Dynamical simulation of energy dissipation in asymmetric heavy-ion induced fission of ^{200}Pb, ^{213}Fr, and ^{251}Es

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The dynamical model based on the asymmetric mass division has been applied to calculate pre-scission neutron multiplicity from heavy-ion induced fusion-fission reactions. Links between the pre-scission neutron multiplicity, excitation energy, and asymmetric mass distribution are clarified based on the Monte Carlo simulation and Langevin dynamics. The pre-scission neutron multiplicity is calculated and compared with the respective experimental data over a wide range of excitation energy and nonconstant viscosity. The analysis indicates a different effect for the application of asymmetric mass division in different energy regions of such processes.

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I. INTRODUCTION

In the recent century the development of nuclear accelerators has been preparing heavy-ion beams for fusion-fission reactions [1] and opened an opportunity to study fission processes induced by a heavy ion. Here we have focused on fusion-fission reactions and energy dissipation of an excited compound nucleus, since that there were many unsolved issues about fission processes, such as the difference between experimental data and theoretical results about rate and the manner of energy dissipation in heavy-ion fusion-fission reactions (a schematic diagram of such reactions is shown in Fig. 1).

We know that aside from the distance between mass centers of nascent fragments, the asymmetry parameter and shape elongation play a crucial role in the study of fusion-fission and deep inelastic processes of especially low energy heavy ion collisions [2]. We have studied the whole dynamics of such a nuclear system by using the Monte Carlo approach to solve the coupled Langevin-type equations of motion to gain access to general features of particle emission and energy dissipation of an energetic heavy-ion compound nucleus. A number of early studies of pre-scission neutron multiplicity and energy dissipation of such reactions can be found in the literature [1–5]. For instance, Frobrich and Gontchar [6], in an approach based on the dissipation effects, used Langevin equations to estimate the energy dissipated in heavy ion fusion-fission reactions and the number of neutrons which were emitted during descent from saddle point to scission point. Later, Chaudhury and Pal discussed [7] the competition of neutron and γ-ray emission during fission based on the dynamical approach. Recently, Lemaire and co-workers developed a theory to study the prompt fission neutron and γ-ray evaporation process, where they are emitted sequentially from fragments [8,9]. The study of neutron emission accompanying asymmetric fission has been highlighting the effective role of dissipation in both statistical and dynamical approaches. Generally it should be noted that for light nuclei with fissilities below the Businaro-Gallone point, asymmetric mass splitting is favored [10,11]. However our Monte Carlo approach allows us to compare our results with various prompt fission neutron observables, such as the energy of emitted neutrons and neutron multiplicity distribution.

The paper is arranged as follows. The theoretical methodology and the input parameters are presented in Sec. II. This is followed by a presentation and discussion of the results for three systems with different fissilities in Sec. III. Our model is applied to the reaction systems ^{30}Si + ^{170}Er, ^{16}O + ^{197}Au, and ^{19}F + ^{232}Th with ^{200}Pb, ^{213}Fr, and ^{251}Es, respectively, as a compound nucleus, in a wide range of the incident energy (corresponding to the excitation energy of the compound nucleus). Finally, the calculated results are discussed in Sec. IV with a brief conclusion.

II. THEORETICAL APPROACH

In this section the Monte Carlo technique is used to simulate compound nucleus deexcitation. Our strategy differs with existing literature in one main respect: we consider the asymmetry issue for systems prior to fission by using a 3D Langevin equation, a general equation of dissipation, and a nonconstant viscosity, jointly.

