Neutron decay widths of excited states of ¹¹Be

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The two-neutron transfer reaction ${}^{9}\text{Be}({}^{16}\text{O},{}^{14}\text{O}){}^{11}\text{Be}[{}^{10}\text{Be} + n]$ has been used to measure the branching ratios for the neutron decay of excited states of ${}^{11}\text{Be}$. The ${}^{14}\text{O}$ ejectile was detected by a Q3D spectrometer at forward angles. The energies and angles of the ${}^{10}\text{Be}$ fragments of the decaying ${}^{11}\text{Be}^*$ recoil were measured in coincidence with the ${}^{14}\text{O}$ ejectile using a double-sided silicon strip detector array at backward angles. This enabled a kinematic reconstruction of the reaction to be performed. Theoretical decay branch ratios were calculated using barrier penetrability factors and were compared to the measured ratios to provide information on the relative reduced widths of the states. The decay widths have been used to link states in ${}^{11}\text{Be}$ with a common structure and structurally to states in the daughter nucleus ${}^{10}\text{Be}$. The $3/2^-$ 8.82-MeV state was identified as a candidate for a molecular band head.

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I. INTRODUCTION

The concepts of nuclear clustering and nuclear molecules are best illustrated with the beryllium isotopes. For example, ⁸Be is an unbound nucleus that has a well-established α - α cluster structure [1,2]. The ⁹Be ground state has a similar two-center structure but, unlike ⁸Be, it is bound by 1.67 MeV. The binding energy that stabilizes the system is provided by the extra neutron that resides in delocalized orbits around the α cores. The signature of such a molecular structure is a rotational band, two of which have been identified in ⁹Be [3–6]; a band built on the $K^{\pi} = 3/2^{-}$ ground state (g.s.) in which the valence neutron is in a π -binding orbital and a band built on the $K^{\pi} = 1/2^{+}$ first excited state in which the valence neutron is in a σ -binding orbital.

The corresponding molecular structures in ¹⁰Be are expected to be dominant just below the $2n + {}^{8}Be$ decay threshold. Thus, the molecular structure of the ¹⁰Be ground state, which is bound with respect to two neutron emission by 8.48 MeV and to single-neutron decay by 6.81 MeV, is unlikely to be strongly developed. The 5.9583-MeV [2⁺], 5.9599-MeV [1⁻], 6.1793-MeV [0⁺], and 6.2633-MeV [2⁻] quartet of states lie much closer to the cluster decay threshold and studies suggest that they may have a molecular structure that is much more pronounced [3-12]. The 1^- and 2^- states have been interpreted as being members of a $K^{\pi} = 1^{-1}$ rotational band [3-6,13] with the two valence neutrons in a σ - π configuration. Such a rotational connection remains to be experimentally demonstrated, but antisymmetrized molecular dynamics (AMD) calculations suggest a 3^{-} to 1^{-} transition rate of $B(E2) = 11.2 e^2 \text{ fm}^4$ [14] that would provide a useful test of the structure. The 2^+ and 0^+ states have been associated with the π^2 and σ^2 molecular configurations, respectively. The 4^+ member of the rotational band built on the 0^+ state

has recently been measured and it was observed [9] to have a well-developed α :2*n*: α molecular structure as predicted.

The structures of the ground states of ⁹Be and ¹⁰Be can be used to examine the structures of the ¹¹Be states with the use of transfer reactions [15-17]. In a 2n transfer reaction the molecular structures of the ⁹Be target nuclei are already preformed and are thus expected to persist in the final nucleus, whereas for a ¹⁰Be target the compact structure of the ground state leads to the population of states with a core plus neutron configuration. The population of ¹¹Be states using the oneneutron transfer, 10 Be $({}^{14}N, {}^{13}N){}^{11}$ Be, and two-neutron transfer, ${}^{9}\text{Be}({}^{14}\text{N},{}^{12}\text{N}){}^{11}\text{Be}$, reactions have been measured [18]. For the one-neutron transfer only three excited states were observed to be populated at 0.32, 1.78, and 2.69 MeV, whereas in the two-neutron transfer reaction almost all known states of ¹¹Be up to 25 MeV were observed. Thus, the authors suggested that a rotational band built on the 3.96-MeV $[3/2^{-}]$ state is populated up to high spins [19]. The band was found to have a large moment of inertia that supports the interpretation of a molecular structure with a very large α - α separation of 5–6 fm.

These molecular configurations may also have a connection with halolike structures, because both imply a core plus valence neutrons. For example, it has been suggested that if the core of a halo nucleus such as ¹¹Li undergoes β decay then it is possible for the halo wave function to retain its features after the β decay, despite the fact that the core may now have a rather different structure [20]. Sarazin et al. [21] suggested that the 8.82-MeV ¹¹Be state, populated in the β decay of ¹¹Li, is a possible candidate for a two-neutron halo structure that can subsequently survive in a halolike configuration after the neutron decay to ¹⁰Be. In this instance the ⁹Li core undergoes β decay to ⁹Be and the two valence neutrons remain in the $2s_{1/2}$ orbital. If the β decay proceeds to the ground state of ⁹Be then this involves a significant structural change of the core; as the rotational band of the ground state of ⁹Be indicates a large deformation and is consistent with a structure based on

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a $2\alpha + n$ cluster configuration [3–6]. Thus, the two valence "halo" neutrons of the 8.82-MeV state of ¹¹Be would orbit a very different core to the two halo neutrons of the ¹¹Li. This would strongly overlap with a $(1p_{3/2})(2s_{1/2})^2$ shell-model configuration or, in the language of molecular orbitals, two α cores with two neutrons in σ -type molecular orbitals and one neutron in a π orbital. In other words, the two-neutron halo state in ¹¹Be may in fact correspond to a molecular configuration with delocalized covalent neutrons exchanged between the α particles. The decay by emission of a valence neutron would be expected to populate similar molecular/halo states in ¹⁰Be. The emission of a $2s_{1/2}$ (σ) neutron from ¹¹Be would be expected to populate the negative-parity states close to 6 MeV in ¹⁰Be. In this way the halo and molecular descriptions of this state are not distinct.

