q)|^2/N^2 (|F_p(q)|^2/Z^2)$ for the neutron (proton) density.

theory changes smoothly, so the derivative of S(r) increases in phase with S(r). However, the present situation is much more complex than that in Ref. [4]. The value of S(r) near the half-density radius of the proton density for the skin-type neutron density is larger than that of the halo-type one, while at very large radius the value of S(r) for the skin-type neutron density is smaller than that of the halo-type one. It means that the magnitude of S(r) does not change in phase with its derivative. Therefore, up to now one can not estimate the amplitude of the parity violating asymmetry qualitatively only according to the magnitude of S(r). The numerical results of the parity violating asymmetries calculated in the relativistic eikonal approximation at E = 850 MeV for these two types of neutron distribution are shown in the upper part of Fig. 2. It is clear that the amplitude of the parity violating asymmetry for the skin-type distribution is much larger than that of the halo-type distribution. In order to explain this phenomenon, we start with the plane wave Born approximation (PWBA). In PWBA, the parity violating asymmetry is approximatively proportional to the ratio of the neutron and proton form factors [8,10]:

$$A_{LR}(q) = \frac{G_F q^2}{4\pi\sqrt{2}\alpha} \left[\frac{F_n(q)}{F_p(q)} + 4\sin^2\theta_W - 1 \right].$$
 (6)

The neutron and proton form factors are nothing but the Fourier transforms of the neutron and proton densities. It is

well known that the form factor can describe the shape of the density distribution. Therefore, the difference in shape between these two types of neutron density can be reflected by the difference between the neutron and proton form factors. In order to see clearly the difference between the neutron and proton form factors for these two types of neutron density, we show them in the middle and bottom parts of Fig. 2. In this figure $|F(q)|^2$ denotes $|F_n(q)|^2/N^2(|F_p(q)|^2/Z^2)$ for the neutron (proton) density. From this figure, one can see that for the halo-type distribution the minima (or zero points) for the neutron and proton form factors are close to each other. While, for the skin-type distribution the minima of the neutron and proton form factors depart from each other. In PWBA, when the zero points for the neutron and proton form factors depart from each other, the amplitude of the parity violating asymmetry will be infinite [see Eq. (6)]. While in the eikonal approximation, the zero points of form factor are broken down by the Coulomb distortion effects. Consequently, the amplitude of the parity violating asymmetry is finite. By comparing the form factors and the parity violating asymmetries, one can find that the amplitude of the parity violating asymmetry increases with the distance between the minima for the neutron and proton form factors. This conclusion is consistent with the one drawn in Ref. [4].

Besides ¹²⁴Sn, ²⁰⁸Pb is also an interesting research object. Up to now, ²⁰⁸Pb is the heaviest stable doubly magic nuclei. The properties of this nucleus play crucial roles in nuclear physics. For example, the direct relationship between the neutron density of ²⁰⁸Pb and the neutron equation of state has been well established [66,67]. Therefore, the measurement of the neutron density for this nucleus by the parity violating electron scattering is very important. We also take the 2pF distribution for the proton and neutron densities of this nucleus. The experimental data $a_p = 0.446$ and $c_p = 6.684$ fm are derived from Ref. [25]. The difference of the rms radii $\Delta r_{np} = 0.16(2)$ fm is taken from Ref. [35]. Because of the large neutron excess, S(r) of ²⁰⁸Pb is taken as ρ_n/ρ_p . The values of S(r) for the two types of neutron density of ²⁰⁸Pb are shown in Fig. 3. One can find clearly that the shapes of S(r) for the skin- and halo-type neutron distributions of ²⁰⁸Pb are very similar to those of 124 Sn. What we are interested in is whether the behavior of the parity violating asymmetries for these two types of neutron density is also similar to that of ¹²⁴Sn. We



FIG. 3. Same as Fig. 1 but for ²⁰⁸Pb.