A. Asymmetric dynamical model (ADM)

The dynamical time evolution of the fission process and energy deexcitation of the compound nucleus during descent from saddle to scission constitutes a complex issue. We used the Langevin equation which by considering “funny hills” parametrization (c, h, α) [12] and its conjugate momentum p as the dynamical variables, gives these equations as [7]

$$\frac{dq_i}{dt} = \frac{p_j}{m_{ij}}, \tag{1}$$

$$\frac{dp_i}{dt} = -\frac{p_i\, p_j}{2}\frac{\partial}{\partial q_i}\left(\frac{1}{m_{ij}}\right) - \frac{\partial F}{\partial q_i} - \gamma_i \frac{dq_i}{dt} + R(t). \tag{2}$$

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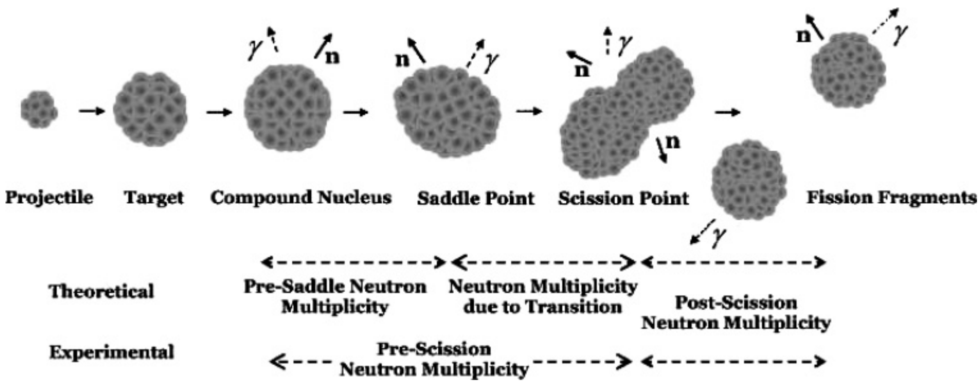

FIG. 1. Schematic diagram of particle emission in a typical heavy-ion induced fusion-fission reaction.

In the above equations $q = (c, h, \alpha)$ are the collective coordinates and $p = (p_c, p_h, p_\alpha)$ are the conjugate momenta. m and γ are the shape-dependent collective inertia and friction coefficients, respectively. Also i and j refer to funny hills parameters. The neck thickness and asymmetry parameter are denoted by h and α as follows [13]:

$$h = -1.047\,c^3 + 4.297\,c^2 - 6.309\,c + 4.073 \tag{3}$$

and

$$\alpha = 0.11937\alpha_{\rm asy}^2 + 0.24720\alpha_{\rm asy}, \tag{4}$$

where $\alpha_{\rm asy} = (A_1 - A_2)/A_{\rm C.N.}$. Also c, F, and $R(t)$ represent the elongation, free energy of the system, and random part of the interactions between the fission degree of freedom and thermal bath [14]. Also A_1 and A_2 refer to nascent fission fragments. Following the work of Frobrich and Gontchar [6], we solved these equations by coupling with the neutron and γ emission at each time step evolution of fission accompanying the Monte Carlo scheme which is often the only practical way to evaluate such difficult calculations. For the parametrization of the nuclear surface we used a parametric family of shapes based on the cylindrical coordinates as follows [13]:

$$\rho_s^2(z) = (c^2 - z^2)(A_s/c^2 + B_{\rm sh}\, z^2/c^2 + \alpha\, z), \tag{5}$$

where ρ_s is the radial coordinate of the nuclear surface and z is the coordinate along the symmetry axis. A_s and $B_{\rm sh}$ are defined in Ref. [13]. Although it was already pointed out earlier that one-body dissipation is the dominant mode of energy damping in nuclear fission in describing fission dynamics [15], for a general description, however, we have used the following expressions to consider both one- and two-body dissipation [13,16,17]:

$$\gamma_i^{\rm TB} = \pi\,\mu\,R_{\rm C.N.}\,f_i \int_{-c}^{+c} \rho_s^2(z)\big[3A_i'^2 + \rho_s^2(z)\,A''^2_i/8\big]\,dz \tag{6}$$

and

$$\gamma_i^{\rm OB} = 2\pi\,\rho_m\,\bar{v}\,R_{\rm C.N.}^2\,f_i \int_{-c}^{+c} \rho_s(z)[A_i\,\rho_s' + A_i'\,\rho_s/2]^2\left[1 + \rho_s'^2\right]^{-1/2} dz. \tag{7}$$

