PHYSICAL REVIEW C **78**, 058201 (2008)

Scaling study of the pion electroproduction cross sections

T. Horn,[1] X. Qian,[2] J. Arrington,[3] R. Asaturyan,[4] F. Benmokthar,[5] W. Boeglin,[6] P. Bosted,[1] A. Bruell,[1] M. E. Christy,[7] E. Chudakov,[1] B. Clasie,[8] M. M. Dalton,[9] A. Daniel,[10] D. Day,[11] D. Dutta,[12] L. El Fassi,[3] R. Ent,[1] H. Fenker,[1] J. Ferrer,[13] N. Fomin,[11] H. Gao,[2] K. Garrow,[14] D. Gaskell,[1] C. Gray,[9] G. M. Huber,[15] M. K. Jones,[1] N. Kalantarians,[16] C. E. Keppel,[1,7] K. Kramer,[2] Y. Li,[16] Y. Liang,[17] A. F. Lung,[1] S. Malace,[7] P. Markowitz,[6] A. Matsumura,[18] D. Meekins,[1] T. Mertens,[19] T. Miyoshi,[18] H. Mkrtchyan,[4] R. Monson,[20] T. Navasardyan,[4] G. Niculescu,[13] I. Niculescu,[13] Y. Okayasu,[18] A. K. Opper,[21] C. Perdrisat,[22] V. Punjabi,[23] A. W. Rauf,[24] V. Rodriguez,[16] D. Rohe,[19] J. Seely,[8] E. Segbefia,[7] G. R. Smith,[1] M. Sumihama,[18] V. Tadevosyan,[4] L. G. Tang,[1,7] V. Tvaskis,[25] A. Villano,[26] W. Vulcan,[1] F. R. Wesselmann,[23] S. A. Wood,[1] L. Yuan,[7] and X. C. Zheng[3]

[1]*Physics Division, TJNAF, Newport News, Virginia 23606, USA*
[2]*Triangle Universities Nuclear Laboratory, Duke University, Durham, North Carolina 27708, USA*
[3]*Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Yerevan Physics Institute, 375036 Yerevan, Armenia*
[5]*University of Maryland, College Park, Maryland 20742, USA*
[6]*Florida International University, Miami, Florida 33119, USA*
[7]*Hampton University, Hampton, Virginia 23668, USA*
[8]*Massachussets Institute of Technology, Cambridge, Massachusetts 02139, USA*
[9]*University of the Witwatersrand, Johannesburg, South Africa*
[10]*University of Houston, Houston, Texas 77204, USA*
[11]*University of Virginia, Charlottesville, Virginia 22904, USA*
[12]*Mississippi State University, Mississippi State, Mississippi 39762, USA*
[13]*James Madison University, Harrisonburg, Virginia 22807, USA*
[14]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada*
[15]*University of Regina, Regina, Saskatchewan S4S 0A2, Canada*
[16]*University of Houston, Houston, Texas 77204, USA*
[17]*The American University, Washington, District of Columbia 20016, USA*
[18]*Tohoku University, Sendai, Japan*
[19]*Basel University, Basel, Switzerland*
[20]*Central Michigan University, Mount Pleasant, Michigan 48859, USA*
[21]*Ohio University, Athens, Ohio 45071, USA*
[22]*College of William and Mary, Williamsburg, Virginia 23187, USA*
[23]*Norfolk State University, Norfolk, Virginia, USA*
[24]*University of Manitoba, Winnipeg, Manitoba, R3T-2N2, Canada*
[25]*Faculteit Natuur-en Sterrenkunde, Vrije Universiteit, NL-1081 HV Amsterdam, The Netherlands*
[26]*Rensselaer Polytechnic Institute, Troy, New York 12180, USA*

(Received 12 July 2007; revised manuscript received 9 May 2008; published 18 November 2008)

The $^1\mathrm{H}(e, e'\pi^+)n$ cross section was measured for arange of four-momentum transfer up to $Q^2 = 3.91$ GeV^2 at values of the invariant mass W above the resonance region. The Q^2 dependence of the longitudinal component was found to be consistent with the Q^2-scaling prediction for hard exclusive processes. This suggests that the QCD factorization theorem is applicable at rather low values of Q^2. The transverse term falls off slower than the naive Q^{-8} expectation and remains appreciable even at $Q^2 = 3.91$ GeV^2.

