Structure of $A = 82$ analogs and isospin-symmetry-breaking effects on superallowed Fermi β decay

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We study the effects of the isospin-symmetry breaking on the superallowed Fermi β decay of the ground state of ^{82}Nb to ^{82}Zr. Results on the analog as well as nonanalog β-decay branches are self-consistently obtained within the *complex* excited VAMPIR approach. The $^{82}\text{Nb} \rightarrow {}^{82}\text{Zr}$ β decay to the first two excited 0^+ states with significant strength is predicted to coexist with the superallowed decay. The structure and electromagnetic properties of the analog and nonanalog low- and high-spin states in ^{82}Nb and ^{82}Zr are compared with the available experimental data.

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I. INTRODUCTION

The investigation of the structure and dynamics of exotic nuclei around the $N = Z$ line in the $A \simeq 80$ mass region is one of the most exciting challenges in low-energy nuclear physics today. Apart from displaying some rather interesting nuclear structure effects, the superallowed $0^+ \rightarrow 0^+$ Fermi β decay of these nuclei is a valuable tool in probing many properties of the weak interaction. Superallowed Fermi β decays between $0^+ T = 1$ analog states provide tests of the validity of the conserved-vector-current (CVC) hypothesis and the unitarity of the Cabibbo-Kobayashi-Maskawa matrix (see Ref. [1] and the references therein). Since the late 1990s, intense theoretical effort has been devoted to the investigation of the superallowed Fermi β decay [1–6] and several experimental programs have been initiated to measure the half-lives and branching ratios for the decays of odd-odd $N = Z$ nuclei with $A \geqslant 62$ where the charge-dependent correction terms are expected to be large.

The β^+-decay half-life for the odd-odd $N = Z$ nucleus ^{82}Nb has been measured following the fragmentation of a primary ^{92}Mo beam at the GANIL laboratory and the reported value is $T_{1/2} = 52(6)$ ms [7]. Extending the mass range over which the CVC hypothesis can be tested this is one of the heaviest $N = Z$ nuclei for which superallowed Fermi β decay was established. In this study we examine the charge-symmetry-breaking effects on the superallowed Fermi β decay of ^{82}Nb to ^{82}Zr using the complex excited VAMPIR variational approach.

The charge independence of nuclear forces requires that the energy spectra in pairs of mirror nuclei are identical. Small differences between energy levels could then be interpreted as isospin-symmety-breaking effects. To illustrate these effects at low and high spins and to test the quality of the complex excited VAMPIR description of the properties of these states we investigated the even-spin positive-parity states up to 20^+ in the mirror nuclei ^{82}Zr and ^{82}Nb.

Calculations based on the variational approaches of the VAMPIR family have been successfully performed for the description of a variety of nuclear structure phenomena in the $A = 60$–90 mass region, not only in nuclei along the valley of β-stability but also in some exotic nuclei close to the proton drip line [8–16]. The complex excited VAMPIR approach allows for a unified description of low- and high-spin states, including in the projected mean fields neutron-proton correlations in both the $T = 1$ and $T = 0$ channels and general two-nucleon unnatural-parity correlations. The oblate-prolate coexistence and mixing, the variation of the deformation with mass number, increasing spin, as well as excitation energy have been compared with the available experimental information. Because the VAMPIR approaches enable the use of rather large model spaces and of general two-body interactions, large-scale nuclear structure studies going far beyond the abilities of the conventional shell-model configuration-mixing approach are possible. Our previous investigations on microscopic aspects of shape coexistence in $N \simeq Z$ nuclei in this mass region indicated the presence of a strong competition between particular configurations based on large and small oblate and prolate quadrupole deformations. Furthermore, as expected, because in $N \simeq Z$ nuclei neutrons and protons fill the same single particle orbits, the neutron-proton pairing correlations were found to play an important role [9]. However, the theoretical results suggest that certain properties of these nuclei are extremely sensitive to small variations of particular parts of the effective Hamiltonian [10,11].

We briefly describe the complex excited VAMPIR variational procedure and define the effective Hamiltonian in the next section. In Sec. III we then discuss the results on isospin-symmetry-breaking corrections for superallowed Fermi β decay of ^{82}Nb to ^{82}Zr. Results on the low- and high-spin states, including electromagnetic properties, will be compared for the ^{82}Zr and ^{82}Nb with the available data in Sec. IV. Finally we present some conclusions in Sec. V.

II. THEORETICAL FRAMEWORK

The complex excited VAMPIR approach uses Hartree-Fock-Bogoliubov (HFB) vacua as basic building blocks, which are restricted only by time reversal and axial symmetry. The underlying HFB transformations are essentially complex and do mix proton with neutron states as well as states of different parity and angular momentum. The broken symmetries of these vacua (nucleon numbers, parity, total angular momentum) are restored by projection techniques

and the resulting symmetry-projected configurations are then used as test wave functions in chains of successive variational calculations to determine the underlying HFB transformations as well as the configuration mixing. The HFB vacua of the above type account for arbitrary two-nucleon correlations and thus simultaneously describe like-nucleon as well as isovector and isoscalar proton-neutron pairing.

