

## Difference between interaction cross sections and reaction cross sections

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We study the validity of the substitution of interaction cross sections for total reaction cross sections for a nucleus incident on a target nucleus at relativistic energies. We show that, for incident *stable* nuclei, the predicted difference between interaction and total reaction cross sections is large enough to probe the nuclear structure, particularly in a mass region of less than around 40. For analyses of the difference, we construct “pseudo data” for the reaction cross sections because empirical data are very limited at high energies. The construction of the pseudo data is based on our assumption that empirically unknown total reaction cross sections are precisely predicted by the phenomenological black-sphere model of nuclei that we developed recently. The comparison with the empirical interaction cross sections suggests a significant difference between the reaction and interaction cross sections for stable projectiles on a carbon target, which is of the order of 0–100 mb.

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Measurements of interaction cross sections have been performed for stable and light unstable nuclei [1] and are planned for heavy unstable nuclei in radioactive ion beam facilities, such as the RIKEN RI Beam Factory. The interaction cross section,  $\sigma_I$ , for a nucleus incident on a target nucleus is defined as the total cross section for all processes associated with proton and/or neutron removal from the incident nucleus [2], which is measured by a transmission-type experiment. In this experiment, the cross section is obtained as

$$\sigma_I = (1/N_t) \log(\gamma_0/\gamma), \quad (1)$$

where  $\gamma$  is the ratio of the number of noninteracting nuclei to the number of incoming nuclei for a target-in run,  $\gamma_0$  is the same ratio for an empty-target run, and  $N_t$  is the number of the target nuclei per  $\text{cm}^2$  [2].

The above definition of  $\sigma_I$  leads to the relation  $\sigma_I = \sigma_R - \sigma_{\text{inel}}$ , where  $\sigma_R$  is the total reaction cross section and  $\sigma_{\text{inel}}$  is the cross section for inelastic channels as will be specified below. The total reaction cross section in turn satisfies the relation  $\sigma_R = \sigma_T - \sigma_{\text{el}}$ , where  $\sigma_T$  is the total cross section and  $\sigma_{\text{el}}$  is the total elastic cross section.

In the measurements of  $\sigma_I$ , only the number of events in which an incoming nucleus has at least one nucleon removed is counted. The following processes are *not* counted in measuring  $\sigma_I$ : (1) incident nuclei are excited without changing the original  $Z$  and  $N$ , and target nuclei can change in any way; (2) target nuclei are excited without changing the original  $Z$  and  $N$ , while incoming nuclei remain in the ground state; and (3) target nuclei break up, while incoming nuclei remain in the ground state. These processes contribute exclusively to  $\sigma_{\text{inel}}$  and thus are included in  $\sigma_R$ . For incident nuclei with no excited bound states, such as  ${}^{11}\text{Li}$  [3], we expect  $\sigma_{\text{inel}} \cong 0$  and thus  $\sigma_I \cong \sigma_R$ .

In the present study, for stable projectiles, we address the question of how much we can rely on the substitution of interaction cross sections for total reaction cross sections and estimate how large the difference between reaction and interaction cross sections could be. We systematically analyze empirical data for the reaction and interaction cross sections

measured at high beam energy,  $\gtrsim 800$  MeV per nucleon. We complement the limited data for  $\sigma_R$  by constructing pseudo data with the help of a black-sphere picture of nuclei [4,5].

Theoretically, Ogawa, Yabana, and Suzuki [6] pointed out that, for reactions of  ${}^{11}\text{Li}$  with several kinds of targets, the contribution of  $\sigma_{\text{inel}}$  to  $\sigma_R$  is negligibly small. Recently, Ozawa *et al.* [7] experimentally estimated  $\sigma_{\text{inel}}$  for  ${}^{34}\text{Cl}$  incident on a C target as less than about 10 mb. Because  $\sigma_I = 1334 \pm 28$  mb, the contribution of  $\sigma_{\text{inel}}$  to  $\sigma_R$  is also negligibly small. In both cases, however, the projectiles are loosely bound systems. For stable nuclei, whether the contribution of  $\sigma_{\text{inel}}$  to  $\sigma_R$  is negligibly small or not is still an open question [8].