DOI: 10.1103/PhysRevC.78.058201 PACS number(s): 14.40.Aq, 25.30.Rw, 11.55.Jy, 13.40.Gp

The partonic picture of the hadron predicts a factorization of short- and long-distance physics, where the factorization scale is given by the four-momentum transfer squared (Q^2). Measurements of inclusive processes, such as deep inelastic scattering (DIS), confirm that in the limit of large Q^2, at fixed values of the Bjorken variable x_B (the longitudinal momentum fraction of the hadron carried by the parton in the infinite momentum frame), such processes can be viewed as scattering from individual partons within the hadronic system. The long-distance physics is summarized in the parton distribution functions, which can be related to the structure of the hadron. A similar factorization of scales may be expected to apply to hard exclusive processes.

In meson electroproduction, a factorization theorem has been proven for longitudinally polarized photons [1]. It predicts that for sufficiently large values of Q^2, at fixed x_B, and fixed momentum transfer to the nucleon, $-t$, the amplitude for hard exclusive reactions can be expressed in terms of a short-distance (hard) process, a distribution amplitude describing the formation of the final state meson, and generalized parton distributions (GPDs). The last two encode the long-distance (soft) physics. Though it is expected that the factorization theorem is valid for $Q^2 > 10$ GeV^2, to date it is unclear whether it may already be approximately valid at moderately high values of Q^2 under certain conditions [2].

Longitudinal/transverse (L/T) separated pion production cross sections are one way to test the predictions of the factorization theorem for hard exclusive processes. To leading order, the meson electroproduction cross section for longitudinally polarized virtual photons scales like $\sigma_L \sim Q^{-6}$ at fixed $-t$ and x_B [3]. The contribution of transversely polarized photons is suppressed by at least an additional power of $1/Q$ in the amplitude.

The goal of the present analysis is to investigate which values of Q^2 are sufficient for factorization to apply in pion electroproduction. The Q^2-scaling prediction can be experimentally tested with measurements of σ_L, while the size of transverse contributions can be quantified via σ_T. The interference terms, σ_{LT} and σ_{TT}, may provide additional information on the applicability of the QCD factorization prediction. Based on the expected $1/Q$ suppression of transverse contributions, one may naively expect the corresponding power laws of $1/Q$ and $1/Q^2$ for $\sigma_{\mathrm{LT}}/\sigma_L$ and $\sigma_{\mathrm{TT}}/\sigma_L$, respectively.

In our analysis, we use the previously published L/T separated $^1\mathrm{H}(e, e'\pi^+)n$ cross sections, as well as newly available results from the pion transparency experiment (E01-107), which was carried out in Hall C at the Thomas Jefferson National Accelerator Facility (JLab). The data were acquired at $Q^2 = 2.15$ and 3.91 GeV^2 for values of $-t$ ranging between 0.145 and 0.450 GeV^2, at a center-of-mass energy of $W = 2.2$ GeV. A more detailed description of the experiment can be found in Ref. [4].

The experimental cross sections are calculated by comparing the experimental yields to a Monte Carlo simulation of the experiment. To take into account variations of the cross section across the acceptance, the simulation uses a $^1\mathrm{H}(e, e'\pi^+)n$ model based on pion electroproduction data. The model was optimized in an iterative fitting procedure to match the t and Q^2 dependence of the data. The W dependence of the data is well described by the empirical form $(W^2 - M_p^2)^2$, where M_p is the proton mass [5]. The resulting model cross section is known to 10%.

The uncorrelated systematic uncertainty, which is amplified by $1/\Delta\epsilon$ in the L/T separation, is 0.6% at $Q^2 = 2.15$ GeV^2 and 3.7% at $Q^2 = 3.91$ GeV^2. The latter is dominated by the model dependence of the cross section. At $Q^2 = 2.15$ GeV^2, the model dependence is <1%, as the data were analyzed at the same values of W and Q^2 at high and low ϵ. This was achieved by placing constraints to equalize the W/Q^2 phase space, and obtaining average central values from datasets over the overlapping region. At $Q^2 = 3.91$ GeV^2, the W/Q^2 kinematic coverage at high ϵ does not overlap with the one at low ϵ, and it does not allow for selecting the bin centers from a common set of values. An explicit correction (on the order of 4%) was required to perform the L/T separation at common values of W and Q^2. Other contributions to the systematic uncertainties are the correlated systematic uncertainty (3.7%), which is dominated by radiative corrections and pion absorption, and the $-t$-correlated systematic uncertainty (1.2%).

