# Shell model formalism for all hypernuclei types: A guide to solving the nonmesonic weak decay puzzle

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We extend to odd-odd core hypernuclei our independent particle shell model (IPSM) formalism developed previously for the evaluation of the $\Gamma_{\rm NM}$, $\Gamma_{n/p}$, and $a_\Lambda$ hypernuclear weak decay observables. The present procedure reproduces the even-odd and even-even core results as particular cases. Adopting the standard strangeness-changing weak $\Lambda N \to NN$ transition potential with exchange of the complete pseudoscalar and vector meson octets ($\pi, \eta, K, \rho, \omega, K^*$) we get simple analytical expressions for all observables. Numerical values for ${}^{4}_{\Lambda}$He, ${}^{5}_{\Lambda}$He, ${}^{11}_{\Lambda}$B, ${}^{12}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{17}_{\Lambda}$O, and ${}^{28}_{\Lambda}$Si hypernuclei are obtained and compared with available experimental data, putting special attention on the asymmetry parameter. We remark that, in the present form, the IPSM gives roughly the same value of $a_\Lambda$ for all hypernuclei in contradiction with experiments. We stress the necessity of introducing configuration mixing to go beyond the IPSM taking into account, in a more realistic way, nuclear structure effects. Moreover, one could to include more relevant degrees of freedom, even within the IPSM framework, like: (i) modifications of the exchange potential (two-pion, $a_1$ meson, $\Delta T = 3/2$ terms of vector mesons, etc.), (ii) final state interactions accounting for the distortion of the plane waves of emitted nucleons, and (iii) two-nucleon induced decay, as possible ways to solve the puzzle.

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## I. INTRODUCTION

Hypernuclear physics adds a new flavor (strangeness) to usual nuclear physics studying the behavior of hyperons ($\Lambda$, $\Sigma$, $\Xi$, $\Omega$) inside the nuclear medium, which is now a bound system of neutrons, protons, and hyperons. In particular, we are interested in hypernuclei with strangeness $S = -1$, produced mainly via strong interactions through the $\pi^+ n \to \Lambda K^+$, $K^- n \to \pi^- \Lambda$, and $K^- p \to \pi^0 \Lambda$ reactions. As known, the free $\Lambda$ hyperon decays mainly through the weak mesonic mode $\Lambda \to N\pi$, but inside the nuclear medium this mode is Pauli blocked and a new nonmesonic mode, $\Lambda N \to NN$, is opened. It can be stimulated either by neutrons ($\Lambda n \to nn$) or protons ($\Lambda p \to np$) with rates $\Gamma_n$ and $\Gamma_p$, respectively.

The nonmesonic hypernuclear weak decay (NMHWD) offers a good opportunity to analyze the $|\Delta S| = 1$ nonleptonic weak interactions between hadrons inside nuclear medium. For many years there has been a great effort to find agreement between theoretical and experimental results for the observables of this mode: the total decay rate, $\Gamma_{\rm NM} = \Gamma_n + \Gamma_p$, the neutron to proton ratio, $\Gamma_{n/p} = \Gamma_n/\Gamma_p$, and the intrinsic asymmetry parameter in polarized hypernuclei, $a_\Lambda$. Between theoretical calculations we have (i) the simplest one-pion exchange model (OPEM) assuming the $\Delta T = 1/2$ isospin rule [1,2]; (ii) models including exchange of heavier mesons like $\eta, K, \rho, \omega$, and $K^*$ [3–7]; (iii) models describing short-range baryon-baryon interaction in terms of quark degrees of freedom [8,9]; (iv) models including correlated (in the form of $\sigma$ and $\rho$ mesons) and uncorrelated two-pion ($2\pi$) exchange [10–13], and also the axial-vector $a_1$ meson [14]; (v) models including interaction terms that violate the isospin $\Delta T = \frac{1}{2}$ rule [15–18]; (vi) analysis of the two-nucleon stimulated process $\Lambda NN \to NNN$ [19–21]; and (vii) calculations taking into account the effect of final state interactions (FSI) involving the ejected nucleons [22–27]. These calculations, performed by some in the nuclear matter framework and by others using shell model (SM), reproduce fairly well the total nonmesonic decay rate. In the light of recent developments [23,24] and new measurements [28–34], the experimental value of $\Gamma_{n/p}$ seems to be small and close to 0.50. However, the majority of calculations predict very similar values for the asymmetry parameter, $a_\Lambda$, for ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C (in the range from −0.73 to −0.19) although measurements favor a negative value for ${}^{12}_{\Lambda}$C and a positive one for ${}^{5}_{\Lambda}$He. These discrepancies constitute an interesting puzzle involving the hypernuclear weak decay that still needs to be solved. Here it is important to remark on the relevance of the recent works from Refs. [12–14] that point to possible solutions via the inclusion of correlated plus uncorrelated chirally motivated $2\pi$ exchange, or $a_1$ meson.

Despite the fact that nuclear matter and SM calculations lead to similar general conclusions about the mentioned observables, in Refs. [5–7] the necessity of using a SM framework to calculate the observables, mainly for light hypernuclei where the nuclear matter approximation cannot be well justified, has been discussed and analyzed carefully. Moreover, Ref. [35] has established the bridge between both formalisms. As usual, a ${}^{A}_{\Lambda}Z$ hypernucleus is represented as a $\Lambda$ particle coupled to a ${}^{A-1}Z$ core. Thus, we can group the hypernuclei according to their even-even, even-odd, and odd-odd cores. The SM formalism developed in our previous works involves a partial wave expansion of the emitted nucleon waves, preserving naturally the antisymmetrization between the escaping particles and the residual core. However, there only the cases of even-even and even-odd hypernuclei, ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C, respectively, were treated. In looking for general conclusions about the theoretical values for the NMHWD observables, it would be very useful to have a valid SM formalism for all types of hypernuclei. This will contribute to our understanding of why a similarity between theoretical values of the observables for different hypernuclei exists; at the same time it could clearly give a guide to finding the origin of the discrepancies between theory and experiments. Motivated by the previously mentioned arguments, here we extend to the odd-odd case the formalism of Refs. [5] and [7] and evaluate the observables for several different hypernuclei.