For nuclei in the $A \simeq 80$ mass region we use a ^{40}Ca core and include the $1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 1d_{5/2}$, and $0g_{9/2}$ oscillator orbits for both protons and neutrons in the valence space. We start with an isospin symmetric basis and then introduce the Coulomb shifts for the proton single-particle levels resulting from the ^{40}Ca core by performing spherically symmetric Hartree-Fock calculations using the Gogny-interaction D1S in a 21 major-shell basis [4].

The effective two-body interaction is constructed from a nuclear matter G matrix based on the Bonn one-boson-exchange potential (Bonn A/Bonn CD). This G matrix was modified by adding short-range (0.707 fm) Gaussians with strength -35 MeV in the $T = 1$ channel and a Gaussian with strength -70 MeV in the $T = 0$ channel to enhance the pairing correlations. In addition the isoscalar interaction was modified by monopole shifts of -500 keV for all $T = 0$ matrix elements of the form $\langle 1p1d_{5/2}; IT = 0|\hat{G}|1p1d_{5/2}; IT = 0\rangle$, where $1p$ denotes either the $1p_{1/2}$ or the $1p_{3/2}$ orbit. For the matrix elements of the form $\langle 0g_{9/2}0f; IT = 0|\hat{G}|0g_{9/2}0f; IT = 0\rangle$ monopole shifts of -400 keV and -250 keV have been added for the ones involving $0f_{5/2}$ and the $0f_{7/2}$ orbitals, respectively. These monopole shifts have been introduced in our earlier calculations to influence the onset of deformation. Previous results indicated also that the oblate-prolate coexistence and mixing at low spins depend on the strengths of the neutron-proton $T = 0$ matrix elements involving nucleons occupying the $0f_{5/2}$, $0f_{7/2}$, and $0g_{9/2}$ single-particle orbits.

III. SUPERALLOWED FERMI β DECAY OF ^{82}Nb

The β^+-decay half-life for the odd-odd $N = Z$ nucleus ^{82}Nb has been measured at GANIL and the reported value is $T_{1/2} = 52(6)$ ms [7]. The *log* ft value of 3.53(5), indicating the superallowed Fermi character, was deduced assuming a single ground-state-to-ground-state transition, 100% branching ratio, and extracting the β-decay Q value from the systematics ($Q_{\mathrm{EC}} = 11.220(0.592)$ MeV [17]).

The isospin-symmetry-breaking correction that has to be applied to the measured "ft value" of a $0^+ \rightarrow 0^+$ β transition between $T = 1$ analog states to obtain the coupling constant G_v is δ_c in the relationship [1]

$$ft(1+\delta_R)(1-\delta_c) = \frac{K}{2G_v^2\left(1+\Delta_R^v\right)},$$

where f is the statistical rate function, t is the partial half-life for the transition, δ_R and Δ_R^v are the radiative corrections, and K is a known constant.

Our strategy for investigating the isospin-symmetry-breaking effects on the superallowed Fermi β decay of ^{82}Nb is as follows. We consider three different Hamiltonians: the H_0 for the charge-symmetric case, where the single-particle levels for protons and neutrons are identical and the above defined effective two-body interaction is based on the Bonn A potential. To investigate the Coulomb effect we added to H_0 the two-body matrix elements of the Coulomb interaction between the valence protons and the Coulomb contributions to the single-particle energies resulting from the ^{40}Ca core. The corresponding Hamiltonian will be denoted by H_1 in the following. To include also the isospin-mixing effects due to charge-symmetry breaking in the strong interaction we replaced the Bonn A by Bonn CD potential and denoted this Hamiltonian by H_2.

To investigate the superallowed Fermi β decay of ^{82}Nb we calculated the lowest 30 0^+ states in ^{82}Zr and ^{82}Nb using the three above defined Hamiltonians. First the VAMPIR solutions, representing the optimal mean-field description of the yrast states by single symmetry-projected HFB determinants, have been obtained. Then the excited VAMPIR approach was used to construct additional excited states by independent variational calculations. The final solutions have been obtained diagonalizing the residual interaction between the successively constructed orthogonal configurations. The variational procedure that we use involves projection before variation on particle number, angular momentum, and parity but not on isospin. Thus, even if the charge symmetric Hamiltonian H_0 is used, good total isospin can be expected only if the configurations form a complete set under isospin rotations. This is obviously not the case, if only the 30 lowest 0^+ states are considered. Furthermore, as has been discussed [4], the symmetry-projected configurations created by the complex excited VAMPIR approach account for arbitrary two-nucleon correlations; however, because of the assumption of time reversal and axial symmetry some four- (and more) nucleon correlations are missing. Thus even if the number of configurations would be drastically increased, there is still some "spurious isospin impurity" to be expected.

The results obtained for the 0^+ states in ^{82}Zr and ^{82}Nb indicate a variable, sometimes very strong, mixing of prolate and oblate deformed configurations in the final wave functions for all three above-defined Hamiltonians. The amount of mixing for the lowest five calculated 0^+ states are presented in Table I for ^{82}Zr and in Table II for ^{82}Nb. In these tables we indicate the amount of mixing for the excited VAMPIR configurations contributing more than 2% of the total amplitude.

