PHYSICAL REVIEW C **78**, 051301(R) (2008)

# Spin distribution in low-energy nuclear level schemes

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(Received 5 September 2008; published 4 November 2008)

The spin-cutoff parameter $\sigma$ has been determined for experimental spin distributions at low excitation energies of 310 nuclei between $^{18}$F and $^{251}$Cf (more than 8000 levels with their spin). The results indicate a weak dependence on the mass number $A$ of the spin-cutoff parameter $\sigma^2 \sim A^{0.28}$, and an even-odd spin staggering in the spin distribution of the even-even nuclei, with a strong enhancement of the number of states with spin zero. A modification of the spin-cutoff distribution formula is proposed in order to describe the even-even nuclei data. These findings are in good agreement with recent predictions of shell-model Monte Carlo calculations.

DOI: 10.1103/PhysRevC.78.051301 PACS number(s): 21.10.Ma, 21.60.−n

The exponential-like increase of the nuclear level density $\rho(E, J)$ with energy $E$ for a given spin $J$ is assumed to be given by the formula

$$\rho(E, J) = f(J)\rho(E), \tag{1}$$

where $f(J)$ is the spin distribution function and $\rho(E)$ is the total level density (summed over all spins). Here we neglect a possible parity dependence of the level densities.

Many studies concentrated on the experimental determination of the total level density and its theoretical description. Very widely used models are the Fermi gas formula (BSFG), which has two free parameters, $a$ and the back-shift energy $E_1$, and the constant temperature formula (CT), also with two parameters, $T$ and $E_0$ [1]. In previous investigations [2] we have empirically determined the two level density parameters for each of the above two models, for 310 nuclei between $^{18}$F and $^{251}$Cf, by fitting the known low excitation energy levels and the level density at the neutron binding energy. In order to find the systematics of these level density parameters we proposed formulas [2] which use pairing energies and shell effect values that can be extracted from the mass tables.

The spin distribution function $f(J)$ is usually assumed to be given by the spin-cutoff model [3]. In this model the individual nucleon spins point in random directions, therefore the spin distribution is a gaussian-like curve which depends on a single parameter, $\sigma$. The spin-cutoff distribution is

$$f(J, \sigma) = e^{-J^2/2\sigma^2} - e^{-(J+1)^2/2\sigma^2} \approx \frac{2J+1}{2\sigma^2} e^{-J(J+1)/2\sigma^2}. \tag{2}$$

There is a small number of experimental determinations of the spin-cutoff parameter $\sigma$. Thus, in Refs. [4–7], $\sigma$ was determined for nuclei in the $A \sim 60$ region and at excitation energies generally between 4 and 9 MeV, from the analysis of angular distributions of particles emitted in compound nucleus reactions. A dependence of $\sigma$ on the mass number $A$ was proposed in [8,9] for a number of nuclei in the mass range 20 to 250, but this was based on a rather limited knowledge of the discrete levels. More recently, $\sigma$ was determined from low-energy level schemes in nuclei in the mass range 20 to 110 [10] (where its dependence on both mass number $A$ and excitation energy is discussed), or for particular nuclei, e.g., $^{112}$Cd [11], and $^{116}$Sn [12].

The spin-cutoff parameter $\sigma$ is generally related to an effective moment of inertia. It is assumed that this parameter may depend on the nuclear mass $A$, the level density parameter $a$, nuclear temperature $t$, and the nuclear moment of inertia. Different formulas were proposed for these dependencies (see, e.g., Refs. [1,12–16]). However, these dependencies are not well established, and a systematic study of their experimental parameters is missing. For this reason, theoretical predictions of $\sigma$ are of great interest, especially those of microscopic model calculations that take into account nucleon correlations (which are missing in the derivation of the spin-cutoff distribution). Two recent such calculations, namely that of Alhassid *et al.* [17], based on shell-model Monte Carlo model, and of Kaneko and Schiller [18], based on the RPA, predict quite interesting features for the variation with mass, excitation energy, and type of nucleus.

