Universality of Mallmann correlations for nuclear band structures

D. Bucurescu, N. V. Zamfir, G. Căta-Danil, M. Ivaşcu, and N. Mărginean
Horia Hulubei National Institute of Physics and Nuclear Engineering, P.O. Box MG-6, Bucharest-Măgurele, Romania
(Received 24 April 2008; published 23 October 2008)

It is shown that the Mallmann's energy ratio correlations, for the first time observed for the ground state band of the even-even nuclei, are universal: various band structures in all collective nuclei obey the same systematics, and consequently the same spin dependence. Based on a second order anharmonic vibrator description, parameter-free recurrence relations between Mallmann-type energy ratios are deduced, which can be used to extrapolate bands to higher spin.

DOI: 10.1103/PhysRevC.78.044322 PACS number(s): 21.10.Re, 21.60.Ev

The understanding of the energies of the positive-parity yrast levels (the "ground state band"—g.s.b.) of even-even nuclei has been a subject of continuous interest since many years. Almost 50 years ago Mallmann [1] observed that the experimental energy ratios $R(6/2) = E(6^+)/E(2^+)$ and $R(8/2) = E(8^+)/E(2^+)$ lie on two "universal" curves when represented as a function of the lowest ratio in the band, $R(4/2) = E(4^+)/E(2^+)$ [where $E(J^+)$ is the excitation energy of the state of spin J]. With increasing number and quality of data (more nuclei and higher observed spin) it was found that Mallmann-type ratios for higher spin states in the g.s.b. define also compact, unique patterns. This implies that all g.s.b. in even-even nuclei have energies with the same spin dependence. We emphasize that the concept of Mallmann-type correlation has been associated until now only with the g.s.b. of even-even nuclei.

The fact that the g.s.b. structure in the even-even nuclei follows a simple systematic incited to the development of many theoretical and phenomenological approaches aiming at understanding this behavior. Among the early various approaches to the description of the yrast excitation energies, important steps are the expansion in the powers of $I(I+1)$ [2], the variable moment of inertia (VMI) model [3], the Holmberg-Lipas expression [4], the anharmonic vibrator (AHV) [5], and the empirical relation of Ejiri [6] (see also [7] and references therein). More recent approaches are, for example, Refs. [8–15].

The AHV-type of approach to the description of the band structures regained a lot of interest after the surprising finding [14,16] that the g.s.b.'s of most of the even-even collective, nonrotational ($2.0 < R(4/2) < 3.15$) nuclei are well represented by a universal anharmonic vibrator:

$$E(I) = nE(2_1^+) + \frac{n(n-1)}{2}\varepsilon_4 \quad (1)$$

with a nearly constant anharmonicity ε_4 (where n, the number of phonons of the state, is $n = I/2$ as a function of spin I). This equation, where $E(2_1^+)$ and ε_4 can be regarded more generally as free parameters, is equivalent with the two-parameter relation empirically proposed by Ejiri [6], $E(I) = aI + bI(I+1)$ (a, b are parameters). Similar energy correlations have subsequently been found for different bands in collective, both rotational and nonrotational odd-A and odd-odd nuclei [17–20]. The extent of applicability of the AHV type relations for $B(E2)$ values is also very interesting, considering the importance of the K quantum number, but this investigation is hindered by the lack of extensive data in odd-A and odd-odd nuclei, and it is outside the goal of this paper.

Equation (1) is only a first order description of the nuclear band structures. It is found experimentally [19] that with increasing n (spin) the plots increase in scattering but with highly correlated deviations around the straight lines (1), a fact which indicates the need of additional anharmonicities. In Ref. [15] the next order AHV expression was studied:

$$E(I) = nE(2_1^+) + \frac{n(n-1)}{2}\varepsilon_4 + \frac{n(n-1)(n-2)}{6}\varepsilon_6 \quad (2)$$

and it was found that it represents the experimental data very well, including the good rotor nuclei ($R(4/2) > 3.15$) as well. As shown in Refs. [18,19], Eq. (2) is a good approximation to a rather smooth and compact correlation empirically found between the experimental energies of four successive levels in the band. As emphasized in Ref. [15], the applicability of Eq. (2) to different types of band structures remains a surprising, empirical finding.