For the charge-symmetric case one expects degenerate isovector excitation spectra for the two nuclei. Furthermore, one expects for the superallowed Fermi transitions from the ground state of ^{82}Nb to the ground states of ^{82}Zr both a total strength of two and (because of orthogonality) vanishing strengths for all the transitions from the ground state of ^{82}Nb to all excited states of ^{82}Zr. Deviations from these values as well as from the degeneracy of the spectra can then be attributed to isospin-mixing effects. In Fig. 1 we present the relative spectra for the lowest 20 calculated 0^+ states, if the charge-symmetric Hamiltonian H_0 is used. The spectrum of odd-odd nucleus ^{82}Nb displays slight deviations of the order of about 300 keV with respect to the even-even nucleus ^{82}Zr and the difference between the ground-state energies amounts to 91 keV. These differences of the energies indicate that the configuration space is not complete with respect to

TABLE I. The amount of mixing for the lowest calculated 0^+ states of ^{82}Zr. The prolate components are indicated in boldface notation.

Charge-symm. Ham. o mixing/**p mixing**	Bonn A + Coulomb o mixing/**p mixing**	Bonn CD + Coulomb o mixing/**p mixing**
4(2)%/**91**%	3%/**94**%	3%/**94**%
38(15)%/**20**(**16**)(**5**)%	7(2)%/**87**(**2**)%	7%/**88**(**2**)%
2%/**74**(**17**)%	41(10)%/**25**(**8**)(**5**)(**5**)%	51(4)%/**22**(**8**)(**6**)(**4**)%
39(34)(2)%/**10**(**6**)(**5**)%	40(10)%/**33**(**10**)(**3**)%	20(14)(2)%/**35**(**18**)(**3**)%
28(7)%/**44**(**7**)(**4**)(**2**)%	23(2)%/**43**(**16**)(**5**)%	9(6)%/**46**(**17**)(**14**)(**2**)%

isospin rotations partly because of truncation, partly because of "missing" four- (and more) nucleon couplings in the excited VAMPIR solutions. As expected, this deficiency is reflected in the strengths of the Fermi transitions, too. For the total strength (S_T) of the transitions from the Nb ground state to all the calculated 30 0^+ states of the Zr nucleus we obtain 1.9740 with 1.9642 for the ground-to-ground transition (S_{g-g}). Thus in between 1.30 [(2-S_T)/2] and 1.79 [(2-S_{g-g})/2]% of the expected strength is not accounted for due to truncation and/or inherent isospin impurity effects. As it is expected the total strength S_T increased continuously from 1.9308 (for one projected determinant per state), adding more and more configurations in the excited VAMPIR basis.

To obtain the Coulomb-induced isospin-mixing effects we calculated the lowest 30 0^+ states in the two nuclei using the H_1 Hamiltonian. The Coulomb interaction removes the degeneracy and induces the energy difference between the ground states of the two nuclei. The corresponding relative excitation energies for the lowest 20 calculated 0^+ states are displayed in the second column of Figs. 2 and 3 for ^{82}Zr and ^{82}Nb, respectively. The calculated 0^+ states are spread in this case over 8.9 MeV excitation energy in ^{82}Zr and 9.4 MeV in ^{82}Nb. The $Q_{\rm EC}$ value obtained using the H_1 Hamiltonian amounts to 10.496 MeV. For the total Fermi-transition strength for the Nb ground-state decay to the Zr states one obtains 1.9761. Thus the total strength is almost identical to that obtained without the Coulomb interaction. For the ground-to-ground transition one now obtains 1.9357. Thus the effect of the Coulomb interaction $\{\alpha = [S_{g-g}(H_0) - S_{g-g}(H_1)]/2\}$ for the Nb to Zr ground-to-ground transition amounts to $\alpha = 1.43\%$. Most of this depleted strength is concentrated in the nonanalog branches to the first two excited states. Estimating the error by summing the missing strengths between all and the analog transitions for the charge symmetric case $\{\epsilon_1 = [S_T(H_0) - S_{g-g}(H_0)]/2\}$ and the missing total strengths between the calculations performed with H_0 and H_1 $\{\epsilon_2 = [S_T(H_0) - S_T(H_1)]/2\}$ one may conclude that for the ground-to-ground transition a depletion of at least 0.83 and at most 1.43% is obtained. A nonanalog branch feeding the first excited 0^+ state at 1.862 MeV in Zr in the range of 0.30–0.90% from the total strength and a weaker one with an upper limit of 0.33% feeding the second excited 0^+ state at 2.178 MeV are obtained.

Finally, the isospin-mixing effects, including the charge-symmetry breaking in the strong interaction, were investigated replacing the Bonn A by Bonn CD potential (H_2). For ^{82}Zr the amount of mixing presented in Table I and the relative spectra for the lowest 20 0^+ states in Fig. 2 indicate differences with respect to the results obtained using the H_1 Hamiltonian. Larger deviations from the charge-symmetric case are also observed comparing the structure of the wave functions and the relative spectra obtained using the Bonn A + Coulomb and Bonn CD + Coulomb potential for ^{82}Nb in Table II and Fig. 3, respectively. Using the H_2 Hamiltonian the lowest 30 calculated 0^+ states are spread over 9.0 MeV excitation energy in ^{82}Zr and 9.7 MeV in ^{82}Nb. The $Q_{\rm EC}$ value obtained using the H_2 Hamiltonian amounts to 10.291 MeV. As it is indicated in Table III the total Fermi β strength for the Nb ground-state decay to the Zr states amounts to 1.9752, almost identical to that obtained without the Coulomb interaction. For the ground-to-ground transition one obtains now 1.9293. As it is expected, the isospin-symmetry-breaking effect is larger if we take into account the charge dependence of the strong force beside the Coulomb-induced isospin mixing. Estimating the error in the same way as for the H_1 Hamiltonian, one obtains a depletion of the ground-to-ground decay from Nb to Zr in between 1.20 and 1.75% of the sum rule strength and two nonanalog branches to the first two 0^+ excited states in Zr, of similar strengths: the nonanalog branch to the first excited state at 1.711 MeV in the

TABLE II. The amount of mixing for the lowest calculated 0^+ states of ^{82}Nb. The prolate components are indicated in boldface notation.