In this work we present an experimental study of the spin distribution, based on the actual knowledge of the discrete levels at low excitation energies. The main purpose of the study was to determine the general evolution of the spin-cutoff parameter with the mass number and possibly other quantities, without considering an explicit dependence on the excitation energy. For this, we employed the collection of discrete levels that was used in the level density study for a number of 310 nuclei from $^{18}$F to $^{251}$Cf [2]. After some re-adjustments of this nuclear level database, according to most recent information in the ENSDF [19], we had a number of 8116 levels (with spin value), in 1556 spin groups (levels with the same spin in a nucleus), up to excitation energies usually of about 1–3 MeV. It is essential to have complete level schemes in the given energy and spin range. In some cases questionable spins were also accepted. The completeness and correctness is assumed to be 90–95%.

The spin-cutoff parameter $\sigma$ was determined by least squares fits performed with the following $\chi^2$ [8,9]:

$$\chi^2 = \Sigma_k \Sigma_J [(n_k(J) - F_k f(J, \sigma))^2 / \delta n_k(J)^2], \tag{3}$$

where $n_k(J)$ is the number of levels of spin $J$ in nucleus $k$. The normalization factor is $F_k = \Sigma_J n_k(J) / \Sigma_J f(J, \sigma)$. The choice of the errors $\delta n_k(J)$ is not a straightforward

 

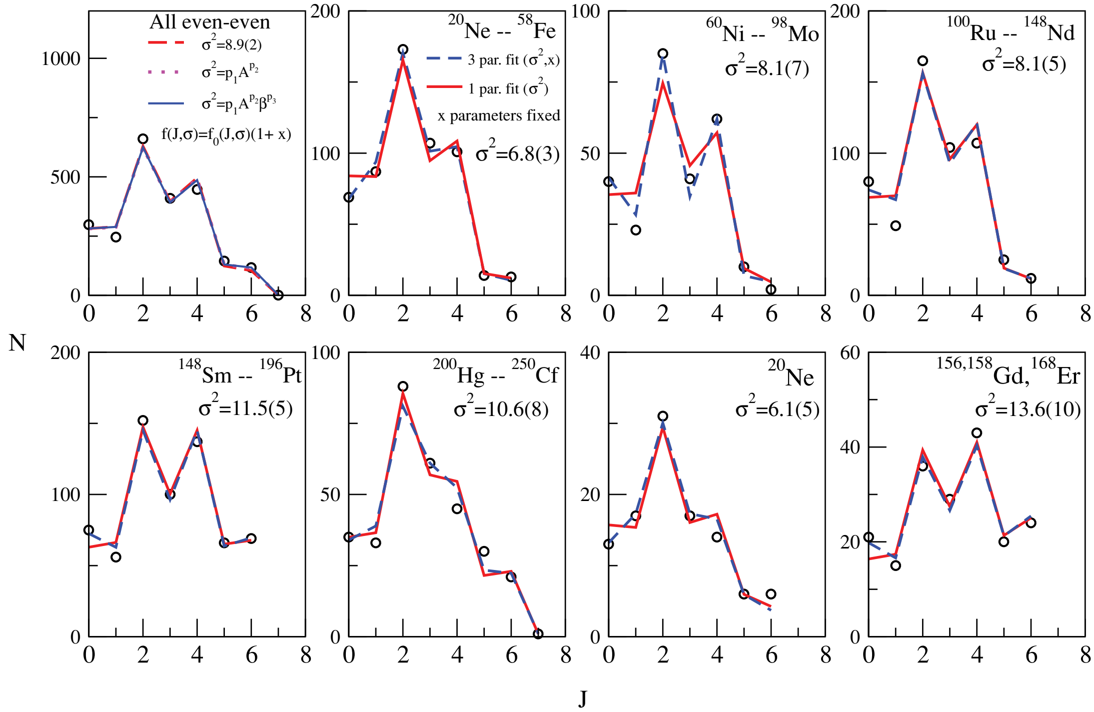


FIG. 1. (Color online) Measured and fitted spin distributions in various groups of even-even nuclei. $\sigma^2$ is always fitted independently. The full line shows the fits with the fixed spin staggering parameters [Eq. (5)], the dashed lines those with individually fitted spin staggering parameters.