FIG. 4. Same as Fig. 2 but for ²⁰⁸Pb.

show the parity violating asymmetries for these two types of neutron densities in the upper part of Fig. 4. From this figure, one can see again that the amplitude of the parity violating asymmetry for the skin-type neutron distribution is much larger than that of the halo-type one. The neutron and proton form factors for these two types of neutron density are shown in the middle and lower parts of Fig. 4. By comparing the parity violating asymmetries with the form factors one can see again that the amplitudes of the parity violating asymmetries correspond to the distances between the minima of the neutron and proton form factors. It means that the parity violating asymmetries of the same type of neutron density for different nuclei have the same property.

Then we will analyze whether the neutron density at the surface can be measured very accurately by the parity violating electron scattering in principle. From the best fit procedure, it has been found that the charge form factor in the low and medium momentum transfer (q) region is sensitive to the outer part of the charge density [29,30]. The charge density near the nuclear center should be sensitive to the form factor in the very large q region. Actually, the form factors in this region cannot be measured because of the sharp decay of the form factor with the increase of q. That is why the error bars in the charge density near the nuclear center are a little larger, and why the density can be fitted quite well by the 2pF distribution. Since the parity violating asymmetry is determined by the cross sections for the right- and left-handed electrons, the experimental data will be available in the low and medium qregion. Therefore, we conclude that once the parity violating asymmetries are measured with high accuracy the neutron densities in the outer part of nuclei can be measured accurately.



FIG. 5. The ratio of the theoretical proton density of ²⁰⁸Pb obtained from RMF with TM1 [42] to the experimental data fitted by the 2pF model [25].

By combining with the discussions given above, one can reach the conclusion that the parity violating electron scattering can be used to verify the type of neutron density distribution in neutron-rich stable nuclei.

In the next, let us turn to the second problem. In this part we will take Ba and Pb isotopic chains as examples to predict the parity violating asymmetries. Due to the lack of experimental charge densities along these isotopic chains from the neutron-rich side to the proton-rich one, we will use the nucleon densities obtained from the RMF theory. The RMF theory [40–42] is a successful tool in describing various properties of nuclei such as the binding energies, and the proton densities. In Fig. 5 we show the ratio of the theoretical proton density of ²⁰⁸Pb obtained from RMF with the parameter TM1 [42] to the experimental data fitted by the 2pF model [25]: $\rho_{p,rmf}/\rho_{p,2pf}$. From this figure one can see clearly that the proton density of ²⁰⁸Pb can be reproduced very well by the RMF theory with TM1 [42]. Such a good agreement is not accidental. It is because in the best fit procedure to obtain the effective interactions of RMF theory, the charge radii and densities are important input parameters. However, due to the lack of accurate experimental data on neutron radii and densities, it is not clear whether the RMF theory can reproduce the neutron densities of nuclei accurately. So systematical test of the RMF theory in calculating the neutron densities of nuclei is needed. In this paper we will predict the parity violating asymmetries for Ba and Pb isotopic chains. Once the experimental parity violating asymmetries for these nuclei are measured, one can test the RMF theory in calculating the neutron densities by comparing experimental data with our results.

Now, let us see the results for Ba isotopes. In calculations, the neutron and proton densities are obtained from the RMF theory with parameter TM1 [42]. The values of $S(r)(=\rho_n/\rho_p)$ for ^{120,128,136,144}Ba are shown in Fig. 6. From this figure one can see that the values of S(r) obtained from the RMF theory change smoothly with r, and that the values of S(r) increase with the neutron excess at large radius. This situation is similar to those of Ca isotopes, N = 50 isotones, and N = Z



FIG. 6. The values of $S(r) (= \rho_n / \rho_p)$ for Ba isotopes obtained from the RMF theory with TM1 [42].

doubly magic nuclei (see Ref. [4]). What we are interested in is whether the amplitude of the parity violating asymmetry still corresponds to the value of S(r) as shown in Ref. [4]. The parity violating asymmetries are shown in the upper part of Fig. 7. From this figure one can see easily that the amplitude of the parity violating asymmetry does increase with the neutron excess. It means that when the values of S(r) change smoothly, they can describe the parity violating asymmetries qualitatively. From the discussions given above we know that besides S(r) the nucleon form factors can also be used to analyze the behavior of the parity violating asymmetry. We also plot the nucleon form factors and the parity violating asymmetries together. By comparing the amplitudes of the parity violating asymmetries and the distances between the minima of the neutron and proton form factors, one can easily find that the amplitude of the parity violating asymmetry



FIG. 7. The parity violating asymmetries (upper part), the neutron form factors (middle part), and the proton form factors (lower part) for Ba isotopes.



