Intensity profiles of superdeformed bands in Pb isotopes in a two-level mixing model

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A recently developed two-level mixing model of the decay out of superdeformed bands is applied to examine the loss of flux from the yrast superdeformed bands in ^{192}Pb, ^{194}Pb, and ^{196}Pb. Probability distributions for decay to states at normal deformations are calculated at each level. The sensitivity of the results to parameters describing the levels at normal deformation and their coupling to levels in the superdeformed well is explored. It is found that except for narrow ranges of the interaction strength coupling the states, the amount of intensity lost is primarily determined by the ratio of γ decay widths in the normal and superdeformed wells. It is also found that while the model can accommodate the observed fractional intensity loss profiles for decay from bands at relatively high excitation, it cannot accommodate the similarly abrupt decay from bands at lower energies if standard estimates of the properties of the states in the first minimum are employed.

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I. INTRODUCTION

One of the most distinctive features of superdeformed (SD) bands is the pattern with which intensity is lost through decays to states of normal deformation (ND levels). After feeding at high spins, followed by a plateau region, complete loss of flux occurs over only two to three SD levels. The similarity of the intensity profiles is maintained over a large number of isotopes in several different mass regions, presumably sampling a wide range of excitation energies, spins, and ND shapes. Any model that seeks to describe the escape from the superdeformed well must therefore be able to account for this characteristic pattern.

In the following, we examine the ability of the two-level mixing model proposed by Stafford and Barrett [1] and further developed by Cardamone, Stafford, and Barrett [2], to describe the intensity profiles of the yrast SD bands in even-even Pb isotopes. We ask two questions: (i) can the model accommodate the abruptness of the onset of the decay out of the SD bands, i.e. the *shape* of the intensity profile, and (ii) can it accommodate the *similarity* of the observed profiles over even this small range of isotopes?

A. Parameters influencing the decay profile

Most models of the decay of SD bands formulate the probability for intensity loss from a particular level in terms of the following properties of the SD and ND states: (i) the width of the SD state for decay within the second well, Γ_S, (ii) the fractional intensity loss from a given level in the SD band, F_N, (iii) the width for decay within the primary well of the ND states at approximately the same excitation energy, Γ_N, and their average separation, D_N, and (iv) a real, positive matrix element V which describes the interaction between the ND and SD states. In the absence of any additional mechanisms coupling states in the two minima, V will reflect the size of the potential barrier separating them in deformation space.

For any initial SD level, F_N can be measured directly by measuring the ratio of the intensities of the preceding and subsequent in-band transitions. With the high statistics data sets obtained using the Gammasphere and Euroball multidetector arrays, such ratios are derived from intensity measurements with uncertainties of only 2–3%, and hence are very well constrained. Where possible, the in-band decay width Γ_S is obtained directly from lifetime measurements of the state of interest. In other cases, it may be estimated either on the basis of measured quadrupole moments for other levels in the same band (or in other bands in the region) or on the basis of calculated deformations. The properties of the ND states, on the other hand, are not accessible to current experimental methods. It is usual to use a cranking model estimate of the level density [3] (and hence the level spacing D_N), and to combine this with the tail of the giant dipole resonance strength function to estimate Γ_N.

B. Choice of isotopes

To make estimates of the ND state properties with any degree of confidence, it is essential that the excitation energy and spin of the SD band at the point of decay are known. In the following, we make use of experimentally determined SD excitation energies to obtain estimates of D_N and Γ_N at the appropriate spins and excitation energies in ^{192}Pb, ^{194}Pb, and ^{196}Pb, the three even-even Pb isotopes where SD excitation energies have been experimentally determined [4–7]. The fraction of intensity leaving levels in each of these bands is shown in Fig. 1.

We focus on the Pb isotopes for several reasons.

(i) The excitation energies of the SD levels relative to the yrast states in the normal well differ by up to 2.5 MeV, leading us to expect significantly different level densities and decay widths in the ND well for states of the same spin in each isotope.

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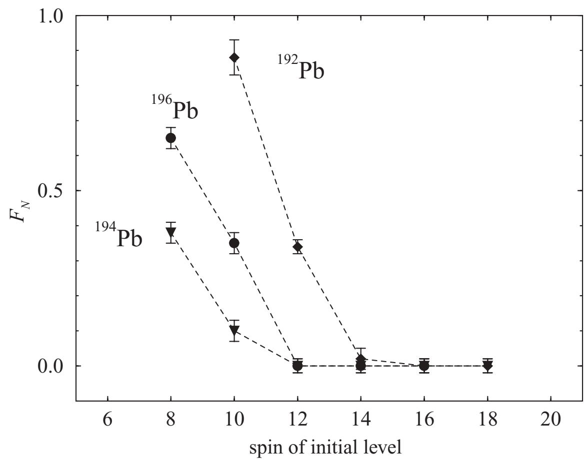

FIG. 1. Fractional intensity lost from each level in the yrast SD bands in ^{192}Pb [4], ^{194}Pb [5,6], and ^{196}Pb [7].

(ii) Despite the difference in excitation energies, the measured intensity profiles have a similar shape, differing mainly in the absolute spins at which the decay occurs ($8\hbar$–$14\hbar$ in ^{192}Pb and $6\hbar$–$10\hbar$ in ^{194,196}Pb).

(iii) The excitation energies at all spins in ^{192}Pb and at spins higher than the region at which decay out of the band is observed in all isotopes are relatively low, and thus the mixing with ND levels might reasonably be expected to be dominated by mixing with a single level. In other cases, the higher excitation energy might in itself render a simple two-level model inappropriate.

II. EXTRACTION OF PROBABILITY DISTRIBUTIONS FOR EXIT FROM THE SD WELL

A. Two-level mixing model

While several models have been proposed to describe the decay from the SD to ND wells (e.g., Refs. [8–13]), in the present paper we consider only the simple, two-level mixing model first proposed by Stafford and Barrett [1]. In subsequent work, Cardamone, Stafford, and Barrett built on this initial proposal to develop a better physical understanding of the parameters of the model and to introduce statistical approaches leading to the extraction of ND-SD coupling strengths [2]. At the same time, they showed that the inclusion of additional levels does not have a significant impact on measured quantities.

