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# Unified approach to structure factors and neutrino processes in nucleon matter

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We present a unified approach to neutrino processes in nucleon matter based on Landau's theory of Fermi liquids that includes one and two quasiparticle-quasihole pair states as well as mean-field effects. We show how rates of neutrino processes involving two nucleons may be calculated in terms of the collision integral in the Landau transport equation for quasiparticles. Using a relaxation time approximation, we solve the transport equation for density and spin-density fluctuations and derive a general form for the response functions. We apply our approach to neutral-current processes in neutron matter, where the spin response function is crucial to the calculation of neutrino elastic and inelastic scattering and neutrino-pair bremsstrahlung and absorption from strongly interacting nucleons. We calculate the relaxation rates using modern nuclear interactions and including many-body contributions, and find that rates of neutrino processes are reduced compared with estimates based on the one-pion exchange interaction, which is used in current simulations of core-collapse supernovae.

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## I. INTRODUCTION

Neutrino emission, absorption, and scattering processes in nucleon matter play a crucial role in the physics of stellar collapse, supernova explosions, and neutron stars [1,2]. Since the leptons in these processes interact weakly, the neutrino rates can be expressed compactly in terms of the response of nuclear matter to axial and vector probes. In many situations, the axial response is the more important, and in this paper we concentrate on this case, which for a system of nonrelativistic nucleons amounts to the spin or spin-isospin response. These responses have been calculated by a number of groups [3–7] allowing for single nucleon quasiparticle-quasihole pair states.[1] However, this is insufficient for rates of neutrino processes involving two nucleons, such as neutrino-pair bremsstrahlung and absorption, and modified Urca reactions, in which two particle-hole pair states are necessary. The possible importance of two particle-hole pair states for neutrino inelastic scattering, in particular for energy exchange and the formation of the neutrino spectra, has been emphasized by Raffelt *et al*. [8–10]. Bounds on the magnitude of the two particle-hole pair weight have been investigated in Ref. [11], and it has been shown how the two-pair response is directly related to the collision term in Landau's transport equation for quasiparticles [12].

Noncentral contributions to nuclear interactions, such as tensor forces from pion exchanges and spin-orbit forces, are essential for the two particle-hole pair response, as is clear from calculations of neutrino-pair bremsstrahlung and the modified Urca processes [13] and from general considerations based on conservation laws [11]. Neutrino-pair bremsstrahlung and absorption change the number of neutrinos and are key for equilibrating muon and tau neutrino number densities in supernovae. The standard rates for bremsstrahlung are based on the one-pion exchange model for nucleon-nucleon interactions [13] (in the context of supernovae, see, for example, Ref. [10]). This is a reasonable starting point, since it represents the long-range part and the leading noncentral contribution in chiral effective field theory for nuclear forces [14]. However, the tensor force from pion exchange is singular at short distances, which in free space requires iteration in the spin-triplet channels [15]. In addition, subleading noncentral contributions to nuclear interactions are important for reproducing nucleon-nucleon scattering for the relevant channels and energies [16].

The aim of this paper is to give a unified treatment of neutrino processes that includes one and two particle-hole pair states as well as mean-field (Fermi liquid) effects consistently, and to present improved rate calculations of these processes based on modern nuclear interactions beyond one-pion exchange and including many-body contributions. A convenient framework for doing this is Landau's theory of normal Fermi liquids. This work represents an extension of Ref. [12], which included two particle-hole pair states only in leading order using diagrammatic perturbation theory. Here we shall use the quasiparticle transport equation. This provides a useful framework for understanding the basic physics and for making detailed calculations. In this paper, we focus on neutral-current processes in normal (nonsuperfluid) neutron matter. We leave for future work the application to mixtures of neutron and protons, charged-current reactions, and the extension to superfluid phases.

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[1]The basic single-particle-like excitations we work with are quasiparticles and quasiholes that have properties quantitatively different from those of free particles or holes. However, for brevity, we shall refer to these excitations simply as particles and holes.

 

This paper is organized as follows. Section II gives an introduction to neutrino processes and the dynamical structure factors. In Sec. III, we discuss Landau Fermi-liquid theory, show that it represents a useful effective theory for neutrino processes in nucleon matter, and introduce the transport equation for quasiparticles. Using a relaxation time approximation, we solve the transport equation for density and spin-density fluctuations and derive a general form for the response functions in Sec. IV. The response function includes contributions from one particle-hole pair (corresponding to elastic scattering of neutrinos from nucleons) and two particle-hole pair states (which enter calculations of inelastic scattering, and neutrino-pair bremsstrahlung and absorption). In Sec. V, we calculate the appropriate relaxation times for the one-pion exchange interaction and for a general operator representation of the quasiparticle scattering amplitude. We present results in Sec. VI based on modern nuclear interactions and including many-body contributions, and contrast these with rates obtained using the one-pion exchange interaction, which is typically used in supernova simulations. Finally, we assess the significance of the improved treatment of nuclear interactions for neutrino mean free paths, energy loss, and energy transfer in supernovae. We summarize the improvements and conclude in Sec. VII.

## II. NEUTRINO PROCESSES AND DYNAMICAL STRUCTURE FACTORS

For neutral-current processes, the weak interaction Lagrangian density for low-energy probes is given by

$$\mathcal{L}(x) = \frac{G_F}{\sqrt{2}} l_\mu(x) j^\mu(x), \tag{1}$$

where $G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$ is the Fermi coupling constant, and the weak neutral currents are $l_\mu(x)$ for leptons and $j_\mu(x)$ for hadrons. The neutrino contribution to the leptonic current is

$$l_\mu(x) = \overline{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu, \tag{2}$$

and for nonrelativistic nucleons the hadronic current can be written as

$$\begin{aligned} j_\mu(x) &= \overline{\psi}_N \gamma_\mu (C_V - C_A \gamma_5) \psi_N \\ &\approx \phi_N^\dagger (C_V \delta_{\mu 0} - C_A \delta_{\mu i} \sigma_i) \phi_N, \end{aligned} \tag{3}$$

where $\psi_\nu$ are neutrino fields, $\psi_N$ nucleon Dirac fields, $\phi_N$ nonrelativistic nucleon spinors, and $\sigma_i$ Pauli matrices. The neutral-current vector coupling constant is $C_V = -1/2$ for neutrons and $C_V = 1/2 - 2\sin^2\theta_W \approx 0$ for protons; $C_A$ is the axial-vector coupling, $C_A = -g_a/2 = -1.26/2$ for neutrons and $C_A = g_a/2$ for protons. While the vector current is conserved, the axial coupling can be modified in a many-body system. As a result, one may expect a reduction of $g_a$ for a nucleon quasiparticle by 5–10% in neutron matter and 10–20% in symmetric nuclear matter [17,18].

Consider neutrinos with incoming energy $\omega_1$ and momentum $\mathbf{q}_1$ that scatter from nuclear matter to a final state with energy $\omega_2$ and momentum $\mathbf{q}_2$. Since neutrinos interact weakly, the rate for neutrino scattering can be expressed in terms of the dynamical structure factors for vector and axial responses of the nuclear medium [1,4]. Because neutron velocities in neutron matter at the densities of interest are nonrelativistic, these reduce to the density and spin responses, which are decoupled if the system is not magnetically polarized.

The dynamical structure factors depend on the energy and momentum transferred to the system, $\omega = \omega_1 - \omega_2$ and $\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$, and are defined for the density response by [4,19]

$$\begin{aligned} S_V(\omega, \mathbf{q}) &= \frac{1}{\pi n} \frac{1}{1 - e^{-\omega/T}} \operatorname{Im} \chi(\omega, \mathbf{q}) \\ &= \frac{1}{n} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle n(t, \mathbf{q}) n(0, -\mathbf{q}) \rangle, \end{aligned} \tag{4}$$

and for the spin response by

$$\begin{aligned} S_{A,ij}(\omega, \mathbf{q}) &= \frac{1}{\pi n} \frac{1}{1 - e^{-\omega/T}} \operatorname{Im} \chi_{ij}(\omega, \mathbf{q}) \\ &= \frac{1}{n} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle \mathbf{s}_i(t, \mathbf{q}) \mathbf{s}_j(0, -\mathbf{q}) \rangle, \end{aligned} \tag{5}$$

where $n$ denotes the neutron number density, $T$ is the temperature, $\mathbf{s} = \phi^\dagger \boldsymbol{\sigma} \phi$ is the spin density, and $\chi(\omega, \mathbf{q})$ and $\chi_{ij}(\omega, \mathbf{q})$ are the density-density and spin-density–spin-density response functions, respectively. We use units with $\hbar = c = k_B = 1$.