matter, since they depend on many factors, some of them uncontrollable: missing or wrongly assigned levels; artificial cut due to the maximum accepted excitation energy; possible slightly different distributions (e.g., depending on the type of nucleus), etc. It is not clear how to distribute the global error that we assume for the completeness onto the numbers of levels in the different spin groups of a nucleus. First we tried an error $\delta n_k(J) = n_k(J)^{1/2}$, which resulted in $\chi^2 \approx 0.5$. After different trials, we have chosen $\delta n_k(J) = n_k(J)^{1/4}$, which provides $\chi^2$ values close to 1. This choice makes also sense since it gives more weight in the fit to the spin groups with a larger number of levels, and provides errors which are never smaller than 1.

We have determined the $\sigma$ parameter both for all nuclei and separately for even-even, odd-mass, or odd-odd nuclei. Also, since in many nuclei the number of levels was not too large, in order to get a rough idea about the mass dependence, we have made five large mass groups of nuclei, with $A$ roughly from 18 to 60, 60 to 100, 100 to 150, 150 to 200, and 200 to 250. The highest excitation energy for which the level scheme was considered complete varies from nucleus to nucleus, being generally higher for the light nuclei. In most of our nuclei we have had complete data available mainly below 2–3 MeV excitation. We did not propose ourselves to investigate the dependence of the spin-cutoff parameter $\sigma$ on the excitation energy; our results must therefore be regarded as average values, representative for the "low excitation energy region" (up to about 3 MeV).

Figure 1 shows the experimental spin distributions for the even-even nuclei, and their different fits. We emphasize that in this figure and in the similar ones which follow, except for particular nuclei which are placed in the lowest right side graph(s), these are not real spin distributions, but just the result of the superposition of all the available data (because in different nuclei we have different excitation energy and spin windows). In Fig. 1 one immediately observes that the smooth spin distribution (1) cannot describe the experimental data of the even-even nuclei, because there is a strong even-odd spin staggering. In order to describe this staggering, the following ad hoc modification of the formula was defined:

$$f_{ee}(J,\sigma) = f(J,\sigma)(1+x), \tag{4}$$

where two values of $x$ were fitted, one value with positive sign for even spin and negative sign for odd spin, and the second value for zero spin. It has to be mentioned that this new formula is not any more normalized to 1. For our fit procedures which use only ratios this is not relevant. In other cases a new normalization has to be introduced for even-even nuclei. However, we note that the deviations from 1 are rather small for large $\sigma$ values, and reach about 6% for smaller values ($\sigma \sim 2.5$). The $x$ values were found to be practically independent of the mass, therefore the following values were finally adopted:

$$x = \begin{cases} +0.227(14), & \text{for even spin values,} \\ -0.227(14), & \text{for odd spin values,} \\ +1.02(9), & \text{for zero spin levels.} \end{cases} \tag{5}$$

The spin staggering was observed in practically all even-even nuclei. It can be clearly observed for the particular nuclei shown in the lower graphs of Fig. 1. Also, the data for $^{112}$Cd (discussed in Ref. [11]) and $^{116}$Sn (Ref. [12]), that is, the levels