To date, no conceptual objections have been raised to the model; however, there has been some debate as to whether the decay can adequately be described in terms of mixing with a single level in the primary minimum [14–16] (or whether mixing with a single compound level is the equivalent of statistical mixing with many levels). The model has been successfully used to extract tunneling widths between the SD and ND wells [17] and to examine the abruptness of the onset of decay [18]. In the present paper, we examine the ability of the model to describe the experimentally observed intensity profiles of the yrast SD bands in the even-even Pb isotopes while employing widely used estimates of the ND state properties.

In this approach, mixing is modeled between one SD and one ND state, and the tunneling part of the decay is described using a Green's function approach. The model provides a closed formula for F_N [2]:

$$F_N = \frac{(1+\Gamma_N/\Gamma_S)V^2}{\Delta^2+\bar{\Gamma}^2(1+4V^2/\Gamma_N\Gamma_S)}, \tag{1}$$

where $\bar{\Gamma} = (\Gamma_N + \Gamma_S)/2$ and Δ is the difference in energy of the two states in the SD and ND wells. This cannot be measured, and so previous investigations [2,17] have assumed the following distribution [2]:

$$\mathcal{P}(\Delta) = \frac{\pi}{2D_N}\mathrm{erfc}\left(\sqrt{\pi}\frac{|\Delta|}{D_N}\right), \tag{2}$$

where erfc(x) is the complementary error function (this distribution follows from the assumption that the ND states are distributed according to the Gaussian orthogonal ensemble).

Equation (1) can be rewritten to provide a relation between Δ and F_N,

$$\Delta = \sqrt{V^2\left(1+\frac{\Gamma_N}{\Gamma_S}\right)\left[\frac{1}{F_N}-\left(1+\frac{\Gamma_S}{\Gamma_N}\right)\right]-\bar{\Gamma}^2}. \tag{3}$$

A distribution $\mathcal{P}(F_N)$ describing the probability for escape from the SD well from any particular level in the SD band can then be obtained through a change of variables,

$$\mathcal{P}(F_N)\,dF_N = \mathcal{P}(\Delta)\,d\Delta, \tag{4}$$

yielding

$$\begin{aligned}\mathcal{P}(F_N) &= \frac{\pi}{2D_N}\frac{V^2}{F_N^2}\left(1+\frac{\Gamma_N}{\Gamma_S}\right)\\ &\times\frac{1}{\sqrt{V^2(1+\frac{\Gamma_N}{\Gamma_S})[\frac{1}{F_N}-(1+\frac{\Gamma_S}{\Gamma_N})]-\bar{\Gamma}^2}}\\ &\times\mathrm{erfc}\left(\frac{\sqrt{\pi}}{D_N}\sqrt{V^2\left(1+\frac{\Gamma_N}{\Gamma_S}\right)\left[\frac{1}{F_N}-\left(1+\frac{\Gamma_S}{\Gamma_N}\right)\right]-\bar{\Gamma}^2}\right),\end{aligned} \tag{5}$$

for values of F_N up to a maximum value $F_{\max}$ (see below). The probability of F_N exceeding $F_{\max}$ is zero.

B. Input parameter values

Equation (5) can be used to calculate probability distributions for intensity loss from the SD well given values for Γ_S, D_N, Γ_N, and V. In the following, we have used the methods described in Ref. [17] to obtain estimates of the ND level densities and decay widths, together with values of Γ_S based on measured lifetimes and measured quadrupole moments [19–23]. Previously, we suggested an angular momentum dependent parametrization of the backshift parameter which characterizes the strength of pairing correlations [17]: in the current work, we examine the effect of neglecting the effects of pairing correlations at low spins or using the variable form. Table I lists the widths and average level spacings used in the calculations presented below.

TABLE I. Values of γ decay widths and average level spacings (in μeV) used to calculate the intensity profiles shown below. Superscripts a and b indicate values calculated without and with a spin-dependent correction for the pairing energy, respectively.

Nucleus	Spin ($\hbar$)	Γ_S	D^a_{ave}	Γ^a_N	Γ_S/Γ^a_N	$\bar{\Gamma}^a$	D^b_{ave}	Γ^b_N	Γ_N/Γ^b_S	$\bar{\Gamma}^b$
^{192}Pb	6	4	159	674	5.93×10^{-3}	339	5480	83	0.0482	44
	8	16	199	578	0.0277	297	4477	101	0.158	59
	10	48	274	499	0.0962	274	3780	114	0.421	81
	12	132	406	412	0.320	269	3476	121	1.091	127
	14	266	656	324	0.821	295	3473	121	2.20	194
	16	487	1162	237	0.419	362	3772	114	4.27	301
	18	815	2293	158	0.355	487	4460	102	7.99	459
	20	1279	5144	92	0.249	686	5766	84	15.2	682
^{194}Pb	6	3	19	1451	2.07×10^{-3}	727	252	501	599×10^{-3}	252
	8	14	25	1308	0.0107	661	223	531	0.0264	273
	10	45	36	1139	0.0395	592	214	539	0.0835	292
	12	125	56	953	0.131	539	226	526	0.238	326
	14	266	98	760	0.350	513	261	493	0.540	380
	16	497	198	570	0.872	534	330	441	1.127	469
	18	843	418	393	2.15	618	459	375	2.25	609
	20	1337	700	226	5.92	782	700	226	5.92	782
^{196}Pb	6	3	11	1705	1.760×10^{-3}	854	135	640	4.69×10^{-3}	322
	8	10	12	1623	6.16×10^{-3}	817	103	721	0.0139	366
	10	34	15	1523	0.0223	779	82	793	0.0428	414
	12	88	18	1406	0.0625	747	68	855	0.1029	472
	14	193	24	1277	0.1511	735	59	904	0.213	549
	16	377	33	1137	0.332	757	54	939	0.401	658
	18	665	47	991	0.671	828	51	960	0.693	813
	20	1120	68	845	1.325	983	55	962	1.164	1041

The SD decay widths have been inferred from lifetime measurements of in-band decays [19–21,23] (the widths given for the in-band decay from the $6\hbar$ levels in ^{194,196}Pb and the $6\hbar$ and $8\hbar$ levels in ^{192}Pb, which have not been observed experimentally, are based on extrapolation of the transition energies from the higher spin states).