In the long-wavelength limit, $q \to 0$, the spin response is in the direction of the applied magnetic field, hence $\chi_{ij} = 0$ for $i \neq j$. This is not the case at nonzero $q$, and the transverse and longitudinal spin responses differ [4]. However, for neutrino processes in supernovae and neutron stars, the momentum transfers are small compared with typical momenta of the nucleons, such as the Fermi momentum or the inverse Compton wavelength, and therefore the spin response is essentially diagonal,

$$\chi_{ij} \approx \chi_\sigma \delta_{ij} \quad \text{and} \quad S_{A,ij} \approx S_A \delta_{ij}\,. \tag{6}$$

The transition probability $\Gamma(Q_1, Q_2)$ for a neutrino with energy and momentum $Q_1 = (\omega_1, \mathbf{q}_1)$ to scatter to a state $Q_2 = (\omega_2, \mathbf{q}_2)$ is fully determined by the density and spin response functions (see, for example, Refs. [1,4]),

$$\begin{aligned} \Gamma(Q_1, Q_2) = 2\pi n G_F^2 \big[ & C_V^2 (1 + \cos\theta) S_V(\omega, \mathbf{q}) \\ & + C_A^2 (3 - \cos\theta) S_A(\omega, \mathbf{q}) \big], \end{aligned} \tag{7}$$

where $\theta = \arccos(\widehat{\mathbf{q}}_1 \cdot \widehat{\mathbf{q}}_2)$ is the scattering angle. The rate for bremsstrahlung of a neutrino with four-momentum $Q_2$ and an antineutrino with four-momentum $Q_1$ is given by $\Gamma(-Q_1, Q_2)$, and for absorption of a neutrino with $Q_1$ and antineutrino with $Q_2$ by $\Gamma(Q_1, -Q_2)$. We note that Eq. (7) neglects corrections of order $\omega/m$ from weak magnetism and other effects [20]. In terms of the transition probability, the rate of change of the neutrino occupation number $n_{\mathbf{q}_1}$ due to interaction with the nuclear medium is given by

$$\begin{aligned} \frac{dn_{\mathbf{q}_1}}{dt} = \int \frac{d\mathbf{q}_2}{(2\pi)^3} \big[ & \Gamma(Q_2, Q_1) n_{\mathbf{q}_2} (1 - n_{\mathbf{q}_1}) \\ & - \Gamma(Q_1, Q_2) n_{\mathbf{q}_1} (1 - n_{\mathbf{q}_2}) \\ & + \Gamma(-Q_2, Q_1) (1 - n_{\mathbf{q}_1}) (1 - \overline{n}_{\mathbf{q}_2}) \\ & - \Gamma(Q_1, -Q_2) n_{\mathbf{q}_1} \overline{n}_{\mathbf{q}_2} \big], \end{aligned} \tag{8}$$

where $\overline{n}_{\mathbf{q}_i}$ is the antineutrino occupation number. The four terms correspond to "in-scattering," "out-scattering," and neutrino-pair bremsstrahlung and absorption, respectively. These differ only by the kinematics in the dynamical structure factors.

## III. LANDAU FERMI-LIQUID THEORY AND QUASIPARTICLE TRANSPORT EQUATION

In supernovae and neutron stars, the neutrino energies are typically $\omega_1, \omega_2 \lesssim 30$ MeV. The corresponding neutrino momenta $q_1, q_2 \lesssim 0.15\ \mathrm{fm}^{-1}$ are therefore small compared to the momenta of neutrons, which are of the order of the Fermi momentum $k_F \sim 1.0\ \mathrm{fm}^{-1}$ for densities $n \sim n_0/10$. Here, $n_0 = 0.16\ \mathrm{fm}^{-3}$ or $\rho_0 = 2.8 \times 10^{14}\mathrm{gcm}^{-3}$ denotes the saturation density of symmetric nuclear matter. Consequently, it is a good first approximation to work only to lowest order in the neutrino momenta. In addition, we focus on situations when the temperature is small compared to the Fermi energy of neutrons. This is the regime in which Landau's theory of normal Fermi liquids may be used [21,22]. Landau theory provides a clear separation between long-wavelength, low-frequency degrees of freedom, which are treated explicitly, and short-wavelength, high-frequency ones, whose effects are included in low-energy constants that incorporate the renormalization of matrix elements of currents and interparticle interactions. Another strength of Landau Fermi-liquid theory is that it brings out clearly the role played by conservation laws. Low-temperature expansions for Fermi liquids are often useful for $T/\varepsilon_F = 1/\eta \lesssim 1/\pi$. We therefore expect our results to be reasonable for degeneracy parameters $\eta \gtrsim 3$, which is typically valid for the relevant densities in supernovae and neutron stars.

Nucleon matter differs from liquid $^3$He, the prototype Fermi liquid, in that the interactions between nucleons have significant noncentral parts. This fact has several consequences. The magnetic moment of a quasiparticle is not equal to the free space value (as discussed above, the same holds for the axial coupling), and it is a tensor, that depends on the orientation of the spin with respect to the momentum of the quasiparticle. In addition, the Landau quasiparticle interaction contains tensor and other noncentral contributions [23], which couple spin and orbital degrees of freedom. For the response to a magnetic field, which is completely equivalent to the case of an axial-vector probe, these effects have been explored in Ref. [24].

In Landau Fermi-liquid theory, one describes the long-wavelength, low-frequency response of the system in terms of quasiparticles. However, if the current of interest is not a conserved quantity, the corresponding response function at long wavelengths contains contributions that cannot be expressed in terms of quasiparticle degrees of freedom. In addition, there are two-body contributions to the effective operators. In Ref. [11], it was shown from sum-rule arguments that the contribution to the response not coming from single particle-hole pairs could be substantial. One class of processes that can be calculated within Landau Fermi-liquid theory corresponds to creating a single particle-hole pair, which subsequently creates a second pair. This is taken into account by including a collision term in the transport equation for quasiparticles, and in Ref. [12] it is described how to do this, starting from diagrammatic perturbation theory.

The general formalism for calculating the rates of kinetic processes from microscopic theory is well developed, but to apply it to specific physical situations is usually complicated. However, if collisions are sufficiently infrequent, one can adopt an approach based on a kinetic equation similar to the Boltzmann equation for dilute gases, in which one introduces a distribution function for the elementary excitations that depends on the momentum of the excitation. More generally, when the width of an excitation becomes comparable to the real part of the energy of an excitation, it is necessary to work in terms of the spectral density for adding a single particle to the system (the imaginary part of the single-particle propagator), which is a function of energy as well as of momentum [25,26]. In this paper, we assume that the widths are sufficiently small that a kinetic equation approach can be used.

Next we describe the quasiparticle transport equation for a single-component Fermi system with spin 1/2. We assume that the system is not magnetically polarized. The generalization to isospin is straightforward. The quasiparticle distribution function is a matrix in spin space and we write it as

$$[n_{\mathbf{p}}]_{\alpha\alpha'} = n_{\mathbf{p}}\delta_{\alpha\alpha'} + \mathbf{s}_{\mathbf{p}} \cdot \boldsymbol{\sigma}_{\alpha\alpha'} \, . \tag{9}$$

Likewise, the quasiparticle energy can be written in the form

$$[\varepsilon_{\mathbf{p}}]_{\alpha\alpha'} = \varepsilon_{\mathbf{p}}\,\delta_{\alpha\alpha'} + \mathbf{h}_{\mathbf{p}} \cdot \boldsymbol{\sigma}_{\alpha\alpha'}, \tag{10}$$

where $\varepsilon_{\mathbf{p}}$ and $\mathbf{h}_{\mathbf{p}}$ are the spin-independent and spin-dependent contributions to the quasiparticle energy. The linearized transport equation in momentum space for the spin response $\delta\mathbf{s}_{\mathbf{p}}$ of quasiparticles with momentum $\mathbf{p}$ is given by [19,22]

$$\begin{aligned}(\omega - \varepsilon_{\mathbf{p}+\mathbf{q}/2} + \varepsilon_{\mathbf{p}-\mathbf{q}/2})\delta\mathbf{s}_{\mathbf{p}} + (n_{\mathbf{p}+\mathbf{q}/2} - n_{\mathbf{p}-\mathbf{q}/2})\delta\mathbf{h}_{\mathbf{p}} \\ = i\,I_\sigma[\mathbf{s}_{\mathbf{p}'}],\end{aligned} \tag{11}$$

where the perturbation to the quasiparticle energy is

$$\delta\mathbf{h}_{\mathbf{p}} = \mathbf{U}_\sigma + 2\int \frac{d\mathbf{p}'}{(2\pi)^3}\, g_{\mathbf{p}\mathbf{p}'}\delta\mathbf{s}_{\mathbf{p}'}, \tag{12}$$

and the dependence of $\delta\mathbf{s}_{\mathbf{p}}(\omega, \mathbf{q})$ and $\delta\mathbf{h}_{\mathbf{p}}(\omega, \mathbf{q})$ on the energy and momentum transfers is implicit. Here, $I_\sigma[\mathbf{s}_{\mathbf{p}'}]$ is the collision integral, the prime on the momentum argument indicating that it generally depends on the distribution function for states other than $\mathbf{p}$, and $\mathbf{U}_\sigma$ is an external field that couples to the nucleon spin. The spin-dependent Landau quasiparticle interaction has a central part, $g_{\mathbf{p}\mathbf{p}'}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, as well as symmetric tensor and antisymmetric terms [23]. Since the latter are generally weaker [24], we keep only the central term in Eq. (12). For the density response, Eq. (11) holds with the spin-dependent contributions replaced by their spin-independent counterparts, and the equation analogous to Eq. (12) is

$$\delta\varepsilon_{\mathbf{p}} = U + 2\int \frac{d\mathbf{p}'}{(2\pi)^3}\, f_{\mathbf{p}\mathbf{p}'}\delta n_{\mathbf{p}'} \, . \tag{13}$$