Charge-symm. Ham. o mixing/**p mixing**	Bonn A + Coulomb o mixing/**p mixing**	Bonn CD + Coulomb o mixing/**p mixing**
4(2)%/**91**%	3%/**94**%	3%/**94**%
48(21)%/**18**(**7**)%	40(9)%/**37**(**5**)(**3**)(**2**)%	71(4)%/**6**(**5**)(**4**)(**3**)(**3**)%
3%/**86**(**3**)(**2**)(**2**)%	22(3)%/**56**(**10**)(**3**)(**2**)%	5%/**88**%
48(31)%/**7**(**6**)(**3**)%	44(18)%/**17**(**5**)(**5**)(**4**)%	42(7)%/**34**(**3**)(**3**)(**2**)%
8(6)(2)%/**61**(**7**)(**6**)(**2**)(**2**)%	18(2)%/**40**(**26**)(**6**)(**2**)%	23%/**38**(**27**)(**5**)%

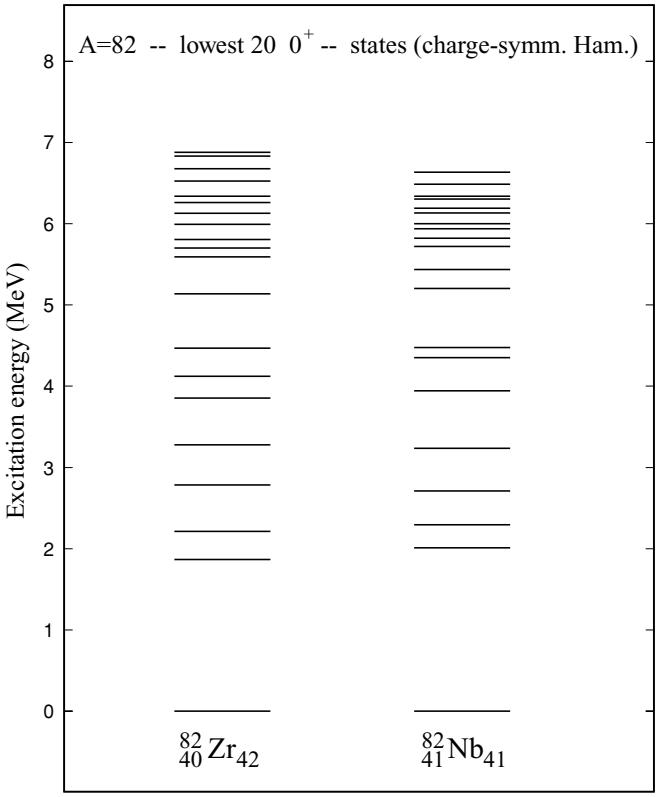

FIG. 1. Comparison of the theoretical spectrum of the lowest 20 0^+ states in ^{82}Zr and ^{82}Nb obtained within the complex excited VAMPIR approach using the charge-symmetric effective Hamiltonian (H_0).

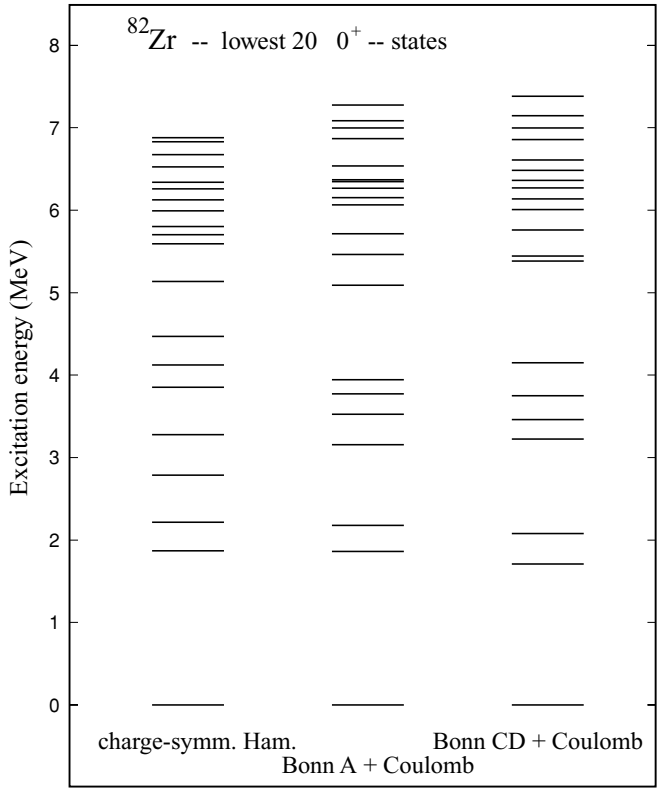

FIG. 2. The relative spectrum of the lowest 20 0^+ states in ^{82}Zr, using charge-symmetric and charge-dependent effective Hamiltonians.

range of 0.08–0.63% and the one to the second excited state at 2.079 MeV in the range of 0.11–0.66% of the total strength. The results on isospin-symmetry-breaking correction for the Fermi β decay of the ground state of ^{82}Nb to the analog state as well as the strengths of the nonanalog branches to the first two excited 0^+ states in ^{82}Zr are summarized in Table IV.