The article is organized as follows. In Sec. II the general formalism is presented and we show that it reproduces in particular our previous results from Refs. [5] and [7]. In Sec. III we give the numerical results for ${}^{4}_{\Lambda}$He, ${}^{5}_{\Lambda}$He, ${}^{11}_{\Lambda}$B, ${}^{12}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{17}_{\Lambda}$O, and ${}^{28}_{\Lambda}$Si hypernuclei, draw our conclusions, and give our final remarks.

## II. FORMALISM

The decay rates $\Gamma_n$ and $\Gamma_p$ for the NMHWD of an initial hypernucleus (with spin $J_I$ and energy $\mathcal{E}_I$) to a residual nucleus (with spin $J_F$ and energy $\mathcal{E}_F$) plus two free nucleons (with total spin $S$ and energies $\epsilon_P \equiv \epsilon$ and $\epsilon_p = \Delta_F - \epsilon$) can be evaluated by means of Fermi's golden rule as [5]

$$\Gamma_N = \frac{16M_N^3}{\pi}\sum_{J_F\nu_{J_F}}\int_0^{\Delta_F} d\epsilon\sqrt{\epsilon(\Delta_F-\epsilon)} \times \sum_{SlL\lambda JT}\left|\left\langle plPL\lambda SJTJ_F\nu_{J_F};J_I|V|J_I\right\rangle\right|^2, \quad (1)$$

where $p=\sqrt{M_N(\Delta_F-\epsilon)}$ and $P=2\sqrt{M_N\epsilon}$ are the relative and center-of-mass momenta and $\Delta_F=\mathcal{E}_I-\mathcal{E}_F-2M_N$ is the released energy, with $M_N$ being the nucleon mass. $\nu_{J_F}$ are the set of labels to specify different states with the same $J_F$ (the final nucleus isospin $M_{T_F}=M_{T_I}-m_{t_N}-m_{t_\Lambda}$ is fixed); $M_T=m_{t_N}+m_{t_\Lambda}$, with $m_{t_N}=\frac{1}{2}(-\frac{1}{2})$ for $N=p\ (n)$ and $m_{t_\Lambda}=-1/2$; and $V$ is the transition potential, where

$$\langle plPL\lambda SJTJ_F\nu_{J_F};J_I|V|J_I\rangle = \hat{J}_I^{-1}\sum_{\mathrm{j}_N} f_J(\mathrm{j}_N J_F\nu_{J_F})\mathcal{M}(plPL\lambda SJT;\mathrm{j}_\Lambda \mathrm{j}_N m_{t_N}), \quad (2)$$

with $\mathrm{j}_N \equiv n_N\, l_N\, j_N\, t_N$ and $\mathrm{j}_\Lambda \equiv n_\Lambda\, l_\Lambda\, j_\Lambda\, t_\Lambda$ being the single-particle states for the nucleon and $\Lambda$, respectively (we assume that the $\Lambda$ particle behaves as a $|\frac{1}{2},-\frac{1}{2}\rangle$ isospin particle in the $1s_{1/2}$ level). Here we define

$$f_J(\mathrm{j}_N J_F\nu_{J_F}) = (-)^{2J_F}\hat{J}\hat{J}_I\begin{Bmatrix} J_C & J_I & j_\Lambda \\ J & j_N & J_F\end{Bmatrix}\langle J_C\|a^\dagger_{\mathrm{j}_N m_{t_N}}\|J_F\nu_{J_F}\rangle, \quad (3)$$

with $J_C$ being the core spin such that $|(J_C j_\Lambda)J_I\rangle$, and the matrix element

$$\mathcal{M}(plPL\lambda SJT;\mathrm{j}_\Lambda \mathrm{j}_N m_{t_N}) = \frac{1}{\sqrt{2}}[1-(-)^{l+S+T}](plPL\lambda SJT|V|\mathrm{j}_\Lambda \mathrm{j}_N m_{t_N}J) \quad (4)$$

corresponds to the two body matrix element between the bounded $\Lambda$-nucleon system and the two final unbounded nucleons.

For the sake of simplicity we will work within the independent particle shell model (IPSM) where the final states are built by single hole states created on $|J_C\rangle$. Figure 1 shows the initial and final states contributing to each case (even-even, even-odd, and odd-odd core). We begin with a ${}^{A}_{\Lambda}Z$ hypernucleus with an odd-odd core being

$$\begin{aligned}|J_C M_C\rangle &= \left(b^\dagger_{\mathrm{j}_{n_c}} b^\dagger_{\mathrm{j}_{p_c}}\right)_{J_C M_C}|0\rangle,\\ |J_F\nu_{J_F} &= \left\{\mathrm{j}_N, J_1, M_{T_F}=M_{T_C}-m_{t_N}\right\}, M_F\rangle\\ &= \delta_{m_{t_N}1/2}\left[b^\dagger_{\mathrm{j}_{n_c}},\left(b^\dagger_{\mathrm{j}_{p_c}} b^\dagger_{\mathrm{j}_N}\right)_{J_1}\right]_{J_F M_F}|0\rangle\\ &\quad + \delta_{m_{t_N}-1/2}\left[b^\dagger_{\mathrm{j}_{p_c}},\left(b^\dagger_{\mathrm{j}_{n_c}} b^\dagger_{\mathrm{j}_N}\right)_{J_1}\right]_{J_F M_F}|0\rangle, \end{aligned} \quad (5)$$

