Conditions for observation of fade out of collective enhancement of the nuclear level density

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The results of two recent papers searching for the disappearance of collective enhancements with energy in nuclear level densities are examined. It is found that the effects of such enhancements are less than has been assumed. The reduction in the size of the effect only partially resolves the disagreement between theory and experiment. This effect also plays a role in explaining the results of an earlier experiment.

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A recent paper by Komarov *et al.* [1] looked at the alpha particle spectra produced in ^{16}O on ^{160}Gd reactions. The purpose of the measurement was to try to observe a transition between the level density for a nucleus assumed to be deformed at low energy and spherical at high excitation. It has been shown [2,3] that the level density for an axially symmetric deformed nucleus is a factor of σ^2 larger than for a spherical nucleus where σ is the spin cutoff factor and is typically close to 10 for a nucleus with $A \approx 150$ at 50 MeV. If the level density changed from having a multiplier of σ^2 to one of 1 over an energy range of a few MeV, the level density should show a kink or at least a plateau at the energy of transition. This was not found in Ref. [1]. In this paper, we review the assumptions of Ref. [1] to try to explain this result.

We adopt the usual terminology and, for a spherical nucleus, denote a state as one of the $(2J+1)J_z$ projections of a level of spin J. For an axial deformed nucleus, the degeneracy in energy is partially lifted. The angular momentum is still denoted J, but the projection of the angular momentum on the symmetry axis is denoted K. States with $+K$ and $-K$ are still degenerate, but do not have the same energy as those with $|K'| \neq |K|$. Thus, a 5/2 level in a spherical nucleus corresponds to one level and six degenerate states. In a deformed basis, three levels are produced: one has $K = \pm 1/2$, one has $K = \pm 3/2$ and the third has $K = \pm 5/2$.

The distribution of spins among levels in a spherical nucleus was first derived by Bethe [4]. If the total number of states $\rho_s(E)$ at a given energy E is assumed to have a Gaussian distribution in J_z, then

$$\rho_s(E, J_z) = \frac{\rho_s(E)}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left[\frac{-J_z^2}{2\sigma^2}\right],$$

where $\sigma = \langle J_z^2 \rangle^{1/2}$. Clearly

$$\rho_L(E, J) = \rho_s(E, J_z = J) - \rho_s(E, J_z = J+1) \quad (1)$$

$$\approx -\left.\frac{d\rho_s}{dJ_z}\right|_{J_z=J+1/2} \quad (2)$$

$$= \frac{\rho_s(E)}{\sqrt{2\pi}} \frac{1}{\sigma} \frac{(2J+1)}{2\sigma^2} \exp\left[\frac{-\left(J+\frac{1}{2}\right)^2}{2\sigma^2}\right] \quad (3)$$

$$= \frac{\rho_s(E)}{\sqrt{8\pi}\sigma^3} (2J+1) \exp\left[\frac{-\left(J+\frac{1}{2}\right)^2}{2\sigma^2}\right]. \quad (4)$$

It can be seen that

$$\rho_L(E) = \sum_J \rho_L(E, J) = \frac{\rho_s(E)}{\sqrt{2\pi}\,\sigma}. \quad (5)$$

In these expressions, $\rho_L(E)$ is the total level density at energy E and $\rho_L(E, J)$ is the level density for spin J at energy E. Thus, the energy dependence of the level density and the state density are similar, differing only by the energy dependence of σ.

For deformed axially symmetric rotors, the level density can be calculated by building rotational bands on each intrinsic level. As is shown in Bohr and Mottelson [2,3], the intrinsic state density is

$$\rho_{\rm int}(E, K) = \frac{1}{\sqrt{2\pi}\sigma_\parallel} \exp\left[-\frac{K^2}{2\sigma_\parallel{}^2}\right]\rho_{\rm int}(E). \quad (6)$$

In this expression, $\rho_{\rm int}(E)$ is the total intrinsic state density at E and $\rho_{\rm int}(E, K)$ is the intrinsic state density at energy E and angular momentum projection K. The level density is

$$\rho_L(E, J) = \frac{1}{2} \sum_{K=-J}^{J} \rho_{\rm int}(E - E_{\rm rot}(K, J), K) \quad (7)$$

$$= \frac{1}{\sqrt{8\pi}} \frac{1}{\sigma_\parallel} \rho_{\rm int}(E) \sum_{K=-J}^{J} \exp\left[-\frac{\left(J+\frac{1}{2}\right)^2}{2\sigma_\perp^2} - \left(\frac{1}{2\sigma_\parallel^2} - \frac{1}{2\sigma_\perp^2}\right)K^2\right]. \quad (8)$$

The 1/2 arises because each level is comprised of two degenerate states.

It is pointed out in Bohr and Mottelson that if J is small, then the magnitude for the last factor in the argument of the exponential is small and the sum is approximately

$$\rho_L(E, J) = \frac{1}{\sqrt{8\pi}} \frac{2J+1}{\sigma_\parallel} \exp\left[-\frac{\left(J+\frac{1}{2}\right)^2}{2\sigma_\perp^2}\right] \rho_{\rm int}(E). \quad (9)$$

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Thus, $\rho_L(E, J)$ is a factor of $\sigma_\perp^2$ larger than the level density for a spherical nucleus. Note that in this case the total state density is twice the level density, while for a spherical nucleus it is $\sqrt{2\pi}\,\sigma$ times larger.

The peak in the spin distribution for the level density is for $J = \sigma$. Although $\sigma_\perp$ is slightly larger than σ (spherical) for prolate deformation and slightly smaller for oblate deformation, the average spin value is rather similar for the spherical and deformed nuclei in a given mass region. An explanation for this is that the enhancement in the level and state density do not come from levels and states created by the deformation. They are from states which were in the original basis but which have been lowered in energy by the deformation [1–3,5].