The results reported here represent the first calculations of this type able to estimate the significant nonanalog branches for the ^{82}Nb to ^{82}Zr decay. To reduce the error in the estimation of the isospin mixing correction more effort will be devoted in the near future for improving the accuracy of the wave functions of the involved 0^+ states increasing the dimension of the excited VAMPIR many-nucleon basis.

It is worthwhile mentioning that in the frame of the excited VAMPIR approach the many-body wave functions for the involved 0^+ states are independently calculated applying the projection before the variation for each considered nucleus. All

TABLE III. The total (S_T) and analog (S_{g-g}) Fermi β-decay strengths for the charge-symmetric, Bonn A + Coulomb, and Bonn CD + Coulomb effective Hamiltonian.

Charge-symmetric		Bonn A + Coulomb		Bonn CD + Coulomb	
S_T	S_{g-g}	S_T	S_{g-g}	S_T	S_{g-g}
1.9740	1.9642	1.9761	1.9357	1.9752	1.9293

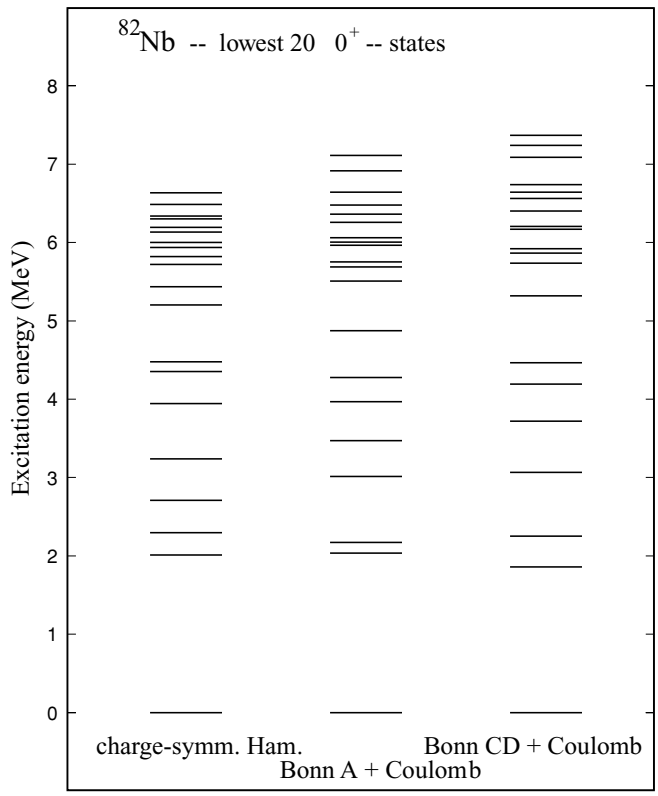

FIG. 3. The same as in Fig. 2 but for ^{82}Nb.

TABLE IV. The isospin-symmetry-breaking correction and nonanalog strengths for the β decay of ^{82}Nb to 0^+ states in ^{82}Zr using Bonn A + Coulomb and Bonn CD + Coulomb effective Hamiltonian (in percentage of the total strength).

β branch	Bonn A + Coulomb		Bonn CD + Coulomb	
	$E_x(0^+)$ (MeV)	δ_c(%)	$E_x(0^+)$ (MeV)	δ_c(%)
$0^+_g \to 0^+_g$		0.83–1.43		1.20–1.75
$0^+_g \to 0^+_1$	1.862	0.30–0.90	1.711	0.08–0.63
$0^+_g \to 0^+_2$	2.178	$\leqslant$0.33	2.079	0.11–0.66

the investigated effects should be described by the corresponding wave functions if the many-nucleon basis is large enough and the effective interaction is adequate for the employed model space in a given mass region. We use the correct isospin operator $\tau_+ = \sum_\alpha a^+_\alpha b_\alpha$, where α represents any single-particle state of the model space and a^+_α creates neutrons and b_α annihilates protons in the state α. Then the Fermi matrix element is given by $M_F = \langle f|\tau_+|i\rangle$ with $|i\rangle$ and $|f\rangle$ the initial and final eigenstates of the Hamiltonian, respectively. To take into account the mixing to one-particle-one-hole isovector monopole states we have to enlarge the model space to allow at least $2\hbar\omega$ excitations, a problem involving huge computational effort that we are currently addressing.

IV. LOW- AND HIGH-SPIN STRUCTURE OF THE ^{82}Zr AND ^{82}Nb MIRROR NUCLEI

To make a comparison of the structure of low- and high-spin isobaric analog states in the ^{82}Zr and ^{82}Nb mirror nuclei we calculated the positive-parity states up to spin 20^+ using the above defined H_1 Hamiltonian.