The calculation of the average level spacings and ND decay widths depend on (i) the level density parameter, which we have taken as $A/9$, and (ii) the excitation energy of the SD states relative to the ND yrast line. To simplify the calculations, this has been estimated by performing variable moment of inertia fits to the energies of the SD and ND yrast levels, with the ND fits restricted to levels above $I = 14\hbar$, where the character of the ND yrast line becomes more regular. Relative excitation energies for any given spin are then estimated as the difference between the two fitted energies. Level spacings and widths calculated without and with a spin-dependent correction to this energy due to low spin pairing correlations are indicated in Table I by superscripts a and b, respectively.

As can be seen from Eq. (5), the probability distribution for decay out of the SD band from a given level is governed by the ratio Γ_S/Γ_N and the average $\bar{\Gamma}$. These quantities are therefore also included in the table. The ratio Γ_S/Γ_N increases rapidly (approximately exponentially) with increasing spin. Inclusion of the spin-dependent pair gap has the effect of increasing the magnitude of the ratio at any given level and making the increase with increasing spin slightly more gradual. Regardless of the treatment of pairing, the ratio is smallest for ^{196}Pb over all spins, as would be expected given the higher excitation energy of states in this nucleus. Similarly, the average (and hence total) γ-decay widths are consistently highest for ^{196}Pb and lowest for ^{192}Pb. The inclusion of the spin-dependent pair gap has the effect of changing the trend in $\bar{\Gamma}$ so that it increases monotonically over the spin range considered.

The only remaining undetermined quantity in Eq. (5) is the interaction strength V. Although the magnitude of V is not known, it is unlikely to be larger than 10 keV, since a larger interaction strength would be expected to result in a significant perturbation of the energy of the levels in the SD band. Although there is some evidence for splitting of the lowest level observed in the yrast SD band in ^{194}Pb [24], the energy difference between the two levels is measured to be $\approx$1 keV, and no evidence for perturbations from the expected rotational behavior of more than 2–3 keV has been observed elsewhere [25]. We have therefore explored the effect on the probability distributions $\mathcal{P}(F_N)$ of varying the interaction strength from 0.01 eV up to 10 keV.

III. CALCULATED PROBABILITY DISTRIBUTIONS

A. Ranges of the distributions $\mathcal{P}(F_N)$: Limits on the maximum fractional intensity loss from each level

Before discussing the detailed features of the probability distributions describing the intensity loss out of the superdeformed bands, it is worth commenting on their ranges.

It has been previously noted [2,17] that the model implies a maximum value of F_N for any given level. As is evident from Eq. (3), the limiting value $F_{\rm max}$ is given by

$$\frac{1}{F_{\rm max}} = \left(1 + \frac{\Gamma_S}{\Gamma_N}\right) + \frac{\bar{\Gamma}^2}{V^2}\frac{1}{\left(1+\frac{\Gamma_N}{\Gamma_S}\right)} \tag{6}$$

for real Δ. In general, $F_{\rm max}$ increases with decreasing spin, since the ratio Γ_S/Γ_N decreases with decreasing spin. If $\bar{\Gamma}/V \lesssim 10^{-2}$, the second term on the right-hand side of Eq. (6) is negligible, and the approximation

$$F_{\rm max} \approx \left(1 + \frac{\Gamma_S}{\Gamma_N}\right)^{-1} \tag{7}$$

can be made. When this approximation holds, the maximum is effectively independent of V. However, since $\bar{\Gamma} \approx 10^{-4}$ eV for most of the levels considered in this work, if $V \leqslant 10^{-2}$ eV, the dependence of the maximum on the interaction strength starts to become more important. For the interaction strengths considered in this work, the effect on $F_{\rm max}$ is less than a 3% reduction for all states in ^{192}Pb, and less than 5% in ^{194}Pb and ^{196}Pb except for levels at the highest spins (from which no loss of intensity is observed).

Figure 2 shows the values of $F_{\rm max}$ obtained from Eq. (6). Filled symbols show the results neglecting low spin pairing, and open symbols show the effect of including a spin-dependent pair gap. The inclusion of a pair gap reduces the maximum fractional intensity loss from a given SD level, with the strongest effect where the excitation energy is lowest, in ^{192}Pb. The range of $F_{\rm max}$ due to the dependence on V is smaller than the symbol size in almost every case. The exceptions are the highest spin levels in ^{196}Pb; the ranges here are indicated by downward error bars.

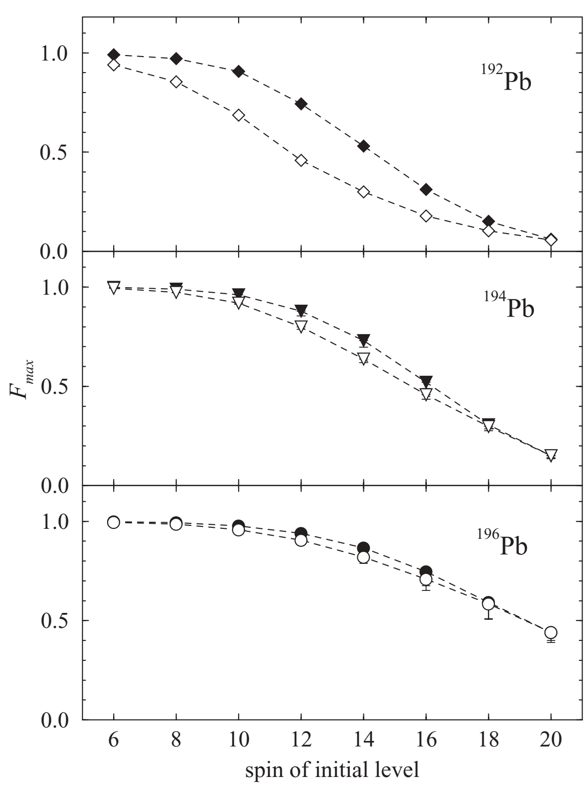

FIG. 2. Maximum fractional intensity loss allowed within the model from each level in the yrast SD bands in ^{192}Pb, ^{194}Pb, and ^{196}Pb. Filled symbols show the values calculated neglecting the effects of low-spin pairing correlations; open symbols show the results of including a spin-dependent parametrization of the pair gap.

