Charge radii in macroscopic-microscopic mass models of reflection asymmetry

H. Iimura[1] and F. Buchinger[2]

[1]Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan

[2]Physics Department, McGill University, Ernest Rutherford Building, 3600 University St., Montréal, Québec, H3A 2T8 Canada

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We show that the charge radii of reflection-asymmetric nuclei calculated in the frame of the finite-range droplet model are in better agreement with measured charge radii when reflection asymmetry is taken into account. However discrepancies between experimental and calculated changes in mean square charge radii still remain for some isotopic chains. These discrepancies cannot be removed by empirically including dynamic contributions to the quadrupole deformation.

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Nuclear mass models which have predictive power and can describe several properties of all nuclei with a consistent set of parameters are required for many applications. Modeling the properties of the thousands of nuclei involved in stellar nucleosynthesis processes, as the rapid-neutron capture process, is just one of the examples. The finite-range droplet model (FRDM) and the finite-range liquid-drop model (FRLDM) [1–5] are presently the most sophisticated and successful macroscopic-microscopic approaches to mass models. Within the FRLDM, Möller and co-workers performed in 1995 the first calculations of ground-state reflection asymmetry for nuclei across the nuclear chart [5]. Their calculation of reflection asymmetry involved two steps: In the initial step the ground-state ε_2 and $\varepsilon_4$deformations were determined from the potential energy surfaces calculated as a function of these deformation parameters; then, in the second step, the energy was minimized with respect to reflection asymmetry (ε_3), both with $\varepsilon_2$and ε_4 held fixed at their previously determined values. Later they also considered axial-asymmetry effects within the same model, and found that, for those nuclei where reflection or axial asymmetry was found, a systematic deviation between calculated and measured masses was removed [6]. Recently they have refined the calculation of reflection asymmetry considerably by identifying the lowest minimum in a four-dimensional deformation space in the coordinates ε_2, ε_3, ε_4, and ε_6. Deformation parameters determined in this fashion are tabulated in Ref. [7].

Since octupole deformation also affects directly the rms charge radius, it is certainly interesting to investigate a possible improvement between predicted and experimental radii using the new extended data set for nuclear ground state deformations from Ref. [7]. Such a comparison serves at the same time as a cross check of the reliability of the models used, since rms charge radii are easily obtained from mass models, but are not included as input data when the optimum model parameters are determined. In our earlier paper [8], we have investigated how the rms charge radii are affected when axial asymmetry is included in the FRDM calculations. In the present report we present the reflection asymmetry effect on charge radii in the same format that we used for the calculation of the axial-asymmetry effects. This type of statistical analysis has not been performed before.

None of the FRDM [4,5] or FRLDM [2,3,5] mass tabulation gave charge radii, although they can easily be calculated by using the prescriptions outlined in the paper of Myers and Schmidt (MS) [9]. Our previous calculations [8,10,11] of FRDM charge radii were based on that paper. In the FRLDM the rms charge radii are given by simplified forms of the MS expressions, both compression and Coulomb-redistribution effects being absent. We have pointed out previously that using the FRLDM gives a significantly worse agreement with measured charge radii [12]. Thus, in this work, we use the FRDM to calculate the new rms charge radii from the tabulated deformation parameters [7] including reflection asymmetry, although these parameters were obtained in the FRLDM. The deformation parameters can be assumed to be not dependent on whether they are found by using the FRDM or FRLDM. This is the same assumption we made in our previous study on axial-asymmetry effects [8], where the FRDM was used to calculate the charge radii from the deformation parameters determined in the FRLDM. Also in mass calculations, Möller and co-workers determined the deformation parameters by using the FRLDM, and then calculated the masses with the FRDM from these deformations [5].