where $|0\rangle = |^{A+1}(Z+1)\rangle$ is the vacuum state and $b_j^\dagger$ ($b_j$) creates (destroys) a hole state. Note that the extra label $\nu_{J_F}$ in this model is associated with the hole state $\mathrm{j}_N^{-1}$, $J_1$, the type of destroyed nucleon $m_{t_N}$, and $M_F=M_I$. Because each $|J_F\nu_F\rangle$ is built for only one hole state $\mathrm{j}_N^{-1}$, consequently the sum in Eq. (2) reduces to only one term and the index $\nu_{J_F}\equiv \mathrm{j}_N J_1$, being $\mathcal{E}_F\equiv\mathcal{E}_C-\epsilon_{\mathrm{j}_N}$ and $\Delta_{\mathrm{j}_N}\equiv M_\Lambda - M_N+\epsilon_{\mathrm{j}_N}+\epsilon_{\mathrm{j}_\Lambda}$, with the single-particle energy $\epsilon_{\mathrm{j}_N}$. This suggests that we rewrite our Eq. (1) as

$$\Gamma_N = \frac{16M_N^3}{\pi}\sum_{\mathrm{j}_N}\int_0^{\Delta_{\mathrm{j}_N}} d\epsilon\sqrt{\epsilon(\Delta_{\mathrm{j}_N}-\epsilon)} \times \sum_{SlL\lambda JT}\mathcal{S}_1(J, j_N m_{t_N}) \times\left|\mathcal{M}(plPL\lambda SJT;\mathrm{j}_\Lambda\mathrm{j}_N m_{t_N})\right|^2 \quad (6)$$

$$\mathcal{S}_1(J,\mathrm{j}_N m_{t_N}) \equiv \hat{J}_I^{-2}\sum_{J_F J_1} f_J(J_F J_1 \mathrm{j}_N m_{t_N})^2, \quad (7)$$

where we see that all the nuclear structure information should be present in the spectroscopic factors $\mathcal{S}_1(J,\mathrm{j}_N m_{t_N})$. After some simple algebra, we can write[1]

$$\begin{aligned} f_J(J_F J_1 \mathrm{j}_N m_{t_N}) &= (-)^{j_{n_c}+j_{p_c}+J_C+2J_F}\delta_{m_{t_N}1/2}\left[1+(-)^{J_1}\delta_{\mathrm{j}_N\mathrm{j}_{p_c}}\right]^{1/2}\hat{J}\hat{J}_I\hat{J}_C\hat{J}_1\hat{J}_F\\ &\times\begin{Bmatrix} J_C & J_I & j_\Lambda\\ J & j_N & J_F\end{Bmatrix}\begin{Bmatrix} j_{n_c} & j_{p_c} & J_C\\ j_N & J_F & J_1\end{Bmatrix} + (p_c, m_{t_N}\leftrightarrow n_c, -m_{t_N}), \end{aligned} \quad (8)$$

[1]It is important to remark that the allowed $j$ values are limited by the fact that, in the $j_1 j_2$ coupling we are using here, one needs to satisfy the condition $j+t=$ odd for configurations with $j_1=j_2$ [36].

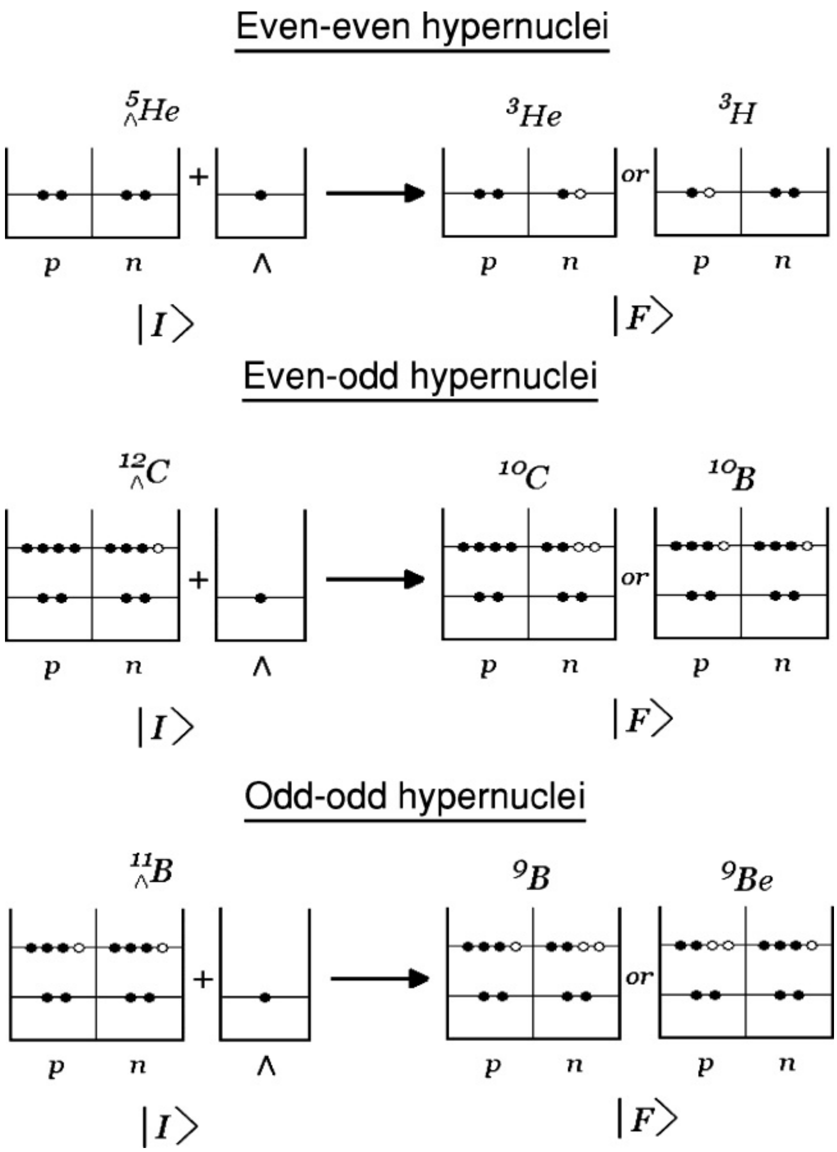


FIG. 1. Initial and final states contributing to the hypernuclear weak decay for even-even, even-odd, and odd-odd core cases.