Because tests of the Q^2 dependence of the cross section require scaling of our data in $-t$ and x_B, it is beneficial to first look into the agreement with the well-known $-t$ dependence. Our results are shown in Fig. 1. The separated cross sections were evaluated at fixed values of W, Q^2, and $-t$. At $Q^2 = 2.15$ GeV^2, σ_L shows the expected almost exponential falloff with $-t$, while σ_T is largely independent of $-t$. The interference terms are not large but clearly contribute to the total cross section.

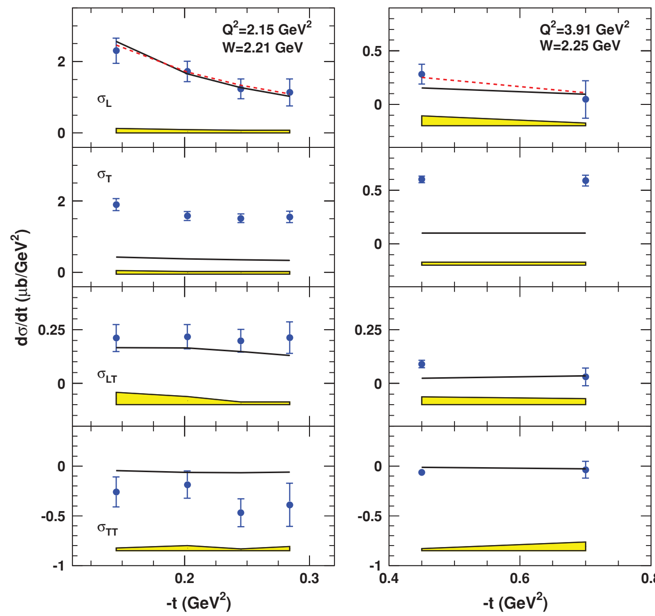

FIG. 1. (Color online) The t dependence of the separated cross sections from E01-107 data. The error bars include the statistical and uncorrelated systematic uncertainty combined in quadrature, and the error band includes the correlated and t-correlated systematic uncertainties. The solid black lines denote the best fit of the VGL/Regge model [6] to the data. The fit parameter is the pion cutoff, which relates to the pion form factor in a monopole form. The fit value for the pion cutoff is $\Lambda_\pi^2 = 0.518$ GeV^2 at $Q^2 = 2.15$ GeV^2, and $\Lambda_\pi^2 = 0.584$ GeV^2 at $Q^2 = 3.91$ GeV^2. The value for the ρ cutoff was taken to be $\Lambda_\rho^2 = 1.7$ GeV^2 at both values of Q^2. The red-dashed curve shows the result of a calculation using the VGG/GPD model [7].

Regge-based calculations such as the one by Vanderhaeghen, Guidal, and Laget (VGL) [6], in which the t dependence is determined by the exchange of Regge trajectories for π- and ρ-like particles, have been well established over a wide kinematic range in $-t$ and W. Most parameters are fixed by pion photoproduction data with the pion and pion transition form factor as the only free parameters. Both form factors are parametrized by a monopole form, $[1 + Q^2/\Lambda_i^2]^{-1}$, where $\Lambda_{\pi,\rho}^2$ is the π (ρ) mass scale. The VGL Regge model does a good job overall in describing the magnitude and t and Q^2 dependences of our σ_L data, but it strongly underestimates the magnitude of σ_T, for which the model seems to have limited predictive power. This emphasizes the need for L/T separations in these kinematics.

Our data are also compared with a GPD calculation by Vanderhaeghen, Guidal, and Guichon (VGG) [7], which uses a factorized ansatz in terms of a form factor and a quark distribution function. The pion pole part of the amplitude is obtained using a parametrization of F_π data including power corrections due to intrinsic transverse momenta and soft overlap contributions. The strong coupling constant between quarks and the exchanged gluon is deduced from the asymptotic freedom expression for α_s and by imposing Q^2 analyticity. This infrared (IR) finite form can be found in Refs. [7,8]. The GPD calculation gives a rather good description of the t dependence of the data, and the Q^2 dependence is also described fairly well.

It is interesting to note that both the Regge and the GPD models describe the Q^2 dependence of σ_L quite well at low $-t$ and low Q^2. The agreement at low values of $-t$ may be expected, because the Regge calculation, which is valid at low $-t$ and low Q^2, overlaps with the region of validity of the GPD calculation. However, this observation suggests that the agreement between data and GPD calculations is a necessary but not sufficient condition for the applicability of QCD factorization in this kinematic regime.