Because the detailed expressions for calculating the rms charge radii are given in our previous papers [10,11], we simply recall that the mean-square charge radius (R^2) is written as

$$R^2 = \langle r^2\rangle_u + \langle r^2\rangle_r + 3b^2 + s_p^2, \qquad (1)$$

where $\langle r^2\rangle_u$ is the contribution from the size of the uniform distribution, $\langle r^2\rangle_r$ the contribution from the Coulomb redistribution, b the surface-diffuseness parameter ($b = \sqrt{2}a_{\rm den}$, where $a_{\rm den}$ is given in Ref. [5]), and $s_p = 0.8$ fm is the rms charge radius of the finite proton. The values of the model parameters used in the calculations are taken from the 1995 mass fit [5] as it was also done in Ref. [11]. Only the surface-diffuseness parameter is reduced from the original value of $b = 0.99$ fm [5] to 0.93 fm, because the radius calculated by the FRDM is always larger than the experimental value [11]. This new b value was determined in our previous work [8] by adjusting R^2 to all of the available experimental charge radii (a total of 782), and was used to calculate the charge radii of axially asymmetric nuclei.

In order to examine how reflection asymmetry influences the rms charge radii, we include octupole deformation (β_3) and describe the shape of the proton distribution (r) by

$$r = nR_Z(1 + \beta_2 Y_{20} + \beta_3 Y_{30} + \beta_4 Y_{40}), \quad (2)$$

where n ensures volume normalization and R_Z is the sharp radius for the proton distribution. We assumed axial asymmetry because the calculated decrease in energy due to triaxiality is less than 0.01 MeV for the nuclei which we study here [7]. By defining the shape as given in Eq. (2), the uniform-distribution part of the mean-square radius can be expressed as

$$\langle r^2\rangle_u = \frac{3}{5}R_Z^2\left(1 + \alpha_2^2 + \frac{10}{21}\alpha_2^3 + \frac{5}{7}\alpha_3^2 - \frac{27}{35}\alpha_2^4 + \frac{10}{7}\alpha_2^2\alpha_4 + \frac{5}{9}\alpha_4^2 + \frac{20}{21}\alpha_2\alpha_3^2\right), \quad (3)$$

where

$$\alpha_l = \sqrt{\frac{2l+1}{4\pi}}\beta_l. \quad (4)$$

Equation (3) is the same as given in Ref. [13], except that we added the α_2^4 term so that it reduces to Eq. (22) of MS [9] for reflection-symmetric nuclei ($\alpha_3 = 0$). For the redistribution part of the mean-square radius, we use the same expression as that of MS [9]. Since in Ref. [7] the deformation parameters are tabulated in terms of ε_2, ε_3, and ε_4, we transform the ε parameters to the α parameters in Eq. (3) by using Eq. (4) and the relations between ε and β given in Refs. [14,15]. By using integration given by Eq. (38) in Ref. [5], Möller also converted ε to β [16]. Since there is no significant difference between the results of our conversion and Möller's, we use the β parameters by Möller [16] to calculate the charge radii.

Möller and co-workers calculated masses for several thousands of nuclei from very light (^{16}O) to the heaviest nuclei, and found that there are about 410 nuclei for which reflection asymmetry lowers the mass ($\Delta E_{\varepsilon 3}$) by $\Delta E_{\varepsilon 3} \geqslant 0.01$ MeV [7]. Among those, we study 53 nuclei for which experimental charge radii are available from the most recent and most comprehensive compilation of charge radii [17]. Our results for the difference between the experimental ($R_{\rm exp}$) and calculated ($R_{\rm th}$) charge radii of these 53 nuclei are shown in Fig. 1. In the upper part of the figure we present the difference $R_{\rm exp} - R_{\rm th}$ when $R_{\rm th}$ is calculated including reflection-asymmetric shapes. For comparison we have also calculated the difference $R_{\rm exp} - R_{\rm th}$ neglecting reflection asymmetry. The deformation parameters used for the calculation of $R_{\rm th}$ were taken from Möller's table [16], which were found in a three-dimensional deformation space (ε_2, ε_4, and ε_6), with ε_3 set to zero. The results are shown in the lower part of Fig. 1.