In local equilibrium, the net collision rate vanishes, and the distribution function is given by the equilibrium Fermi function for quasiparticle energy $\varepsilon_{\mathbf{p}}$, evaluated at the values of the local chemical potential, temperature, and flow velocity corresponding to the local number, energy, and momentum densities. The quasiparticle energy that occurs in the local-equilibrium distribution function includes contributions from quasiparticle interactions, so the quasiparticle energy is not the one for the equilibrium state. This choice is physically the most meaningful, because in the energy conservation condition the quasiparticle energies that appear must include the effect of the nonequilibrium quasiparticle distribution. From linear response theory and for $\omega = 0$, the local-equilibrium response then follows from Eq. (11) and is given by

$$\delta \mathbf{s}_{\mathbf{p}}\big|_{\mathrm{le}} = R_{\mathbf{p}} \delta \mathbf{h}_{\mathbf{p}} \quad \text{with} \quad R_{\mathbf{p}} = \frac{n_{\mathbf{p}+\mathbf{q}/2} - n_{\mathbf{p}-\mathbf{q}/2}}{\varepsilon_{\mathbf{p}+\mathbf{q}/2} - \varepsilon_{\mathbf{p}-\mathbf{q}/2}}, \tag{14}$$

where the subscript "le" denotes the value of the quantity for local equilibrium.

## IV. RELAXATION TIME APPROXIMATION

In general, it is difficult to solve the transport equation for the full collision integral. We therefore approximate the collision integral as

$$I_\sigma[\mathbf{s}_{\mathbf{p}'}] = -\frac{\delta \mathbf{s}_{\mathbf{p}} - \delta \mathbf{s}_{\mathbf{p}}\big|_{\mathrm{le}}}{\tau_\sigma}, \tag{15}$$

where $\tau_\sigma$ is an average relaxation time. In this section, we focus on the spin response, but analogous expressions hold for the density and isospin responses. Equation (15) amounts to the assumption that all angular harmonics of the spin-dependent part of the quasiparticle distribution function relax at the same rate, and this form ensures that the collision term vanishes when $\delta \mathbf{s}_{\mathbf{p}} = \delta \mathbf{s}_{\mathbf{p}}|_{\mathrm{le}}$. In addition, the relaxation time is assumed to be independent of the quasiparticle momentum. However, consideration of the scattering process in detail shows that in order to obtain agreement with rates in the collisionless limit, $|\omega|\tau_\sigma \to \infty$, the relaxation time must depend on the energy transfer (see Ref. [12] and Sec. V). For the spin response, $\tau_\sigma$ corresponds to the rate of change of the nucleon spin through collisions with other nucleons, and by solving the transport equation, we include multiple-scattering effects.

More generally, one could have allowed for changes in the temperature of the two different spin components, but for Fermi systems at low temperatures, this effect, which corresponds to thermoelectric phenomena for charged systems, is relatively unimportant. For most condensed matter systems, Eq. (15) is a rather poor approximation, since the total spin, which corresponds to the component of the deviation function having angular symmetry corresponding to $l = 0$, is conserved to a good approximation because noncentral forces generally play little role, while higher-$l$ components of the spin deviation function can decay on a much shorter timescale. For example, in liquid $^3$He, the lack of spin conservation is due to the interaction between the nuclear magnetic dipole moments, which is very weak compared to the central parts of the interatomic interaction. However, in nuclear systems, noncentral contributions to nuclear interactions, especially those from tensor forces due to pion exchanges, are strong, and the single relaxation time approximation is expected to be better. The approximate form for the collision term in the transport equation for the density response must have a more general form, since particle number conservation ensures that the $l = 0$ component of the distribution function does not relax, and for a single-component system, momentum conservation ensures that the $l = 1$ component does not relax either (see, for example, Ref. [27]). For a multicomponent system, such as a mixture of neutrons and protons, the number of particles of each component is conserved, and consequently the $l = 0$ components cannot relax, but the $l = 1$ components can relax, because momentum may be transferred from one component to another.

### A. Calculation of the response function

With the approximation Eq. (15), the linearized transport equation can be rewritten in the form

$$\left(\omega + \frac{i}{\tau_\sigma} - \mathbf{v}_{\mathbf{p}} \cdot \mathbf{q}\right) \delta \mathbf{s}_{\mathbf{p}} + \left(\mathbf{v}_{\mathbf{p}} \cdot \mathbf{q} - \frac{i}{\tau_\sigma}\right) R_{\mathbf{p}} \delta \mathbf{h}_{\mathbf{p}} = 0, \tag{16}$$

with $\varepsilon_{\mathbf{p}+\mathbf{q}/2} - \varepsilon_{\mathbf{p}-\mathbf{q}/2} \approx \mathbf{v}_{\mathbf{p}} \cdot \mathbf{q}$. In the expansion of the quasiparticle interaction in Legendre polynomials, the $l = 0$ term $g_0$ is the dominant spin-dependent contribution in neutron matter [28], and therefore we neglect the higher-$l$ terms. With this assumption, the perturbation to the quasiparticle energy, Eq. (12), is given by

$$\delta \mathbf{h}_{\mathbf{p}} = \mathbf{U}_\sigma + g_0 \mathbf{s} \quad \text{with} \quad \mathbf{s} = 2 \int \frac{d\mathbf{p}'}{(2\pi)^3}\, \delta \mathbf{s}_{\mathbf{p}'}\,. \tag{17}$$

As in Eq. (5), $\mathbf{s}$ is the Fourier transform of the spin deviation. We then solve the transport equation and find

$$\mathbf{s} = -\chi_\sigma(\omega, \mathbf{q}) \mathbf{U}_\sigma, \tag{18}$$

where the response function $\chi_\sigma$ is given by

$$\chi_\sigma = \frac{X_\sigma}{1 + g_0 X_\sigma}$$

and

$$X_\sigma = 2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{\mathbf{v}_{\mathbf{p}} \cdot \mathbf{q} - i/\tau_\sigma}{\omega + i/\tau_\sigma - \mathbf{v}_{\mathbf{p}} \cdot \mathbf{q}} R_{\mathbf{p}}\,. \tag{19}$$

Here $X_\sigma$ is the response function in the absence of mean-field effects. Provided the temperature is low relative to the Fermi energy, the main contributions to the integral in Eq. (19) come from the vicinity of the Fermi surface, which leads to

$$X_\sigma = N(0)\left[1 - \frac{\omega}{2 v_F q} \ln\left(\frac{\omega + i/\tau_\sigma + v_F q}{\omega + i/\tau_\sigma - v_F q}\right)\right], \tag{20}$$

where $N(0) = m^* k_F/\pi^2$ is the density of states at the Fermi surface for both spin populations, $m^*$ being the nucleon effective mass and $v_F = k_F/m^*$ the Fermi velocity. For the imaginary part of $\chi_\sigma$ we have

$$\mathrm{Im}\chi_\sigma = \frac{\mathrm{Im} X_\sigma}{|1 + g_0 X_\sigma|^2} = N(0) \frac{\mathrm{Im}\widetilde{X}_\sigma}{|1 + G_0 \widetilde{X}_\sigma|^2}, \tag{21}$$

with dimensionless Landau parameter $G_0 = N(0)g_0$, and $\widetilde{X}_\sigma = X_\sigma/N(0)$, whose imaginary part is

$$\mathrm{Im}\widetilde{X}_\sigma = \frac{\omega}{2v_F q}\big[\arctan[(\omega + v_F q)\tau_\sigma] - \arctan[(\omega - v_F q)\tau_\sigma]\big]. \quad (22)$$

The branch of the arctangent to be used is that lying between $-\pi/2$ and $+\pi/2$. For $\tau_\sigma \to \infty$, the form for $\mathrm{Im}\chi_\sigma$ given by Eqs. (21) and (22) reproduces the results of Ref. [4] for single particle-hole pair states, with

$$\mathrm{Im}\widetilde{X}_\sigma \to \frac{\pi\omega}{2v_F q}\,\Theta(v_F q - |\omega|), \quad (23)$$

where $\Theta(x)$ is the step function. Our results generalize earlier work by taking into account effects due to nonzero wavelengths and recoil of the nucleons. A direct inspection shows that the resulting dynamical structure factor satisfies the detailed balance condition $S(-\omega) = S(\omega)e^{-\omega/T}$. In contrast to Ref. [12], where calculations were made to leading order in the scattering rate, Eq. (21) contains contributions of higher order and thereby takes into account the Landau-Pomeranchuk-Migdal effect [29,30].