To determine the structure of ¹¹Be excited states it is important to study their decay schemes and transition probabilities. Thus, the branching ratios for the neutron decay of excited states of ¹¹Be to states of ¹⁰Be has been measured. *R*-matrix penetrabilities were used to calculate theoretical decay widths for each decay channel available to the excited states. The theoretical and experimentally determined widths are compared to provide information on the relative reduced widths of the states. The decay modes, relative reduced widths, population characteristics, and structures of the ¹⁰Be daughter states were used to determine the structure of the ¹¹Be states.

The levels and decay scheme of ¹¹Be have previously been studied through measurements of the β -delayed neutron emission of ¹¹Li [21–27]. Where available, the branching ratios are compared with these measurements. Note that the β -delayed neutron decay spectroscopy method relies on a complicated analysis procedure with a number of ambiguities, whereas the method used here is independent of any measurement of the β -decay branching ratios and relies on the detection of the ¹⁰Be fragments, allowing their excitation energy to be obtained so that the decay branches can be measured directly.

II. EXPERIMENTAL PROCEDURE

The two-neutron transfer reaction ${}^{9}\text{Be}({}^{16}\text{O},{}^{14}\text{O}){}^{11}\text{Be}^*$, ${}^{11}\text{Be}^* \rightarrow {}^{10}\text{Be} + n$ was measured at a beam energy of 234 MeV at the Ionen-Strahl-Labor facility, at the Hahn-Meitner-Institut, Berlin. This reaction has a ground-state Q value of $Q_0 = -21.57$ MeV.

The measurements were performed with a Q3D magnetic spectrometer [28–30] and an array of charged-particle detectors. The angular acceptance of the spectrometer was defined by the entrance slits giving a horizontal range of laboratory scattering angles from 8.5° to 11.5° and a vertical range of $\pm 1.0^{\circ}$. The ¹⁴O ejectile was identified in the Q3D focal plane detector using the energy loss (ΔE) of the particles in a gas-filled detector, the energy signal (*E*) from a scintillator, and the time-of-flight (TOF) of the particles through the spectrometer. The dependence of the TOF of the ¹⁴O ejectile on its scattering angle enabled the scattering angle to be measured, see Ref. [31]. Due to the excellent time resolution an angular resolution of 0.3° was achieved. The position of the ¹⁴O along the focal plane was measured using the

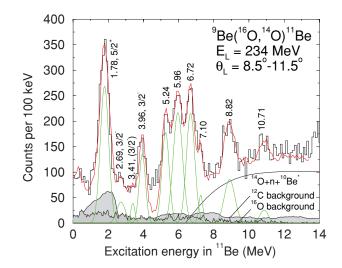


FIG. 1. (Color online) The ¹¹Be excitation energy spectrum fitted with Gaussian peaks (centroids indicated) and a smooth background related to three bodied processes ($^{14}O + n + {}^{10}Be$). The peaks ranging from 0 to 11 MeV represent the ¹¹Be states populated in the two-neutron transfer reaction ${}^{9}Be({}^{16}O,{}^{14}O){}^{11}Be$. Background contributions from ${}^{12}C$ and ${}^{16}O$ target contaminations are labeled. The angular range of the Q3D acceptance aperture is indicated.

delay-line read-out technique. The ¹⁴O nucleus has no bound excited states and thus its position along the focal plane yields the excitation of the ¹¹Be nucleus, as shown in Fig. 1. The resolution [full width at half maximum (FWHM)] of the isolated peak at 1.78 MeV is 0.7 MeV. Hence, the widths observed in the present measurements are dominated by the experimental resolution (see Table I). The reaction target (⁹Be $200 \,\mu g \,\mathrm{cm}^{-2}$) had a significant ¹²C and ¹⁶O content. Thus, the measurements of the ${}^{12}C({}^{16}O,{}^{14}O){}^{14}C$ and ${}^{16}O({}^{16}O,{}^{14}O){}^{18}O$ reactions were performed on separate targets for background subtraction. The remaining continuous background, in the Q3D focal plane, is due to a three-body process associated with the sequential ${}^{9}\text{Be}({}^{16}\text{O}, {}^{15}\text{O}^* \rightarrow {}^{14}\text{O} + n){}^{10}\text{Be}^*$ reaction. More details regarding the backgrounds and the methods used to fit them to the experimental data can be found in Ref. [31] and the references therein. The scattering angle of the ¹⁴O and the excitation energy of the ¹¹Be were used to determine the kinetic energy of the ¹⁴O with high precision in the focal plane and thus a complete reconstruction of the binary reaction was performed.