Coexistence of two or more stable shapes at comparable excitation for a given spin and parity has been known in the $A \simeq 80$ mass region. The most deformed nuclei in the region have been observed for the charge numbers 38 and 40. High-spin states in ^{82}Zr have been investigated experimentally with the Gammasphere array [18] and lifetimes of low- and intermediate-spin states have been measured [18–20]. The first experimental information on the first excited states in the odd-odd nucleus ^{82}Nb were obtained at GSI [21].

TABLE V. $B(E2; I \to I-2)$ values (in e^2 fm^4) for the yrast states of ^{82}Zr and ^{82}Nb.

$I^\pi(\hbar)$	^{82}Zr		^{82}Nb
	Experiment [20]	Theory	theory
2^+	2328(1058)	1322	1274
4^+	1672(360)	1970	1897
6^+	2539(1058)	2138	2064
8^+	2328(635)	2174	2042
10^+	1926(487)	2129	1912
12^+	1904(635)	1974	1824
14^+	>610	1733	1696
16^+		1472	1479
18^+		809	774
20^+		808	

The available data suggest that ^{82}Zr exhibits shape coexistence. We investigate self-consistently the influence of the oblate-prolate shape coexistence and mixing with increasing spin in the frame of the complex excited VAMPIR approach. We calculated the lowest 14 excited VAMPIR configurations for the states up to spin 4^+ and 9 orthogonal configurations up to spin 20^+ in ^{82}Zr and up to 18^+ in ^{82}Nb. In ^{82}Zr as well as in ^{82}Nb the lowest two states up to spin 14^+ are prolate deformed and the third orthogonal projected configuration is oblate deformed in the intrinsic system. After the diagonalization of the residual interaction the solution for the yrast state at each investigated spin is built out of the lowest excited VAMPIR configuration, but for the yrare states a variable, sometimes strong, mixing was obtained. The states are organized in bands according to the $B(E2)$ values connecting them. In Table V the $B(E2)$ values for the yrast band in ^{82}Zr are compared with the available data [20] and with the results for ^{82}Nb.

In Fig. 4 we present the spectrum for the lowest three calculated bands in ^{82}Zr. The yrast spectrum for ^{82}Zr reveals a rather good agreement between theory and experiment. In Fig. 5 we present the calculated spectrum for ^{82}Nb and the lowest states of positive parity assigned experimentally. A direct comparison of the yrast positive-parity band in the two mirror nuclei is presented in Fig. 6. The irregularities observed in the data for ^{82}Zr are also seen in the theoretical spectrum at spins 12^+ and 18^+, as well as in ^{82}Nb. Strong similarities can be observed for the $B(E2)$ values of the yrast band in the two nuclei as can be observed from Table V. The spectroscopic quadrupole moments for the bands calculated in ^{82}Zr are presented in Table VI. The effective charge is 0.4 for neutrons and 1.4 for protons.

TABLE VI. Spectroscopic quadrupole moments $Q_2^{\rm sp}$ (in e fm^2) for the states of the lowest bands in ^{82}Zr.

I^π	p(po)	p(po)	po(op)
2^+	−74.9	−77.2	−38.7
4^+	−95.8	−98.0	22.5
6^+	−106.0	−106.9	−15.7
8^+	−109.7	−105.3	64.6
10^+	−109.5	−94.7	70.6
12^+	−105.1	−82.1	68.2
14^+	−98.6	−10.8	49.3
16^+	−71.4	−62.1	7.3
18^+	−37.4	1.0	55.2
20^+	−23.44	−52.55	57.43

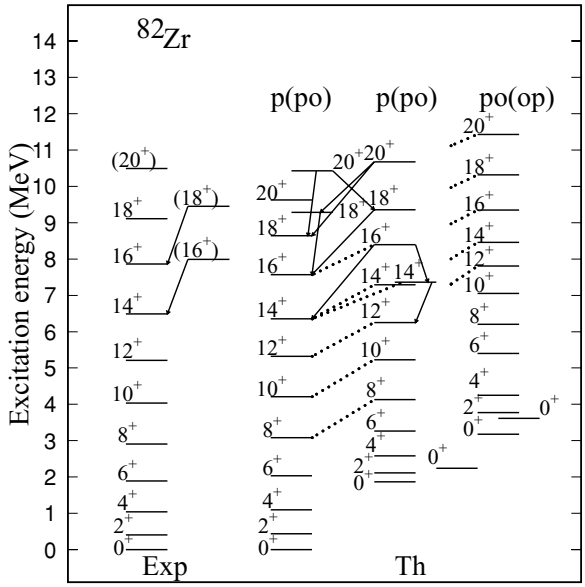

FIG. 4. The theoretical spectrum of the positive-parity states of ^{82}Zr obtained within the complex excited VAMPIR approach using the effective Hamiltonian H_1 is compared with the available data [18]. The dotted lines indicate M1, $\Delta I = 0$ transitions.