Recently, for the purpose of deducing nuclear size from proton-nucleus elastic scattering and reaction cross sections, we proposed a model in which a nucleus is viewed as a “black” sphere of radius  $a$  [4,5]. Here we assume that the target nucleus is strongly absorptive to the incident proton and hence acts like a black sphere. Another requirement for the black-sphere picture is that the proton wavelength is considerably smaller than the nuclear size. For proton incident energies higher than about 800 MeV, these requirements are basically satisfied.

In this scheme, we first evaluate the black-sphere radius,  $a$ , from the measured elastic diffraction peak and then identify  $a$  as a typical length scale characterizing the nuclear size [4]. The center-of-mass (c.m.) scattering angle for proton elastic scattering is generally given by  $\theta_{\text{c.m.}} = 2 \sin^{-1}(q/2p)$  with the momentum transfer,  $q$ , and the proton incident momentum in the c.m. frame,  $p$ . For the proton diffraction by a circular black disk of radius  $a$ , we can calculate the value of  $\theta_{\text{c.m.}}$  at the first peak as a function of  $a$ . (Here we define the zeroth peak as that whose angle corresponds to  $\theta_{\text{c.m.}} = 0$ .) We determine  $a$  in such a way that this value of  $\theta_{\text{c.m.}}$  agrees with the first peak angle for the measured diffraction in proton-nucleus elastic scattering,  $\theta_M$ . The radius,  $a$ , and the angle,  $\theta_M$ , are then related by

$$2pa \sin(\theta_M/2) = 5.1356 \dots \quad (2)$$

For scattering of protons having energies higher than  $\sim 800$  MeV with stable nuclei, we obtained the following

results [4,5]: (1) the absorption cross section,  $\pi a^2$ , agrees with the empirical total reaction cross section within error bars. Therefore,  $a$  can be regarded as a “reaction radius,” inside which the reaction with incident protons occurs. (2)  $\sqrt{3/5}a$  ( $= r_{BS}$ ) almost completely agrees with the empirically deduced values of the root-mean-square matter radius for nuclei having mass  $A \gtrsim 50$ , while it systematically deviates from the deduced values for  $A \lesssim 50$ . We also found that, for stable nuclei ranging from He to Pb, the black-sphere radius scales as [5]

$$a \simeq 1.2135A^{1/3} \text{ (fm)}. \quad (3)$$

From the scale  $a$  determined above, we calculate nucleus-nucleus absorption cross sections, which are to be compared with empirical total reaction cross sections,  $\sigma_R$ . We simply set

$$\sigma_{BS} = \pi(a_P + a_T)^2, \quad (4)$$

where  $a_P$  ( $a_T$ ) is the black-sphere radius of a projectile (target) nucleus. Here we assume that the incident protons are point particles as in Ref. [5]. By substituting the values of  $a_P$  and  $a_T$  determined by Eq. (2) into Eq. (4), we evaluate  $\sigma_{BS}$  for various sets of stable nuclei. That  $\sigma_{BS}$ , Eq. (4), works as pseudo data for  $\sigma_R$  of nucleus-nucleus reactions is merely an assumption, but several available data support its validity to within some uncertainties as we show later.

Now we concentrate on the reactions of stable projectile nuclei on a carbon target. Then, Eq. (4) reduces to

$$\sigma_{BS} = \pi(a_P + a(C))^2, \quad (5)$$

where  $a(C)$  is the black-sphere radius of the target C nucleus obtained from the measured angle of the first diffraction maximum in proton elastic scattering [5]. For proton incident energy higher than  $\sim 800$  MeV,  $a(C) = 2.69 \pm 0.07$  fm. For later convenience, we introduce the interaction radius,  $a_I$ , through the expression

$$\sigma_I = \pi(a_I + a(C))^2. \quad (6)$$

Here we have adopted the black-sphere radius  $a(C)$  as the size of the target nucleus because this radius can be viewed as a critical radius inside which reactions with the projectile occur in whatever states the target nucleus ends up.