FIG. 8. Same as Fig. 6 but for Pb isotopes.

increases with the distances between the minima of the neutron and proton form factors.

We are also interested in Pb isotopes. The importance of the accurate neutron density of ²⁰⁸Pb has been discussed in the above. This isotopic chain is also a candidate for the atomic parity nonconservation experiment. The numerical results of S(r) obtained from the RMF theory with parameter TM1 [42] are shown in Fig. 8. For this isotopic chain one can find that the values of S(r) also increase with the neutron excess. The parity violating asymmetries and nucleon form factors are shown in Fig. 9. From this figure we can see again that the amplitudes of the parity violating asymmetries increase with the neutron excess. Though the value of S(r) for ¹⁹²Pb is quite large, the amplitude of the parity violating asymmetry is quite small. This phenomenon can be interpreted by comparing the minima of the neutron and proton form factors. It means that the amplitude of the parity violating asymmetry is not determined by the abstract magnitude of S(r) but rather by the



FIG. 9. Same as Fig. 7 but for Pb isotopes.

distance between the minima of the neutron and proton form factors.

Finally, it is interesting to discuss the feasibility of the parity violating electron scattering from nuclei at existing facilities. From the discussions given above, we know that the amplitude of the parity violating asymmetry is mainly determined by the difference between the cross sections for the right- and lefthanded electrons near the diffraction minima. So the precision of the parity violating electron scattering experiments mainly depends on the accuracy of the cross sections for the left- and right-handed electrons in these q regions. Experimental results [29,30] have shown that the cross sections near the diffraction minima are smaller than those at the neighboring maxima by about an order of magnitude. From the numerical results given above, one can see that the parity violating asymmetry is about a magnitude of 10^{-5} . It means that the precision of the cross sections in the parity violating electron scattering experiment should be improved by about 6 orders of magnitude respect to that of the Coulomb electron scattering. How to improve the precision is a great challenge for experimental physicists. In the past several decades, experimental physicists have made great efforts [43–48] to improve the experimental conditions. Up to now, the highly stable electron beam facilities, for example the facility at Jefferson Lab [47,48], make it possible to carry out the parity violating electron scattering experiments from stable nuclei. Actually, the parity violating asymmetries for some light nuclei [47,48] (i.e., ¹H, ²H, and ⁴He) have been measured. Therefore, the precise measurement of the neutron densities of heavy stable nuclei, such as ¹²⁴Sn and ²⁰⁸Pb, is likely to be achievable in the upcoming decade or so with further development in the technology of accelerator and detector. In contrast, it appears to be beyond the capabilities of any of the present technology to perform the parity violating electron scattering experiments from exotic nuclei at the electron-ion colliders.

IV. SUMMARY

In summary, two investigations by the parity violating electron scattering are performed from theoretical point of view. One is to verify the type of neutron density for neutronrich stable nuclei such as ¹²⁴Sn and ²⁰⁸Pb, and the other is to predict the neutron densities of some isotopic chains, such as Ba and Pb, which have been suggested to perform the atomic PNC experiments to test the standard model. The parity violating asymmetries are calculated in the relativistic eikonal approximation. For the first topic, the proton and neutron densities are taken to be the 2pF distribution, and the proton densities are taken as experimental data. Results show that the amplitude of the parity violating asymmetry for the skin-type neutron distribution is much larger than that of the halo-type distribution. That is to say, the parity violating electron scattering can be used to determine the type of neutron density in neutron-rich stable nuclei. For the second topic, the neutron and proton densities are obtained from the RMF theory with the parameter TM1 [42]. Since the proton densities of some nuclei, for instance ²⁰⁸Pb (see Fig. 5), can be reproduced quite well, our results can provide a new groundwork of testing the validity of the RMF theory in calculating the neutron densities of nuclei. By combining the results of these two topics we find that the amplitudes of the parity violating asymmetries increase with the distances between the minima of proton and neutron form factors, though the values of S(r) for these two cases differ greatly from each other. It means that S(r) cannot describe the amplitude of the parity violating asymmetry in any case. In this sense, this paper can be seen as a supplement of Ref. [4].