The third band manifests oblate-prolate mixing at spins 0^+, 2^+, 4^+, and 6^+ but at higher spins the oblate components dominate the structure of the wave functions in both ^{82}Zr and ^{82}Nb as can be seen from Tables VII and VIII where we present the amount of mixing for the two nuclei. However, the structure of the wave functions along the lowest two bands, as well as the quadrupole moments, indicate that above spin 14^+ in the yrast band and above 12^+ in the first excited band in ^{82}Zr significant oblate mixing is present. The same behavior is obtained for ^{82}Nb above spin 16^+ in both the yrast and the first excited band. It is worthwhile mentioning that significant $M1$ transitions compete with the in-band and intraband $E2$ transitions for the

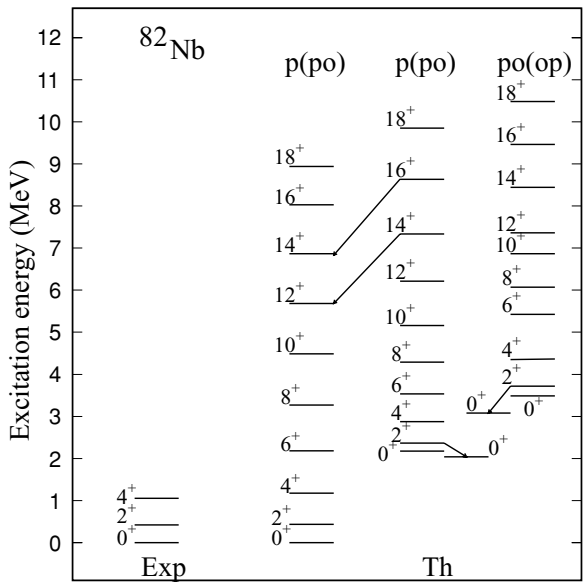

FIG. 5. The same as in Fig. 4 but for the ^{82}Nb nucleus.

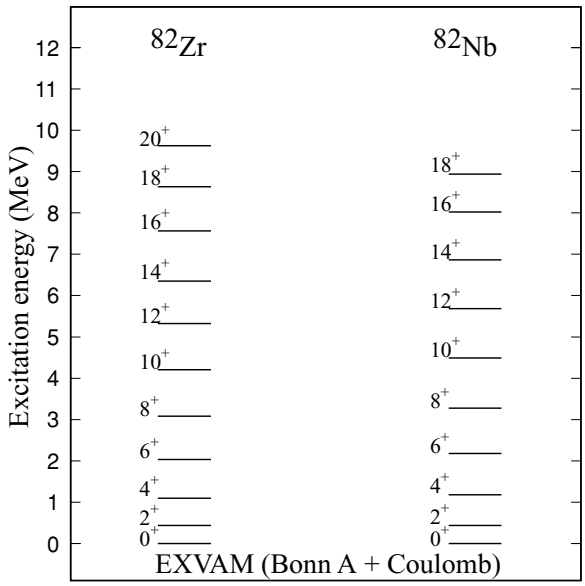

FIG. 6. A direct comparison of the yrast positive-parity spectrum of the mirror nuclei ^{82}Zr and ^{82}Nb.

intermediate- and high-spin states. The $B(E2)$ strengths for the secondary branches indicated in Fig. 4 represent at least 5% of the total $E2$ decaying strength for a given state in ^{82}Zr. The $B(M1; \Delta I = 0)$ strengths for the transitions connecting the second with the yrast band vary from 0.73 μ_N^2 for the 12^+ state up to 0.31 μ_N^2 for the decay of the 16^+ state. Significant $M1$ transitions manifest the highest states of the third band with $B(M1; \Delta I = 0)$ varying from 0.17 μ_N^2 to 0.66 μ_N^2.

In Figs. 7 and 8 we present the alignment plot indicating the contribution of the neutrons and protons occupying the $0g_{9/2}$ orbital in the direction of the total angular momentum for ^{82}Zr and ^{82}Nb, respectively. The alignment for ^{82}Zr indicates that protons are aligning first from spin 10^+ to 12^+ and above spin 12^+ also neutrons are aligning fast, while in ^{82}Nb for the yrast states neutrons and protons manifest the same alignment.

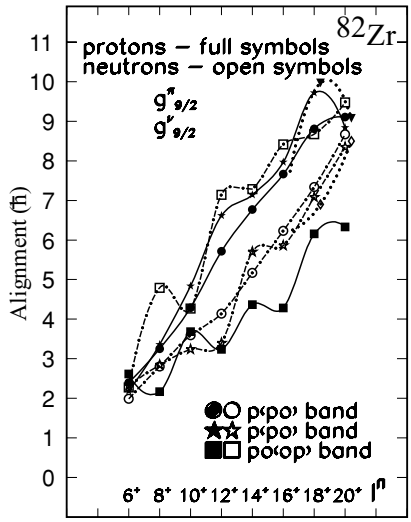

FIG. 7. The alignment plot for the lowest bands of ^{82}Zr presented in Fig. 4.

TABLE VII. The amount of mixing for the states of the lowest calculated bands of ^{82}Zr. The prolate components are indicated in boldface notation.