It is important to verify that the assumptions made in evaluating the sum over K in Eq. (7) do not break down badly at high spins. For mass 160, $\sigma \approx 10$. For a deformation of 0.25, $\sigma_\parallel$ will be about 8.75 and $\sigma_\perp$ about 11.25 if the deformation is prolate. At $J = 40$, it can be shown that

$$\sum_{K=-J}^{J} \exp - \left[\frac{1}{2\sigma_\parallel^2} - \frac{1}{2\sigma_\perp^2}\right] K^2 \tag{10}$$

differs from $2J + 1$ by about a factor of 2.5, lowering the level density by this factor for $J = 40$. On the other hand, the factor $\exp[-(J + 1/2)^2/2\sigma_\perp^2]$ is about a factor of four larger than the corresponding factor for a spherical nucleus $\exp[-(J + 1/2)^2/2\sigma^2]$. Combining these two factors produces about a factor of two enhancement beyond $\sigma_\perp^2$ for a deformed nucleus relative to a spherical one at $J = 40$. If the deformed nucleus were oblate, the factor would be about 1/2 for $J = 40$. In either case, the enhancement factor would be about $\sigma_\perp^2$ for the level density, since most of the levels will have $J \ll 40$.

For the reaction $^{16}O + {}^{160}Gd$ at 60 MeV, the maximum angular momentum is about $60\hbar$. This is well beyond the $\sim 10\hbar$ characterizing the maximum in the level density. A typical alpha emission will remove $\sim 15\hbar$ in angular momentum. If a sharp cutoff model is assumed for the entrance channel transmission coefficients, about 25% of the compound states formed will have $0 \leqslant J \leqslant 30\hbar$, about 25% will have $30\hbar \leqslant J \leqslant 45\hbar$ and about 50% will have $45\hbar \leqslant J \leqslant 60\hbar$. For states with $J \leqslant 30\hbar$, it is likely that the inequality $J > L > J'$ applies, where J is the compound level spin, L is the emitted orbital angular momentum and J' is the final state spin. In this case, there will be $(2J' + 1)$ values of L coupling a level of spin J to a final state of level J'. Essentially, the final spectrum will be proportional to the *state* density rather than the *level* density. For initial states with $45\hbar \leqslant J \leqslant 60\hbar$ or $30\hbar \leqslant J \leqslant 45\hbar$, it will take two or one decay, respectively, to put the spin in the range $J \leqslant 30\hbar$, but in reactions where four to six emissions occur, it can be seen that the majority of decays will occur from states with J below $30\hbar$. Thus, emission spectra will be dominated by decays which are proportional to the state density, rather than the level density.

This distinction is important because the change in state densities between deformed and spherical nuclei is only a factor of $\sqrt{2/\pi}\,\sigma$ rather than σ^2 as for the level density. If we assume a nuclear temperature of 1.6 MeV, the disappearance of the σ^2 factor would reduce the level density to that of an energy 7 MeV lower, while the $\sqrt{2/\pi}\,\sigma$ factor would reduce the state density to that at an energy 3 MeV lower. Obviously, if all nuclei in a given calculation are deformed or all are spherical, this distinction becomes unimportant. Finally, the effects described here will be automatically incorporated in a Hauser-Feshbach calculation.

As is pointed out in Ref. [1], if the fade out of collective enhancement is so rapid that it gives a reduction in level density with increasing energy, the heat capacity goes negative. We point out here that this argument should be applied to the state density, rather than the level density. This again causes a relaxation of the condition on the region over which fade out of collective effects occurs.

Finally, the consequences of this analysis for the experiment of Junghans *et al.* [6] are not completely clear. In looking at reactions where fission decay competed with other channels, they found that the fade out of rotational enhancement did not depend on ground state deformation. These considerations suggest that effects of rotational enhancement fade out could appear to be different for various channels in the same compound nucleus if some channels are fed primarily with large J compound states and others with low J compound states.

It should be pointed out that a much earlier experiment by Chenevert *et al.* [7] produced results that may be explained in part by these arguments. The study reported in Ref. [7] looked at (α, α') reactions on Ta, Au, and Pb. It was found that at back angles the spectra were largely compound nuclear. The data were found to be consistent with the disappearance of shell effects in nuclear level density at energies above 60 MeV. The results did not show the fade out of collective effects for the nuclei populated in the deformed region near Ta. Since the reactions studied in this paper were induced by alpha particles, the average angular momentum was less than in the work of Komarov *et al.* In the case of the (α, α') reactions, a much larger fraction of the compound states than for ^{16}O-induced reactions have J values in the range where decays populate final nuclei based on the state density rather than the level density. This would, as previously argued, make the fade out more subtle and help explain why it was not seen in the results of Ref. [7].

The present analysis does not indicate that fade out effects do not occur. To the extent that the final nuclei are populated based on state density rather than level density, the signature of the transition between deformed and spherical states will be more subtle than previously thought, making the lack of evidence for such a change in some experiments less surprising. The effect should be properly accounted for in Hauser-Feshbach calculations, leaving some discrepancy between theoretical calculations and experiment in Ref. [1].

It is also possible that the expectation that deformation effects die out completely is unrealistic. Since both prolate and oblate deformations cause enhancement in state and level densities, even when the calculations indicate that the average deformation is zero, any dispersion in the shape about the spherical limit will give some enhancement. Further, the enhancement at low energies comes at the expense of the level density at higher energy. This in principle causes a reduction of the level density for high energies. Depending on the region from which the levels are removed, the combination of the effects of the state density versus level density change, the fact that some levels will still be deformed even if the average level is spherical and the level reduction mentioned above could make the transition very difficult to observe.

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