- [1] A. Baker, Phys. Rev. 134, B240 (1964).
- [2] Zaijun Wang and Zhongzhou Ren, Phys. Rev. C 70, 034303 (2004); 71, 054323 (2005); Zaijun Wang, Zhongzhou Ren, and Ying Fan, *ibid.* 73, 014610 (2006).
- [3] Zaijun Wang and Zhongzhou Ren, Nucl. Phys. A794, 47 (2007).
- [4] Tiekuang Dong, Zhongzhou Ren, and Zaijun Wang, Phys. Rev. C 77, 064302 (2008).
- [5] T. W. Donnelly, J. Dubach, and I. Sick, Nucl. Phys. A503, 589 (1989).
- [6] Parity Violation in Electron Scattering, edited by E. J. Beise and R. D. Mckeown (World Scientific, Singapore, 1990).
- [7] M. J. Musolf, T. W. Donnelly, J. Dubach, S. J. Pollock, S. Kowalski, and E. J. Beise, Phys. Rep. 239, 1 (1994).
- [8] T. Suzuki, Phys. Rev. C 50, 2815 (1994).
- [9] W. E. Ormand, Phys. Rev. Lett. 82, 1101 (1999).
- [10] C. J. Horowitz, Phys. Rev. C 57, 3430 (1998).
- [11] C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels, Phys. Rev. C 63, 025501 (2001).
- [12] E. D. Cooper and C. J. Horowitz, Phys. Rev. C 72, 034602 (2005).
- [13] D. Vretenar, P. Finelli, A. Ventura, G. A. Lalazissis, and P. Ring, Phys. Rev. C 61, 064307 (2000).
- [14] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis, and P. Ring, Phys. Rep. 409, 101 (2005).
- [15] S. J. Pollock and M. C. Welliver, J. Phys. G: Nucl. Part. Phys. 27, 787 (2001).
- [16] M. C. Welliver and S. J. Pollock, J. Phys. G: Nucl. Part. Phys. 29, L21 (2003).
- [17] A. I. Milstein and O. P. Sushkov, Phys. Rev. C 71, 045503 (2005).
- [18] A. Bohr and B. R. Mottelson, *Nuclear Structure* (W. A. Benjamin, New York, 1975), Vol. 1.
- [19] R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).
- [20] T. W. Donnelly and J. D. Walecka, Annu. Rev. Nucl. Part. Sci. 25, 329 (1975).
- [21] T. W. Donnelly and I. Sick, Rev. Mod. Phys. 56, 461 (1984).
- [22] E. Moya de Guerra, Phys. Rep. 138, 293 (1986).
- [23] I. Sick, Prog. Part. Nucl. Phys. 47, 245 (2001).
- [24] H. De Vries, C. W. De Jager, and C. De Vries, At. Data Nucl. Data Tables 36, 495 (1987).
- [25] G. Fricke, C. Bernhardt, K. Heilig, L. A. Schaller, L. Schellenberg, E. B. Shera, and C. W. De Jager, At. Data Nucl. Data Tables 60, 177 (1995).
- [26] J. Cavedon, J. B. Bellicard, B. Frois, D. Goutte, M. Huet, P. Leconte, X.-H. Phan, S. K. Platchkov, and I. Sick, Phys. Lett. B118, 311 (1982).
- [27] J. R. Ficenec, L. A. Fajardo, W. P. Trower, and I. Sick, Phys. Lett. B42, 213 (1972).