In Fig. 1, we plot the empirical  $\sigma_R$  and  $\sigma_I$  data for incident energy per nucleon above  $\sim 800$  MeV. For comparison, we also plot  $\sigma_{BS}$ . Because the number of the  $\sigma_R$  data is very limited in the energy region of interest here [11], we consider  $\sigma_{BS}$  as pseudo data for  $\sigma_R$ .  $\sigma_{BS}$  is useful for predicting  $\sigma_R$  for nuclides for which proton elastic scattering data are available while no data are available for  $\sigma_R$ . The dashed curve in the figure shows the scaling cross section,  $\sigma_{\text{scaling}}$ , for a nucleus- $^{12}\text{C}$  reaction, defined on the basis of Eq. (3) as

$$\sigma_{\text{scaling}} = \pi(1.2135A^{1/3} + a(C))^2 \text{ (fm}^2\text{)}, \quad (7)$$

where  $a(C)$  is fixed at 2.6930 fm. When the data for proton elastic scattering are not available, we adopt this  $\sigma_{\text{scaling}}$  as the pseudo data.

In fact, for incident energies per nucleon higher than  $\sim 800$  MeV, only a few data are available for nucleus-carbon total reaction cross sections. For the  $\sigma_R$  data for  $^{12}\text{C} + ^{12}\text{C}$  at 870 MeV per nucleon, one finds  $939 \pm 17$  mb and

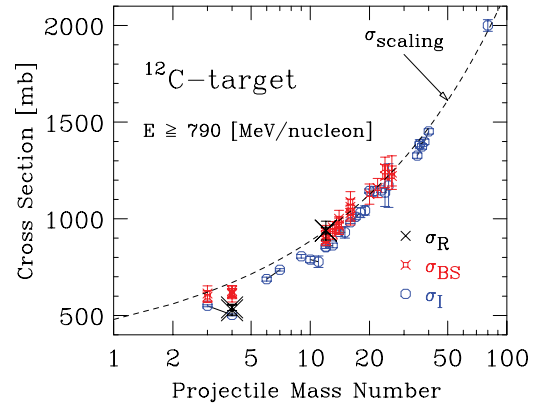


FIG. 1. (Color online) Comparison of the total reaction cross sections,  $\sigma_R$  ( $\times$ ), and their substitutes  $\sigma_{BS}$  (squares) and  $\sigma_{\text{scaling}}$  (dashed curve) with the interaction cross sections,  $\sigma_I$  ( $\circ$ ), for stable projectiles of  $A \leq 80$  and a  $^{12}\text{C}$  target. The absorption cross section,  $\sigma_{BS}$ , defined by Eq. (5) and the scaling cross section,  $\sigma_{\text{scaling}}$ , defined by Eq. (7) act as the pseudo data for  $\sigma_R$ . The empirical data for  $\sigma_R$  are taken from Ref. [9] (see text), and those for  $\sigma_I$  are taken from Refs. [1,7,10] for a projectile of  $^{3,4}\text{He}$ ,  $^{6,7}\text{Li}$ ,  $^9\text{Be}$ ,  $^{10,11}\text{B}$ ,  $^{12,13}\text{C}$ ,  $^{14,15}\text{N}$ ,  $^{16,17,18}\text{O}$ ,  $^{19}\text{F}$ ,  $^{20,21}\text{Ne}$ ,  $^{23}\text{Na}$ ,  $^{24,25}\text{Mg}$ ,  $^{35,37}\text{Cl}$ ,  $^{36,38,40}\text{Ar}$ , and  $^{80}\text{Kr}$  at energy per nucleon  $\gtrsim 800$  MeV. This figure is the updated version of Fig. 4 of Ref. [5].

$939 \pm 49$  mb from Ref. [9]. By substituting these values into  $\sigma_{BS}$  in Eq. (4), one obtains  $a_P = a_T = 2.73 \pm 0.03$  fm and  $2.73 \pm 0.07$  fm, respectively. Note that this result is consistent with the value of  $a(C)$  determined from proton elastic scattering data. For the  $\sigma_I$  data for the same reacting system, on the other hand, one finds  $856 \pm 9$  mb at 790 MeV per nucleon and  $853 \pm 6$  mb at 950 MeV per nucleon from Ref. [1]. From Eq. (6) one then obtains  $a_I = 2.53 \pm 0.10$  fm and  $2.52 \pm 0.09$  fm. The difference between  $a(C)$  and this  $a_I$  is about 0–0.3 fm, which is typically of the order of neutron skin thickness for stable nuclei [12]. When we discuss the nuclear surface structure, therefore, such a difference should be considered seriously.