Here $\gamma_i^{\rm OB}$ and $\gamma_i^{\rm TB}$ refer to one- and two-body dissipation, respectively, and the value of the viscosity coefficient μ used in the present calculation may be expressed in terms of a nonlinear function of E/A and mass of the compound system ($A_{\rm C.N.}$) as the following form [18]:

$$\mu = a\frac{E}{A} + bA_{\rm C.N.}^3. \tag{8}$$

The values of the parameters $a = (1.80 \pm 0.23) \times 10^{-24}$ [sec fm^{-3}] and $b = (3.57 \pm 0.26) \times 10^{-30}$ [MeV sec fm^{-3}] have been obtained through least square fitting of the viscosity coefficients for all the systems studied in this paper. Also $R_{\rm C.N.}$ is the radius of compound nucleus and f_i defined as follows [13]:

$$f_i = \left(\frac{\partial\, q_i}{\partial\, x}\right)^2 + 2\,\frac{\partial\, q_i}{\partial\, x}, \tag{9}$$

where $x = r_{\rm c.m.}/R_{\rm C.N.}$ and the parameter $r_{\rm c.m.}$ is defined as the center to center distance between the two parts of the fissioning system. Also $A_i(z)$ defined as follows:

$$A_i(z) = -\frac{1}{\rho^2(z)}\,\frac{\partial}{\partial q_i}\int_{-c}^{z} \rho^2(z')dz'. \tag{10}$$

The quantities A_i' and A_i'' are the first and second derivatives of $A_i(z)$ with respect to z [13]. Also ρ_m is the nuclear density, $\bar{v}$ is the average nucleon speed inside the nucleus [13], and ρ_s' is the first derivative of ρ_s with respect to z, also i refers to funny hills parameters. Based on Eqs. (6) and (7) we calculate the general dissipation as follows:

$$\gamma_i = \gamma_i^{\rm TB} + \gamma_i^{\rm OB}. \tag{11}$$

To compare the competition between neutron emission, γ-ray emission, and fission we need their decay widths. Several approaches have been used in earlier works to describe the emission of particles such as neutrons from a highly excited nucleus, here we used Weisskopf's conventional evaporation theory for neutrons [20] and Lynn's theory for the emission of giant dipole γ-ray [19]. The neutron decay width is calculated as follows [20]:

$$\Gamma_n = \frac{2m_n}{(\pi\hbar)^2\rho_m(E_{\rm int})}\int_0^{E_{\rm int}-B_n} d\varepsilon\rho_d(E_{\rm int} - B_n - \varepsilon)\varepsilon\sigma_{\rm inv}, \tag{12}$$

where ε is the energy of the emitted neutron and $E_{\rm int}$ is the intrinsic excitation energy of the parent nucleus. Also ρ_m and ρ_d are the level densities of the compound and residual nuclei that are defined by [21]

$$\rho(E_{\text{int}}, A, l) = \frac{(2l+1)\sqrt{a}}{12E_{\text{int}}^2}\left[\frac{\hbar^2}{2J_0}\right]^{3/2}\exp(2\sqrt{aE_{\text{int}}}) \quad (13)$$

Where l is the angular momentum of the compound or residual nuclei, σ_{inv} is the inverse cross section [22], B_n is the binding energy of neutron and the level density parameter denoted by a. Also J_0 is the moment of inertia [6]. The γ-ray decay width at each time step is calculated using the following relation [6]

$$\Gamma_\gamma = \frac{3}{\rho_m(E^*)}\int_0^{E_{\text{int}}} d\varepsilon \rho_d(E_{\text{int}} - \varepsilon) f(\varepsilon), \quad (14)$$

where ε is the energy of the emitted γ-ray and $f(\varepsilon)$ is defined by [6]

$$f(\varepsilon) = \frac{0.74e^2NZ}{A\hbar mc^3}\frac{5\varepsilon^4}{(5\varepsilon)^2 + (\varepsilon^2 - (80A^{-1/3})^2)^2}. \quad (15)$$

Here A, Z, and N refer to the compound nucleus.