The decay products of the excited ¹¹Be recoil were detected, in coincidence with the oxygen ejectile, using an array of four 50×50 mm double-sided silicon strip detectors (DSSSD), approximately 500 μ m thick. The center of the detectors, with respect to the center of the reaction target, were placed at distances of 142.7, 141.2, 167.3, and 165.8 mm and at angles of 48.9° , 48.3° , 27.3° , and 26.3° , respectively. Each detector face was subdivided into 16 independent 3-mm-wide strips, with the direction of the strips on the front face being perpendicular to those on the back. The detectors had an angular resolution of $\Delta \theta_{lab} \leq 1.3^{\circ}$ and a typical energy resolution of 150 keV (FWHM). The DSSSD also acted as a stop for the TOF of the decay fragments. This technique allowed a kinematic reconstruction of the reaction as well as providing enhanced

TABLE I. Experimental branching ratios for the decay of various ¹¹Be states via the ¹⁰Be + *n* decay channel. The branching ratios are shown for the neutron decay to the 0⁺ ground state, the 2⁺, 3.3680-MeV first excited state, the quartet of states at ~6 MeV, and the 7.371-MeV [3⁻] and 7.542-MeV [2⁺₃] states of ¹⁰Be. The total widths are taken from Ref. [19].

$ \begin{array}{l} E_x(^{11}\text{Be}) \\ (\text{MeV}) \end{array} $		Total width			
	0_{1}^{+}	2_{1}^{+}	6-MeV group	$3_1^-, 2_3^+$	(keV)
1.78	1.05 ± 0.14				100
3.96	0.48 ± 0.06	0.54 ± 0.07			15
5.24	< 0.27	0.81 ± 0.16			40
5.96	< 0.13	0.97 ± 0.16			400
6.72	< 0.18	0.88 ± 0.15	0.09 ± 0.04		40
8.82	< 0.15	< 0.06	0.52 ± 0.16	0.59 ± 0.28	200

background suppression. More details about the experimental technique can be found in Ref. [32].

III. ANALYSIS AND RESULTS

Below the two-neutron threshold at 7.316 MeV the only available decay channel for the excited ¹¹Be nucleus is the neutron decay to ¹⁰Be ($E_{\rm thresh} = 0.503$ MeV). Thus, a particle detected in the DSSSD array, in coincidence with a ¹⁴O ejectile in the Q3D, was assumed to be a ¹⁰Be fragment of the decaying ¹¹Be* nucleus. Momentum conservation was then used to reconstruct the energy and the angle of the undetected neutron and the three-body Q value, Q_3 , was calculated as

$$Q_3 = E_{^{14}\text{O}} + E_{^{10}\text{Be}} + E_n - E_{\text{beam}}.$$
 (1)

The excitation energy of the ¹⁰Be was calculated by comparing Q_3 with the three-body Q value in which the decay products

are produced in their ground states, Q_{3_0} :

$$E_x({}^{10}\text{Be}) = Q_{3_0} - Q_3.$$
 (2)

The excitation energies of the ¹⁰Be fragments were calculated for each individual ¹¹Be state by gating on the appropriate peak in the ¹¹Be excitation energy spectrum (Fig. 1) and a ¹⁰Be excitation energy spectrum (Fig. 2) was produced for each decaying ¹¹Be^{*} state. It should be observed that the dominant contribution to the calculated ¹⁰Be excitation energy resolution arises from the energy and angular resolution of the silicon array and energy and angular straggling in the target of the recoil-like particles. The missing momentum contribution of the unobserved γ -ray from the decay of the ¹⁰Be excited states is small in comparison.

The detection efficiencies of the various neutron decay channels of each individual ¹¹Be state were calculated using Monte Carlo simulations of the reaction and detection processes. The simulations reproduced the experimental

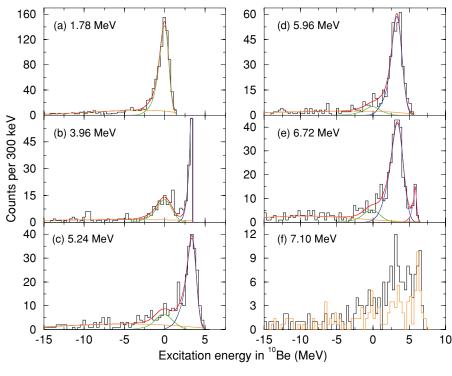


FIG. 2. (Color online) (a)-(e) ¹⁰Be states populated in the decay of the excited states of ¹¹Be. The spectra are produced by gating on the appropriate peak in the ¹¹Be excitation energy spectrum (Fig. 1) following the coincident detection of the ¹⁴O ejectile and the ¹⁰Be fragment and assuming a missing neutron. The data are fitted with the simulated peak line shapes and a smooth polynomial background. In each spectrum the excitation energy of the decaying ¹¹Be state is indicated. (f) The ¹⁰Be excitation energy spectrum gated on the 7.10-MeV peak (black histogram) together with the spectrum corresponding to a gate immediately above the 7.10-MeV peak in Fig. 1 (orange histogram).

acceptances and smeared the energies and angles in accordance with the experimental resolutions. The outputs of the simulations were subsequently processed as if they were real events using the same analysis procedures as those used for the experimental data. These simulations permitted the theoretical peak line shapes in the decay products excitation energy spectra to be reconstructed. Figure 2 shows the ¹⁰Be states populated in the decay of various ¹¹Be states. The black histograms correspond to the experimental data that has been fitted with the normalized theoretical peak line shapes and smooth polynomial backgrounds. The integrated areas of the normalized peak line shapes were compared with the number of counts in the associated states in the Q3D, ¹¹Be, excitation energy spectrum (Fig. 1) to deduce branching ratios for the neutron decay of the excited states of ¹¹Be to the states of ¹⁰Be. The peaks in Fig. 1 were fitted with Gaussian line shapes with widths reflecting the experimental resolutions.