and thus, making the angular momentum sums present in Eq. (7), we get

$$\mathcal{S}_1(J, \mathrm{j}_N m_{t_N}) = \delta_{m_{t_N} 1/2} \hat{J}^2 \left[ \hat{j}_\Lambda^{-2} - \delta_{\mathrm{j}_N \mathrm{j}_{p_c}} \hat{J}_C^2 \begin{Bmatrix} J_C & j_N & j_{n_c} \\ J_I & j_\Lambda & J_C \\ j_\Lambda & J & j_N \end{Bmatrix} \right] + (p_c, m_{t_N} \leftrightarrow n_c, -m_{t_N}). \tag{9}$$

It is important to note that when the extracted nucleon comes from a full shell ($\mathrm{j}_N \neq \mathrm{j}_{N_c}$), the factor $\mathcal{S}_1$ is independent of the initial hypernucleus and also on the level $\mathrm{j}_N$. For the case of the even-odd core hypernucleus considered here we have

$$|J_C M_C\rangle = \left(b^\dagger_{\mathrm{j}_{n_c}}\right)_{J_C M_C} |0\rangle, \quad |J_F M_F\rangle = \left(b^\dagger_{\mathrm{j}_{n_c}} b^\dagger_{\mathrm{j}_N}\right)_{J_F M_F} |0\rangle, \tag{10}$$

where $|0\rangle = |^{A+1}Z\rangle$ is now the vacuum state, and the results in this case can be obtained from those of the odd-odd core by choosing $j_{p_c} = 0$ (which means $j_{n_c} = J_C$ and $J_1 = J_F$ $(J_1 = j_N)$ for neutron (proton)-induced decay).[2] Similarly, because

$$|J_C M_C\rangle = |0\rangle, \quad |J_F M_F\rangle = b^\dagger_{\mathrm{j}_N} |0\rangle, \tag{11}$$

for the even-even core hypernucleus, being now $|0\rangle = |^A Z\rangle$, results for this case are reproduced with $j_{p_c} = j_{n_c} = 0$ (which means $J_C = 0$ and $J_1 = J_F = j_N$) in the odd-odd core formulas. In conclusion, we have arrived at the general expression (8) for the $f_J(J_F J_1 \mathrm{j}_N m_{t_N})$ coefficients of odd-odd hypernuclei, which includes the even-odd and even-even hypernuclei as particular cases.

[2]Alternatively, for an odd-even core hypernucleus the final state of which is described as $|J_F M_F\rangle = (b^\dagger_{\mathrm{j}_{p_c}} b^\dagger_{\mathrm{j}_N})_{J_F M_F}|0\rangle$, the results can be obtained from odd-odd formulas with $j_{n_c} = 0$.

Analogously, the intrinsic asymmetry parameter can be calculated as [7]

$$a_\Lambda = \frac{\omega_1}{\omega_0}, \qquad (12)$$

where

$$\omega_\kappa(J_I) = 2^8\pi^5\sqrt{2}\begin{Bmatrix} j_\Lambda & J_I & J_C \\ J_I & j_\Lambda & \kappa^{-1}\end{Bmatrix}\int d\cos\theta_{p_1} \times \int dF \sum_{STT'}(-)^{T+T'}\sum_{lL\lambda J}\sum_{l'L'\lambda'J'} i^{-l'-L'-l-L} \times (-)^{\lambda+S+J_C-\frac{1}{2}+2J_I+J+J'+\kappa+1+J_F}\,\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{\lambda}\hat{\lambda}'\hat{J}\hat{J}' \times \sum_{kK}(l0l'0|k0)(L0L'0|K0)[Y_k(\theta_p,\pi) \otimes Y_K(\theta_P,0)]_{\kappa 0}\begin{Bmatrix} J_I & \kappa & J_I \\ J & J_F & J'\end{Bmatrix}\begin{Bmatrix} \kappa & J' & J \\ S & \lambda & \lambda'\end{Bmatrix} \times \begin{Bmatrix} l & l' & k \\ L & L' & K \\ \lambda & \lambda' & \kappa\end{Bmatrix},\ \langle plPL\lambda SJTJ_F; J_I|V|J_I\rangle \times \langle pl'PL'\lambda'SJ'T'J_F; J_I|V|J_I\rangle^*, \qquad (13)$$

where we use the short notation ($\hbar = c = 1$)

$$\int dF \ldots = 2\pi\sum_{\nu_{J_F}J_F}\int\frac{p_2^2\,dp_2}{(2\pi)^3}\int\frac{p_1^2dp_1}{(2\pi)^3}\delta \times\left(\frac{p_1^2}{2M}+\frac{p_2^2}{2M}+\frac{|\mathbf{p}_1+\mathbf{p}_2|^2}{2M_F}-\Delta_{\nu_{J_F}J_F}\right)\ldots. \qquad (14)$$