In the long-wavelength limit, $q \to 0$, we have

$$\widetilde{X}_\sigma(\omega, q \to 0) = \frac{1}{1 - i\omega\tau_\sigma}$$

and

$$\widetilde{\chi}_\sigma(\omega, q \to 0) = \frac{1}{1 + G_0 - i\omega\tau_\sigma}, \quad (24)$$

with imaginary part

$$\mathrm{Im}\widetilde{\chi}_\sigma(\omega, q \to 0) = \frac{\omega\tau_\sigma}{(1 + G_0)^2 + (\omega\tau_\sigma)^2}. \quad (25)$$

In the absence of mean-field effects, this has the same form as the *Ansatz* used by Raffelt *et al.* to account for multiple scattering at low $\omega$ [8–10]. Equation (25) shows that the characteristic frequency for the response is $\sim(1 + G_0)/\tau_\sigma$. The factor $1 + G_0$ indicates that near the transition to a ferromagnetic state, $G_0 \to -1$, the characteristic time becomes long, corresponding to what is referred to as critical slowing down. For neutrons, one has $G_0 > 0$ [28], and the spin response is pushed to higher frequencies.

## V. RELAXATION TIMES

To begin, we consider the time for an excess population of quasiparticles in a particular momentum, energy, and spin state (denoted by $\mathbf{p}_1$, $\varepsilon_1$, and $\boldsymbol{\sigma}_1$) to relax when the distribution function for all other states is that for equilibrium. It is convenient to consider the general case when the quasiparticles of the excess population are not on the energy shell, since this is the quantity that naturally enters calculations of the response functions at high frequency [12]. The relaxation time can be written in operator form as

$$\frac{1}{\tau(\varepsilon_1 + \omega, \boldsymbol{\sigma}_1 \cdot \widehat{\mathbf{p}}_1)} = \frac{1}{\tau(\varepsilon_1 + \omega)}(1 + \alpha\,\boldsymbol{\sigma}_1 \cdot \widehat{\mathbf{p}}_1), \quad (26)$$

where $\alpha$ is a coefficient that characterizes the strength of noncentral contributions to the relaxation rate. Unlike in systems with only central interactions ($\alpha = 0$), the relaxation rate depends on the spin orientation of the quasiparticle, because spin and momentum are coupled.

By generalizing the standard theory of relaxation rates [22] to the case of noncentral interactions, we have [12]

$$\frac{1}{\tau(\varepsilon_1 + \omega)} = \frac{3}{4}C\left[T^2 + \frac{(\varepsilon_1 + \omega)^2}{\pi^2}\right], \quad (27)$$

where the factor 3/4 is included so that energy-averaged relaxation rates have a simple form [see Eqs. (33) and (35)], and the coefficient $C$ is given by

$$C = \frac{4\pi^3}{3N(0)^2}\prod_{i=2,3,4}\left(\frac{m^*}{k_F}\int\frac{d\mathbf{p}_i}{(2\pi)^3}\delta(p_i - k_F)\right) \times (2\pi)^3\delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \times \frac{1}{4}\mathrm{Tr}\big[\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\mathcal{A}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')\big]\big|_{p_1=k_F}. \quad (28)$$

Here we have taken $\mathbf{p}_1$ to lie on the Fermi surface, $\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')$ denotes the quasiparticle scattering amplitude in units of the density of states, $\mathbf{k} = \mathbf{p}_1 - \mathbf{p}_3$ and $\mathbf{k}' = \mathbf{p}_1 - \mathbf{p}_4$ are the momentum transfers,[2] and we have neglected the neutrino momenta in the $\delta$ function that expresses momentum conservation, since they are small compared to the Fermi momentum. The factor 1/4 in Eq. (28) is the symmetry factor.[3] Since we work with antisymmetrized amplitudes, one factor of 1/2 is necessary to avoid double counting of final states, and a second factor of 1/2 comes from taking the average over initial spin states of particle 1. On the Fermi surface, the momentum transfers are orthogonal, and we can express Eq. (28) as

$$C = \frac{\pi^3 m^*}{6k_F^2}\left\langle \frac{1}{4}\mathrm{Tr}\big[\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\mathcal{A}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')\big]\right\rangle, \quad (29)$$

where the average is over the Fermi surface. In terms of $k$, $k'$, this can be written as [31]

$$\langle F(k,k')\rangle = \frac{1}{\pi}\int_0^{2k_F}\frac{dk}{k_F}\int_0^{2k_F}\frac{dk'}{k_F}\,\frac{k_F\Theta\big(4k_F^2 - k^2 - k'^2\big)}{\sqrt{4k_F^2 - k^2 - k'^2}}\,F(k,k'). \quad (30)$$

With this average, the coefficient $\alpha$ can be written as

$$\alpha = \frac{1}{2}\frac{\big\langle \mathrm{Tr}\big[\boldsymbol{\sigma}_1 \cdot \widehat{\mathbf{p}}_1\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\mathcal{A}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')\big]\big\rangle}{\big\langle \mathrm{Tr}\big[\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\mathcal{A}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')\big]\big\rangle}. \quad (31)$$

More general disturbances of the quasiparticle distribution function will depend both on the direction of the quasiparticle momentum on the Fermi surface and on the spin of the quasiparticle, and the relaxation time for the disturbance will

[2] We use $\mathbf{k}$ and $\mathbf{k}'$ for the momentum transfers between nucleons to distinguish them from the momentum transfer $\mathbf{q}$ in the structure factors. This differs from the notation used in Refs. [23,28,31], and they should also not be confused with relative momenta.

[3] We note that Refs. [13,31] use a symmetry factor of 1/2 instead of 1/4 and consequently overestimate rates by a factor of 2.

depend on an average of the scattering rate over the Fermi surface and over quasiparticle spins, weighted by functions of the direction of the quasiparticle momentum and of the spin. In general, the eigenstates of the collision operator will have a definite value of the total angular momentum, which is made up of an orbital component coming from the dependence of the quasiparticle distribution on the angle on the Fermi surface and of the spin of the quasiparticle.

The most important case for relaxation of long-wavelength spin fluctuations is a disturbance of the distribution function corresponding to a spin polarization that is independent of direction on the Fermi surface. For long wavelengths $|\omega| \gg v_F q$ and for frequencies large compared to the relaxation rate $|\omega| \gg 1/\tau_\sigma$, the appropriate average relaxation time for the transport equation and the spin response is given by [12]

$$\frac{1}{\tau_\sigma} = \frac{1}{\omega N(0)} \sum_{m_{s_1}} \int \frac{d\mathbf{p}_1}{(2\pi)^3} \frac{n(\varepsilon_1) - n(\varepsilon_1+\omega)}{\tau_\sigma(\varepsilon_1+\omega, \boldsymbol{\sigma}_1\cdot\widehat{\mathbf{p}}_1)}, \quad (32)$$

$$= \frac{1}{\omega}\int d\varepsilon_1 \frac{n(\varepsilon_1) - n(\varepsilon_1+\omega)}{\tau_\sigma(\varepsilon_1+\omega)}, \quad (33)$$

where the noncentral term in the spin relaxation rate [$\alpha_\sigma$ in the operator form analogous to Eq. (26)] averages to zero. Following Refs. [12,22], one has for the coefficient $C_\sigma$ for the spin relaxation rate

$$C_\sigma = \frac{\pi^3 m^*}{6k_F^2}\left\langle \frac{1}{12}\sum_{j=1,2,3} \mathrm{Tr}\{\mathcal{A}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\boldsymbol{\sigma}_1^j \times [(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)^j, \mathcal{A}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')]\}\right\rangle. \quad (34)$$

The commutator with the two-body spin operator demonstrates that only noncentral terms in the scattering amplitude contribute. The factor $1/12$ in Eq. (34) includes the symmetry factor $1/4$ and a factor $1/3$, because we have summed over all possible directions of the spin component $j$.