As is evident from Fig. 2, the general behavior of $F_{\rm max}$ is similar in the three isotopes, although the range and absolute values of the spins over which it increases from $\sim$0 to $\sim$1 increase with increasing mass (i.e., increasing excitation energy of the states). When the pair gap is included in the calculations, $F_{\rm max}$ changes from <0.05 to >0.95 over approximate ranges of $28\hbar$–$12\hbar$, $26\hbar$–$10\hbar$, and $18\hbar$–$6\hbar$ in ^{196}Pb, ^{194}Pb, and ^{192}Pb respectively.

B. Characteristics of the distributions $\mathcal{P}(F_N)$

We have obtained probability distributions for levels in the yrast SD bands in ^{192}Pb, ^{194}Pb, and ^{196}Pb, both while neglecting low spin pairing correlations and while including an angular momentum dependent pair gap. Although the details of the distributions depend on the properties of the ND and SD states under consideration, the distributions can be described in terms of three regimes. If the interaction strength is sufficiently small, the probability for any intensity loss from the SD band is negligible. If the interaction strength is sufficiently large, the probability distributions display a single very narrow peak at $F_N \approx F_{\rm max}$. For intermediate interaction strengths, the distributions become somewhat flat with either single peaks at $F_N \approx 0$ or $F_N \approx F_{\rm max}$ or with some enhanced probability at both extreme values.

These characteristics are illustrated by the cumulative probability distributions shown in Fig. 3. These have been calculated for the $I = 10\hbar$ level in the yrast SD band in ^{194}Pb, neglecting low spin pairing correlations, for a variety of interaction strengths. For very small interaction strengths ($V \leqslant 10^{-2}$ eV, as shown in the top panel of the figure), there is almost no probability for any loss of intensity from the SD band. The decay to ND states is effectively turned off in this region. As the interaction strength increases, the probability for some intensity loss starts to increase, until at $V = 1$ eV the probability for fractional intensity loss equal to F_N is almost constant, resulting in the smoothly increasing cumulative probability shown in the middle panel of the figure. For the precise values of Γ_S, Γ_N, and V shown, the probability distribution exhibits a very narrow peak at $F_{\rm max} = 0.962$ resulting in the slight increase in the slope at the end of the cumulative distribution; varying the conditions can result in a small peak for low F_N followed by a fairly flat distribution up to $F_N \approx F_{\rm max}$, or a distribution with peaks at both $F \approx 0$ and $F_N \approx F_{\rm max}$. In these cases, the probability for intermediate F_N is *not* negligible, meaning the outcome is highly uncertain. Of course, for any given level in a real nucleus, F_N has a fixed, definite value; the flat $\mathcal{P}(F_N)$ indicates that whether some intensity is lost from the band, and if so how much, depends on precisely how close the mixing SD and ND levels are in energy. Finally, as the interaction strength increases still further

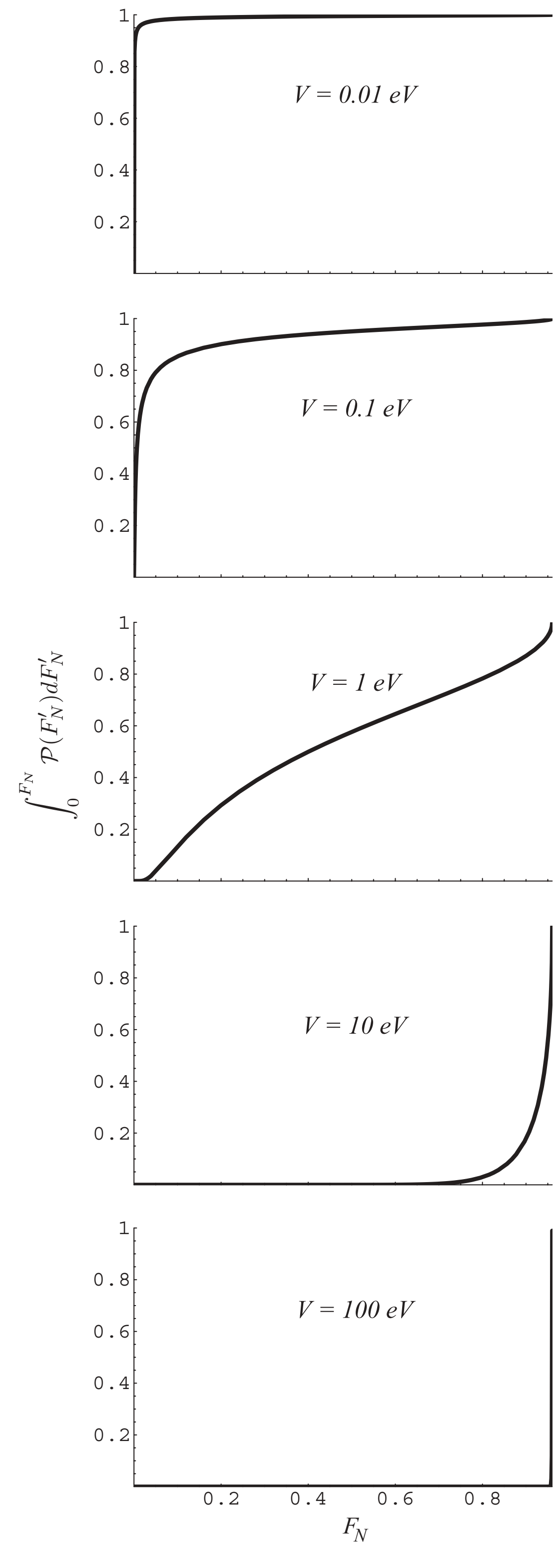

FIG. 3. Representative examples of cumulative fractional intensity loss probability distributions (calculated for the $10\hbar$ level in ^{194}Pb, neglecting pairing correlations, for which $F_{\max} = 0.9642$).