In Fig. 1 the rms deviation σ and mean deviation ε for ($R_{\rm exp} - R_{\rm th}$) of the 53 nuclei considered here is indicated. If we calculate $R_{\rm th}$ without taking into account reflection asymmetry, we obtain an rms deviation of $\sigma = 0.0220$ fm and a mean deviation of $\varepsilon = 0.0137$ fm. When reflection asymmetry is included in the calculation of $R_{\rm th}$, these deviations reduce to $\sigma = 0.0181$ fm and $\varepsilon = 0.0050$ fm as shown in Fig. 1, implying an improved agreement with the experimental radii. However it is evident from Fig. 1 that systematic deviations are still present for nuclei with mass around $A = 150$. The Pearson-Goriely collaboration has constructed a series of mass models based on the Hartree-Fock-Bogoliubov method with Skyrme forces, the force parameter being fitted to the mass data. Their latest model is HFB-15 [18], within which they have calculated the charge radii for several thousands of nuclei across the nuclear chart. For the 53 nuclei considered here, the deviations corresponding to this model are found to be $\sigma = 0.0159$ fm and $\varepsilon = -0.0001$ fm. It is surprising that such a good agreement is achieved since the octupole degree of freedom is not included in the HFB-15 calculation.

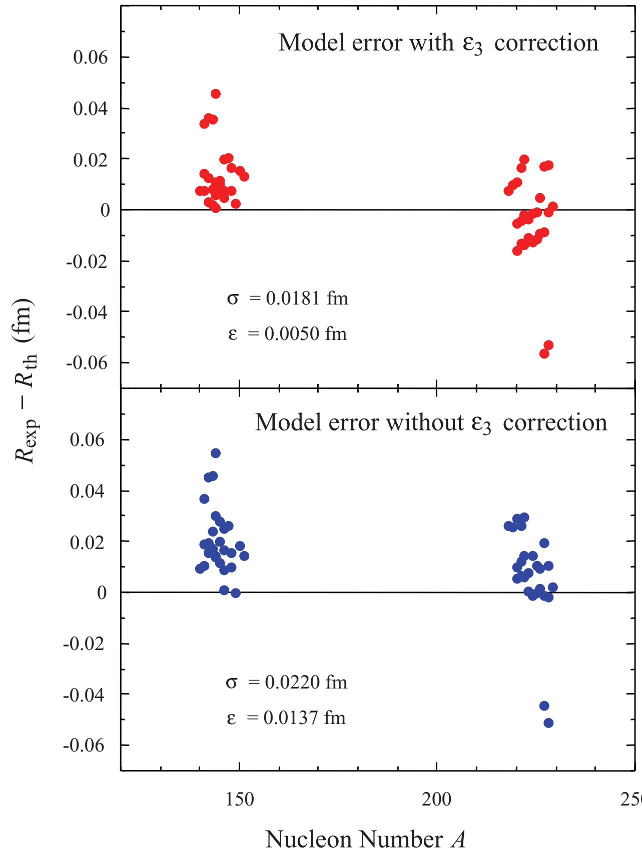

FIG. 1. (Color online) Difference between experimental ($R_{\rm exp}$) and calculated ($R_{\rm th}$) rms charge radii, when the FRDM calculation for $R_{\rm th}$ (i) does not include reflection-asymmetric shape (bottom panel) or (ii) does account for reflection-asymmetric shape (top panel), versus A. The plots include 53 nuclei where reflection asymmetry lowers the ground-state mass by more than 0.01 MeV [7].

For an additional comparison of the predictions of the FRDM calculations with the experimental values of the charge radii, we now consider relative values δR^2 along isotopic chains. Since in the FRDM the surface-diffuseness does not have any isotopic dependence, and since the rms charge radius of the finite proton, s_p is constant too, such a comparison is independent of these parameters. In Fig. 2 we compare experimental changes of mean-square charge radii (δR^2) for several isotopic chains to the results of the FRDM calculations with and without reflection asymmetry. The experimental data is obtained from Ref. [17]. We take the isotope with $N = 82$ or 126 as the reference isotope, which are the