A question which occurs often in practice is how to extrapolate a band structure to higher spins by using the known levels. Practical methods employed by experimenters to do this imply following the evolution of different quantities related to the band, such as the excitation energy as a function of spin $E(I)$; the in-band transition (or gamma-ray) energies—or the first derivative $dE(I)/dI$; the differences of gamma-ray energies (the second derivative $d^2E(I)/dI^2$, which is proportional to the inverse of the dynamical moment of inertia). These methods usually work very well in the good rotational cases. In using the AHV relation (2) for different types of nuclei, best results are obtained if the model parameters are determined for each case considered; note that even when looking at the gamma-ray differences (second derivative) one still has two parameters. It is well known that the structure of a band can change at a certain spin (e.g., backbending). This can be easily seen [21] by a comparison with the AHV predictions, and the change is marked by a difference in the parameters [for example, a rotational sequence is described by Eq. (1) with $\varepsilon_4 = \frac{4}{3}E(2_1^+)$].

The universality of the AHV approach (2), that is, the fact that just *one single* formula describes very well any nuclear band structure, leads to the idea of using it in a parameter-independent way. In this respect, its universality should clearly be related to that of the Mallmann correlations for the ground state bands of the even-even nuclei. To this end, in the present work we show that: (i) experimental Mallmann correlations for all band structures in collective even-even, odd-mass, and odd-odd nuclei are universal, implying that all collective bands, in all nuclei, have the same spin dependence; (ii) parameter-free relations resulting from the second order AHV relation (2), which connect different Mallmann-type energy ratios, are rather suitable tools for extrapolating band energies to higher spin.

In this work we study experimental band structures in "collective" nuclei, as extracted from the ENSDF database [22]. For the even-even nuclei the condition applied was $R(4/2) \geqslant 2.0$, which eliminates the nuclei with magic numbers and some of the close-to-magic nuclei. For the odd-A and odd-odd nuclei, the criterion was that their even-even neighbors (core nuclei) are collective. In these nuclei we selected different band structures, i.e., sequences of states of increasing spins $j, j+2, j+4, \ldots$ connected by strong $E2$ transitions, which were reasonably well known (at least three transitions). We have not restricted the set of bands only to the yrast states. In the even-even nuclei, besides the g.s.b., bands built on intrinsic excitations, such as the "beta" quasiband (usually built on the first excited 0^+ state), or the "gamma" quasiband, built on the 2^+_γ excited state were also considered [19,20]. We have 306 even-even nuclei between ^{42}Ti and ^{254}No for which the g.s.b. is known at least up to the 6^+ state. For the odd-A and odd-odd nuclei, the collection of bands is that used in Ref. [19]: 695 different bands (most of them based on one-quasiparticle configurations, both signatures of a band structure considered when available) in nuclei between ^{73}As and ^{185}Hg, and 400 bands (mainly based on two-quasiparticle states) in nuclei between ^{72}As and ^{194}Au, respectively. Data for a band were considered as long as the band is "unperturbed," that is, we consider only levels bellow up(back)-bending (in the g.s.b. of even-even nuclei, this usually means a maximum spin of 12 or 14$\hbar$). For each band we define the Mallmann-type ratios $R(j+2n/j+2) = [E(j+2n) - E(j)]/[E(j+2) - E(j)]$ between excitation energies relative to the "bandhead" ($n = 0$, energy $E(j)$).

Figure 1 shows two representative examples of usual Mallmann correlation plots for our collection of bands: $R(j+6/j+2)$ and $R(j+12/j+2)$ versus $R(j+4/j+2)$, respectively. For the collective even-even nuclei $R(j+4/j+2)$ varies from 2.0 (vibrational nuclei) to 3.33 (good rotor nuclei) while for the odd-A and odd-odd nuclei the plots continue to higher values, following rather closely the "Ejiri" lines. The highest $R(j+4/j+2)$ values in the odd-A nuclei correspond to bands with strong Coriolis mixing where, in some cases, the state of spin $j+2$ has very low energy relative to the bandhead j (these bands were discussed in detail in Ref. [19]). In Fig. 1, the points for the odd-A and odd-odd nuclei were translated upward in order to clearly observe the pattern for each case. To ease the comparison with the even-even plot, the continuous line which is a spline interpolation to the even-even data is also drawn (correspondingly shifted) through the odd-A and odd-odd data. One can see that the three correlation patterns are absolutely similar. The (parameter-