Now, if we assume the IPSM as before, in the case of an odd-odd core the Eqs. (13) and (14) can be rewritten, respectively, as (remember that $N = p$ because the asymmetry is measured in the proton-induced decay)

$$\omega_\kappa(J_I) = 8\sqrt{2}\sum_{\mathrm{j}_p}\int d\cos\theta_{p_1}\int dF_{\mathrm{j}_p}\sum_{STT'}(-)^{T+T'} \times \sum_{lL\lambda J}\sum_{l'L'\lambda'J'} i^{-l'-L'-l-L}\,(-)^{\lambda+S-\frac{1}{2}}\,\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{\lambda}\hat{\lambda}'\hat{J}\hat{J}' \times\sum_{kK}(l0l'0|k0)(L0L'0|K0)[Y_k(\theta_p,\pi) \otimes Y_K(\theta_P,0)]_{\kappa 0}\begin{Bmatrix} \kappa & J' & J \\ S & \lambda & \lambda'\end{Bmatrix}\begin{Bmatrix} l & l' & k \\ L & L' & K \\ \lambda & \lambda' & \kappa\end{Bmatrix} \times\mathcal{S}_2(J,J',\mathrm{j}_p,\kappa)\mathcal{M}(plPL\lambda SJT;\mathrm{j}_\Lambda\mathrm{j}_p) \times\mathcal{M}^*(pl'PL'\lambda'SJ'T';\mathrm{j}_\Lambda\mathrm{j}_p), \qquad (15)$$

and

$$\int dF\ldots = \frac{1}{(2\pi)^5}\sum_{\mathrm{j}_p}\int dF_{\mathrm{j}_p}\sum_{J_1J_F}\ldots, \qquad (16)$$

with

$$\int dF_{\mathrm{j}_p}\ldots = \int p_2^2dp_2\int p_1^2dp_1\delta \times\left(\frac{p_1^2}{2M}+\frac{p_2^2}{2M}+\frac{|\mathbf{p}_1+\mathbf{p}_2|^2}{2M_F}-\Delta_{\mathrm{j}_p}\right)\ldots. \qquad (17)$$

Here

$$\mathcal{S}_2(J,J',\mathrm{j}_p,\kappa) \equiv \hat{J}_I^{-2}(-1)^{J_C+2J_I+J+J'+\kappa+1}\begin{Bmatrix} j_\Lambda & J_I & J_C \\ J_I & j_\Lambda & \kappa\end{Bmatrix}^{-1} \times\sum_{J_1J_F}(-)^{J_F}\begin{Bmatrix} J_I & \kappa & J_I \\ J & J_F & J'\end{Bmatrix} \times f_J(J_FJ_1\mathrm{j}_p1/2)f_{J'}(J_FJ_1\mathrm{j}_p1/2), \qquad (18)$$

which, after using Eq. (8) for $f_J$, can be reduced as

$$\mathcal{S}_2(J,J',\mathrm{j}_p,\kappa) = \hat{J}\hat{J}'\left[(-)^{-j_p}\begin{Bmatrix} \kappa & j_\Lambda & j_\Lambda \\ j_p & J & J'\end{Bmatrix}+\delta_{\mathrm{j}_p\mathrm{j}_{p_c}}\hat{J}_C^2(-1)^{J_C+2J_I+j_{n_c}} \times\begin{Bmatrix} j_\Lambda & J_I & J_C \\ J_I & j_\Lambda & \kappa^{-1}\end{Bmatrix}\begin{Bmatrix} j_\Lambda & J_I & J_C & j_p \\ J & \kappa & J' & J_C \\ j_p & J_I & j_{n_c} & j_\Lambda\end{Bmatrix}\right], \qquad (19)$$

valid for odd-odd hypernuclei and containing as special cases the even-odd (with $j_{p_c} = 0$) and even-even (with $j_{p_c} = j_{n_c} = 0$) ones. Here also we can realize that for a full shell in the core, in which case only contributes the first term in Eq. (19), the contribution of each $\mathrm{j}_N$ to the asymmetry is independent of the hypernucleus, i.e., on $J_C$ and $J_I$.

## III. NUMERICAL RESULTS AND SUMMARIZING CONCLUSIONS

We have performed numerical computation for ${}^4_\Lambda$He, ${}^5_\Lambda$He, ${}^{11}_\Lambda$B, ${}^{12}_\Lambda$C, ${}^{16}_\Lambda$O, ${}^{17}_\Lambda$O, and ${}^{28}_\Lambda$Si hypernuclei, with a standard strangeness-changing weak $\Lambda N \to NN$ transition potential comprising the exchange of the complete pseudoscalar and vector meson octets ($\pi, \eta, K, \rho, \omega, K^*$) [5–7]. In our calculation, we have taken into account corrections due to kinematical effects related to the $\Lambda$-nucleon mass difference and the first-order nonlocal terms, which are carefully discussed in Ref. [6]. Also, we have included both (i) finite nucleon size (FNS) effects that are phenomenologically implemented by a monopole form factor $(\Lambda_M^2-\mu_M^2)/(\Lambda_M^2+q^2)$, $\Lambda_M$ being the cutoff for the meson $M$ [5], and (ii) initial and final short-range correlations (SRC) that are simulated, respectively, by means of the correlation functions

$$g_{\Lambda N}(r) = \left(1-e^{-r^2/\alpha^2}\right)^2+\beta r^2e^{-r^2/\gamma^2}, \quad g_{NN}(r) = 1-j_0(q_cr), \qquad (20)$$

with $\alpha = 0.5$ fm, $\beta = 0.25$ fm$^{-2}$, $\gamma = 1.28$ fm, and $q_c = 3.93$ fm$^{-1}$. Tables I and II show the numerical results for the decay rates and the intrinsic asymmetry parameter, respectively, which can be compared with the available experimental data for each observable exhibited in Table III.