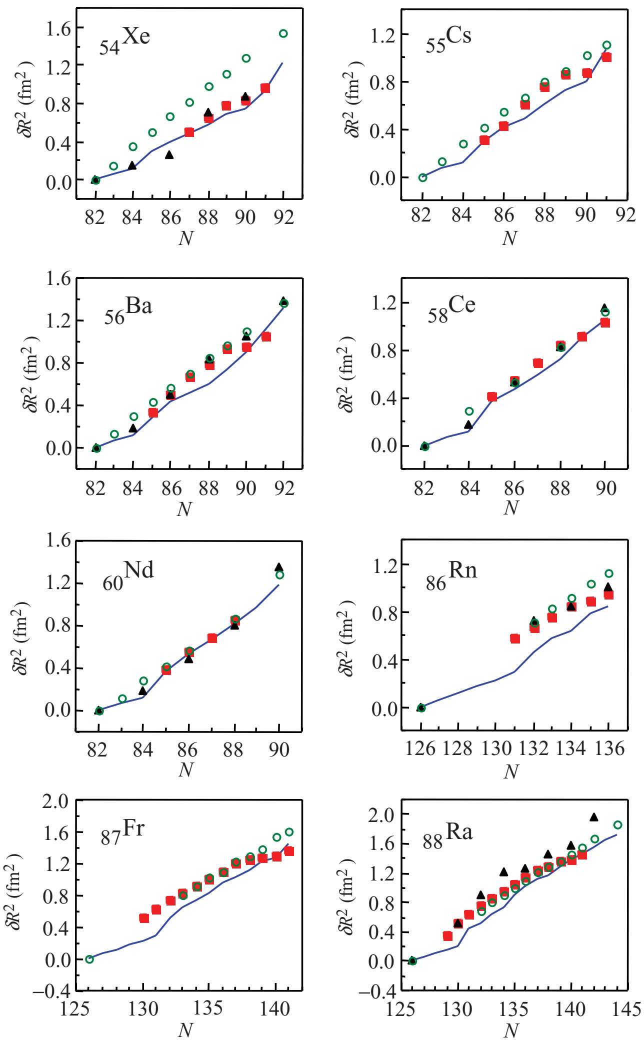

FIG. 2. (Color online) Experimental results for changes of mean-square charge radii, together with two FRDM predictions for the Xe, Cs, Ba, Ce, Nd, Rn, Fr, and Ra isotope chains. The open circles represent the experimental data points. The predictions not considering reflection-asymmetric shape are shown by the solid line. The filled squares represent the predictions when the octupole degree of freedom is taken into account in the calculations. Only nuclei for which reflection asymmetry lowers the mass by more than 0.01 MeV are included in the plots. For some even-even nuclei, where β_2 was obtained from the $B(E2)$ values, the predictions are indicated as filled triangles.

same reference isotopes as chosen in the compilation of the experimental data in Ref. [17] except for ^{137}Cs and ^{213}Fr. As before, we have considered only nuclei where the masses are lowered by more than 0.01 MeV when reflection-asymmetric shapes are taken into account. Overall, when the effect of reflection asymmetry is taken into account, the agreement of the theoretical predictions with the experimental data becomes better. For some nuclides such as the Xe isotopes, the Rn isotopes with $N > 134$, and the Fr isotopes with $N > 139$, there still remain discrepancies between experimental and calculated δR^2. Often those discrepancies are attributed to the omission of dynamic contributions to mean-square charge radii in the calculations, since the β parameters from the FRLDM calculations contain only static deformations. However the reduced electric quadrupole transition probability, $B(E2)$, from the ground state to the first excited 2^+ state of even-even nuclides provide direct information on deformation including shape fluctuation, and have been measured for many nuclei. In order to include the dynamic properties in the radius calculations, we replace the β_2 parameters determined in the FRLDM, with those deduced from the experimental $B(E2)$ values using