Since the dependence on the quasiparticle energy factorizes from the nuclear interaction part, we can directly calculate the average relaxation time of Eq. (33) and finally obtain

$$\frac{1}{\tau} = C[T^2 + (\omega/2\pi)^2] \quad \text{and} \quad \frac{1}{\tau_\sigma} = C_\sigma[T^2+(\omega/2\pi)^2]. \quad (35)$$

### A. One-pion exchange interaction

For the one-pion exchange (OPE) interaction, the direct and exchange contributions to the scattering amplitude in Born approximation are given by

$$\mathcal{A}^{\mathrm{OPE}}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}') = -N(0)\left(\frac{g_a}{2F_\pi}\right)^2\left[\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}}{k^2+m_\pi^2} - \frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}'\boldsymbol{\sigma}_2\cdot\mathbf{k}' + k'^2(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/2}{k'^2+m_\pi^2}\right], \quad (36)$$

with pion decay constant $F_\pi = 92.4$ MeV and neutral pion mass $m_\pi = 134.98$ MeV. The spin trace in the relaxation time for the spin response, Eq. (34), leads to

$$\frac{1}{12}\sum_{j=1,2,3}\mathrm{Tr}\{\mathcal{A}^{\mathrm{OPE}}_{\sigma_1,\sigma_2}(\mathbf{k},\mathbf{k}')\boldsymbol{\sigma}_1^j[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)^j, \mathcal{A}^{\mathrm{OPE}}_{\sigma_1,\sigma_2}(-\mathbf{k},\mathbf{k}')]\}$$
$$= \frac{4}{3}N(0)^2\left(\frac{g_a}{2F_\pi}\right)^4\left[\frac{k^4}{(k^2+m_\pi^2)^2} + \frac{k'^4}{(k'^2+m_\pi^2)^2} + \frac{k^2k'^2}{(k^2+m_\pi^2)(k'^2+m_\pi^2)}\right]. \quad (37)$$

For $m_\pi = 0$, each of the three terms in the square bracket of Eq. (37) yields 1 when averaged over the Fermi surface according to Eq. (30); for nonzero $m_\pi$, this integral can be calculated analytically, and one finds for the spin relaxation rate from one-pion exchange [13]

$$C_\sigma^{\mathrm{OPE}} = \frac{2\pi^3 m^*}{3k_F^2}N(0)^2\left(\frac{g_a}{2F_\pi}\right)^4 G\left(\frac{m_\pi}{2k_F}\right), \quad (38)$$

where the factor $G(x)$ takes into account the effects of a nonzero pion mass,

$$G(x) = 1 - \frac{5x}{3}\arctan\left(\frac{1}{x}\right) + \frac{x^2}{3(1+x^2)} + \frac{x^2}{3\sqrt{1+2x^2}}\arctan\left(\frac{\sqrt{1+2x^2}}{x^2}\right). \quad (39)$$

For $|\omega|\tau_\sigma \gg 1$, the imaginary part of the spin response function in the long-wavelength limit is given by $N(0)/(\omega\tau_\sigma)$ [see Eq. (25)]. In this limit, when multiple-scattering effects are small, our result for the dynamical structure factor using the spin relaxation time of Eq. (38) agrees with the result of Raffelt *et al.* [8,9] using $f/m_\pi \approx g_a/2F_\pi$.

We can compare the spin relaxation time $\tau_\sigma^{\mathrm{OPE}}$ with the relaxation time corresponding to decay of an excess of quasiparticles in a particular momentum state $\tau^{\mathrm{OPE}}$. For the latter, the spin trace of Eq. (37) has to be replaced by the one in the brackets $\langle\ldots\rangle$ of Eq. (29), which yields exactly the same result as the right-hand side of Eq. (37) up to the factor 4/3. As a result, we find that the spin relaxation rate and thus spin-flipping collisions obtained from the one-pion exchange interaction are comparable to the relaxation rate for decay of an excess population in one momentum state, with

$$\frac{1}{\tau_\sigma^{\mathrm{OPE}}} = \frac{4}{3}\frac{1}{\tau^{\mathrm{OPE}}}. \quad (40)$$

This result highlights the importance of noncentral contributions to nuclear interactions and encourages us to perform more systematic calculations of these rates beyond one-pion exchange. Next, we calculate the contributions to the relaxation times from a general representation of the quasiparticle scattering amplitude and present results in Sec. VI.

### B. General operator representation

For neutron matter, using the general operator representation of the scattering amplitude on the Fermi surface in the notation of Refs. [23,31], we find for the spin trace of

Eq. (34)[4]

$$\frac{1}{12}\sum_{j=1,2,3}\mathrm{Tr}\{\mathcal{A}_{\sigma_1,\sigma_2}\sigma_1^j[(\sigma_1+\sigma_2)^j,\mathcal{A}_{\sigma_1,\sigma_2}]\}$$
$$=\frac{4}{3}\big[\widetilde{\mathcal{A}}^2_{\text{tensor}}+\widetilde{\mathcal{A}}^2_{\text{exch. tensor}}-\widetilde{\mathcal{A}}_{\text{tensor}}\widetilde{\mathcal{A}}_{\text{exch. tensor}}+\mathcal{A}^2_{\text{spin-orbit}}+\mathcal{A}^2_{\text{diff. vector}}+\mathcal{A}^2_{\text{cross vector}}\big], \tag{41}$$

where the amplitudes on the right-hand side are functions of $k$ and $k'$. The scattering amplitudes on the Fermi surface $\widetilde{\mathcal{A}}_{\text{tensor}}, \widetilde{\mathcal{A}}_{\text{exch.tensor}}, \mathcal{A}_{\text{spin-orbit}}, \mathcal{A}_{\text{diff.vector}}$, and $\mathcal{A}_{\text{crossvector}}$ are real and characterize the momentum-dependent strengths (in units of the density of states) of the tensor operator $S_{12}(\widehat{\mathbf{k}})$, the exchange tensor $S_{12}(\widehat{\mathbf{k}}')$, the spin-orbit operator $i(\sigma_1+\sigma_2)\cdot\widehat{\mathbf{k}}\times\widehat{\mathbf{k}}'$, the spin difference vector $i(\sigma_1-\sigma_2)\cdot\widehat{\mathbf{k}}\times\widehat{\mathbf{P}}$ (or antisymmetric spin-orbit), and the cross vector operator $(\sigma_1\times\sigma_2)\cdot(\widehat{\mathbf{k}}'\times\widehat{\mathbf{P}})$, respectively, with two-body center-of-mass momentum $\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_3+\mathbf{p}_4$ (for details, see Refs. [23,31]). The latter two operators do not conserve the spin of the interacting particle pair and are induced in the medium due to screening by particle-hole excitations [23]. Finally, the tilde on the tensor parts of the scattering amplitude indicates that they take into account induced center-of-mass tensor operator contributions, since this is not a linearly independent operator on the Fermi surface, as discussed in Refs. [23,31].

For the spin trace of Eq. (29) corresponding to the relaxation rate for decay of an excess population in one momentum state, we have

$$\tfrac{1}{4}\mathrm{Tr}[\mathcal{A}_{\sigma_1,\sigma_2}\mathcal{A}_{\sigma_1,\sigma_2}]$$
$$=\mathcal{A}^2_{\text{scalar}}+3\mathcal{A}^2_{\text{spin}}+\tfrac{2}{3}\big[\widetilde{\mathcal{A}}^2_{\text{tensor}}+\widetilde{\mathcal{A}}^2_{\text{exch. tensor}}-\widetilde{\mathcal{A}}_{\text{tensor}}\widetilde{\mathcal{A}}_{\text{exch. tensor}}\big]+2\mathcal{A}^2_{\text{spin-orbit}}+2\mathcal{A}^2_{\text{diff. vector}}+2\mathcal{A}^2_{\text{cross vector}}, \tag{42}$$

where in addition the central parts of the scattering amplitude, $\mathcal{A}_{\text{scalar}}$ and $\mathcal{A}_{\text{spin}}$, contribute. These correspond to the spin-independent amplitude and the spin-spin operator $\sigma_1\cdot\sigma_2$, respectively. We note that all contributions in Eqs. (41) and (42) are positive. The minus sign of the direct-exchange tensor interference term is canceled by a relative minus sign in the exchange tensor amplitude.

## VI. RESULTS

We calculate the contributions beyond one-pion exchange based on low-momentum interactions $V_{\text{low}\,k}$ [32,33], which are obtained by evolving nuclear forces to low momentum using the renormalization group. The resulting two-nucleon interactions become universal at momentum scales $\Lambda \lesssim 2\ \text{fm}^{-1}$ and provide a basis for model-independent predictions of low-energy processes. The renormalization-group evolution preserves the long-range parts from pion exchanges, and $V_{\text{low}\,k}$ includes subleading noncentral contributions, so that all low-energy nucleon-nucleon scattering observables and deuteron properties are reproduced. In this first study, we have not included contributions from low-momentum three-nucleon interactions [34]. Their effects are generally weaker in neutron matter, but calculations of the equation of state show that three-nucleon interactions become important for $k_F \gtrsim 1.5\ \text{fm}^{-1}$ [35]. We will study their contributions to neutrino processes in future work.