$I^\pi(\hbar)$	p(po) band o mixing/**p mixing**	p(po) band o mixing/**p mixing**	po(op) band o mixing/**p mixing**
0^+	3(1)%**94**%	4(1)%**91**(**1**)(**1**)%	38(9)%**40**(**10**)(**1**)%
2^+	**98**%	**97**%	20(4)%**30**(**21**)(**18**)(**4**)(**2**)%
4^+	**98**(**1**)%	**97**%	64(2)(2)%**16**(**11**)(**2**)(**1**)(**1**)%
6^+	**98**(**1**)%	**93**(**5**)(**2**)%	36%**58**(**4**)%
8^+	**98**(**1**)%	**75**(**22**)(**2**)%	52(41)(1)%**4**(**2**)%
10^+	**99**%	**64**(**34**)%	76(19)(1)%**2**%
12^+	**99**%	1%**83**(**15**)%	71(25)%**4**%
14^+	**96**(**2**)%	43(2)%**48**(**3**)(**2**)%	64(13)(3)%**12**(**6**)(**2**)%
16^+	14%**77**(**8**)%	5(3)%**79**(**12**)%	93(2)%**4**%
18^+	11(11)(2)%**43**(**31**)(**2**)%	47(12)(5)(2)%**24**(**8**)(**1**)%	84%**8**(**6**)(**1**)%
20^+	24%**55**(**14**)(**4**)%	9(2)%**59**(**26**)(**2**)%	80(3)(2)%**7**(**4**)(**4**)%

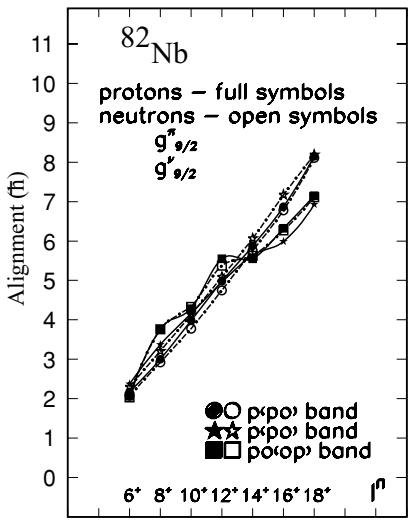

FIG. 8. The same as in Fig. 7 but for the ^{82}Nb nucleus.

Based on the alignment plots in the two mirror nuclei we cannot conclude on the importance of the neutron-proton pairing correlations. We can conclude that the prolate-oblate coexistence found in the structure of the lowest 30 0^+ states involved in the analog and nonanalog superallowed Fermi β decay of ^{82}Nb to ^{82}Zr is corroborated by the presence of this mixing at low, intermediate, and high spins in the positive-parity spectrum of both nuclei.

V. CONCLUSIONS

In this report we presented the first results on the effect of isospin mixing on superallowed Fermi β decay of the odd-odd ^{82}Nb nucleus to ^{82}Zr calculating the lowest 0^+ states in these nuclei in the frame of the complex excited VAMPIR approach using the best effective interaction we obtained up to now for this mass region. The first theoretical estimation of the strengths of the significant nonanalog branches is presented. For the first time we estimated the isospin-symmetry-breaking correction taking into account both the Coulomb interaction and the charge-symmetry breaking in the strong force as it is considered by the Bonn CD potential. To reduce the error in the estimated isospin-symmetry-breaking correction we have to increase the dimension of the many-nucleon basis used to describe the 0^+ states in both nuclei and to address the

TABLE VIII. The amount of mixing for the states of the lowest calculated bands of ^{82}Nb. The prolate components are indicated in boldface notation.

$I^\pi(\hbar)$	p(po) band o mixing/**p mixing**	p(po) band o mixing/**p mixing**	po(op) band o mixing/**p mixing**
0^+	3(1)%**95**%	30(4)%**44**(**12**)(**3**)(**1**)(**1**)%	15(3)%**43**(**23**)(**7**)(**2**)(**2**)(**1**)%
2^+	**98**%	1(1)%**94**%	46(1)%**27**(**13**)(**5**)(**4**)(**2**)(**1**)%
4^+	**98**%	1%**97**%	58(3)%**15**(**12**)(**6**)(**3**)(**2**)%
6^+	**98**(**2**)%	**95**(**3**)(**2**)%	70%**28**%
8^+	**95**(**4**)(**1**)%	**95**(**3**)(**2**)%	63(25)(6)(2)%**3**%
10^+	**98**%	**96**(**2**)%	62(24)(8)%**5**%
12^+	**99**%	2%**93**(**5**)%	58(32)(3)%**5**(**1**)%
14^+	**97**(**2**)%	3%**84**(**10**)(**2**)%	80(8)(5)%**4**(**2**)(**1**)%
16^+	2(1)%**89**(**5**)(**3**)%	4%**85**(**7**)(**2**)%	84(3)%**10**(**2**)%
18^+	14(4)(2)(2)%**72**(**4**)(**2**)%	26(2)(1)(1)%**57**(**7**)(**6**)%	77%**23**%

problem of possible missing four- and more-nucleon correlations originating in the time-reversal and axial-symmetry restrictions using the general VAMPIR approach [22]. Also we are currently involved in obtaining the renormalization of the nucleon-nucleon interaction adequate for an enlarged model space. These improvements require a considerable computational effort.

We compared for the first time the isobaric analog states in the ^{82}Nb and ^{82}Zr. The structure of the low-, intermediate- and high-spin yrast states are similar for the mirror nuclei. The comparison with the available data indicates good agreement for the yrast states. The prolate-oblate coexistence and mixing found in the structure of the investigated 0^+ states is also present in the yrast bands at the highest calculated spins and in the structure of the nonyrast calculated bands of the two nuclei.

Obviously the present study can only be considered as a first step toward the problem. More refined investigations concerning the isospin impurity inherent in the model and the dependence of the radial matrix elements on the size of the model space are needed in the future.

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