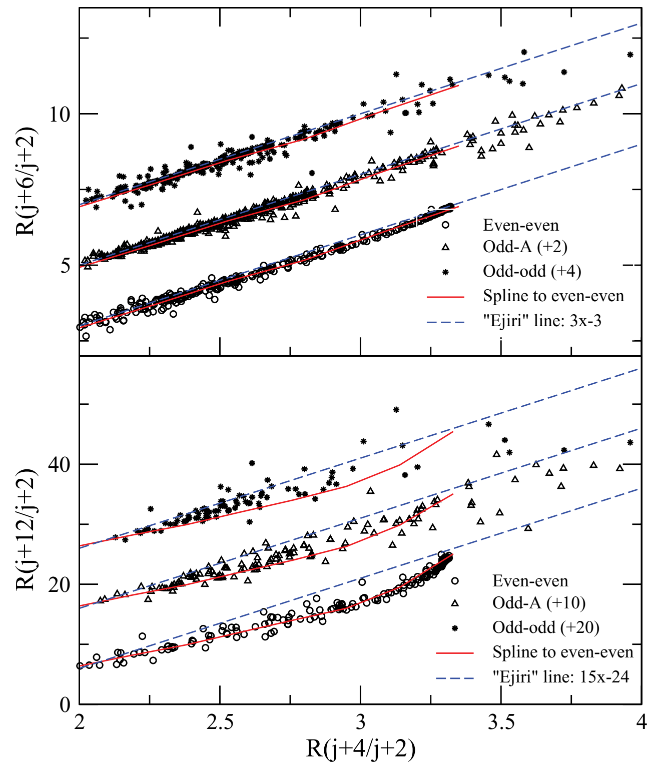

FIG. 1. (Color online) Representative experimental Mallmann correlation plots for the band structures of different nuclei: even-even (the g.s.b.), odd-A, and odd-odd, as indicated (note the vertical shift of the odd-A and odd-odd data). The continuous lines through the even-even data are spline interpolations, and are drawn (correspondingly shifted) also through the odd-A and odd-odd data to guide a comparison with the even-even case. The dashed lines are the "Ejiri" straight lines [prediction of the Ejiri formula or of Eq. (1)]. Note that the data for odd-A nuclei contain points with $R(j+4/j+2)$ values up to 78 (not shown), which are close to the Ejiri lines.

free) predictions of the Ejiri formula [or of Eq. (1)], are also shown for comparison. Other bands, such as bands based on intrinsic excitations (beta and gamma) in the even-even nuclei [19,20] show absolutely similar correlations.

We conclude therefore that the Mallmann correlations are *universal* in the sense that all band structures in collective nuclei (even-even, odd-mass, or odd-odd) show exactly the same patterns. This was effectively checked on a large collection of band structures, as described above, comprising mainly one- and two-quasiparticle bands for the odd-A and odd-odd nuclei, respectively. Some checks were made for bands built on other intrinsic excitations as well, so this statement may be generalized to any band structure (with the condition that it stays non perturbed—e.g., by the interaction with other bands). This implies that the energies of *any* band, and in *all* nuclei, are described by the *same* functional spin dependence.

Equation (2) gives a good description of all these bands [19] (here $n = 0$ for the first state ("bandhead") of spin $j, n = 1$ corresponds to the first excited state in the band (spin $j+2$), etc.). From Eq. (2) adapted to a band built upon a state of spin