The absolute branching ratio, BR, is the ratio between the decay rate, N_A , of an individual decay channel of a particular ¹¹Be state and the total decay rate, N_T , of that state and is

$$BR = \frac{N_A/\varepsilon_A}{N_T} = \frac{\text{No. of counts in DSSSD}/\varepsilon_A}{\text{No. of counts in Q3D detector}},$$
 (3)

where ε_A is the detection efficiency obtained from the Monte Carlo simulations. Note that the detection efficiency is affected only by the dead areas between the detectors as the entire angular distribution of the ¹⁰Be fragments falls within the limits of the DSSSD array. Typically the detection efficiencies were around 60%. The value of N_T , for the ¹¹Be state being studied, is given by the number of counts in the relevant peak, in the ¹¹Be excitation energy spectrum (Q3D counts). The value of N_A is given by the number of counts in the associated peak, in the decay fragments excitation energy spectrum (DSSSD counts). The number of counts in a peak was calculated by integrating the normalized function that corresponds to the peak line shape. The branching ratios of ¹¹Be states are shown in Table I.

The 1.78-MeV ¹¹Be state has sufficient excitation energy only to neutron decay to the ground state of ¹⁰Be. Thus, this state provides a good method for testing the validity of the procedure used to calculate the branching ratios. Using Eq. (3), the absolute branching ratio for this decay is $1.05 \pm$ 0.14. Thus, within errors, the absolute branching ratio is equal to unity, as expected.

The 3.96-MeV peak in the ¹¹Be excitation energy spectrum of Fig. 1 can be associated with the 3.89- and 3.96-MeV states. However, Bohlen *et al.* [33] showed that the state populated in the ⁹Be(¹⁶O,¹⁴O)¹¹Be reaction is the 3.96-MeV state of ¹¹Be. This state has a substantial width for neutron decay to both the ¹⁰Be 0_1^+ ground state and the 2_1^+ first excited state.

The 5.24-, 5.96-, and 6.72-MeV states favor the neutron decay to the 2_1^+ state. For these states the decay to the ground state is relatively weak and the associated peak is poorly resolved. In addition to this the ${}^{12}C({}^{16}O, {}^{14}O){}^{14}C[{}^{13}C + n]$ reaction contributes to small even backgrounds in the regions of the ground-state peaks of Fig. 2. Consequently, these branching ratios are listed as upper limits and are expected to be smaller. The 6.72-MeV state also has sufficient excitation

energy to decay to the 2_2^+ , 1_1^- , and 0_2^+ trio of 10 Be states at ~ 6 MeV. The state is seen to decay with a small branch to a state (or states) at ~ 6 MeV. However, the decay to the individual states is unresolved due to the experimental resolution.

From Fig. 1 it is clear that the 8.82-MeV peak lies on a three-body background associated with the ${}^{9}\text{Be}({}^{16}\text{O}, {}^{15}\text{O}^* \rightarrow {}^{14}\text{O} + n)^{10}\text{Be}^*(\sim 6 \text{ MeV})$ sequential reaction. An analysis of the coincidence data reveals that the ${}^{14}\text{O} + n$ decays come from a region close to the associated ${}^{15}\text{O}^* \rightarrow {}^{14}\text{O} + n$ decay threshold. This three-body background leads to events in which the ${}^{10}\text{Be}$ recoil is detected in the DSSSD array in coincidence with an ${}^{14}\text{O}$ in the Q3D. These events cannot be distinguished from those of the desired reaction channel in which a ${}^{10}\text{Be}$ is detected following the decay of the excited ${}^{11}\text{Be}$ recoil. Thus, for the 8.82-MeV state it is necessary to subtract the background in the ${}^{10}\text{Be}$ excitation energy spectrum that results from the coincident detection of background events in the Q3D and background events in the DSSSD array.

The approximate shape of this background contribution was determined by analyzing the region of the ¹¹Be excitation energy spectrum that lies between the 8.82- and 10.71-MeV peaks and contains only background events. This region was analyzed as a 9.97-MeV ¹¹Be state (mean energy of events in this region) and coincident hits in the DSSSD array were used to produce a ¹⁰Be excitation energy spectrum. The histogram showed the shape of the background contribution that arises from the coincident detection of background events. The line shape of this histogram was normalized to fit the 8.82-MeV data by comparing the number of Q3D background events in the 8.82-MeV peak with the number of Q3D background events in the region that was analyzed as a 9.97-MeV peak. This enabled the background in the ¹⁰Be excitation energy spectrum to be subtracted.

The 7.10-MeV state also lies in a region where the ${}^{9}\text{Be}({}^{16}\text{O}, {}^{15}\text{O}^* \rightarrow {}^{14}\text{O} + n){}^{10}\text{Be}^*(\sim 6 \text{ MeV})$ sequential reaction contributes. In this case the magnitude of the backgrounds compared with the peak strength means that the extraction of branching ratios is more problematic. Figure 2(f) shows the spectrum corresponding to the decay of the 7.10-MeV state (black histogram) together with an orange histogram that corresponds to gating on the {}^{11}\text{Be} spectrum immediately above the 7.10-MeV peak (with a gate of the same width). There appears to be an excess of counts in the region corresponding to the {}^{10}\text{Be} 2^+_1 state indicating that this decay branch is preferred.

the ¹⁰Be 2_1^+ state indicating that this decay branch is preferred. Figure 3 shows the ¹⁰Be excitation energy spectrum associated with the decay of the 8.82-MeV ¹¹Be state after background subtraction. This state has sufficient excitation energy to neutron decay to the 3_1^- 7.37-MeV and 2_3^+ 7.54-MeV states of ¹⁰Be, which lie above the ⁹Be + *n* decay threshold (6.8121 MeV). A ¹⁰Be nucleus emitted in one of these excited states will subsequently neutron decay to ⁹Be. In such decays, the ⁹Be nuclei detected in the DSSSD array are analyzed as ¹⁰Be nuclei. Thus, these events contribute to a ⁹Be background in the ¹⁰Be excitation energy spectrum. Monte Carlo simulations were used to reproduce the line shape of the ⁹Be background, in the ¹⁰Be excitation energy spectrum, which results from these sequential two neutron decays. The line shape was normalized to fit the experimental data and