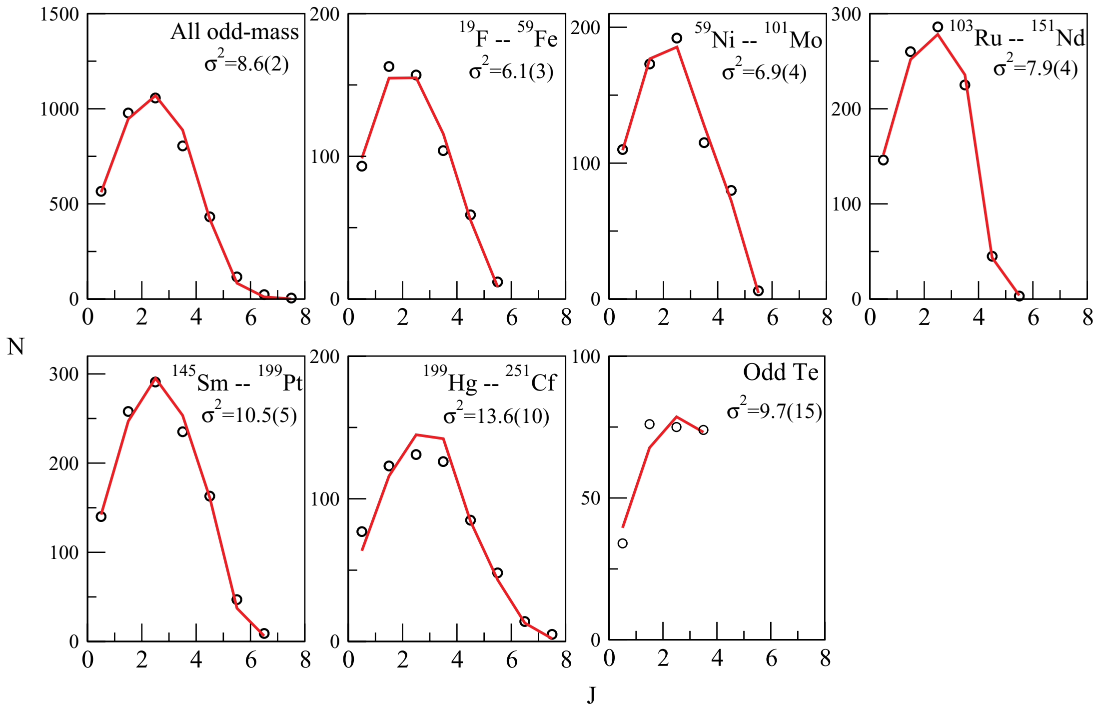


FIG. 2. (Color online) Measured and fitted spin distributions in various mass groups of odd-mass nuclei.

observed up to the excitation energy of 3.2 and 3.9 MeV, respectively, show an even-odd spin staggering effect which is in very good agreement with the predictions of formulas (4) and (5). The correction (4) and (5) was always applied in the case of the even-even nuclei. One remarks that the spin-cutoff model prediction for the zero spin levels requires an especially strong enhancement (a factor of 2) to fit the experimental values. The even-odd spin staggering of the spin distribution was not observed in the odd-odd nuclei. Figures 2 and 3 show the fits for the odd-mass and odd-odd nuclei, respectively, while Fig. 4 shows the distributions for mass groups of all nuclei together. By treating separately the