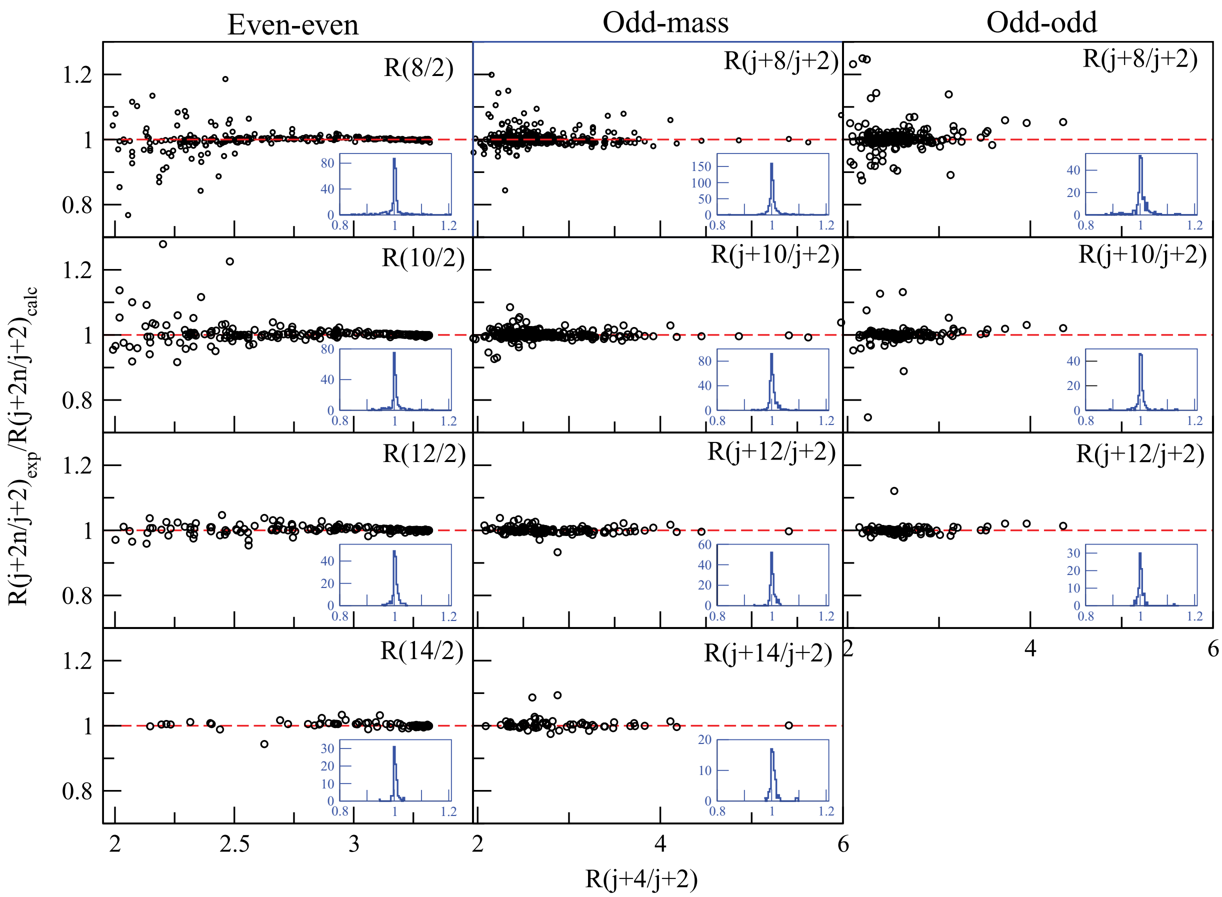

FIG. 2. (Color online) Comparison between the experimental Mallmann ratios and those calculated with Eq. (4). The insets show the distributions of the $R(j+2n/j+2)_{\mathrm{exp}}/R(j+2n/j+2)_{\mathrm{calc}}$ values, counted with a bin value of 0.005.

j we write the Mallmann-type energy ratios as

$$R(j+2n/j+2) = n + \frac{n(n-1)}{2}\frac{\varepsilon_4}{E(j+2)} + \frac{n(n-1)(n-2)}{6}\frac{\varepsilon_6}{E(j+2)}. \quad (3)$$

We can apply this relation to different states n in the band, and by eliminating the parameters $\frac{\varepsilon_4}{E(j+2)}$ and $\frac{\varepsilon_6}{E(j+2)}$ one obtains recurrence relations which express the energy ratio of a state as a function of other two ratios in the band. For example, one can write

$$R(j+2n/j+2) = \frac{n}{n-2}\left[\frac{2}{n-3} + 2R(j+2n-2/j+2) - \frac{n-1}{n-3}R(j+2n-4/j+2)\right] \quad (4)$$

which gives the Mallmann ratio for the nth state as a function of those of the $(n-1)$-th and $(n-2)$-th excited states in the band. By similar manipulations one can get for a certain state different other relations, always expressing its energy ratio as a function of the ratios for any other lower two states. We note that the same recurrence relations can also be obtained from the empirical energy recurrence relations recently derived by Buck *et al.* [23] for ground state band spectra of even-even nuclei; indeed, the recurrence relations found in this work were shown to be satisfied by a general solution for the energies in the band of the type $E(I) = \alpha I + \beta I^2 + \gamma I^3$ which is identical with the spin dependence of the second order AHV Eq. (2).