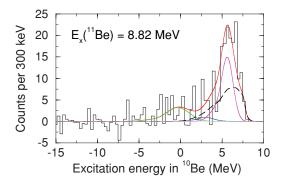


FIG. 3. (Color online) The ¹⁰Be excitation energy spectrum produced for the decay of the 8.82-MeV excited state of ¹¹Be. The data is fitted with peak line shapes and a smooth polynomial background. The dashed line represents the events in which the 8.82-MeV state neutron decayed to either the 7.371- or 7.542-MeV ¹⁰Be states, which subsequently neutron decayed to the ground state of ⁹Be.

is shown as a dashed line in Fig. 3. It should be noted that the line shape is not significantly different for the 7.371- and 7.542-MeV decay paths. The state has a substantial width for decay via the sequential emission of two neutrons as well as a strong branch for decay to a ¹⁰Be state (or combination of states) at ~6 MeV. The decay branch to the ground state and the 2^+_1 state are weak and are given as upper limits in Table I.

Table II shows the branching ratios for the neutron decay of various ¹¹Be states taken from the β -delayed neutron decay spectroscopy measurement of ¹¹Li by Hirayama *et al.* [27]. The two sets of measurements are broadly consistent with a few exceptions.

The branching ratios for the 3.96-MeV state do not agree with the current measurement. The present data indicate a stronger decay branch to the ground state of ¹⁰Be. Due to the large number of ambiguities in the β -delayed neutron decay spectroscopy method as opposed to the current method, it is suggested that those calculated by Hirayama *et al.* [27] may not be correct for this state. We note that the branching ratios of Hirayama *et al.* [27] cannot be reproduced even if some contribution from the proposed branching ratio of the 3.89-MeV state were to be included in the present data. The neutron decays to the ¹⁰Be ground state for the 5.24- and 8.82-MeV states were not observed in Refs. [24,25,27]. The decay of the 8.82-MeV state to the 2⁺ state was also not observed in Refs. [24,25,27], consistent with the present observations.

A. Decay widths

An estimate of the theoretical decay widths of the various decay channels were calculated using *R*-matrix penetrabilities [34]. For neutron decay from an initial ¹¹Be state feeding a ¹⁰Be final state, the decay width is given by

$$\Gamma_{^{10}\text{Be}[J_f]} = \sum_{j_n \ell_n} \Gamma_{J_f j_n \ell_n}^{J_i} \tag{4}$$

Here $\Gamma_{J_f j_n \ell_n}^{J_i} = 2\gamma^2 P_\ell$, where P_ℓ is the penetration factor of Ref. [34] (which includes a phase-space factor) and γ^2 is the reduced width. For the purpose of comparison γ^2 has been assumed to be 1 MeV and a radius of 4.4 fm was used in the calculation of the penetrabilities. J_i is the total angular momentum of the parent nucleus and J_f is the angular momentum of the daughter nucleus. The permissible angular momentum combinations $(j_n \ell_n)$ for the outgoing neutron are subject to the condition $\vec{J}_i = \vec{J}_f + \vec{j}_n$ and ℓ_n is either even or odd depending on the transition. The decay branches were calculated for each allowed neutron decay channel of the excited ¹¹Be states of interest. The ratios of the decay branches were then calculated over a range of J_i^{π} values.

The decay-branch ratio for the neutron decay of ${}^{11}\text{Be}^*$ to the ${}^{10}\text{Be}[0^+]$ ground state as a fraction of the neutron decay to the 2⁺ first excited state is given by

$$R = \frac{\Gamma_{^{10}\text{Be}[0^+]}}{\Gamma_{^{10}\text{Be}[2^+]}}.$$
(5)

The ratios were calculated for $J_i = 1/2$ to 9/2 for both the even- and odd-parity states and are shown in Table III. Also shown are the associated experimental relative branching ratios calculated from the absolute branching ratios of Table I. The theoretical and the experimentally determined ratios were then compared to provide information on the relative reduced widths of the decaying ¹¹Be state.

Table IV shows the ratio of the decay widths for the neutron decay to the 2^+ first excited state as a fraction of the neutron decay to the quartet of state at ~6 MeV for the 6.72- and 8.82-MeV ¹¹Be states.

TABLE II. The spins, parities, and branching ratios of various ¹¹Be states, taken from the β -delayed neutron decay spectroscopy measurement of ¹¹Li by Hirayama *et al.* [27]. The branching ratios are shown for the neutron decay to the 0₁⁺ ground state, the 2₁⁺ first excited state, the 6.263-MeV [2₁⁻] state, and the 7.371-MeV [3₁⁻] state of ¹⁰Be.

¹¹ Be state	J^{π}	${}^{10}{ m Be} + n$				
(MeV)		0_{1}^{+}	2_{1}^{+}	2_{1}^{-}	3_1^-	
3.89	$5/2^{-}$	0.38 ± 0.01	$0.62^{+0.14}_{-0.21}$			
3.96	$3/2^{-}$	0.22 ± 0.04	$0.78^{+0.65}_{-0.12}$			
5.24	$5/2^{-}$		1.00			
7.03	$(5/2^{-})$		1.00			
8.82	3/2-			0.25 ± 0.05	0.75 ± 0.02	

TABLE III. The theoretical ratios of the neutron decay width of ¹¹Be^{*} to the ¹⁰Be[0⁺] ground state as a fraction of the neutron decay width to the 2_1^+ first excited state. The ratios are shown for $J_i = 1/2$ to 9/2 for both even and odd parities. Also shown are the associated experimental relative branching ratios provides information on the relative reduced widths of the decaying ¹¹Be state.