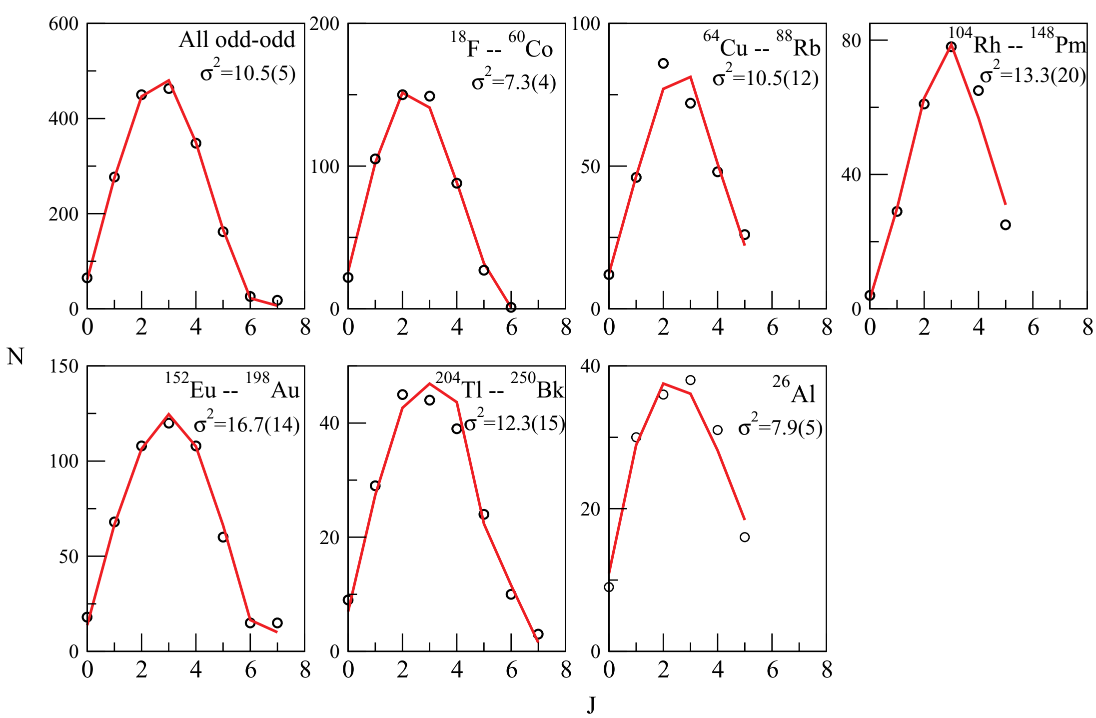


FIG. 3. (Color online) Measured and fitted spin distributions in various mass groups of odd-odd nuclei.

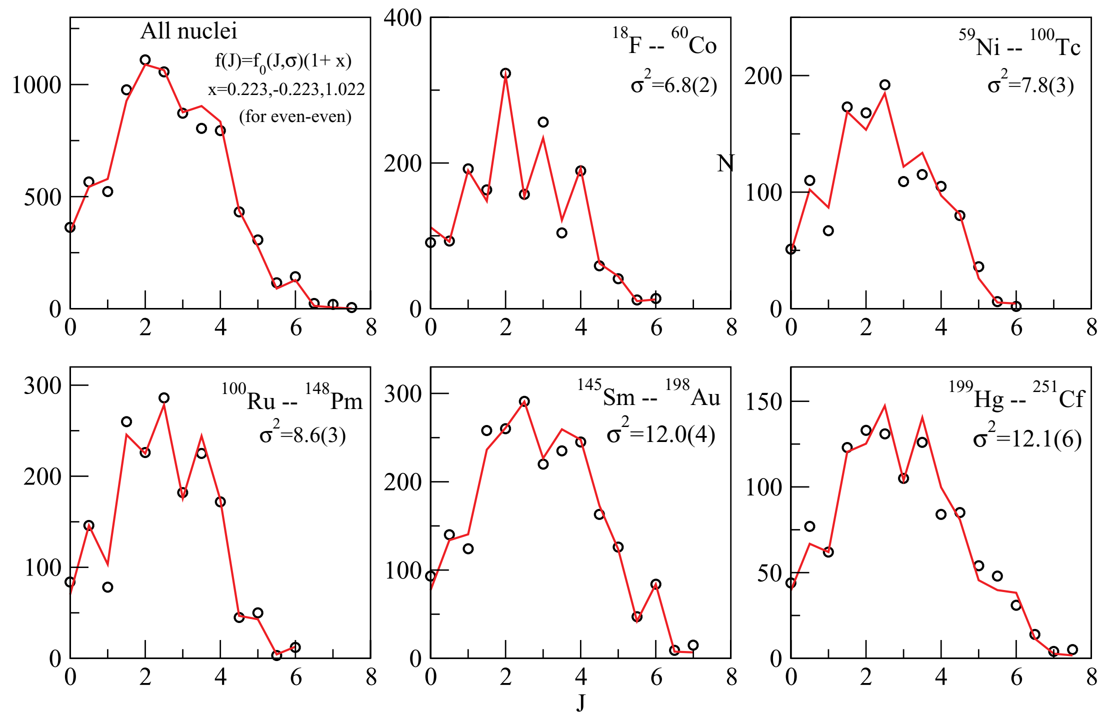


FIG. 4. (Color online) Measured and fitted spin distributions in different mass groups. The spin staggering correction given by Eqs. (4) and (5) was always applied to the even-even nuclei. The first graph shows the fit with $\sigma^2$ varied according to Eq. (7).

even-even, odd-mass, and odd-odd nuclei, one should be able to evidence effects due to the different pairing energies.