In addition, we include many-body noncentral and central correlations from second-order particle-particle (plus hole-hole) and particle-hole contributions using the same $V_{\text{low}\,k}$ interactions. The resulting quasiparticle scattering amplitudes are discussed in detail in Ref. [23] and have been used to calculate the neutrino emissivity from pair bremsstrahlung for neutron star cooling [31]. Based on our results and general arguments [36], second-order corrections become reasonable for low-momentum interactions. The intermediate states include all possible excitations for interacting particles on the Fermi surface. We use the effective mass obtained from the lowest-order $V_{\text{low}\,k}$ for all results, including for the estimates based on the one-pion exchange interaction. The effective mass varies from $m^*/m = 0.95$ at $k_F = 1.0\ \text{fm}^{-1}$ to $m^*/m = 0.78$ at $k_F = 2.0\ \text{fm}^{-1}$, and in this range it is well approximated by a linear dependence on the Fermi momentum. We note that one expects an increase of the effective mass due to polarization effects, but this is compensated for by the reduction of the quasiparticle strength $z_{k_F}$, as can be seen from the results of the renormalization-group calculation of induced interactions in neutron matter [28]. We emphasize that a second-order calculation cannot give final results, but it provides a range for the effects due to many-body correlations.

Finally, we note that the effect of particle-particle correlations on neutrino-pair bremsstrahlung and other neutrino processes has been investigated previously in Refs. [37–40].

### A. Relaxation times

Our results for the spin relaxation coefficient $C_\sigma$ of Eq. (34) are shown in Fig. 1. For energies $\omega = 0$ and $T = 5$–$10$ MeV, the value of $C_\sigma = 0.1\ \text{MeV}^{-1}$ corresponds to spin relaxation rates $1/\tau_\sigma = 2.5$–$10$ MeV. We find that the OPE model significantly overestimates the strength of noncentral contributions, compared to low-momentum interactions $V_{\text{low}\,k}$, for all considered densities. Beyond the $V_{\text{low}\,k}$ results, we find that second-order many-body contributions reduce the spin relaxation rate, especially at lower densities (note that $C_\sigma$ is proportional to the square of the quasiparticle scattering amplitude). These effects are due to second-order particle-hole interference of tensor with strong central interactions, which are driven by large scattering lengths at very low densities. The band in Fig. 1 from $V_{\text{low}\,k}$ to including second-order contributions provides a range for the effects due to many-body correlations. In addition, we observe that the spin relaxation rate depends only weakly on density, and the rate obtained from $V_{\text{low}\,k}$ plus second-order contributions is dominated by the tensor terms in Eq. (41).

[4]We note that the factor 3 in front of the cross vector amplitude in Eq. (7) of Ref. [31] should be 1.

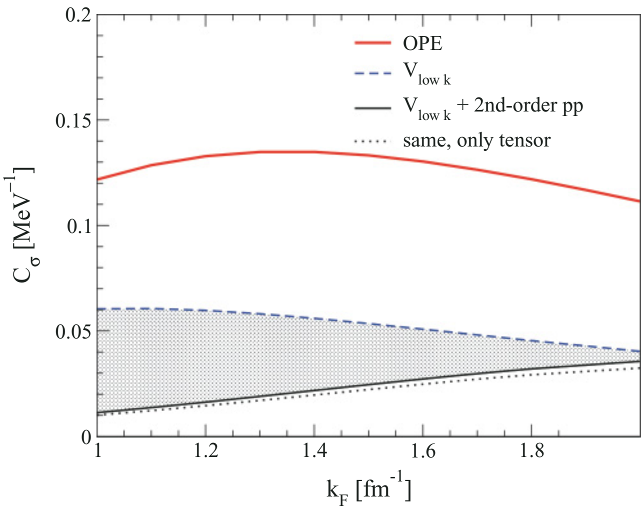


FIG. 1. (Color online) Spin relaxation rate given by $C_\sigma$ of Eq. (34) as a function of Fermi momentum $k_F$ obtained from the OPE interaction, from low-momentum interactions $V_{\text{low}\,k}$, and including second-order many-body contributions. In addition, we show that the result obtained from $V_{\text{low}\,k}$ plus second-order contributions is dominated by tensor interactions (dotted vs solid line).

For the relaxation coefficient $C$ of Eq. (29) corresponding to decay of an excess of quasiparticles in a particular momentum state, we obtain rates in Fig. 2 that are of similar magnitude as the spin relaxation rate. While the OPE rate is approximately independent of density, the OPE model underestimates the relaxation rate at low densities. This is because the central part of the OPE interaction $\sim k^2$ and $\sim k'^2$ does not capture the central shorter-range physics in nuclear forces. This deficiency of the OPE model is most prominent at low densities, in

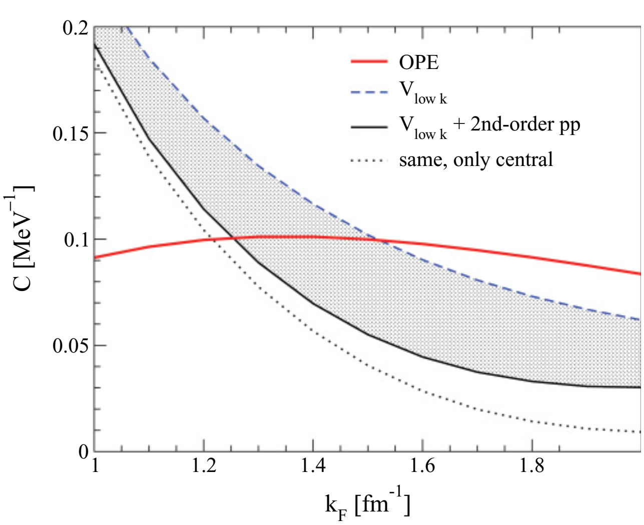


FIG. 2. (Color online) Relaxation rate for decay of an excess of quasiparticles in a particular momentum state given by $C$ of Eq. (29) as a function of Fermi momentum $k_F$ obtained from the OPE interaction, from low-momentum interactions $V_{\text{low}\,k}$, and including second-order many-body contributions. In addition, we show that the result obtained from $V_{\text{low}\,k}$ plus second-order contributions is dominated by central interactions (dotted vs solid line).

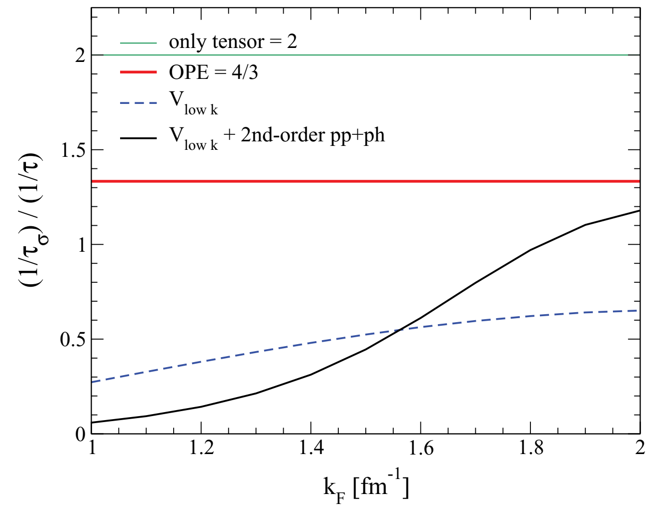


FIG. 3. (Color online) Ratio of the spin relaxation rate to the relaxation rate for an excess of quasiparticles in a single momentum state $(1/\tau_\sigma)/(1/\tau)$ as a function of Fermi momentum $k_F$ for purely tensor scattering amplitudes (in which case the value is 2), for the OPE interaction (which gives the value 4/3), from low-momentum interactions $V_{\text{low}\,k}$, and including second-order many-body contributions.

comparison to the increasing $V_{\text{low}\,k}$ rate. Similar to the spin response, we find a reduction of $C$ due to second-order many-body contributions, where the band in Fig. 2 again indicates a range for the effects due to many-body correlations. Finally, as expected, the relaxation rate obtained from $V_{\text{low}\,k}$ plus second-order contributions is now dominated by the central terms in Eq. (42).

In Fig. 3 we show the ratio $(1/\tau_\sigma)/(1/\tau)$ of the spin relaxation rate to the relaxation rate for an excess of quasiparticles in a single momentum state as a function of Fermi momentum $k_F$. This is a very useful measure of the strength of noncentral interactions relative to central ones. For purely tensor scattering amplitudes, the ratio of the corresponding spin traces in Eqs. (41) and (42) gives $(1/\tau_\sigma)/(1/\tau) = 2$, while for the OPE interaction, which has a central part in Eq. (36), this ratio is $(1/\tau_\sigma)/(1/\tau) = 4/3$, see Eq. (40). While the ratio obtained from $V_{\text{low}\,k}$ and including second-order many-body contributions is considerably smaller at low densities, the relative strength of noncentral interactions increases with momentum and thus with density, as can be seen in the results of Fig. 3 based on modern nuclear forces.