J_i^π	E_x (MeV)				
	3.96	5.24	5.96	6.72	
1/2-, 3/2-	66.21	2.74	2.05	1.67	
$5/2^{-}, 7/2^{-}$	5.16	0.39	0.37	0.38	
9/2-	575.09	0.25	0.14	0.10	
$1/2^{+}$	10096.55	19.08	9.28	5.62	
$3/2^+, 5/2^+$	2.03	0.74	0.67	0.62	
$7/2^+, 9/2^+$	43.57	0.24	0.18	0.17	
Expt.	0.88(0.18)	< 0.33	< 0.13	< 0.20	

IV. DISCUSSION OF ¹¹Be STATES

In two-neutron transfer reactions four dominant ¹¹Be configurations are expected to be populated. These are the positive-parity states that can be described, in the shell-model limit, as an *sd*-shell neutron coupled to ¹⁰Be(g.s.) or ¹⁰Be(2⁺), the low-lying negative-parity states that are purely *p* shell in character and additional negative-parity states based on the ⁹Be \otimes (*sd*)² configuration. A summary of the ¹¹Be states populated in one- and two-neutron transfer reactions and in the β decay of ¹¹Li is shown in Table V. Also shown are the proposed configurations from Refs. [17,35]. It should be noted that the shell-model orbitals associated with *p* character form the basis for π -type molecular orbitals and *sd*-character σ -orbitals.

TABLE IV. The theoretical decay branch ratios for the neutron decay of the 6.72- and 8.82-MeV states of ¹¹Be to the ¹⁰Be[2⁺] first excited state as a fraction of the neutron decay to the quartet of states at ~6 MeV (2_2^+ , 1_1^- , 0_2^+ , 2_1^-). The numbers in the brackets represent the strength of the 2_2^+ , 1_1^- , 0_2^+ , and 2_1^- decay widths as a percentage of their total width. The ratios are shown for $J_i = 1/2$ to 9/2 for both the even- and odd-parity states. Also shown are the associated experimental relative branching ratios.

J_i^{π}	E_x (MeV)		
	6.72	8.82	
1/2-	2.12 (15, 85, 01, 00)	0.50 (26, 45, 23, 06)	
$3/2^{-}$	2.12 (15, 85, 01, 00)	0.37 (19, 34, 17, 30)	
$5/2^{-}$	14.05 (97, 03, 00, 00)	0.67 (34, 10, 01, 54)	
$7/2^{-}$	14.05 (97, 03, 00, 00)	1.25 (64, 20, 02, 15)	
$9/2^{-}$	3080.62 (100, 00, 00, 00)	0.96 (12, 01, 00, 87)	
$1/2^{+}$	1.73 (01, 34, 65, 00)	0.27 (09, 27, 39, 25)	
$3/2^{+}$	3.63 (85, 15, 00, 00)	0.79 (45, 26, 07, 23)	
$5/2^{+}$	3.63 (85, 15, 00, 00)	0.79 (45, 26, 07, 23)	
$7/2^+$	193.38 (99, 01, 00, 00)	0.80 (25, 03, 00, 72)	
$9/2^+$	193.38 (99, 01, 00, 00)	2.68 (84, 09, 00, 06)	
Expt.	9.78(4.65)	<0.12	

The 3.96- and 5.24-MeV states are both populated in the β decay of ¹¹Li [21–27] and have been assigned spins of $3/2^-$ and $5/2^-$, respectively [27]. Neither state is populated in one-neutron transfer reactions, whereas they are both populated in two-neutron transfer reactions [17,18]. This confirms their ⁹Be(g.s.) \otimes (*sd*)² structure. From Table III, for the 3.96-MeV [3/2⁻] state, the simple barrier penetrability argument indicates that it is 66 times more likely to neutron decay to the ¹⁰Be ground state than to the 2⁺₁ first excited state, whereas the 5.24-MeV [5/2⁻] state would neutron decay slightly less to the ground state than to the 2⁺₁ state. The

TABLE V. ¹¹Be states populated in 1*n* and 2*n* transfer reactions up to an excitation energy of 11 MeV [15,17,18] and in the β decay of ¹¹Li [27]. Also shown are the proposed configurations from Refs. [17,35].

2 <i>n</i> transfer		1n transfer	β decay of ¹¹ Li		Configuration
$\overline{E_x}$ (MeV)	J^{π}	E_x (MeV)	E_x (MeV)	J^{π}	
0.00	$1/2^{+}$	0.00			10 Be(g.s.) \otimes ($s_{1/2}$)
0.32	1/2-	0.32	0.32	$1/2^{-}$	10 Be(g.s.) \otimes ($p_{1/2}$)
1.78	$5/2^{+}$	1.78			10 Be(g.s.) \otimes ($d_{5/2}$)
2.69	3/2-	2.69	2.69	$3/2^{-}$	10 Be(2 ⁺) \otimes ($p_{1/2}$)
3.41	$(3/2^{-})$		3.41	$(3/2^{-})$	9 Be(g.s.) \otimes $(sd)^{2}$
3.89	5/2-		3.89	5/2-	$^{10}\text{Be}(2^+) \otimes (p_{1/2})$
3.96	3/2-		3.96	3/2-	9 Be(g.s.) \otimes $(sd)^{2}$
5.24	5/2-		5.24	5/2-	9 Be(g.s.) \otimes $(sd)^{2}$
5.96	$(5/2^{-}, 7/2^{+})$,	
6.72	$(7/2^{-})$				9 Be(g.s.) \otimes $(sd)^{2}$
7.10			7.03	$(5/2^{-})$	
8.04	$(3/2^{-}, 5/2^{-})$		8.02	3/2-	
8.82	$(9/2^{-})$		8.82	$3/2^{-}$	9 Be(g.s.) \otimes $(sd)^{2}$
9.23	• • •			,	
10.73	$(11/2^{-})$		10.6	$5/2^{-}$	⁹ Be(g.s.) \otimes (<i>sd</i>) ²