As for the case of  $^4\text{He} + ^{12}\text{C}$ , both  $\sigma_{BS}$  and  $\sigma_{\text{scaling}}$  significantly overestimate the empirical values of  $\sigma_R$  ( $542 \pm 16$  mb and  $527 \pm 26$  mb) [9] and hence are not acceptable as the pseudo data for  $\sigma_R$ . This exceptional behavior is attributable to the fact that excitations associated with internucleon motion are highly suppressed in  $\alpha$  particles [13].

One can see from Fig. 1 that  $\sigma_I$  is close to  $\sigma_{BS}$  in magnitude for the whole range of the projectile mass, but some deviations do exist. In Ref. [5], we simply stressed a good agreement of  $\sigma_{BS}$  with  $\sigma_I$ , while here we focus on the difference between these two.

To clarify the difference, we plot  $\sigma_I - \sigma_R$ , and also plot  $\sigma_I - \sigma_{BS}$  or  $\sigma_I - \sigma_{\text{scaling}}$ , according to whether proton scattering data are available or not, for stable projectiles of  $A \leq 80$  and a  $^{12}\text{C}$  target in Fig. 2.

From the figure, as expected, we find that the difference is mostly negative. The average of the difference over various projectiles is about  $-60.4$  mb. This is the major finding of this work. Interestingly, the difference seems to decrease in magnitude with the projectile mass  $A$  although the plotted

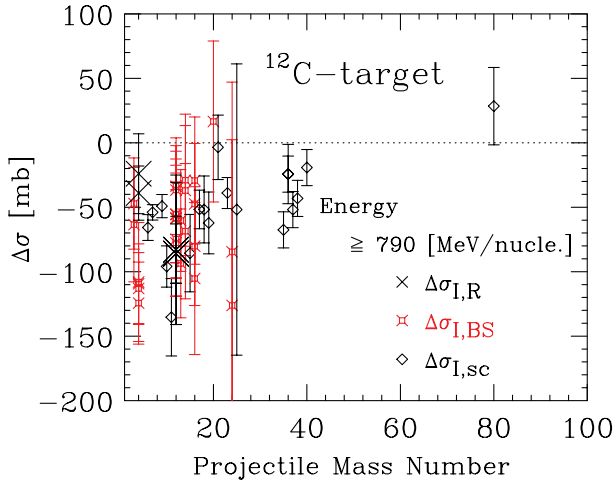


FIG. 2. (Color online)  $\Delta\sigma_{I,R} = \sigma_I - \sigma_R$  (crosses),  $\Delta\sigma_{I,BS} = \sigma_I - \sigma_{BS}$  (squares), and  $\Delta\sigma_{I,sc} = \sigma_I - \sigma_{scaling}$  (diamonds) as functions of projectile mass. The values of  $\Delta\sigma_{I,R}$  are only plotted for  ${}^4\text{He} + {}^{12}\text{C}$  and  ${}^{12}\text{C} + {}^{12}\text{C}$ .

data are rather sparse and often accompanied by large error bars.

Strictly speaking, our conclusion above that the difference of  $\sigma_I - \sigma_R$  is finite is tentative because of the large uncertainties of currently available proton elastic scattering data that are used for generating pseudo data for  $\sigma_R$ . In fact, the uncertainties of the pseudo data themselves are comparable to the predicted values of  $|\sigma_I - \sigma_R|$ . Because these uncertainties originate from those of the angle of the first diffraction maximum of proton elastic scattering data, more precise measurements of the angle should be necessary to pin down the difference between  $\sigma_I$  and  $\sigma_R$  [14]. Provided that the systematic deviation of the pseudo data from the real data for  $\sigma_R$  is small enough, however, the above prediction is expected to be valid. We show later some comparisons between these two.

Let us proceed to analyze the  $A$  dependence of  $\sigma_I - \sigma_{BS}$  under the assumption that  $\sigma_R = \sigma_{BS}$ . Using Eqs. (5) and (6), we can express the difference as

$$\sigma_I - \sigma_{BS} \simeq -2\pi (a_P + a(C)) \Delta a, \quad (8)$$

where  $\Delta a \equiv a_P - a_I$ . The above assumption ensures  $\Delta a > 0$ . If  $\Delta a$  were independent of  $A$ ,  $|\sigma_I - \sigma_{BS}|$  would grow like  $A^{1/3}$  as  $a_P$  behaves as in Eq. (3). As Fig. 2 suggests, however,  $|\sigma_I - \sigma_R|$  approaches a vanishingly small value or at least does not increase with  $A$ . This implies that  $\Delta a$  decreases no slower than  $A^{-1/3}$  as  $A$  increases.