### B. Dynamical structure factor

Motivated by the importance for neutrino rates, we focus on the spin response in this section. The dynamical structure factor is determined by the imaginary part of the spin response function $\text{Im}\chi_\sigma$, which is given by Eq. (21) in the relaxation time approximation. In units of the density of states, the imaginary part $\text{Im}\widetilde{\chi}_\sigma$ is a function of $v_F q \tau_\sigma$ and $\omega/(v_F q)$ or of $v_F q \tau_\sigma$ and $\omega\tau_\sigma$. In the long-wavelength limit, $q \to 0$, we have already found that this is proportional to $\omega$ times a Lorentzian function of $\omega$, see Eq. (25). Therefore, we plot in Fig. 4 the imaginary part of the spin response function versus

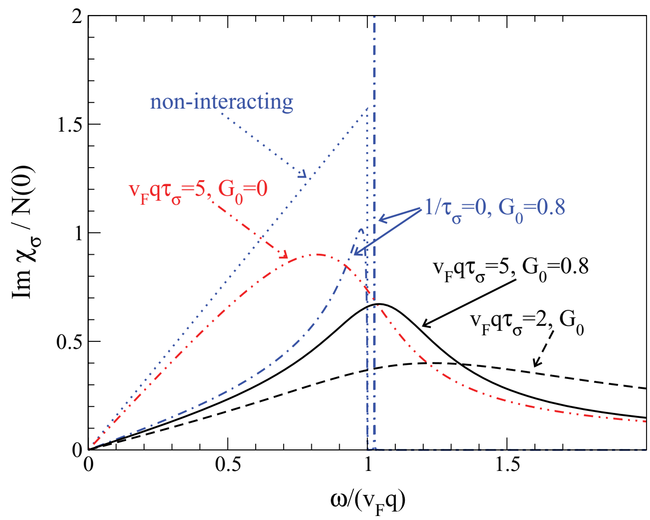


FIG. 4. (Color online) Imaginary part of the spin response function $\mathrm{Im}\chi_\sigma/N(0)$ of Eq. (21) in units of the density of states vs $\omega/(v_F q)$. Results are shown for the noninteracting system, without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$, respectively, and for different values of the spin relaxation rate $1/\tau_\sigma = 0$, $v_F q \tau_\sigma = 2$ and $v_F q \tau_\sigma = 5$.

$\omega/(v_F q)$. Results are shown for the noninteracting system, without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$, respectively, and for different values of the spin relaxation rate $1/\tau_\sigma = 0$, $v_F q/5$, and $v_F q/2$. We have taken the Landau parameter from renormalization-group calculations of induced interactions in neutron matter [28], which yield $G_0 \approx 0.8$ over the densities considered in Sec. VI A. The values of $v_F q \tau_\sigma =$ 25 correspond to spin relaxation rates based on Fig. 1 for typical momentum transfers $q \sim \omega$ over the range $T = 5$–10 MeV and $k_F = 1.0$–1.7 fm$^{-1}$. With $1/\tau_\sigma$ comparable to $v_F q$, these estimates also show that recoil effects may be important.

In the noninteracting case, $G_0 = 0$ and $1/\tau_\sigma = 0$, the imaginary part of the spin response function is given by $\pi\omega/(2v_F q)$ times a step function, see Eq. (23). With single-pair mean-field effects, $G_0 = 0.8$, a collective spin-zero-sound mode appears as a pole contribution at $\omega/(v_F q)|_{\mathrm{zs}} > 1$, where the position of the pole is given by [4]

$$1 + G_0 \widetilde{X}_\sigma(\omega/(v_F q)|_{\mathrm{zs}}, 1/\tau_\sigma = 0) = 0\,. \tag{43}$$

As the spin relaxation rate increases, going from $1/\tau_\sigma = 0$ to $v_F q/5$ and $v_F q/2$, the response is pushed to higher frequencies, and the spin-zero-sound peak disappears already for these moderate spin relaxation rates. For comparison, we also show the effects due to single-pair states at $v_F q \tau_\sigma = 5$, where interactions ($G_0 = 0.8$) decrease the response at low $\omega/(v_F q)$ and also move the strength to higher frequencies.

### C. Neutrino mean free paths, energy loss, and energy transfer

We next assess the significance of the improved rates for neutrino mean free paths, energy loss, and energy transfer. For derivations of Eqs. (44)–(47), see Refs. [1,10]. All rates are for one neutrino flavor. We emphasize that the OPE results are based on the solution to the transport equation in the relaxation time approximation, and do not correspond directly to OPE rates used in supernova simulations. For simple estimates, we use the dynamical structure factor for spin fluctuations in the long-wavelength limit, $S_\sigma(\omega) = S_\sigma(\omega, q \to 0)$, given by Eqs. (5) and (25), without further approximations or *Ansätze* for the structure factor. Effects due to the finite wavelength and recoil of the nucleons will be studied in future work.

In Table I, we present results for an average inverse neutrino mean free path $\langle\lambda^{-1}\rangle$,

$$\langle\lambda^{-1}\rangle = \frac{C_A^2 G_F^2}{20\pi}\,\frac{n}{T^3}\int_0^\infty d\omega\omega^5 e^{-\omega/T} S_\sigma(\omega), \tag{44}$$

for characteristic temperatures and Fermi momenta. This result applies for a Maxwellian initial distribution of neutrinos, and Pauli blocking in the final state has been ignored. We consider structure factors without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$ respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1. With the spin relaxation rates obtained from $V_{\mathrm{low}\,k}$ and including second-order many-body contributions, the mean free paths are significantly longer than in the OPE model. This follows the reduction of $C_\sigma$ seen in Fig. 1. For OPE, the effects of interactions ($G_0 = 0.8$ compared with $G_0 = 0$) reduce the neutrino scattering rate, especially at higher temperature. In contrast, with the rates based on low-momentum interactions, $\omega\tau_\sigma$ is larger and the imaginary part of the spin response function approaches $\mathrm{Im}\chi_\sigma(\omega, q \to 0) \to N(0)/(\omega\tau_\sigma)$. As a result, mean-field effects are weak for $|\omega|\tau_\sigma \gg 1$ in the long-wavelength limit.

TABLE I. Thermally averaged inverse neutrino mean free path $\langle\lambda^{-1}\rangle$ in km$^{-1}$ calculated from Eq. (44) for characteristic temperatures and Fermi momenta. Results are given without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$, respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1.

| $G_0$ | | 0 | 0.8 | 0 | 0.8 | 0 | 0.8 |
|---|---|---|---|---|---|---|---|
| $k_F$ (fm$^{-1}$) | $T$ (MeV) | $C_\sigma$ from OPE | | $V_{\mathrm{low}\,k}$ | | $V_{\mathrm{low}\,k}$+ 2nd order | |
| 1.0 | 5 | 0.0770 | 0.0697 | 0.0397 | 0.0386 | 0.00754 | 0.00753 |
| | 10 | 1.08 | 0.798 | 0.612 | 0.554 | 0.120 | 0.120 |
| 1.7 | 5 | 0.119 | 0.107 | 0.0476 | 0.0468 | 0.0296 | 0.0294 |
| | 10 | 1.66 | 1.21 | 0.744 | 0.700 | 0.470 | 0.457 |

TABLE II. Energy-loss rate $Q$ of Eq. (45) due to neutrino-pair bremsstrahlung, $nn \to nn\nu\bar{\nu}$, for characteristic temperatures and Fermi momenta. Results are given without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$, respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1. The energy-loss rates are in units of $10^{33}$erg cm$^{-3}$ s$^{-1}$ for $T = 5$ MeV and $10^{35}$erg cm$^{-3}$ s$^{-1}$ for $T = 10$ MeV.

| $G_0$ | | 0 | 0.8 | 0 | 0.8 | 0 | 0.8 |
|---|---|---|---|---|---|---|---|
| $k_F$ (fm$^{-1}$) | $T$ (MeV) | $C_\sigma$ from OPE | | $V_{\text{low}\,k}$ | | $V_{\text{low}\,k}$+ 2nd order | |
| 1.0 | 5 | 1.77 | 1.62 | 0.911 | 0.888 | 0.173 | 0.172 |
| | 10 | 4.02 | 3.00 | 2.25 | 2.06 | 0.441 | 0.440 |
| 1.7 | 5 | 2.75 | 2.49 | 1.09 | 1.07 | 0.679 | 0.675 |
| | 10 | 6.18 | 4.55 | 2.73 | 2.57 | 1.72 | 1.68 |

The energy-loss rate $Q$ due to neutrino-pair bremsstrahlung, $nn \to nn\nu\bar{\nu}$, of neutron matter transparent to neutrinos is given by

$$Q = \frac{C_A^2 G_F^2 n}{20\pi^3} \int_0^\infty d\omega \omega^6 e^{-\omega/T} S_\sigma(\omega) . \tag{45}$$

Our results for the energy-loss rate $Q$ are listed in Table II for characteristic temperatures, Fermi momenta, and the different cases of the structure factor. They follow the same general pattern as the inverse mean free paths in Table I: a reduction of the energy loss calculated with modern nuclear forces compared to that from OPE and consequently weak mean-field effects.