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- [28] J. Heisenberg, R. Hofstadter, J. S. McCarthy, I. Sick, B. C. Clark, R. Herman, and D. G. Rarenhall, Phys. Rev. Lett. 23, 1402 (1969).
- [29] G. C. Li, M. R. Yearian, and I. Sick, Phys. Rev. C 9, 1861 (1974).
- [30] I. Sick and J. S. McCarthy, Nucl. Phys. A150, 631 (1970).
- [31] B. B. P. Sinha, G. A. Peterson, R. R. Whitney, I. Sick, and J. S. McCarthy, Phys. Rev. C 7, 1930 (1973).
- [32] R. F. Frosch, R. Hofstadter, J. S. McCarthy, G. K. Nöldeke, K. J. van Oostrum, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. 174, 1380 (1968).
- [33] J. W. Lightbody, Jr., J. B. Bellicard, J. M. Cavedon, B. Frois, D. Goutte, M. Huet, Ph. Leconte, A. Nakada, Phan Xuan Ho, S. K. Platchkov, S. Turck-Chieze, C. W. de Jager, J. J. Lapikás, and P. K. A. de Witt Huberts, Phys. Rev. C 27, 113 (1983).
- [34] J. D. Walecka, *Electron Scattering for Nuclear and Nucleon Structure* (Cambridge University Press, Cambridge, 2001).
- [35] A. Trzcińska, J. Jastrzębski, P. Lubiński, F. J. Hartmann, R. Schmidt, T. von Egidy, and B. Kłos, Phys. Rev. Lett. 87, 082501 (2001).
- [36] B. Kłos, A. Trzcińska, J. Jastrzębski, T. Czosnyka, M. Kisieliński, P. Lubiński, P. Napiorkowski, L. Pieńkowski, F. J. Hartmann, B. Ketzer, P. Ring, R. Schmidt, T. von Egidy, R. Smolańczuk, S. Wycech, K. Gulda, W. Kurcewicz, E. Widmann, and B. A. Brown, Phys. Rev. C 76, 014311 (2007).
- [37] E. N. Fortson, Y. Pang, and L. Wilets, Phys. Rev. Lett. 65, 2857 (1990).
- [38] S. J. Pollock, E. N. Fortson, and L. Wilets, Phys. Rev. C 46, 2587 (1992).
- [39] M. G. Kozlov, S. G. Porsev, and I. I. Tupitsyn, Phys. Rev. Lett. 86, 3260 (2001).
- [40] J. D. Walecka, Ann. Phys. (NY) 83, 491 (1974).
- [41] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
- [42] Y. Sugahara and H. Toki, Nucl. Phys. A579, 557 (1994).
- [43] C. Y. Prescott, W. B. Atwood, R. L. A. Cottrell, H. DeStaebler, E. L. Garwin, A. Gonidec, R. H. Miller, L. S. Rochester, T. Sato, D. J. Sherden, C. K. Sinclair, S. Stein, R. E. Taylor, C. Young, J. E. Clendenin, V. W. Hughes, N. Sasao, K. P. Schüler, M. G. Borghini, K. Lübelsmeyer, and W. Jentschke, Phys. Lett. B84, 524 (1979).
- [44] W. Heil, J. Ahrens, H. G. Andresen, A. Bornheimer, D. Conrath, K.-J. Dietz, W. Gasteyer, H.-J. Gessinger, W. Hartmann, J. Jethwa, H.-J. Kluge, H. Kessler, T. Kettner, L. Koch, F. Neugebauer, R. Neuhausen, E. W. Otten, E. Reichert, F. P. Schäfer, and B. Wagner, Nucl. Phys. B327, 1 (1989).
- [45] P. A. Souder, R. Holmes, D.-H. Kim, K. S. Kumar, M. E. Schulze, K. Isakovich, G. W. Dodson, K. A. Dow, M. Farkhondeh, S. Kowalski, M. S. Lubell, J. Bellanca, M. Goodman,

S. Patch, Richard Wilson, G. D. Cates, S. Dhawan, T. J. Gay, V. W. Hughes, A. Magnon, R. Michaels, and H. R. Schaefer, Phys. Rev. Lett. **65**, 694 (1990).