($V \gtrsim 10$ eV, see lower panels), the only probable outcome is the maximum possible loss of intensity from the band.

For other levels and other isotopes, the values of V required to ensure no loss of in-band intensity, or to guarantee the maximum intensity loss, vary, but the same general pattern is seen in each case. Thus, for all three isotopes, if V is sufficiently small the probability for escape from the SD well is negligible, regardless of the separation Δ of the SD and ND states. If the interaction strength is sufficiently large, on the other hand, the maximum possible fractional intensity loss [restricted by Eq. (6)] occurs, again regardless of Δ. Between these two extremes lies a region in which whether intensity is lost from the band, and if so how much is lost, is governed by Δ.

C. Implications for the possible outcomes of decay from in-band SD levels

The probability distributions $\mathcal{P}(F_N)$ can therefore be roughly divided into three groups: those for which loss of intensity is effectively impossible, those for which it is effectively guaranteed to be the maximum possible, and those for which the decay is sensitive to the SD-ND state separation Δ. Figure 4 gives a schematic illustration of the possible outcomes for the decay from SD levels in ^{194}Pb as a function of interaction strength and spin. The figure shows the approximate division of the (V, I) plane into three regions. Solid lines indicate calculations neglecting pairing correlations; dashed lines show the effect of including a spin-dependent correction. The upper and lower lines indicate approximate interaction strengths for which $F_N \geqslant 0.9F_{\max}$ and $F_N \leqslant 0.1F_{\max}$, respectively, at the 95% confidence level.

The figure illustrates how as the angular momentum, and hence Γ_S/Γ_N, decreases, the interaction strength required to ensure intensity loss decreases. It suggests that for ^{194}Pb, an interaction strength $V \gtrsim 1$ eV is sufficient to guarantee that $F_N \approx F_{\max}$ for $I = 6\hbar$ when pairing is neglected, compared

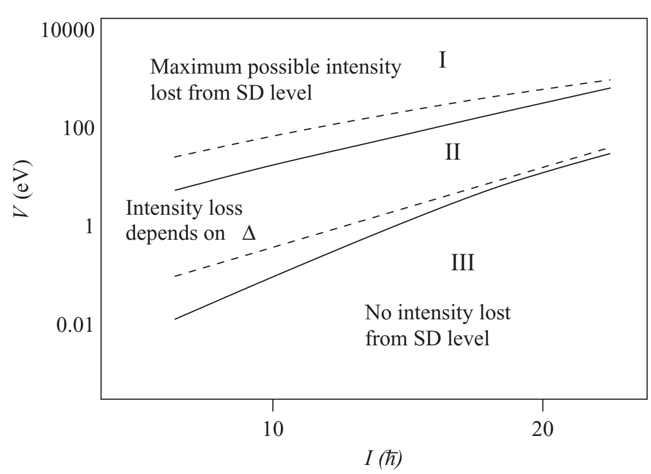

FIG. 4. Illustration of the approximate division of the (V, I) plane into three regions, using the parameters appropriate for ^{194}Pb: (I) the maximum possible fractional intensity loss occurs, regardless of the SD-ND state separation Δ; (II) whether intensity is lost, and the amount lost, is sensitive to Δ; or (III) all the intensity remains within the band, again regardless of Δ. Solid lines show the regions calculated neglecting pairing; dashed lines show the effects of including a spin-dependent pairing correction.

to $V \gtrsim 1000$ eV at $I = 20\hbar$. When pairing is considered, the interaction strength to ensure $F_N \approx F_{\max}$ increases to $\sim$100 eV at $I = 6\hbar$, but remains at $\sim$1000 eV at $I = 20\hbar$. Similarly, as Γ_S/Γ_N increases, the interaction strength required to allow measurable intensity loss increases, so that while it is sufficient that $V \lesssim 10$ eV to ensure no loss of SD flux at $I = 20\hbar$, V must be less than 0.01 eV (neglecting pairing) or 0.1 eV (including pairing) to ensure no loss of SD flux at $I = 6\hbar$.

The other two isotopes exhibit almost identical behavior, differing only in the scales of the interaction strengths required to block and ensure intensity loss from the SD band; larger interaction strengths are required to allow intensity loss from levels in ^{192}Pb (where Γ_S/Γ_N is largest), while smaller interaction strengths are sufficient for ^{196}Pb (where Γ_S/Γ_N is smallest).

It is worth commenting on the sensitivity of the results to the two parameters describing the ND states, Γ_N and D_N. The interaction strengths for which the decay is either ensured or blocked depend primarily on the ratio Γ_S/Γ_N, which decreases rapidly with decreasing angular momentum for all three of these nuclei. For the SD levels considered here, these "threshold" values of V do not depend strongly on D_N or on Γ_N alone. When no account is taken of pairing (or a fixed pair gap is used), Γ_N decreases and D_N increases with decreasing spin. However, as shown in Table I, this decrease may be slowed and even reversed by the inclusion of an angular momentum gap. A linear dependence may result in a somewhat counterintuitive *decrease* in relative excitation energy with decreasing spin. Yet there is little substantial difference between the results obtained with or without pairing. The in-band width Γ_S increases almost exponentially with increasing spin; a fixed or linearly decreasing pair gap thus has only a minor effect on the dependence of the ratio Γ_S/Γ_N on spin, and it seems to be this (together with V) that controls the probability for decay more strongly than any other factor.