Finally, we consider the rate of energy transfer $\Delta Q/\Delta T$ from neutron matter at temperature $T$ to a neutrino fluid at temperature $T_\nu$, with $\Delta T = T - T_\nu$ and $|\Delta T| \ll T$. The energy transfer due to neutrino-pair bremsstrahlung and absorption, $nn \leftrightarrow nn\nu\bar{\nu}$, is given by

$$\frac{\Delta Q}{\Delta T} = \frac{C_A^2 G_F^2}{20\pi^3} \frac{n}{T^2} \int_0^\infty d\omega \omega^7 e^{-\omega/T} S_\sigma(\omega), \tag{46}$$

and for inelastic scattering, $\nu nn \leftrightarrow \nu nn$, one has

$$\frac{\Delta Q}{\Delta T} = \frac{30 C_A^2 G_F^2 n T^3}{10\pi^3} \int_0^\infty d\omega \omega^2 (12 + 6\omega/T + (\omega/T)^2) \times e^{-\omega/T} S_\sigma(\omega) . \tag{47}$$

Our rates for the energy transfer are shown in Table III for characteristic temperatures, Fermi momenta, and the various cases for the structure factor. The pattern of these rates is similar to what we found for other rates in Tables I and II. In addition, for all cases we find that the energy transfer due to inelastic scattering is less than a factor of 2 larger than the contributions from neutrino-pair bremsstrahlung and absorption. In contrast, Hannestad and Raffelt estimated this

TABLE III. Rate of energy transfer $\Delta Q/\Delta T$ due to neutrino-pair bremsstrahlung and absorption, $nn \leftrightarrow nn\nu\bar{\nu}$, of Eq. (46) and due to inelastic scattering, $\nu nn \leftrightarrow \nu nn$, of Eq. (47) for characteristic temperatures and Fermi momenta. Results are given without and with mean-field effects, $G_0 = 0$ and $G_0 = 0.8$, respectively, and for different spin relaxation rates $1/\tau_\sigma$ based on Fig. 1. The rates are in units of $10^{33}$erg cm$^{-3}$ s$^{-1}$ MeV$^{-1}$ for $T = 5$ MeV and $10^{35}$erg cm$^{-3}$ s$^{-1}$ MeV$^{-1}$ for $T = 10$ MeV.

| $G_0$ | | 0 | 0.8 | 0 | 0.8 | 0 | 0.8 | |
|---|---|---|---|---|---|---|---|---|
| $k_F$ (fm$^{-1}$) | $T$ (MeV) | $C_\sigma$ from OPE | | $V_{\text{low}\,k}$ | | $V_{\text{low}\,k}$+ 2nd order | | |
| 1.0 | 5 | 2.48 | 2.26 | 1.27 | 1.24 | 0.241 | 0.241 | $nn \leftrightarrow nn\nu\bar{\nu}$ |
| | | 3.46 | 2.81 | 1.94 | 1.76 | 0.401 | 0.394 | $\nu nn \leftrightarrow \nu nn$ |
| | 10 | 2.81 | 2.10 | 1.58 | 1.44 | 0.308 | 0.307 | $nn \leftrightarrow nn\nu\bar{\nu}$ |
| | | 3.41 | 2.24 | 2.20 | 1.79 | 0.502 | 0.485 | $\nu nn \leftrightarrow \nu nn$ |
| 1.7 | 5 | 3.85 | 3.48 | 1.53 | 1.50 | 0.949 | 0.943 | $nn \leftrightarrow nn\nu\bar{\nu}$ |
| | | 5.33 | 4.30 | 2.38 | 2.20 | 1.53 | 1.46 | $\nu nn \leftrightarrow \nu nn$ |
| | 10 | 4.32 | 3.18 | 1.91 | 1.80 | 1.21 | 1.18 | $nn \leftrightarrow nn\nu\bar{\nu}$ |
| | | 5.21 | 3.37 | 2.76 | 2.35 | 1.84 | 1.67 | $\nu nn \leftrightarrow \nu nn$ |

ratio to be 10 [10]. However, in making this estimate they used Eqs. (46) and (47) with $\mathrm{Im}\chi_\sigma(\omega, q \to 0) \sim 1/\omega^2$, while we find $\mathrm{Im}\chi_\sigma(\omega, q \to 0) \sim 1/\omega$.

## VII. CONCLUDING REMARKS

We have developed a unified treatment for neutrino processes in nucleon matter based on Landau's theory of Fermi liquids that consistently includes one and two particle-hole pair states. The contributions from two particle-hole pair states are crucial for neutrino-pair bremsstrahlung and absorption, for inelastic scattering, modified Urca reactions, and axion emission. In supernovae, neutrino-pair bremsstrahlung and absorption dominate the neutrino-number changing reactions and are key to the production of muon and tau neutrinos.

Neutrino rates involving two nucleons can be calculated in terms of the collision integral in the Landau transport equation for quasiparticles. Using a relaxation time approximation, we have solved the transport equation for density and spin-density fluctuations and derived a general form for the response functions. The solution includes multiple-scattering effects and effects due to nonzero wavelengths and recoil of the nucleons. We have applied our approach to neutral-current processes in neutron matter, but the generalization to isospin is straightforward. Our results for the spin response are summarized by Eqs. (5), (20), (21), (22), (35) and the values of $C_\sigma$ of Fig. 1.

We have calculated the relaxation times based on the OPE model and for a general representation of the quasiparticle scattering amplitude. For OPE, the spin relaxation rate is comparable to the quasiparticle relaxation rate, $\tau/\tau_\sigma = 4/3$. This highlights the importance of noncentral contributions to nuclear interactions. We therefore performed more systematic calculations of these rates. In addition, for $|\omega|\tau_\sigma \gg 1$ and in the long-wavelength limit, our result for the dynamical structure factor agrees with Raffelt *et al.* [8,9].

Beyond OPE, we have calculated the relaxation times based on low-momentum interactions $V_{\mathrm{low}\,k}$ and including second-order many-body contributions. The effects of three-nucleon interactions are generally weaker in neutron matter [35], but need to be included in future work. The OPE model significantly overestimates the strength of noncentral contributions, compared to results for low-momentum interactions $V_{\mathrm{low}\,k}$, for all considered densities. Beyond the $V_{\mathrm{low}\,k}$ results, we have found that second-order many-body contributions reduce the spin relaxation rate, especially at lower densities. This provides a range in Figs. 1 and 2 for the effects due to many-body correlations. By using spin relaxation times that incorporate both "in-scattering" and "out-scattering" terms in the transport equation, effects corresponding to vertex corrections in the microscopic theory are automatically taken into account.

Using the spin response in the long-wavelength limit, but without further approximations or *Ansätze* for the structure factor, we have estimated the significance of the improved rates for neutrino mean free paths, energy loss, and energy transfer. We have found a reduction of these rates using modern nuclear forces compared to OPE and consequently weak mean-field effects. In addition, for all cases we find that the energy transfer due to inelastic scattering is not significantly larger than that due to neutrino-pair bremsstrahlung and absorption.

One may ask how good the relaxation time approximation is. Our choice of spin relaxation time is designed to agree with microscopic theory in the collisionless limit, $|\omega|\tau_\sigma \gg 1$, and at long wavelengths. For the hydrodynamic limit, $|\omega|\tau_\sigma \ll 1$, and long wavelengths, exact solutions of the transport equation have been obtained, and one finds [22,41,42]

$$\frac{\tau_\sigma|_{\mathrm{hydro}}}{\tau} = \frac{4}{3}\sum_{\nu=1,3,5,\ldots} \frac{2\nu+1}{\nu(\nu+1)[\nu(\nu+1)-2+2\,\tau/\tau_\sigma]}\,. \tag{48}$$

The relaxation time for the hydrodynamic limit is always greater than or equal to that for the collisionless limit. For $\tau_\sigma/\tau \gg 1$, $\tau_\sigma|_{\mathrm{hydro}} = \tau_\sigma$, while for $\tau_\sigma/\tau = 1$, $\tau_\sigma|_{\mathrm{hydro}} = (\pi^2/9)\,\tau_\sigma$. Since for realistic nuclear interactions, $\tau_\sigma$ is significantly larger than $\tau$, this indicates that differences between spin relaxation times in the collisionless and hydrodynamic limits are expected to be on the order of a few percent. Consequently, uncertainties due to the use of the relaxation time approximation are small compared with other uncertainties in the calculation.

The use of the quasiparticle transport equation with a collision term allows us to include some two particle-hole pair states, but not all. Among contributions not included are terms that correspond to the incoherent parts of the propagator for a particle-hole pair, that is, to contributions that do not correspond to an intermediate state containing a well-defined quasiparticle together with a well-defined quasihole. Moreover, there are intrinsic two-body contributions to hadronic weak currents. Further work is needed to determine how important these additional contributions are.

There are numerous directions for future work. One is to explore mixtures of neutrons and protons. A second is to extend the calculations to situations when matter is less degenerate. As one sees from our results, there is significant uncertainty in the effects of the medium on quasiparticle scattering amplitudes, since there are sizable differences between rates obtained with $V_{\mathrm{low}\,k}$ and those that include many-body contributions to second order, and an important task is to reduce these uncertainties.

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