B. Monte Carlo dynamical simulation

In this reaction process, we first specify the entrance channel through which a compound nucleus is formed by assuming complete fusion of the target with the projectile. The initial spin of the compound nucleus will be obtained by sampling the fusion spin distribution [6]. As the fully equilibrated compound nucleus is formed at a certain instant that is fixed as the origin of our dynamical trajectory calculation and following earlier literature, we assumed that the initial distribution of the coordinates and momenta are chosen from sampling random numbers following the Maxwell-Boltzmann distribution [6,13,14]. The process of neutron and γ emission from a compound nucleus is governed by the emission rate such as Eqs. (12) and (14). The widths of neutron, γ, and fission decay rates depend upon the excitation energy, spin, and the mass number of the compound nucleus and hence are to be evaluated at each interval of time evolution of the fissioning nucleus due to Langevin equations. The Monte Carlo algorithm used to calculate the competition between neutron emission, γ-ray emission, and fission. To do this we first choose a random number r on the half open interval [0, 1) by using Monte Carlo techniques. The random number is a numerical characteristic assigned to an element of the sample space. Then we define the probability of emission of a neutron as $x = \tau/\tau_n$, where τ_n is the neutron decay time and τ is the time step of the calculation. If $r < x$, it will be interpreted as a particle emission. Following the same procedure the type of emitted particle is decided by the Monte Carlo selection based on the law of radioactive decay for the emitted particles. After each emission the intrinsic excitation energy of residual mass and spin of the compound nucleus recalculated due to the energy that was released based on the one particle emission. This circle of calculations is repeated for typically 50000 Langevin trajectories and until it reaches a scission point ($c = c_{\text{sci}}$):

$$c_{\text{sci}} = -2.0\,\alpha^2 + 0.032\,\alpha + 2.0917. \quad (16)$$

As stated earlier we calculate the pre-scission neutron multiplicity for each α, finally the average of these multiplicities

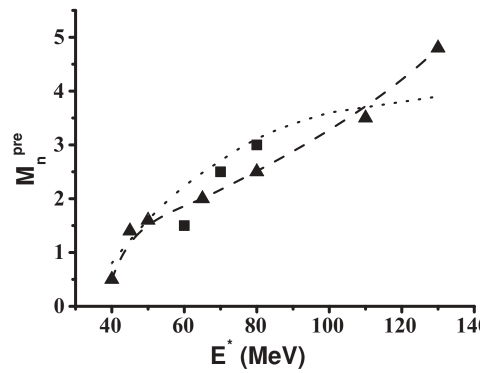

FIG. 2. Variation of pre-scission neutron multiplicity versus excitation energy in our dynamical (dashed line) and statistical (dotted line) calculations is compared with earlier experimental and theoretical results [6,24] that are shown with filled squares and triangles for ^{200}Pb.

shows the pre-scission neutron multiplicity for that system. Therefore the average pre-scission neutron multiplicity is given by

$$\langle M_n^{\text{Pre}}\rangle = \frac{\sum_\alpha \sum_l \langle M_n^{\text{Pre}}\rangle_{l,\alpha}(2l+1)P_l}{\sum_l (2l+1)P_l}, \quad (17)$$

where the probability to cross the fission barrier which depends upon angular momentum is denoted by P_l:

$$P_l = \frac{N_l}{N}. \quad (18)$$

Here N and N_l are the total number of trajectories and the number of trajectories which undergo fission, respectively. Summation over α is defined in an interval $[0, \alpha_f]$ and summation over l is defined in an interval $[0, l_f]$, where α_f and l_f refer to maximum asymmetry and critical angular momentum for fusion, respectively.