The characteristics of the $\mathcal{P}(F_N)$ distributions calculated for most interaction strengths suggest that except for in the restricted region II of the (V, I) plane shown in Fig. 4, the coupling between the SD and ND states and their separation Δ effectively combine to either allow the decay to occur or block it, but they do not control the amount of decay, F_N. This is instead governed by the ratio Γ_S/Γ_N through the constraint given in Eq. (7). This yields the somewhat surprising result that, once critical values of V are exceeded and decay to ND states is guaranteed, the fractional intensity loss is independent of both V and Δ. Since these are the only two quantities in the formalism that are both experimentally unknown and difficult to make reliable model-based estimates of, this feature allows the results presented in the following to be remarkably strongly constrained.

IV. FRACTIONAL INTENSITY LOSS PROFILES

The calculated fractional intensity loss profiles shown in this section have been constructed by assuming that, in the regions where no decay is observed experimentally, V is in region III of the (V, I) plane as illustrated for ^{194}Pb in Fig. 4. The interaction strengths required to guarantee no decay are small even at the highest spins and must be less than $\sim$0.1 to $\sim$1 eV, depending on the treatment of pairing, at the levels just above the decay-out region in ^{194}Pb and ^{196}Pb, and less than $\sim$1 to $\sim$10 eV for the corresponding levels in ^{192}Pb. Given the drastically different structures supporting the superdeformed and normal (near spherical) states in the Pb isotopes, this may be reasonable.

A. Maximally steep profiles: Comparison with data

For the ranges of V and Γ_S/Γ_N defining region II in Fig. 4, the probability distributions are not strongly peaked, suggesting that F_N should depend fairly sensitively on the actual value of Δ, which is governed by the detail of each specific level scheme. In such circumstances, it is not clear that the observed similarity across isotopes or even the universally monotonic increase in the fraction of intensity leaving the band with decreasing spin should be expected. Because of this, as a first attempt to predict intensity loss from the SD bands, one might therefore require two conditions to be fulfilled: (i) the interaction strength must be below the threshold for intensity loss in the regions where no decay out of the band is observed, and (ii) the interaction strength must be above the threshold for ensuring intensity loss in the region where decay *is* observed. The fractional intensity loss profiles calculated in this way are "maximally steep," in the sense that they represent the most rapid increase in F_N allowed within the model.

In this limit, the possible values of F_N are restricted to either $F_N \approx 0$ for case (i) and $F_N \approx F_{\max}$ for case (ii), leading to the result that calculation of $F_{\max}$ indicates what fraction of the intensity is likely to be lost from any given level, and the results are independent of the precise value of V, as long as it is in region I of the (V, I) plane. The fractional intensity loss profiles obtained in this manner are compared with the data in Fig. 5.

The first question that we set out to address in this work was whether the model could accommodate the steepness of the intensity profiles of the yrast SD bands in even-even Pb nuclei. Figure 5 shows that unless the bands sample region II of the (V, I) plane, it proves difficult to predict a *gradual* enough loss of intensity in ^{194}Pb and ^{196}Pb. The fractional intensity loss profile predicted for ^{192}Pb, on the other hand, is close to that seen experimentally, although a little smoother.

It is difficult to improve the agreement between the calculated and measured values for the heavier isotopes without introducing interaction strengths in region II. When $V > 10^{-2}$ eV, as is required to be the case for in-band intensity to be lost from any of the levels, $F_{\max}$ is governed by Eq. (7), and hence by Γ_S/Γ_N alone. Because Γ_S is fairly well constrained by lifetime measurements, the only parameter that can be adjusted to improve the agreement between the data and the maximally steep profiles is Γ_N. To increase the calculated value of F_N, Γ_N must be increased; while to decrease the calculated value, Γ_N must also be decreased. In fact, Γ_N would need to be reduced by up to two orders of magnitude for good agreement to be obtained between the data and the maximally steep profiles for ^{194}Pb and ^{196}Pb. On the other hand, the ND decay width at the $10\hbar$ level in ^{192}Pb would need to be increased

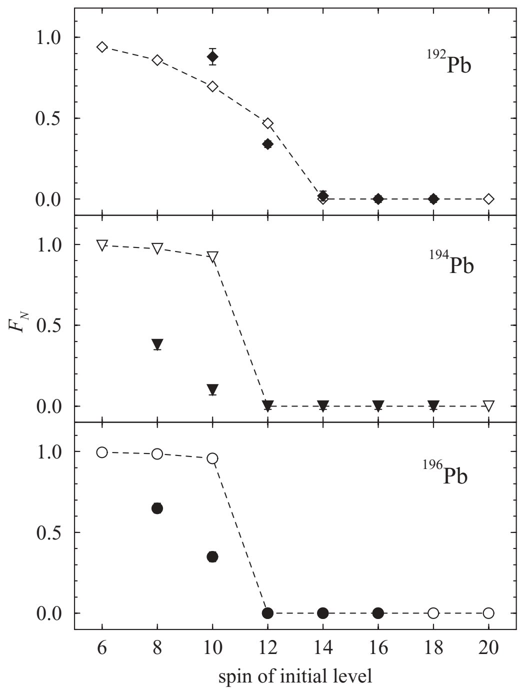

FIG. 5. Maximally steep fractional intensity loss profiles obtained using a spin-dependent backshift parameter (open symbols) compared with the experimental data (closed symbols).

by a factor of 5. It therefore seems unlikely that the model can account for the shapes of the observed decay profiles if the onset of intensity loss does not occur in region II of Fig. 4.

B. Fractional intensity loss from levels in region II: Probability for nonmonotonic decay

The model may allow for more gradual loss of intensity in ^{194}Pb and ^{196}Pb if the bands in these nuclei sample region II of Fig. 4 at at least the first two levels from which out-of-band decay is observed. It is reasonable to assume that the interaction strength will not decrease with decreasing spin, since the SD minimum is partly stabilized by rotation, and the barrier separating it from the ND minimum is not expected to increase with decreasing spin. Together with the ranges spanning region II of the (V, I) plane, this places some constraints on the possible interaction strengths coupling SD and ND wells in the regions where intensity less than F_{max} is lost from the bands. To ensure that the three isotopes sample region II at the onset of decay and at the subsequent level, V must range between ~0.1 and 1 eV (~1 and ~10 eV) for ^{194}Pb and ^{196}Pb, and between ~1 and ~10 eV (~10 and ~100 eV) for ^{192}Pb neglecting (including) pairing.