TABLE I. Numerical results for the decay rates (in units of $\Gamma_0 = 2.5 \times 10^{-6}$ eV). See text for detailed explanation.

| | ${}^{4}_{\Lambda}$He | ${}^{5}_{\Lambda}$He | ${}^{11}_{\Lambda}$B | ${}^{12}_{\Lambda}$C | ${}^{16}_{\Lambda}$O | ${}^{17}_{\Lambda}$O | ${}^{28}_{\Lambda}$Si |
|---|---|---|---|---|---|---|---|
| $\Gamma_n(1s_{1/2})$ | 0.042 | 0.137 | 0.119 | 0.117 | 0.111 | 0.107 | 0.082 |
| $\Gamma_n(1p_{3/2})$ | – | – | 0.092 | 0.089 | 0.122 | 0.117 | 0.088 |
| $\Gamma_n(1p_{1/2})$ | – | – | – | – | 0.027 | 0.055 | 0.042 |
| $\Gamma_n(1d_{5/2})$ | – | – | – | – | – | – | 0.058 |
| $\Gamma_n$ | 0.042 | 0.137 | 0.211 | 0.206 | 0.260 | 0.279 | 0.271 |
| $\Gamma_p(1s_{1/2})$ | 0.564 | 0.553 | 0.468 | 0.462 | 0.437 | 0.429 | 0.327 |
| $\Gamma_p(1p_{3/2})$ | – | – | 0.340 | 0.508 | 0.481 | 0.471 | 0.359 |
| $\Gamma_p(1p_{1/2})$ | – | – | – | – | 0.229 | 0.224 | 0.173 |
| $\Gamma_p(1d_{5/2})$ | – | – | – | – | – | – | 0.281 |
| $\Gamma_p$ | 0.564 | 0.553 | 0.808 | 0.970 | 1.148 | 1.124 | 1.140 |
| $\Gamma_{NM}$ | 0.606 | 0.690 | 1.019 | 1.176 | 1.408 | 1.403 | 1.411 |
| $\Gamma_{n/p}$ | 0.071 | 0.248 | 0.261 | 0.212 | 0.226 | 0.248 | 0.238 |

In Table I we generalize the results previously discussed in Ref. [5] for ${}^{5}$He and ${}^{12}$C to other even-even and even-odd nuclei and include results for the odd-odd case, accordingly with Eqs. (6), (9), (12), (15), and (19) developed in the previous section within the IPSM. One first view shows that $\Gamma_p$ is large in relation to $\Gamma_n$, which is due to the tensor interaction contributing to the parity-conserving part in the proton-induced decay. As a second important observation we note that the partial contributions $\Gamma_{p,n}(\mathrm{j}_N)$ and $\omega_{0,1}(\mathrm{j}_N)$ of each $\mathrm{j}_N$ coming from full shells ($\mathrm{j}_N \neq \mathrm{j}_{N_c}$) are roughly independent of the considered hypernucleus, very small differences coming form the different oscillator parameters used in each case to generate the single-particle wave functions (see paragraph below Eq. (5.2) in Ref. [5] for a discussion about the adopted values for this parameter). This fact can be understood from the factors in Eqs. (9) and (19) that contain the nuclear structure information for the $\Gamma_{p,n}(\mathrm{j}_N)$ and $\omega_{0,1}(\mathrm{j}_N)$, respectively, and are independent of $J_C$ and $J_I$ for full shells. Comparison with the closed shell case indicates a notable change (reduction) in these contributions when the nucleon is promoted from an open shell, as happens for neutrons in ${}^{4}_{\Lambda}$He, ${}^{12}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, and ${}^{28}_{\Lambda}$Si and for both protons and neutrons in ${}^{11}_{\Lambda}$B. This is produced because of the change in the occupancies, accounted for by the second term in brackets in mentioned factors. Nevertheless, the contribution for an open shell is also hypernucleus independent, as can be seen in the case of $\Gamma_n(1s_{1/2})$ for ${}^{11}_{\Lambda}$B and ${}^{12}_{\Lambda}$C. It is important to note that Eqs. (9) and (19) are only valid within the IPSM [see Eq. (5)] where the sums in Eq. (2) reduce to only one term. This enables a contraction between different angular momentum recoupling (geometrical) coefficients present in $f_J$, through the sum on $J_F$ and $J_1$. The third and very important result is that, within the IPSM, we get approximately the same value for the neutron to proton ratio $\Gamma_{n/p} \sim 0.25$ (except for ${}^{4}_{\Lambda}$He) and for the asymmetry parameter $a_\Lambda \sim -0.5$ (see Table II) of all hypernuclei. This can be understood noting that for each hypernucleus $\Gamma_{p,n}(\mathrm{j}_N) \simeq \alpha_{\mathrm{j}_N}\Gamma_{p,n}(1s_{1/2})$ and $\omega_1(\mathrm{j}_N) \simeq \alpha_{\mathrm{j}_N}\omega_1(1s_{1/2})$ (remember that $\omega_0(\mathrm{j}_N) = \Gamma_p(\mathrm{j}_N)$) with $\alpha_{1p_{3/2}} = 1$, $\alpha_{1p_{1/2}} = 0.5$, and $\alpha_{1d_{5/2}} = 0.9$ for initial hypernuclei where the $\mathrm{j}_N$ shell is closed, while, for example, $\alpha_{1p_{3/2}} = 0.75$ for ${}^{11}_{\Lambda}$B where this shell is open for neutrons and protons.In this way $\Gamma_{n/p} \simeq \Gamma_{n/p}(1s_{1/2}) \simeq \Gamma_{n/p}({}^{5}_{\Lambda}\mathrm{He}) \simeq 0.25$ for all the even-even

TABLE II. Numerical results for the intrinsic asymmetry parameter. See text for detailed explanation.