III. RESULTS

We have mainly considered heavier mass nuclei since it is for these nuclei that the neutron and fission widths become

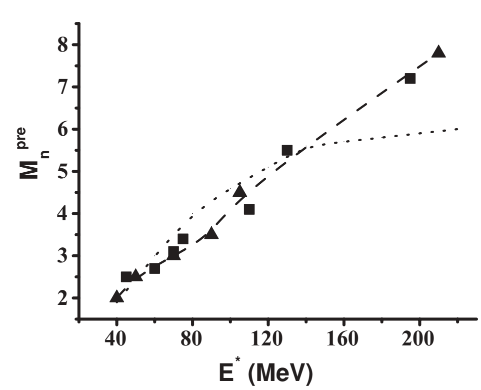

FIG. 3. Pre-scission neutron multiplicity of ^{213}Fr as function of excitation energy. Results of our dynamical (dashed line) and statistical (dotted line) calculations are compared with earlier experimental and theoretical results (filled squares and triangles) [5,6].

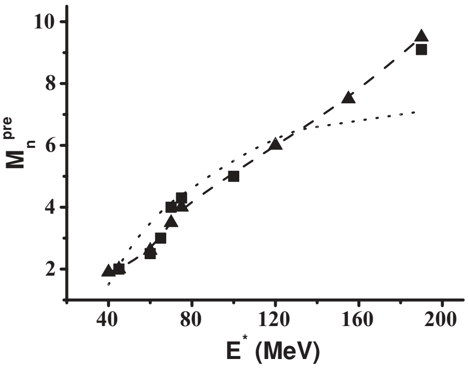

FIG. 4. Variation of pre-scission neutron multiplicity due to excitation energy. Results of our dynamical (dashed line) and statistical (dotted line) calculations are compared with earlier experimental and theoretical results (filled squares and triangles) [5,6] for ^{251}Es.

comparable and their competition strongly dictates the final observables. By considering the alternative Monte Carlo, and directly comparing the corresponding neutron multiplicity of various systems, we study the influence of the asymmetry parameter on the deexcitation of energy. The systems typically chosen for the present work are ^{30}Si + ^{170}Er, ^{16}O + ^{197}Au, and ^{19}F + ^{232}Th in the mass range of 200–260. By considering ^{200}Pb, ^{213}Fr, and ^{251}Es as our expected compound nucleus, the calculated values of pre-scission multiplicities are shown along with the respective experimental data for comparison in Figs. 2, 3, and 4. However, as earlier literature mentioned in-medium excitation energy the pre-scission neutron multiplicities for light and medium heavy nuclei could be reasonably well explained without any considerable revision in the asymmetric statistical model. With an increase in bombarding energy, theoretical calculations without considering viscosity (dotted lines in Figs. 2–4) systematically underpredict the experimental pre-scission neutron multiplicities. Thus the values of the viscosity needed to reproduce the pre-scission neutron multiplicity data are found to have a strong system dependence for ^{200}Pb, ^{213}Fr, and ^{251}Es. Earlier studies show that the angular distributions together with the pre-scission neutron multiplicities could be one of

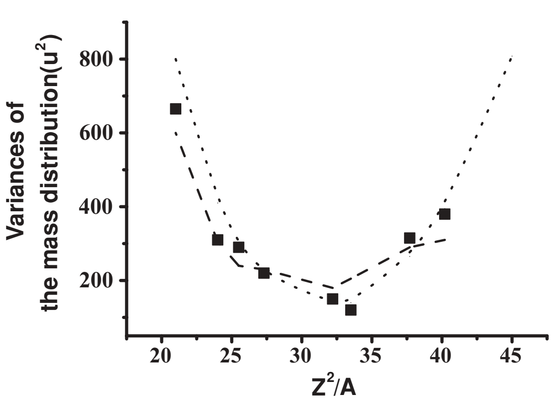

FIG. 5. The variances of the mass distributions of the fission fragments versus the parameter Z^2/A. Results of our calculation (dashed line for asymmetric mode and dotted line for symmetric mode) are compared with experimental data (squares) [24].