experimental relative branching ratio is $\Gamma_{0_1^+}/\Gamma_{2_1^+} = 0.88 \pm 0.18$ for the 3.96-MeV state and $\Gamma_{0_1^+}/\Gamma_{2_1^+} < 0.33$ for the 5.24-MeV state. Thus, the 3.96-MeV state has a significantly larger width for decay to the 2_1^+ , or a much smaller width for decay to the 0_1^+ , than expected from the penetrability calculations. For the 5.24-MeV state the decay to the 2_1^+ is slightly larger, or the decay to the 0_1^+ is slightly smaller, than expected from the penetrability calculations.

The 5.96-MeV state is prominent in two-neutron transfer reactions and is not observed in one-neutron transfer reactions [17,18]. Thus, it does not have a ¹⁰Be(g.s.) + *n* configuration. It is not populated by the β decay of ¹¹Li [21–27] and is therefore either a positive-parity state or a negative-parity state with a spin greater than 5/2. It strongly favors the decay to the 2⁺₁ state of ¹⁰Be over the decay to the ground state.

The 6.72-MeV state has previously been observed in two-neutron transfer reactions [15,17]. In Ref. [19] it was interpreted as being the 7/2⁻ member of a molecular rotational band built on the 3.96-MeV [3/2⁻] state. The spin is unknown, but it is not populated in the β decay of ¹¹Li [21–27] and is therefore either a positive-parity state or a negative-parity state with a spin greater than 5/2. The state is observed to neutron decay to the 2⁺₁ first excited state and to a state (or states) of ¹⁰Be at ~6 MeV. The three possible ¹⁰Be states, at an excitation energy of ~6 MeV, are the 2⁺₂, 1⁻₁, and 0⁺₂. However, due to the finite experimental resolution, decays to these individual states were not resolved.

If the spin of the state was firmly assigned and if the branching ratios of the 2^+_2 , 1^-_1 , and 0^+_2 states at ~6 MeV were measured individually, barrier penetrability calculations could then be used to provide evidence on the structure of the state. For example, if the state equally decayed to all three of the states at ~ 6 MeV, then Table IV could be used to show that for a spin of $7/2^-$ the neutron decay to the ${}^{10}\text{Be}[2^+_1]$ first excited state is 14.05 times more favorable than to the trio of states at ~ 6 MeV. This would suggest that the state decays as one would expect from the barrier penetrabilities. However, if the state decayed only to the 2^+_1 first excited state and the 1_1^- state, then the barrier penetrabilities could be used to show that for a spin of $7/2^-$ the neutron decay to the 2^+_1 state is 468 times more favorable than to the 1_1^- state. In this case the experimental ratio ($\Gamma_{2_1^+} / \Gamma_{1_1^-} = 9.78$) would show that the decay to the 1_1^- is much larger (or the decay to the 2_1^+ much smaller) than expected, indicating that the 6.72-MeV state has a structure similar to that of the 1_1^- state.

The 6.72-MeV state strongly decays to the 2_1^+ state but only weakly decays to the 0_1^+ state. However, the penetrability calculations of Table III show that the state should have a measurable width for decay to the 0_1^+ state. This characteristic is again similar to that of the 3.96- and 5.24-MeV states. This is interesting as the 0_1^+ and 2_1^+ states of ¹⁰Be have similar structures [3–5,7,8]. This suggests that all of these three ¹¹Be states may have a similar structure whose wave function has a stronger overlap with the 2_1^+ state than the 0_1^+ state. The commonality of the decay paths would be in general agreement with the interpretation of the three states being members of a molecular rotational band [19] (3.96 MeV 3/2⁻, 5.24 MeV 5/2⁻, and 6.72 MeV 7/2⁻). The 8.82-MeV state is populated in the β decay of ¹¹Li [21–27] and has been assigned a spin of $3/2^-$ [27]. This is different to Ref. [19], where the state is proposed to be the $9/2^-$ member of a molecular rotational band built on the 3.96-MeV $3/2^-$ state. In the latter instance, the state is proposed to have a ${}^9\text{Be}(\text{g.s.}) \otimes (d_{5/2})^2_{4^+}$ configuration that is expected to be strongly populated in two-neutron transfer reactions in accordance with dynamic matching conditions [36], as is experimentally observed. Thus, it is possible that this peak is associated with more than one state and that this region is occupied by both a $3/2^-$ and a $9/2^-$ state.

However, as with the β -decay studies, in the current measurement, the state is observed to have a strong decay branch to a 10 Be state (or states) at ~6 MeV and it is also observed to decay via the sequential emission of two neutrons via either or both of the 7.371- and 7.542-MeV ¹⁰Be states. The decay to the four states at ~ 6 MeV is unresolved as is the decay to the 7.37- and 7.54-MeV states. However, in the β -delayed neutron decay measurement of ¹¹Li by Hirayama *et al.* [27] the coincident detection of β 's, γ 's, and neutrons enables the branches to the individual states to be determined. Here, it was found that the state decays to the 6.26-MeV $[2_1^-]$ and 7.37-MeV $[3_1^-]$ states. Hence, the decay branches measured here for the 8.82-MeV state are very similar to those measured in β decay. However, this alone is insufficient to assign the spin and it remains a possibility that the state populated here is a member of the 3.96-MeV band.