| | ${}^{4}_{\Lambda}$He | ${}^{5}_{\Lambda}$He | ${}^{11}_{\Lambda}$B | ${}^{12}_{\Lambda}$C | ${}^{16}_{\Lambda}$O | ${}^{17}_{\Lambda}$O | ${}^{28}_{\Lambda}$Si |
|---|---|---|---|---|---|---|---|
| $\omega_0(1s_{1/2})$ | 0.564 | 0.553 | 0.468 | 0.462 | 0.437 | 0.429 | 0.327 |
| $\omega_0(1p_{3/2})$ | – | – | 0.340 | 0.508 | 0.481 | 0.471 | 0.359 |
| $\omega_0(1p_{1/2})$ | – | – | – | – | 0.229 | 0.224 | 0.173 |
| $\omega_0(1d_{5/2})$ | – | – | – | – | – | – | 0.281 |
| $\omega_0$ | 0.564 | 0.553 | 0.808 | 0.970 | 1.148 | 1.124 | 1.140 |
| $\omega_1(1s_{1/2})$ | −0.298 | −0.297 | −0.258 | −0.255 | −0.242 | −0.233 | −0.178 |
| $\omega_1(1p_{3/2})$ | – | – | −0.169 | −0.260 | −0.248 | −0.240 | −0.186 |
| $\omega_1(1p_{1/2})$ | – | – | – | – | −0.119 | −0.114 | −0.090 |
| $\omega_1(1d_{5/2})$ | – | – | – | – | – | – | −0.139 |
| $\omega_1$ | −0.298 | −0.297 | −0.427 | −0.515 | −0.609 | −0.587 | −0.593 |
| $a_\Lambda$ | −0.529 | −0.538 | −0.529 | −0.530 | −0.530 | −0.522 | −0.520 |

TABLE III. Experimental data for the NMHWD observables (total width is given in units of $\Gamma_0 = 2.5 \times 10^{-6}$ eV).

| | $\Gamma_{NM}$ | $\Gamma_{n/p}$ | $a_\Lambda$ |
|---|---|---|---|
| ${}^{4}_{\Lambda}$He | $0.17 \pm 0.05$ [38] | $0.06^{+0.28}_{-0.06}$ [38] | – |
| | $0.20 \pm 0.03$ [39] | $0.25^{+0.05}_{-0.13}$ [39] | – |
| | $0.177 \pm 0.029$ [40] | $\leqslant 0.19$ [40] | – |
| ${}^{5}_{\Lambda}$He | – | $0.39 \pm 0.11$ [24,25] | – |
| | $0.41 \pm 0.14$ [41] | $0.93 \pm 0.55$ [41] | – |
| | – | – | $0.24 \pm 0.22$ [42] |
| | – | $(0.45 - 0.51) \pm 0.15$ [43] | $0.07 \pm 0.08^{+0.08}_{-0.00}$ [43,44] |
| | $0.424 \pm 0.024$ [45] | $0.45 \pm 0.11 \pm 0.03$ [45,46] | $0.11 \pm 0.08 \pm 0.04$ [45,47] |
| ${}^{11}_{\Lambda}$B | – | $1.04^{+0.59}_{-0.48}$ [41] | – |
| | – | – | $-0.16 \pm 0.28^{+0.18}_{-0.00}$ [44] |
| | – | – | $-0.20 \pm 0.26 \pm 0.04$ [47] |
| | – | $0.59 \pm^{+0.17}_{-0.14}$ [48] | – |
| | – | – | $0.28 \pm 0.14$ [49] |
| | $0.95 \pm 0.13 \pm 0.04$ [50] | $2.16 \pm 0.58^{+0.45}_{-0.95}$ [50] | – |
| | $1.33 \pm 0.08$ [51] | – | – |
| | $0.861 \pm 0.063 \pm 0.073$ [52] | – | – |
| ${}^{12}_{\Lambda}$C | – | $0.51 \pm 0.13 \pm 0.04$ [32] | – |
| | $1.14 \pm 0.2$ [41] | $1.33^{+0.12}_{-0.81}$ [41] | – |
| | – | $0.87 \pm 0.09 \pm 0.21$ [43] | $-0.24 \pm 0.26^{+0.08}_{-0.00}$ [43] |
| | – | – | $-0.16 \pm 0.28^{+0.18}_{-0.00}$ [44] |
| | $0.940 \pm 0.035$ [45] | $0.56 \pm 0.12 \pm 0.04$ [45] | $-0.20 \pm 0.26 \pm 0.04$ [45,47] |
| | – | – | $0.02 \pm 0.20$ [49] |
| | $0.828 \pm 0.056 \pm 0.066$ [52] | $0.87 \pm 0.09 \pm 0.21$ [52] | – |
| ${}^{16}_{\Lambda}$O | $2.80^{+1.07}_{-0.84}$ [53] | – | – |
| ${}^{17}_{\Lambda}$O | – | – | – |
| ${}^{28}_{\Lambda}$Si | $1.30 \pm 0.10$ [51] | – | – |
| | $1.125 \pm 0.067 \pm 0.106$ [52] | $0.79^{+0.13+0.25}_{-0.11-0.24}$ [52] | – |
| | – | $1.38^{+0.13+0.27}_{-0.11-0.25}$ [54] | – |

and odd-odd hypernuclei, as a consequence of $\alpha_{j_N}$ being the same for neutron and proton shells and the independence of the hypernuclei mentioned above. This relation is also approximately fulfilled by even-odd hypernuclei (except ${}^{4}_{\Lambda}$He where $\Gamma_n(1s_{1/2})$ is the only contribution to neutrons) where $\alpha_{j_N}$ is a little bit smaller for neutrons than for protons in the valence shell. The mentioned factorization has been already explicitly shown for the case of ${}^{12}_{\Lambda}$C in Ref. [5], but now we see that can be extended to other orbitals and other hypernuclei. The analysis achieved for $\Gamma_{n/p}$ can be easily repeated for $a_\Lambda$ to explain the independence shown in Table II. Here, as the asymmetry is calculated from the ratio $\omega_1/\omega_0$ for the *same* kind of decaying particles (protons) the differences in the factors $\alpha_{j_N}$ between closed and open shell does not affect the ratio and the result $a_\Lambda \sim -0.5$ is fulfilled by all the hypernuclei with great precision even for ${}^{4}_{\Lambda}$He.