Then, what is the physical implication of the difference,  $\Delta a$ ? From the aforementioned interpretation of the black-sphere radius as a reaction radius for incident protons and the fact that the  $np$  total cross section is similar to the  $pp$  total cross section at high incident energy above  $\sim 800$  MeV, we may assume that  $a_P$  corresponds to a critical radius of a projectile nucleus inside which reactions occur with nucleons in a target  $C$  nucleus. Note that  $a_P$  is located in the surface region. At a radius of  $a_I$ , which is only slightly smaller than  $a_P$ , transfer of incident energy into excitations of nucleons inside

the projectile nucleus must be more effective than at a radius of  $a_P$  because of more frequent reactions and, eventually, enough to induce nucleon emission. We thus expect that  $\Delta a$  has relevance to the energy scale characterizing breakup of the projectile nucleus, such as single-particle level spacing and separation energies. In fact, the  $A$  dependence of  $\Delta a$  mentioned above could be a key to clarifying what energy scale controls nucleon emission.

We remark that the present discussion on the behavior of  $\sigma_I - \sigma_{BS}$  with increasing  $A$  is not always applicable when projectiles are deformed nuclei [15]. In this case,  $\sigma_I$  could be appreciably smaller than  $\sigma_R$  even if  $A$  is relatively large. This is because a significant part of  $\sigma_{inel} (= \sigma_R - \sigma_I)$  comes from the low-lying rotational excitations of the projectile nucleus. Candidates for heavy stable nuclei that are deformed in the ground state are  ${}^{80}\text{Kr}$ ,  ${}^{154}\text{Sm}$ ,  ${}^{176}\text{Yb}$ , etc., but at least for  ${}^{80}\text{Kr}$ , the effect is not seen as long as we assume  $\sigma_R = \sigma_{scaling}$ .

One may wonder if our arguments based on the assumption  $\sigma_R \simeq \sigma_{BS}$  and Eq. (4) are valid because of a severe shortage of the real  $\sigma_R$  data at incident energy above  $\sim 800$  MeV per nucleon. Even for the available data for  ${}^{12}\text{C} + {}^{12}\text{C}$  at 870 MeV per nucleon [9], which we have to rely heavily on, its validity remains to be checked.

To lessen such a concern, we proceed to show that  $\sigma_{BS}$  given by Eq. (5) works as pseudo data for  $\sigma_R$  to within uncertainties of the empirical data underlying  $a_P$  and  $a(C)$ . On the basis of the fact that  $\sigma_R$  for proton-nucleus reactions agrees with  $\pi a^2$  within error bars [5] and that this tendency persists for proton incident energy down to about 100 MeV [16], we rederive  $a_P$  and  $a(C)$  from the corresponding  $\sigma_R$  [17,18] as  $(\sigma_R/\pi)^{1/2}$ , rather than from proton elastic scattering data. (Note that one cannot determine  $a_P$  for C and lighter projectile nuclei and  $a(C)$  from elastic scattering angular distributions measured for the proton incident energies less than  $\sim 400$  MeV, which lack the peak structure.) When calculating  $a_P$  and  $a(C)$  to obtain  $\sigma_{BS}$  for projectile-carbon reactions at a given incident energy per nucleon,  $T_P$ , we adopt the values of  $\sigma_R$  measured at proton incident energies within  $\sim 5$  MeV of  $T_P$ . The obtained values of  $\sigma_{BS}$  are to be compared with the  $\sigma_R$  data taken with a  ${}^{12}\text{C}$  target, which are, at high incident energy, presently limited to such projectiles as  ${}^9\text{Be}$ ,  ${}^{12}\text{C}$ , and  ${}^{27}\text{Al}$  [11,19,20].