A more stringent and model-independent test of QCD factorization is the leading order Q^2 power law scaling of the separated cross sections. The Q^2 dependences of σ_L and σ_T, where results from this experiment have been combined with other recent results from JLab [9,10], are shown in Fig. 2. The data were scaled to constant values of $-t$ and x_B using the separated VGL/Regge cross section predictions. The additional uncertainty due to this kinematic scaling was determined from comparisons of the resulting cross section with one obtained using the GPD prediction (for σ_L only) and a parametrization based on pion electroproduction data. The kinematic scaling uncertainty ranges from 7% to 13% in σ_L for both x_B kinematics, while the σ_T uncertainty is larger by a factor of 3.

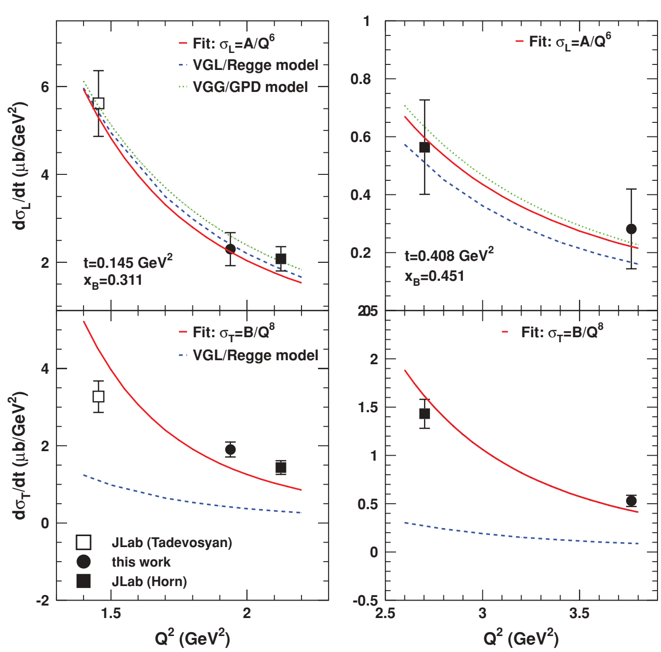

FIG. 2. (Color online) The Q^2 dependence of the separated cross sections at fixed values of $-t$ and x_B. The error bars denote the statistical and systematic uncertainties combined in quadrature. The red, solid curve shows a fit of the form Q^{-6} for σ_L and Q^{-8} for σ_T. The green dotted line is a GPD calculation from Ref. [7]. In this calculation, power corrections to the leading order are included. The blue dashed line is a VGL/Regge [6] calculation using Λ_π^2 from a global fit to F_π.

The "hard scattering" predictions for $\sigma_L(\sim Q^{-6})$ and σ_T $(\sim Q^{-8})$ are indicated by the red lines in Fig. 2. We have also fitted σ_L and σ_T at each value of x_B to the forms $\sigma_L \sim Q^n$ and $\sigma_T \sim Q^m$, where n and m are free parameters. The experimental fit values are listed in Table I together with the χ^2 for $n=-6$ and $m=-8$. The fit values for σ_L are consistent with the hard scattering prediction within the uncertainty. In fact, fitting the Q^2-scaling prediction, $\sigma_L \sim Q^{-6}$, also results in a good description of the data. Note that the fit at $x_B=0.451$ is not well constrained because of the precision of the available data. While the scaling laws are reasonably consistent with the Q^2 dependence of the σ_L data, they fail to describe the Q^2 dependence of the σ_T data. The Q^2 dependence of σ_T does, however, provide less conclusive evidence for having reached the hard scattering regime, as the factorization theorem was proven rigorously only for longitudinal photons [1].

It has been suggested that additional information about QCD factorization may be obtained through the interference terms [11]. However, the small size of these components may complicate the interpretation of the experimentally fitted scaling power. A fit to the interference terms from Ref. [12] suggests that the Q^2 dependence is reasonably well described by a functional form $1/Q(\chi^2=0.94$, probability$=0.62)$ for $\sigma_{\mathrm{LT}}/\sigma_L$, while a functional form of $1/Q^2(\chi^2=1.34$, probability$=0.51)$ does a reasonable job describing the Q^2 dependence of $\sigma_{\mathrm{TT}}/\sigma_L$ at $x_B=0.311$. Because of the small size of σ_{LT}, the Q^2 dependence of the exponent can only be determined at the $\pm 17\%$ level.