Once we assume interaction strengths in these ranges, it is possible to calculate a probability for nonmonotonic change in the fractional intensity loss from sequential levels sampling region II. If the fractional intensity loss from level 1 (spin I) is F_1 and the fractional intensity loss from level 2 (spin $I - 2$) is F_2, the probability that F_2 is less than F_1 is given by

$$\mathcal{P}(F_2 < F_1) = \int_0^{F_{1,\max}} \mathcal{P}(F_1)\, dF_1 \int_0^{F_1} \mathcal{P}(F_2)\, dF_2, \quad (8)$$

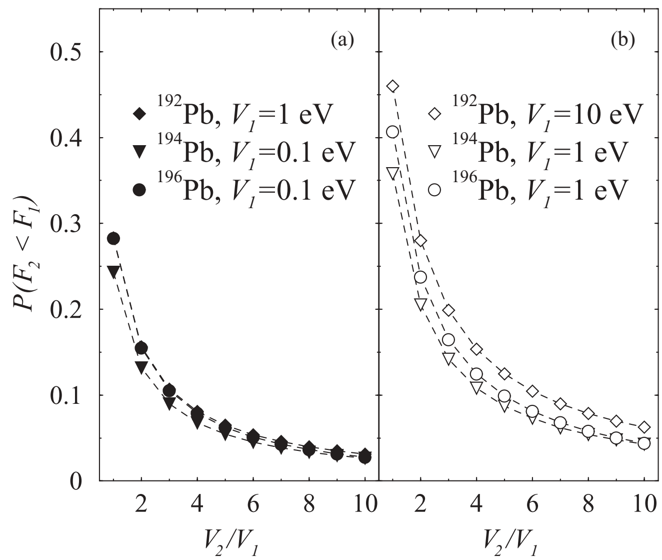

FIG. 6. Probabilities that the fractional intensity loss from the highest spin level from which out-of-band decay is observed is greater than the fractional intensity loss from the subsequent level. (a) Neglecting pairing; (b) with the pair gap correction.

where $F_{1,\max}$ is the maximum fractional intensity loss from level 1, obtained from Eq. (6), and the probability distributions $\mathcal{P}(F_1)$ and $\mathcal{P}(F_2)$ are calculated using Eq. (5) and the appropriate values of Γ_S, Γ_N, $D_{\rm ave}$, and V for levels 1 and 2, respectively.

Probabilities for nonmonotonic decay have been calculated in this way for the $I = 8{,}10\hbar$ levels in ^{194}Pb and ^{196}Pb, and for the $I = 10{,}12\hbar$ levels in ^{192}Pb. Figure 6(a) shows the results obtained neglecting pairing; Fig. 6(b) shows the effect of including a spin-dependent correction for the pair gap. The strength of the interaction coupling the ND and SD states at level 1, V_1, has been chosen for each isotope so that it is within region II of Fig. 4. The strength of the interaction coupling the ND and SD states at level 2, V_2, has been varied from V_1 to $10V_1$; this range has been chosen to ensure that V_2 does not creep into region I, since we are considering levels for which $F_N < F_{\rm max}$.

The results are very similar for all three isotopes. For $V_2 \approx V_1$, there is a significant probability for nonmonotonic decay: approximately 30% if pairing is neglected rising to approximately 40% if the pair gap correction is included in the calculations. As V_2 increases relative to V_1, the probability decreases rapidly, until for $V_2 \approx 10V_1$ (toward the upper limit of the region of indeterminate F_2 for each isotope) there is only a 3–5% probability that $F_2 < F_1$.

It therefore seems that both the shape and similarity of the decay profiles for ^{194}Pb and ^{196}Pb can be accounted for by the model. The observed profiles place quite strong constraints on both the absolute values and trend in V for this to be possible: the bands must sample region II of the (V, I) plane at the onset of decay, and also at the subsequent level. The probability for nonmonotonic decay is not negligible, but it is small, particularly if V increases rapidly with decreasing spin. A rapid increase in V is not unlikely, since a linear decrease in the size of the barrier separating the SD and ND wells might be expected to result in an approximately exponential increase in V.

The experimentally observed fractional intensity loss profile of ^{192}Pb requires $F_N < F_{\max}$ only at the onset of decay, and not at subsequent levels. The model also suggests that the interaction strength required to allow out-of-band decay for this isotope is an order of magnitude larger than in the two heavier neighbors. This is consistent with the expectation that the barrier separating the SD and ND states decreases with decreasing neutron number.

The second question we set out to address in this work was whether the model could account for the similarity of the observed fractional intensity loss profiles. It appears that it can successfully reproduce the similarity among all three isotopes, provided the interaction strengths vary in an appropriate way. Out of the levels considered above, it is only the intensity loss from the $I = 10\hbar$ level in ^{192}Pb that cannot be accounted for by judicious choice of V.

C. Consideration of other isotopes

The three isotopes of Pb considered in the above are those in which the excitation energies of the yrast SD bands are known. However, SD bands have also been observed in other Pb isotopes, including ^{190}Pb [26] and ^{198}Pb [27]. Several different predictions of the excitation energies of SD bands in Pb have been made, but only a small number have considered all five even-even isotopes [28,29]. While no calculation has successfully reproduced the excitation energies in all measured cases [30], the excitation energies of the SD bands in this mass region are generally predicted to increase with neutron number. The yrast SD band in ^{190}Pb is therefore expected to be 0.5–1 MeV lower than the yrast SD band in ^{192}Pb, and the yrast SD band in ^{198}Pb $\approx$ 1 MeV higher than that in ^{196}Pb. Since the ND yrast lines in all Pb isotopes are similar, these differences represent similar differences in excitation energy relative to yrast.