Comparison with the available experimental results from Table III is very useful to remark on the puzzle involving the NMHWD when we use the IPSM with the complete octet meson decay potential. In fact, discrepancies between theoretical and the more recent experimental data can be observed, mainly for the intrinsic asymmetry parameter. Particularly, it is very easy to see that the central values of $\Gamma_{n/p}$ and $a_\Lambda$ do not exhibit the hypernucleus independence obtained within the IPSM. We remember at this point that FSI effects are not included in the present plain model. Thus, certain dependence with the considered hypernuclei could be obtained by adding their contribution [22,27,37].

On the other hand, one of the most serious objections that we could have to the IPSM, without considering the additional effects of the FSI between the ejected nucleons, the three-body $\Lambda NN \to NNN$ decay contribution, the $|\Delta T| = 3/2$ contribution of vector mesons, or other modifications in the exchange potential, is that it does not consider configuration mixing in the final nucleus. For example, suppose that in place of the final states shown in Eq. (5) we adopt the combination

$$|J_F \nu_{J_F}\rangle = \sum_{j_N J_1} C_{j_N, J_1}(J_F \nu_{J_F}) \left[ b^\dagger_{j_{nc}}, \left( b^\dagger_{j_{pc}} b^\dagger_{j_N} \right)_{J_1} \right]_{J_F M_F} |0\rangle \quad (21)$$

within the Hilbert space generated by states with a given $J_F$, obtained by adding one hole to the core,[3] for $m_{t_N} =$

[3]We could enlarge the Hilbert space and consider more complex configurations of one particle-two holes or two particles-three holes coupled to the core, which could in principle interact with the configurations in Eq. (21), without changing the conclusions.

$1/2$ or $-1/2$. Here $C_{j_N,J_1}(J_F \nu_{J_F})$ should be obtained from diagonalization of the *full* Hamiltonian including the residual interaction. Now, the sum in the Eq. (2) does not reduce to only one term, the summation on $J_F$ cannot skip as before the kinematical factors containing $\mathcal{E}_F \equiv \mathcal{E}_{J_F \nu_{J_F}}$, and besides each contribution is weighted by $C_{j_N,J_1}(J_F \nu_{J_F})$. As a consequence of this, Eqs. (6) and (15), responsible for the independence on the initial nucleus in the IPSM, will be no more valid.

For this reason, our calculation, based in the IPSM formalism, should be considered the zero-order approximation for the correct treatment of the nuclear structure, mainly for light hypernuclei. It is also important to mention here the three following aspects that could help to solve the puzzle. First, in view of the large error bars exhibited by the data, it will be very useful to have precise experimental values for the observables of different hypernuclei (more than ${}^5_\Lambda$He and ${}^{12}_\Lambda$C). Thus, if one introduces a more evolved nuclear structure treatment, as for example the effect of configuration mixing, the experimental results should be capable of giving enough information to differentiate the predictions of the different models. Second, in the light of very recent contributions [12–14] it seems that proper modifications of the exchange potential and inclusion of FSI would lead to substantial improvements in the coincidence between the calculations and the experimental results, mainly for the asymmetry parameter. For example, our values in Table II, $a_\Lambda({}^5_\Lambda\text{He}) = -0.538$, $a_\Lambda({}^{11}_\Lambda\text{B}) = -0.529$, and $a_\Lambda({}^{12}_\Lambda\text{C}) = -0.530$, are strongly changed in Ref. [14] to 0.083, 0.078, and 0.045, respectively, by the addition of the $a_1$ meson, all these values being within the error bars (see Tables II, V, and VI from Ref. [14]). Another modification has been the inclusion of correlated plus uncorrelated $2\pi$ in the exchange potential, which is capable of breaking the uniformity of the values for $a_\Lambda$, giving $a_\Lambda({}^5_\Lambda\text{He}) = 0.028$, $a_\Lambda({}^{11}_\Lambda B) = -0.111$, and $a_\Lambda({}^{12}_\Lambda C) = -0.126$ (see Tables 1 and 3 from Ref. [12]) in complete agreement with experiment. Third, any one of the available theoretical evaluations in the literature introduces the contribution of two-nucleon-induced decay to the intrinsic asymmetry parameter. As happens with the neutron to proton ratio [23,24], it could modify the results for this observable.

In summary, we have developed a general IPSM formalism, valid for even-even, even-odd, and odd-odd core hypernuclei, which is an extension of that presented in our previous works [5,7]. We have shown that, in the present form, it predicts approximately the same value for the neutron to proton ratio, $\Gamma_{n/p}$, and asymmetry parameter, $a_\Lambda$, of all hypernuclei and cannot reproduce the available experimental data. These nearly constant values obtained for all considered hypernuclei are due to the fact that nuclear structure detailed information seems to have been removed because of the dropping of configuration mixing in the final states and to the factorization of the $1s_{1/2}$ contribution, which lead to a cancellation of factors between numerator and denominator for both observables approximately. We suggest that one should go beyond the IPSM and consider configuration mixing in the final nucleus. Finally, we have stressed the necessity of evaluating, with the IPSM framework as a starting point, the effects of (i) modifications of the exchange potential ($2\pi$, $a_1$ meson, $\Delta T = 3/2$ terms of vector mesons, etc.), (ii) final state interactions, which could be included on the same footing as in Refs. [22] and [27], and (iii) two-nucleon-induced decay as possible ways to solve the puzzle. We plan to include these contributions and to improve the IPSM model in the future by analyzing why the factorization effect should not be present in such cases.

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