Since, as remarked above, theoretical models propose that $\sigma$ depends on mass, level density parameter $a$, nuclear temperature $t$, or the moment of inertia (or deformation $\beta$), we have tried also fits with the following ansatz:

$$\sigma^2 = p_1 A^{p_2} X^{p_3}, \tag{6}$$

where $X$ stands for $a, t$, or $\beta$. The inclusion of a third parameter did not improve the quality of the fits, therefore it was concluded that within the present limits of precision a dependence on $a, t$, or $\beta$ is not necessary.

Table I gives the results of the fits for $\sigma^2$ values for different types of nuclei and mass regions. One remarks a surprisingly weak dependence on $A$: for a mass variation by a factor of 10, $\sigma^2$ varies only by a factor of at most 2. A general fit, which accepts only a dependence of $\sigma^2$ on the mass number, gives

$$\sigma^2 = 2.61(21)A^{0.28(2)}. \tag{7}$$

TABLE I. Values of $\sigma^2$ (see also Figs. 1–4) fitted in different mass groups.

| Nuclei | All | Even-even | Odd-$A$ | Odd-odd |
|---|---|---|---|---|
| $^{18}$F–$^{60}$Co | 6.8(2) | 6.8(3) | 6.1(3) | 7.3(4) |
| $^{59}$Ni–$^{100}$Tc | 7.8(3) | 8.1(7) | 6.9(4) | 10.5(12) |
| $^{100}$Ru–$^{148}$Pm | 8.6(3) | 8.1(5) | 7.9(4) | 13.3(20) |
| $^{145}$Sm–$^{198}$Au | 12.0(4) | 11.5(5) | 10.5(5) | 16.7(14) |
| $^{199}$Hg–$^{251}$Cf | 12.1(6) | 10.6(8) | 13.6(10) | 12.3(15) |
| All | 9.1(2) | 8.9(3) | 8.6(2) | 10.5(5) |

The $\sigma^2$ values (Table I) for the even-even nuclei [with spin staggering both fitted or fixed to the average values given by Eq. (5)] and odd-mass nuclei are very similar and are close to those calculated with Eq. (7). The odd-odd nuclei seem to require generally larger $\sigma^2$ values, but the statistics and accuracy are not high enough for a firm, quantitative conclusion. Our average mass dependence (7) differs both from the $\sigma^2 \sim A^{7/6}$ behavior predicted by the model of Ref. [3], and from that proposed in Ref. [10] for nuclei in the mass 20 to 110 region, which, depending on the formula used, is roughly $\sigma^2 \sim A^{1.21}$.

In the following, we compare our results with those of the shell-model Monte Carlo calculations of Alhassid *et al.* [17]. In this work, the $\sigma^2$ values of Eq. (2) were calculated for three nuclei: $^{55}$Fe, $^{56}$Fe, and $^{60}$Co. In Fig. 5, we compare these theoretical predictions for the spin distributions with those based on our formulas (4), (5), and (7). The even-odd spin staggering observed by us in the even-even nuclei, was also predicted for $^{56}$Fe [17]. The spin staggering correction—Eq. (5) determined by us, is in very good agreement with that resulting from the SMMC calculations, including the strong enhancement for the spin zero states. In Fig. 5 we show the spin-cutoff distributions both for the $\sigma^2$ values resulting from Eq. (7), and for $\sigma^2$ values which describe the SMMC predictions. One can observe that for all three nuclei these later values are larger than those given by Eq. (7). This is not unexpected, since our values correspond to lower excitation energies (usually below about 3 MeV), whereas the lowest excitation energies for the SMMC calculations (indicated in Fig. 5 for each case) are larger. The SMMC calculations predict indeed an increase of $\sigma^2$ with the energy, which corresponds