As was mentioned in Sec. IIIA, the range and absolute values of the spins over which $F_{\max}$ increases from close to 0 to close to 1 increases with increasing excitation energy. In ^{198}Pb, we therefore expect $F_{\max}$ to increase over a somewhat larger spin range than in ^{196}Pb, and for significant decay to become possible at still higher spins. This means that the model predictions for ^{198}Pb are very similar to those for ^{196}Pb. Thus in order to reproduce the experimentally observed decay profile [27], the band in this nucleus must sample region II of Fig. 4 at the onset of decay and at at least one subsequent level.

In the case of ^{190}Pb, similar arguments suggest that in the model, the fractional intensity loss should occur still more gradually than in ^{192}Pb. Figure 7 compares the measured fractional intensity losses to the values of $F_{\max}$ predicted assuming that the SD band has an excitation energy 0.5 MeV lower than in ^{192}Pb, the same quadrupole moment, and a similar ND yrast line. It appears that the model *cannot* allow the fractional intensity losses observed from levels below $14\hbar$. This is an extension of the problem already encountered for the $I = 10\hbar$ level in ^{192}Pb.

It may be that the method used to estimate Γ_N, which is based on a cranking model prediction for the level densities, is

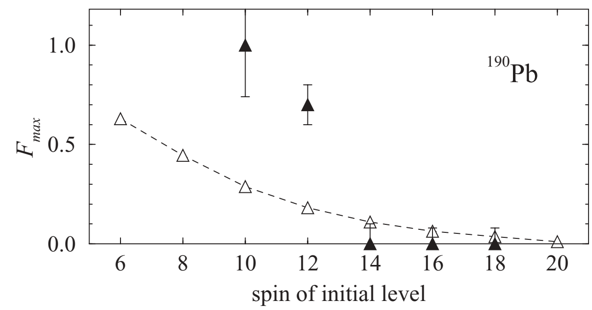

FIG. 7. $F_{\max}$ estimated for ^{190}Pb (open symbols) assuming that the SD band in this nucleus is 0.5 MeV closer to the yrast line than the band in ^{192}Pb, and experimental fractional intensity loss profile (close symbols) for this nucleus.

not appropriate in these nuclei. Because $Z = 82$ is semimagic, the Pb isotopes in this mass region tend to adopt spherical or small oblate deformations. The heaviest Pb isotope in which rotational behavior at normal deformations has been observed is ^{190}Pb [31]; the yrast and near-yrast level schemes in the heavier isotopes considered here are well established up to $I \gtrsim 40\hbar$, and no evidence for strongly collective behavior has been observed. This may mean that both the level density and ND decay strength estimates used in the present work are overestimates. However, a reduction in these estimates can only serve to reduce $F_{\max}$, which only worsens the agreement with the data. Indeed, to obtain values of $F_{\max}$ consistent with the observed fractional intensity losses in ^{192}Pb and ^{190}Pb, our estimates of Γ_N would need to be *increased* by at least an order of magnitude.

The extension of the application of the model to ^{190}Pb and ^{198}Pb highlights an important feature. Despite the fact that decay from a given SD level can be effectively ensured or blocked by choosing an appropriate interaction strength, and nonmonotonic decay can be rendered unlikely by appropriate variation in V with I, this does not in itself guarantee an abrupt loss of flux from the band. The abruptness varies depending on the relative excitation energy at which the decay occurs, so that abrupt decay in ^{190,192}Pb can only be reproduced if it occurs at lower angular momenta (where $F_{\max}$ is closer to 1.0) than in the heavier isotopes. Thus it appears that the answer to our second question—can the model accommodate the similarity of the observed profiles?—is a qualified no. It can do so for those bands at relatively high excitation energies, where Γ_S/Γ_N is small, but fails when Γ_S/Γ_N is large.

V. CONCLUSION

We have investigated the probability for decay from levels in the yrast SD bands of even-even Pb isotopes within the two-level mixing model presented by Cardamone, Stafford, and Barrett. In summary, there are three possible scenarios for decay from any given SD level:

(i) For sufficiently small V and large Γ_S/Γ_N, there is no possibility for decay out of the band, regardless of the proximity of the nearest ND level.

(ii) For intermediate interaction strengths, the outcome is uncertain and can only be precisely determined if Δ is known.

(iii) For sufficiently large V and small Γ_S / Γ_N, the maximum possible loss of intensity occurs, again regardless of Δ.

We have found that the model can accommodate the shape of the fractional intensity loss profiles for the bands in ^{194,196,198}Pb, but that to do so places constraints on the absolute values and variation of the SD-ND interaction strength over the spin range from which intensity loss is observed. In each case, the interaction strength at the first two levels from which decay is observed must lie in the region for which the intensity loss is uncertain. Importantly, we have also shown that it is likely, within the model, for the intensity loss profile to remain monotonic even when the SD-ND interaction strength at two successive levels samples this region of uncertain outcome, as long as the interaction strength increases with decreasing spin. The model also places strong constraints on the interaction strengths coupling SD and ND states at spins *above* the region where out-of-band decay is observed, typically requiring them to be less than 1 eV.

We have also reiterated the fact that there is a strong constraint on the maximum fractional intensity loss from any given level. Strikingly, this maximum intensity loss is effectively independent of V, so increasing V in those cases where the experimentally observed loss is greater than the calculated loss will have no effect. The limit on F_N arises from the relationship between F_N and Δ and is therefore an intrinsic feature of the model. Because of this, the failure of the model to reproduce the data for the lighter isotopes cannot be ascribed to the choice of the distribution of Eq. (2) to describe Δ; indeed, similar results are obtained with other smoothly varying continuous distributions. It may be that the ND state properties, particularly the electromagnetic decay widths, are not well understood, and that in fact better agreement could be achieved if a different parametrization were employed. To clarify the comparison of models with the data, and hence obtain a better understanding of the processes contributing to the loss of intensity from SD bands, it seems essential that more reliable estimates of the properties of the ND states be obtained.

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