TABLE I. Calculated variance of the mass distributions of fission fragments for the fission (σ_M^2) and the mean pre-scission neutron multiplicity $\langle M_n^{\mathrm{pre}} \rangle$ of ^{215}Fr and ^{256}Fm formed in the reactions: ^{18}O + ^{197}Au ($E_{\mathrm{lab}} = 159$ MeV) and ^{18}O + ^{238}U ($E_{\mathrm{lab}} = 159$ MeV).

C.N.	E^*(MeV)	k_s	$\sigma_M^2(u^2)$	$\langle M_n^{\mathrm{pre}} \rangle$	Ref.
^{215}Fr	111	0.25	331 ± 13	3	[23]
		0.5	276 ± 6	4.3	[23]
			272	4.1	[25]
			243	4.5	This work
^{256}Fm	101	0.25	353 ± 12	2	[23]
		0.5	283 ± 10	3.1	[23]
			543	5.1	[5]
			310	3.9	This work

the sensitive probes not only to the magnitude but also to the mechanism of nuclear viscosity [23] and the calculated mass distribution becomes narrower when viscosity increases, this effect has been explained in detail early in Ref. [23]. But we remarked that the pre-scission neutron multiplicities are sensitive to the viscosity magnitudes in the high-energy region of asymmetric heavy-ion fusion-fission reactions. In Fig. 5 the calculated values of the variances of the fission-fragment mass distributions σ_M^2 in both symmetric and asymmetric modes are compared with experimental data as a function of the parameter Z^2/A. This figure shows good quantitative agreement between the asymmetric mode and experimental data for the interval $20 < Z^2/A < 30$. But for nuclei with $Z^2/A > 30$ our calculations in the asymmetric mode did not reproduce the observed experimental growth of variance. Since, as earlier studies showed, if the descent occurs during a finite time, the fissioning system preserves the memory of the large values of σ_M^2. In order to verify our hypothesis we carried out such calculations for a few typical systems. Results are summarized in Table I. The extension of our theoretical scheme for rotating compound nuclei will improve the agreement between the experimental data and theoretical results.

IV. SUMMARY AND CONCLUSION

We have developed a dynamical model for fission where trajectories are generated by solving Langevin equations of motion using both one- and two-body dissipations jointly. The choice of a Monte Carlo simulation to describe this processes allows us to infer important physical quantities that could not be assessed otherwise, and can also be used in assessing in particular the validity of assumptions about how the available total excitation energy gets distributed among the asymmetric fragments. Among the various physical inputs required for solving the Langevin equation with a Monte Carlo algorithm, we paid more attention to the asymmetry parameter. As shown in Figs. 2–4 in medium energy and for heavier systems with larger fissility we see agreements between our calculation and earlier results. To conclude, in the present model and in medium excitation energy with the generalized shape parametrization and general dissipation, both symmetric and asymmetric splittings of the compound nucleus can be treated on the same footing. Also despite the strong friction on the collective motions of nucleons before scission both earlier statistical calculations and our approach (ADM) give good equality with experimental neutron multiplicity especially at small and medium energies and large fissility. By increasing the energy the pre-scission neutron multiplicity is found to be higher than those calculated using a statistical model, which is consistent with the experimental observations. Finally the theoretically pre-scission neutron multiplicity, averaged over the wide range of mass distributions and energy, is found to be significantly higher than those calculated using a statistical saddle point model. The calculations seem to prove that considering the nonconstant viscosity effect, its effect will appreciate by increasing the energy aside from the effect of the asymmetry parameter that plays a crucial role in achieving these results. By increasing the amount of fissility, we see less difference between ADM, statistical calculations, and experimental observations in high energies. With all the experimental progress in the heavy-ion fusion-fission reactions, what can we anticipate for the future? Of course there will be a steady improvement in the precision and confidence with which we can determine the appropriate fission model and its parameters.

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