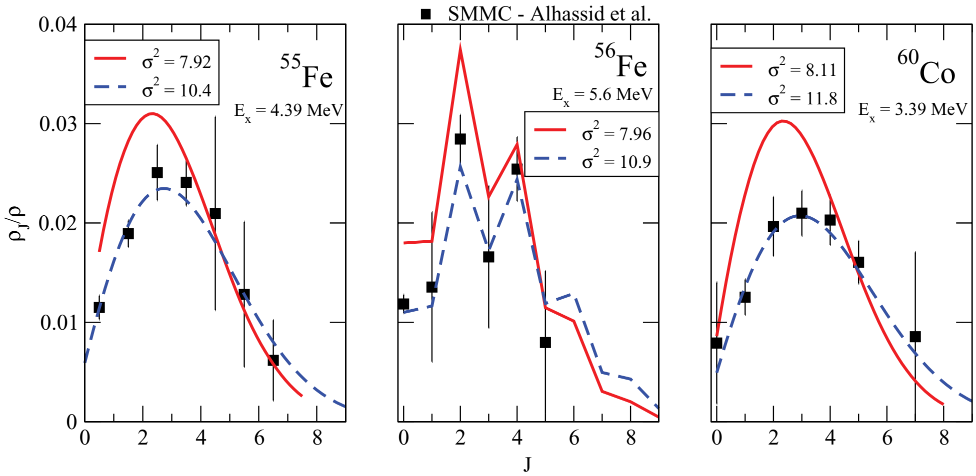


FIG. 5. (Color online) Comparison of spin distributions calculated with spin-cutoff values of the present formula (7) (full lines) and the predictions of SMMC calculations [17] (symbols), for the excitation energies indicated in each case. The dashed lines correspond to $\sigma^2$ values which describe the SMMC values. For the even-even nuclei we applied the correction given by Eqs. (4) and (5). The curves correspond to the total state density (see [17]).

approximately to $E_x^{1/2}$ [17], in agreement with other earlier predictions [3,10]. The SMMC calculations also predict for the odd-odd nucleus $^{60}$Co a $\sigma^2$ value about 10% larger than that of the odd or even-even nuclei, a fact which is also hinted by our experimental values in Table I.

The weak $A$ dependence (7) of $\sigma^2$ may be attributed to a moment of inertia lower than the rigid body value at low excitation energies (below 6–8 MeV according to the calculations), which, especially for the even-even nuclei, is a result of the pairing correlations [17,18]. The even-odd spin staggering of the spin distribution for the even-even nuclei is, according to the SMMC calculations [17], an effect which decreases with increasing excitation energy and vanishes at energies above $E_x \approx 9$ MeV.

In conclusion, we have determined the spin-cutoff parameter $\sigma$ of the spin distribution by examining the low-excitation (0–∼3 MeV) levels of 310 nuclei between $^{18}$F and $^{251}$Cf, a data set containing more than 8000 levels with their spin. The main results of this study are the following. In contrast to previous assumptions, the spin-cutoff parameter $\sigma$ corresponding to the lowest energy excitations depends mainly on the mass number by a surprisingly weak dependence, the average behavior is $\sigma^2 \sim A^{0.28}$. The even-even nuclei show an even-odd spin staggering of the spin distribution, which cannot be described by the spin-cutoff distribution (2), and have a strong enhancement of the density of the states with spin zero. At the low excitation energies studied in this work, the spin distribution for the even-even nuclei has been described by an ad-hoc modification of the spin-cutoff distribution, as given by Eqs. (4), (5). The present experimental findings are in good agreement with results of the shell-model Monte Carlo calculations [17]. Very important for an accurate description of the nuclear spin distribution are the energy dependence of both the spin-cutoff parameter $\sigma$ and of the spin staggering parameter(s) (in the even-even nuclei case). Both experimental data and microscopic model predictions are highly desired in this respect. A more detailed publication is in preparation.

## ACKNOWLEDGMENTS

This work was partially funded by the Romanian National Authority for Scientific Research under PNCDI2 programme, contract No. ID-117/01.10.2007. Fruitful discussions with F. Becvar and Y. Alhassid are greatly appreciated.

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