In Fig. 3, we plot  $\sigma_{BS}$  and  $\sigma_R$  for the reactions of  ${}^9\text{Be} + {}^{12}\text{C}$  and  ${}^{27}\text{Al} + {}^{12}\text{C}$ . The agreement between  $\sigma_{BS}$  and  $\sigma_R$  is fairly good for incident energies per nucleon ranging from  $\sim 100$ – $400$  MeV. Although the uncertainties in  $\sigma_{BS}$  are still large, due mainly to the uncertainties in the measured values of proton-nucleus total reaction cross sections, such agreement strongly supports the effectiveness of Eq. (5) at predicting  $\sigma_R$  for energies per nucleon higher than  $\sim 100$  MeV. Apparently, the corresponding ratio of  $\sigma_{BS}$  to  $\sigma_R$  fluctuates within  $\sim 10\%$  of unity. To clearly confirm a finite difference between  $\sigma_I$  and  $\sigma_R$ , the agreement of  $\sigma_{BS}$  with  $\sigma_R$  within  $\sim 5\%$  will be required. This implies that the black-sphere radius should be determined with uncertainties of less than a few percent, or, equivalently, the data for proton-nucleus total reaction cross sections should be obtained with uncertainties of less than 5%.

Another example is  $\sigma_R$  versus  $\sigma_{BS}$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions. In the black-sphere approximation,  $\sigma_{BS}$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions is expressed as  $\sigma_{BS} = 4\pi a(C)^2$ , because in this case  $a_P = a(C)$

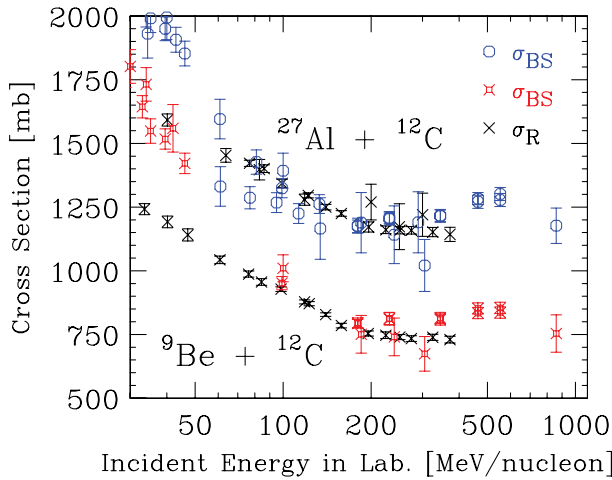


FIG. 3. (Color online) Comparison of  $\sigma_{BS}$  (squares for  ${}^9\text{Be} + {}^{12}\text{C}$  and circles for  ${}^{27}\text{Al} + {}^{12}\text{C}$ ) with  $\sigma_R$  ( $\times$ ) for the reactions of  ${}^9\text{Be} + {}^{12}\text{C}$  (lower) and  ${}^{27}\text{Al} + {}^{12}\text{C}$  (upper) as a function of incident energy per nucleon. The data of  $\sigma_R$  are taken from Refs. [11,19,20].

in Eq. (5). Let us assume that  $\pi a(C)^2$  is equal to  $\sigma_R(p+C)$ , where  $\sigma_R(p+C)$  is the empirical total reaction cross section for protons incident on a carbon target [17,18]. Then, if  $\sigma_R$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions is equal to  $4\sigma_R(p+C)$ , we can show that Eq. (5) works also for the case of  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions.

In Fig. 4, we compare  $4\sigma_R(p+C)$  with  $\sigma_R$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions as a function of incident energy per nucleon. For incident energies per nucleon higher than  $\sim 100$  MeV, although the uncertainties of the data are still large also in this case, we obtain an excellent agreement between them. This implies the validity of the black-sphere picture based on the empirical relation  $\sigma_R \cong \sigma_{BS}$  for these energies. The requirement for the data precision of  $\sigma_R(p+C)$  so as to obtain agreement of  $\sigma_{BS}$  with  $\sigma_R$  within  $\sim 5\%$  is quite similar to the cases of  ${}^9\text{Be} + {}^{12}\text{C}$  and  ${}^{27}\text{Al} + {}^{12}\text{C}$ .