For practical purposes (prediction of higher levels within a band), it is interesting to check the accuracy of these recurrence relations on the existing experimental data. Figure 2 shows how formula (4) works for our entire collection of bands, by displaying the ratio between the experimental Mallmann ratios [for states of spin $(j+8)$ and above] and those calculated with Eq. (4) in which for the energy ratios occurring in the right side of the formula experimental values were used. One can see that for the existing experimental data, covering nuclei from pure vibrators to good rotors, the recurrence relations (4) work, in general, very well. This is valid also for the bands with higher $R(j+4/j+2)$ values not shown in Fig. 2. The inset of each graph in Fig. 2 shows the distribution of the values of the ratio between the experimental and calculated Mallmann ratios (which is 1.0 for perfect agreement), constructed with a bin of 0.005. An inspection of these distributions shows that for the overwhelming majority of the considered cases (bands) the deviation between the experimental and calculated value is within $\pm 1\%$. In a small number of nuclei (compared to the total number) one can observe larger deviations, most of them for transitional nuclei with $R(4/2) < 2.5$. As an example, in the case of the g.s.b. of the even-even nuclei for the $R(8/2)$ ratio, there are 39 nuclei (out of the total of 290) for which the deviation is more than 5%. An inspection of these cases shows that most of them are nuclei with numbers of nucleons differing by two from a magic number (Zn, Cd, Te, Hg isotopes), or nuclei from shape coexistence regions ($^{70-74}$Ge, ^{76}Se, ^{74}Kr), therefore the deviations can be explained by non collective effects or different perturbations of the band. Thus, most of the large deviations shown in Fig. 2 would disappear if the criteria chosen for the nuclear collectivity were more strict. Among all possible recurrence relations for the ratio of the nth state, relation (4) is the most accurate, which is expected because the states $n-1$ and $n-2$ collect the maximum information about the anharmonicities of the band.

One more remark is that the values n and j that we use to characterize a band in an AHV manner [e.g., by formula (3)] are not necessarily related to the actual spin of the states $I = j + 2n$. One may take as *reference* any other state in the band: then, that state becomes by convention the zero-phonon state ($n = 0$) and the energies of the higher states in the band (defined by $n = 1, 2, \ldots$) must be taken relative to this state. This aspect was discussed in Ref. [19]. The observed energy or Mallmann-type ratio correlations are similar with those presented above, where the reference was always the bandhead.

We stress that the present study addressed only the question of the applicability of the AHV-2 description [Eq. (2) and those thereafter derived from it] to the description of different band structures in any collective nucleus—it did not concern the intrinsic structure of these bands (or of the bandheads). Whether the AHV model describes well other nonyrast states (or intrinsic excitations) was investigated in Ref. [20] for a particular case (the γ-quasibands in even-even nuclei), and still remains a problem for further study.

In conclusion, we have shown that the correlations between energy ratios first time observed by Mallmann many years ago for the states of the ground state bands in the even-even nuclei have a *universal* character: they are prevalent in the band structures of all collective nuclei. This unique, universal behavior, indicates a common underlying structure of all collective band structures. Mallmann type correlations constitute a compact and concise way of showing, without detailed fits, that all band structures in collective nuclei obey a simple dependence on spin of the relative excitation energies, which is similar to that predicted by the anharmonic vibrator model. The relevance of a phonon model description of all nuclear band structures remains a challenge for microscopic investigations. Based on the second order anharmonic vibrator formula which is empirically found to describe well all types of band structures that we meet in collective nuclei, we deduce parameter-free recurrence relations for the energy ratios within the band. These recurrence relations predict well higher states in the band, on the basis of the (known) lower lying states. Thus, they may allow to extrapolate, more precisely than other methods, without any fit, band structures for which we know minimum three excited states, with at least one more state, which is of practical interest for the future spectroscopy of exotic nuclei.

ACKNOWLEDGMENTS

This work was partially funded by the Romanian National Authority for Scientific Research under contract No. CEX05-D11-30 and CEX05-D11-50.

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