By considering only the barrier penetrabilities, Table IV can be used to show that for a spin of $3/2^{-1}$ [27] the ratio for the decay to the 2_1^+ first excited state to the decay to the $2_1^$ is 1.23. Experimentally this ratio is less than 0.12. Thus, the decay to the 2_1^- is much larger than expected, or the decay to the 2_1^+ first excited state is much smaller than expected, from the penetrability calculations. In addition, the decay branches to the more compact ¹⁰Be ground state is small, if it exists at all. It is also interesting that the state does not decay to the other three states of ¹⁰Be at ~6 MeV. This suggests that the 8.82-MeV state and the 2_1^- and 3_1^- ¹⁰Be states may have comparable structures. For example, the 8.82-MeV state has a likely ⁹Be(g.s.) \otimes (sd)² configuration and the 2⁻₁ and 3⁻₁ states have likely ${}^{9}Be(g.s.) \otimes (sd)$ configurations, which are reached by the emission of a neutron from the sd shell. In a description that involves molecular orbitals this would be equivalent to a $(\pi)(\sigma)^2$ configuration in ¹¹Be decaying to $(\pi)(\sigma)$ in ¹⁰Be.

This is in agreement with previous theoretical models that have predicted that the 2_1^- and 3_1^- states are members of a molecular rotational band built on the 5.96-MeV $1_1^$ state [3–12]. This would suggest that the 8.82-MeV state is also a possible molecular structure. Furthermore, this state lies close to the $2\alpha + 3n$ decay threshold (8.89 MeV). Clearly, a definitive determination of the structure of the negative-parity ¹⁰Be states at ~6 MeV is key to understanding these structures, as is a firm spin assignment of the 8.82-MeV state populated in 2n transfer.

Recent calculations performed by Ito *et al.* on the ¹²Be system indicate that a variety of molecular or cluster structures appear close to the decay threshold [37]. For example, in the $2\alpha + 4n$ system, the ⁴He + ⁸He, ⁵He + ⁷He, and ⁶He + ⁶He

partitions were all found to be important. It is likely that ¹¹Be will have similar features, with the appearance of ${}^{4}\text{He} + {}^{7}\text{He}$ and ${}^{5}\text{He} + {}^{6}\text{He}$ configurations, which will give rise to cluster bands close to the α -decay threshold. It is possible that the 3.96- and 8.82-MeV $3/2^-$ states are associated with such structures. For example, the covalent exchange of neutrons has been argued to lower the kinetic energy, i.e., increasing the binding energy of the nucleus [3-6]. This may mean that the 3.96-MeV state is associated most strongly with the molecular structure. The appearance of the 8.82-MeV state close to the decay threshold may indicate that the influence of the covalent exchange is diminished, with one or more neutrons located more at one center than the other, and the state has become more a cluster state (as described in Ref. [37]). To summarize, the 3.96-MeV 3/2⁻, 5.24-MeV 5/2⁻, and 6.72-MeV 7/2⁻ states appear to have similar decay properties and thus may form a rotational band.

For the 8.82-MeV state, a $3/2^{-}$ assignment [27] has been made in the β -decay measurements and it has also been suggested from 2n transfer that a state at this energy has spin and parity $9/2^{-}$ [19] and is a continuation of the 3.96-MeV $3/2^{-}$ band. Thus, there are two possibilities. First, the states observed in the 2n transfer and β decay are different and the similarity of the decay patterns is coincidental. In this case the β decay populates a $(2s_{1/2})^2$ structure $(J^{\pi} = 3/2^{-})$ and the 2n populates a $(1d_{5/2})^2$ structure $(J^{\pi} = 9/2^{-})$. Two-neutron transfer would be better matched to populating *d* orbitals. The $3/2^{-}$ state would be a new bandhead and the $9/2^{-}$ state the continuation of the molecular band. Second, the two states are the same and have $J^{\pi} = 3/2^{-}$ as reflected in the decay branches measured here and in Ref. [27]. In either case a state

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at 8.82 MeV with $J^{\pi} = 3/2^{-}$ exists and has most likely a ${}^{9}\text{Be}_{\text{g.s.}} \otimes (2s_{1/2})^{2}$ structure.

V. SUMMARY

The two-neutron transfer reaction ${}^{9}\text{Be}({}^{16}\text{O}, {}^{14}\text{O})$ ${}^{11}\text{Be}[{}^{10}\text{Be} + n]$ has been used to measure the branching ratios of the excited states of ${}^{11}\text{Be}$. The theoretical decay branches of these states have been calculated under the consideration of the *R*-matrix barrier penetrabilities. Differences between the theoretical and experimental widths have been used to provide information on the relative reduced widths of the states. The decay widths and the structures of the excited states of the ${}^{10}\text{Be}$ daughter nucleus have been used to predict the structures of the decaying ${}^{11}\text{Be}$ states.

The 3.96-MeV $3/2^-$, 5.24-MeV $5/2^-$, and 6.72-MeV $7/2^-$ states appear to have similar decay properties and thus may form a rotational band. The 8.82-MeV $[3/2^-]$ state has been identified as a likely molecular/cluster bandhead. The 8.82-MeV state has also been identified in β decay of ¹¹Li and has been assigned a halo-type structure. In this instance there appears to be a close connection between molecular/cluster and halo states.

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