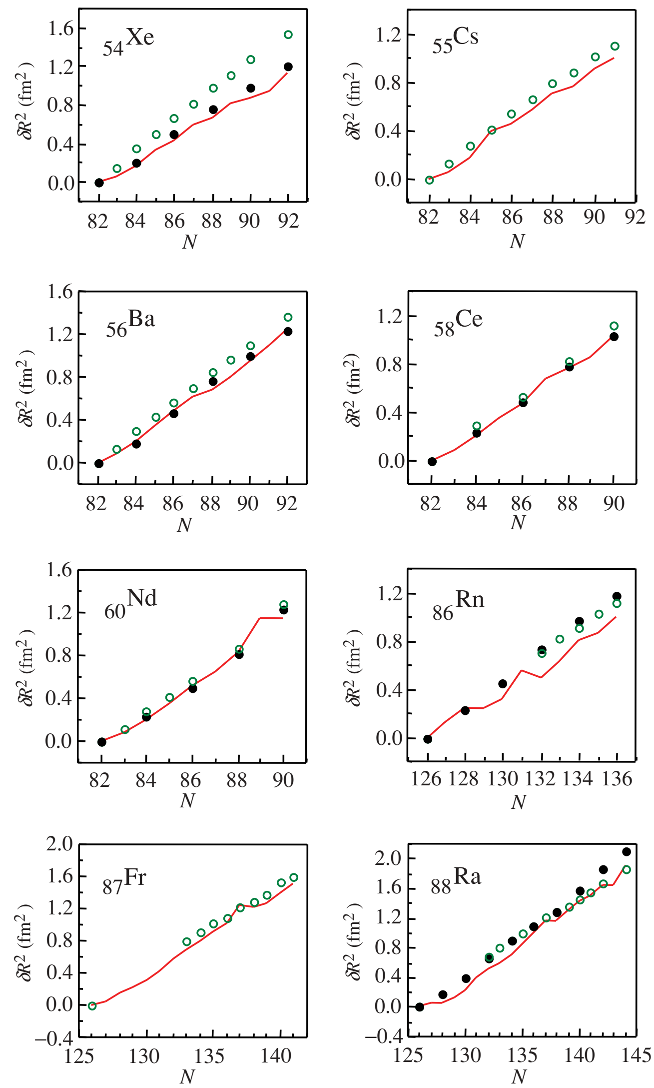

FIG. 3. (Color online) Changes of mean square charge radii for the Xe, Cs, Ba, Ce, Nd, Rn, Fr, and Ra isotopes given by the HFB- 15 (solid line) and RMF (closed circle) calculations. Open circles refer to measured values.

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\sqrt{B(E2)} \tag{5}$$

with $R_0 = 1.2A^{1/3}$. Experimental $B(E2)$ values are taken from the compilation in Ref. [19]. The $B(E2)$ values for ^{138,142,144}Xe, ^{148}Ba, ^{212,218}Rn, and ^{214,220,230}Ra are predictions based on an empirical relation between the excitation energy of the first-excited 2^+ state and $B(E2)$ [19]. As to β_3 and β_4, we use the static deformation determined with the FRLDM mass formula [5,7]. The results are shown by the filled triangles in Fig. 2. The difference between the predictions from the $B(E2)$ and those using the β_2 parameters determined within the FRLDM is small. This means that the remaining discrepancies between experimental and calculated δR^2 are not due to dynamical-deformation effects but represent rather a limitation of the FRDM model.

In Fig. 3 we compare the changes of mean-square charge radii calculated by the HFB-15 mass model [18] to the experimental values. The agreement is comparable to that obtained by the FRDM including reflection asymmetry. Also shown in Fig. 3 for even-even nuclei are the results of the relativistic mean-field (RMF) calculations using the effective force NL3 [20]. The changes of mean-square charge radii predicted from this model are close to the prediction from the HFB-15. As stated for the absolute charge radii, it is surprising that the HFB-15 and RMF calculations reproduce well the experimental data because these models do not take into account reflection asymmetry. It would be interesting how much the mean-square charge radii are affected when reflection asymmetry is included into these calculations.

In conclusion, we have calculated the FRDM charge radii using the deformation parameters determined with the FRLDM mass formula considering reflection-asymmetric shape. Absolute charge radii, calculated including the octupole degree of freedom are in better agreement with experimental results than those calculated without taking into account reflection asymmetry. The changes of charge radii along isotope chains also show that the inclusion of reflection asymmetry in the radius calculations better reproduces the experimental data. By using the β_2 parameters deduced from the experimental $B(E2)$ data, we confirmed that the remaining discrepancies between experimental and calculated relative charge radii can not be accounted for by shape fluctuations. It is also found that charge radii calculated by the HFB-15 and RMF are in good agreement with experimental data, although these models do not consider octupole deformations.

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