An interesting observation is that at first glance, σ_L appears to scale in a manner consistent with the Q^{-6} hard scattering prediction. However, an observation of the "correct" power law behavior of the cross section is not proof that QCD factorization is already applicable, and it is necessary to examine several characteristics of the reaction before drawing conclusions about the reaction mechanism.

High-energy experiments using the ${}^1\mathrm{H}(e,e'\rho){}^1\mathrm{H}$ reaction have produced data that agree with the Q^2-scaling expectation for the cross section ratio σ_L/σ_T under the assumption of s-channel helicity conservation. Data have been obtained at DESY [13,14], Cornell [15], and Fermilab [16] for values of Q^2 ranging from 0.3 to 5.0 GeV^2 at center-of-mass energies W ranging from 1.3 to 6 GeV, and more recently at JLab [17,18].

The Q^2-scaling expectation was also tested using unseparated ω cross section data from JLab for Q^2 values up to 5 GeV^2. The t-channel process was found to dominate, but considerable transverse contributions complicated the isolation of contributions from hard and soft processes from these unseparated data [19].

Higher order (soft) corrections play an important role at experimentally accessible energies [2,20] in electroproduction and may mimic the expected Q^2-scaling behavior characteristic for the hard pQCD term [21] in σ_L. Recall that the fitted scaling power for σ_L could only be determined to ± 0.95, and the experimental uncertainties are rather large. In fact, deviations from the hard scattering prediction comparable to those shown in Table I have been observed in earlier measurements

TABLE I. Central fit values for the Q^2 scaling laws $\sigma_L \sim Q^{-n}$ and $\sigma_T \sim Q^{-m}$ for both x_B settings. Also shown are the χ^2 values for fitting the data with the hard scattering predictions $\sigma_L \sim Q^{-6}$ and $\sigma_T \sim Q^{-8}$, and the corresponding probability P for finding this value of χ^2 or larger in the sampling distribution assuming Poisson statistics.

x_B	$-t$	$\sigma_L \sim Q^{-n}$ n	$\sigma_L \sim Q^{-6}$ χ^2/ν (P)	$\sigma_T \sim Q^{-m}$ m	$\sigma_T \sim Q^{-8}$ χ^2/ν (P)
0.31	0.15	5.08 ± 0.95	0.45 (0.64)	4.20 ± 0.78	10.7($<10^{-3}$)
0.45	0.41	4.17 ± 2.95	0.24 (0.62)	6.01 ± 0.90	4.5 (0.034)

of the energy dependence of large-angle Compton scattering performed at CEA [22]. While the Compton data are in good agreement with the predicted energy dependence from pQCD, at least one other theoretical model describes the data equally well, including the contribution of subleading logarithms at values of $Q^2 < 10$ GeV^2 [23]. Furthermore, recent data from Jefferson Lab [24], covering the same kinematic range at higher precision, show good agreement with the hard scattering prediction, while the extracted scaling power at fixed center-of-mass angles differs considerably from the one predicted by pQCD. If similar effects are significant in σ_L for charged pion production, the observed "scaling" behavior may be accidental. The measurements will thus have to be extended to higher values of Q^2, for instance, at JLab after the 12 GeV upgrade, or to even higher energies, for instance, at an electron ion collider.

In summary, the Q^2 dependence of the separated pion electroproduction cross sections plays a large role in the determination of the Q^2 regime for which the factorization of long- and short-distance physics needed for the extraction of GPDs from hard exclusive processes is valid. The Q^2 dependence of separated ^{1}H$(e, e'\pi^+)n$ longitudinal cross sections from Jefferson Lab is in good agreement with the Q^{-6}-scaling prediction within the experimental uncertainty. This would suggest that GPDs can be accessed at relatively low values of Q^2. However, higher order contributions may still be significant in this kinematic regime and mimic the expected Q^2-scaling behavior. Transverse contributions to the cross section drop considerably slower than the naive Q^{-8} scaling expectation.

We thank S. Brodsky and A. Radyushkin for helpful discussions. This work was supported in part by the US Department of Energy. The Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility for the US Department of Energy under Contract DE-AC05-84150. We acknowledge additional research grants from the US National Science Foundation and the Natural Sciences and Engineering Research Council of Canada.

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