Note that  $4\sigma_R(p+C)$  is appreciably larger than  $\sigma_R$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions for incident energies per nucleon less than about 100 MeV. This implies that the classical picture, the geometrical description of the cross section underlying Eq. (5), breaks down. If we could adopt  $4\sigma_R(p+C)$  as a basis in assessing  $\sigma_R$  for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions, we need a certain “transparency” effect [19] to reproduce the data. We remark that the Coulomb effect on  $\sigma_R$  would be hard to resolve the disagreement alone, because the cross-section reduction due to the Coulomb repulsion between the projectile and target nuclei is stronger for proton-carbon reactions than for carbon-carbon ones at given incident energy per nucleon. Similar deviations also appear for  ${}^9\text{Be} + {}^{12}\text{C}$  and  ${}^{27}\text{Al} + {}^{12}\text{C}$  cases of incident energy per nucleon lower than about 100 MeV as in Fig. 3.

We should be careful about a deviation of  $\sigma_{BS}$  from  $4\sigma_R(p+C)$  that is appreciable around 500 MeV per nucleon in Fig. 4. It does not imply a flaw in the black-sphere model, but simply reflects the fact that the measured values of  $\sigma_R(p+C)$  at proton incident energies of 220–570 MeV [27] are larger than expected from the systematics [13]. Note that a similar tendency appears in the values of  $\sigma_{BS}$  for  ${}^9\text{Be} + {}^{12}\text{C}$

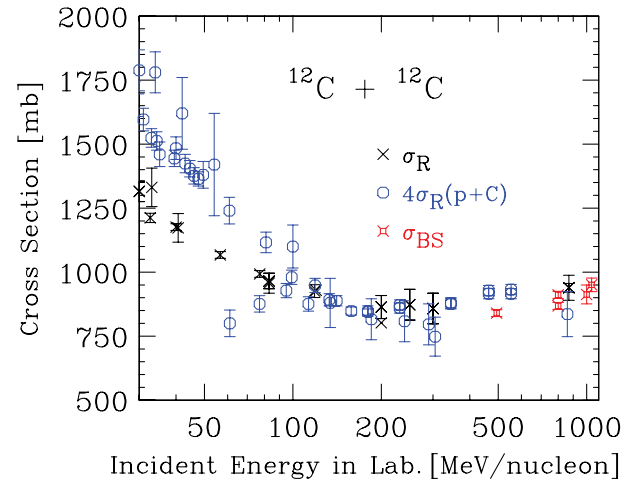


FIG. 4. (Color online) Comparison of  $4\sigma_R(p+C)$  ( $\circ$ ) with  $\sigma_R$  ( $\times$ ) for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions ( $\times$ ) as a function of incident energy per nucleon. The  $\sigma_R(p+C)$  data are taken from Refs. [17,18], while the  $\sigma_R$  data for  ${}^{12}\text{C} + {}^{12}\text{C}$  reactions are taken from Refs. [9,11,19–26]. For reference, we also plot  $\sigma_{BS} = 4\pi a(C)^2$ , where  $a(C)$  is determined from the measured angle of the first diffraction peak of proton elastic scattering, by squares.

and  ${}^{27}\text{Al} + {}^{12}\text{C}$  reactions derived from the same values of  $\sigma_R(p+C)$  (see Fig. 3).

In summary, we have pointed out that, for stable nuclei incident on a carbon target, there is a significant difference between real  $\sigma_I$  data and pseudo  $\sigma_R$  data even at relativistic energies. This difference would lead to possible uncertainties of about 0–0.3 fm in estimates of nuclear matter radii, if relying on the  $\sigma_I$  data alone. We have found that this difference is consistent with the fact that  $\sigma_I < \sigma_R$  and generally stays within 0–100 mb. The difference is clear for small  $A$  while it is less clear for larger  $A$ . This implies that the scale  $\Delta a$  characterizing the difference between the black-sphere and interaction radii of the projectile nucleus decreases no slower than  $A^{-1/3}$  as  $A$  increases, a feature relevant to the problem of what energy scale controls the breakup of the projectile. Of course, the above implications strongly depend on the validity of our black-sphere picture, which is based on  $\sigma_R \cong \sigma_{BS}$ . The presently available  $\sigma_R$  data for  ${}^9\text{Be}$ ,  ${}^{12}\text{C}$ , and  ${}^{27}\text{Al}$  incident on  ${}^{12}\text{C}$  support the above relation although the uncertainties of the data are still large. To confirm more clearly the finite difference between  $\sigma_I$  and  $\sigma_R$ , more precise measurements of proton-nucleus total reaction cross sections and elastic scattering are inevitable.

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