- [46] B. Mueller, D. H. Beck, E. J. Beise, E. Candell, L. Cardman, R. Carr, R. C. DiBari, G. Dodson, K. Dow, F. Duncan, M. Farkhondeh, B. W. Filippone, T. Forest, H. Gao, W. Korsch, S. Kowalski, A. Lung, R. D. McKeown, R. Mohring, J. Napolitano, D. Nilsson, M. Pitt, N. Simicevic, B. Terburg, and S. P. Wells, Phys. Rev. Lett. **78**, 3824 (1997).
- [47] K. A. Aniol et al., Phys. Rev. Lett. 82, 1096 (1999).
- [48] K. A. Aniol *et al.*, Phys. Rev. C **69**, 065501 (2004); K. A. Aniol *et al.*, Phys. Rev. Lett. **96**, 022003 (2006).
- [49] T. Suda, K. Maruyama, and I. Tanihata, RIKEN Accel. Prog. Rep. 34, 49 (2001).
- [50] M. Wakasugi, T. Suda, and Y. Yano, Nucl. Instrum. Methods A 532, 216 (2004).
- [51] T. Suda and M. Wakasugi, Prog. Part. Nucl. Phys. 55, 417 (2005).
- [52] M. Wakasugi, T. Emoto, Y. Furukawa, K. Ishii, S. Ito, T. Koseki, K. Kurita, A. Kuwajima, T. Masuda, A. Morikawa, M. Nakamura, A. Noda, T. Ohnishi, T. Shirai, T. Suda, H. Takeda, T. Tamae, H. Tongu, S. Wang, and Y. Yano, Phys. Rev. Lett. 100, 164801 (2008).
- [53] An International Accelerator Facility for Beams of Ions and Antiprotons, GSI Report, 2002.
- [54] H. Simon, in Proceedings of the International Workshop XXXII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Austria, 2004, edited by M. Buballa, J. Knoll, W. Nörenberg, B.-J. Schaefer, and J. Wambach (GSI, Darmstadt, 2004), p. 290.

- [55] H. Simon, Nucl. Phys. A787, 102c (2007).
- [56] C. A. Bertulani, Phys. Lett. B624, 203 (2005).
- [57] S. Karataglidis and K. Amos, Phys. Lett. B650, 148 (2007).
- [58] E. Moya de Guerra, E. Garrido, and P. Sarriguren, in *Proceedings* of the Seventh International Spring Seminar on Nuclear Physics, edited by A. Covello (World Scientific, Singapore, 2002), p. 63.
- [59] A. N. Antonov, M. K. Gaidarov, D. N. Kadrev, P. E. Hodgson, and E. Moya de Guerra, Int. J. Mod. Phys. E 13, 759 (2004).
- [60] A. N. Antonov, D. N. Kadrev, M. K. Gaidarov, E. Moya de Guerra, P. Sarriguren, J. M. Udias, V. K. Lukyanov, E. V. Zemlyanaya, and G. Z. Krumova, Phys. Rev. C 72, 044307 (2005).
- [61] Tiekuang Dong, Zhongzhou Ren, and Yanqing Guo, Phys. Rev. C 76, 054602 (2007).
- [62] A. Krasznahorkay, M. Fujiwara, P. van Aarle, H. Akimune, I. Daito, H. Fujimura, Y. Fujita, M. N. Harakeh, T. Inomata, J. Jänecke, S. Nakayama, A. Tamii, M. Tanaka, H. Toyokawa, W. Uijen, and M. Yosoi, Phys. Rev. Lett. 82, 3216 (1999).
- [63] L. Ray, Phys. Rev. C 19, 1855 (1979).
- [64] G. W. Hoffmann, L. Ray, M. Barlett, J. McGill, G. S. Adams, G. J. Igo, F. Irom, A. T. M. Wang, C. A. Whitten Jr., R. L. Boudrie, J. F. Amann, C. Glashausser, N. M. Hintz, G. S. Kyle, and G. S. Blanpied, Phys. Rev. C 21, 1488 (1980).
- [65] V. E. Starodubsky and N. M. Hintz, Phys. Rev. C 49, 2118 (1994).
- [66] S. Typel and B. A. Brown, Phys. Rev. C 64, 027302 (2001).
- [67] B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000).