Identification of triaxial strongly deformed bands in ^{168}Hf

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Possible decay pathways associated with three candidates for triaxial strongly deformed (TSD) bands in ^{168}Hf have been investigated. The spin and excitation energy of the strongest band, TSD1, were determined approximately based on γ-ray coincidence relationships. Discrete links were established for the second band. The overall agreement between the observed properties of the bands and cranking calculations using the ULTIMATE CRANKER code provides strong support for an interpretation where band TSD1 is associated with a TSD minimum, $(\varepsilon_2, \gamma) \sim (0.43, 20^\circ)$, involving the $\pi(i_{13/2})^2$ and the $\nu(j_{15/2})$ high-j orbitals. This constitutes the first identification of a TSD band in Hf isotopes, which has been long-predicted by theoretical studies. The second band is understood as being associated with a near-prolate shape and a deformation enhanced with respect to the normal deformed bands. It is proposed to be built on the $\pi(i_{13/2}h_{9/2}) \otimes \nu(i_{13/2})^2$ configuration.

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I. INTRODUCTION

The search for experimental signatures of triaxial nuclear shapes has proved to be very challenging. Potential energy surface (PES) calculations using different approaches (see, e.g., Ref. [1,2]) predict that nuclei with $Z \sim 72$ and $N \sim 94$ constitute a region where such exotic shapes coexist with others associated with normal deformed (ND) prolate shape. More systematic subsequent cranking calculations using the ULTIMATE CRANKER (UC) code [3,4] predict high-spin triaxial strongly deformed (TSD) minima with $(\varepsilon_2, \gamma) \sim (0.40, \pm 20^\circ)$ for nuclei in this region. These TSD minima are caused by large single-particle shell gaps associated with proton numbers $Z = 71$ and 72, and neutron numbers $N = 94$ and 97 [5,6]. Indeed, TSD structures have been identified in several Lu isotopes and the wobbling motion, a low-lying collective excitation mode characteristic of nuclei with stable triaxiality [7], has been established in ^{163,165,167}Lu [8–11] and, possibly, in $^{161}_{71}$Lu [12]. Further theoretical investigations based on the particle-rotor model [13,14] and on the cranked shell model plus random phase approximation [15] pointed to the essential role of the rotation-aligned $i_{13/2}$ quasiproton, which allows wobbling to compete in energy with quasiparticle excitations in these Lu nuclei.

An extensive search for TSD bands in Hf ($Z = 72$) nuclei has been carried out without success in proving triaxiality. The strongly deformed bands observed in $^{170-175}$Hf [16–19] were suggested to fall into two groups, labeled as ED and SD, based on their rotational properties and theoretical studies using the UC calculations and the cranked relativistic mean-field (CRMF) approach [17]. The ED bands are likely built on the proton $i_{13/2}h_{9/2}$ configuration and are associated with near prolate shapes with $\varepsilon_2 \sim 0.3$ (i.e., deformations enhanced with respect to the normal deformed nuclear shapes, $\varepsilon_2 \sim 0.22$, characterizing the ground states). For the bands in the SD group, only band-2 in ^{175}Hf is linked to known structures [19]. The suggested intrinsic configurations of this band, and likely similar SD bands in $^{172-174}$Hf, involve the $\pi i_{13/2}$ (proton), as well as the $\nu j_{15/2}$ (neutron) orbitals originating above the $N = 126$ spherical shell closure. The SD bands are associated with superdeformed prolate shapes (CRMF calculated $Q_t \sim 11.6\,e$ b, compared to experimental values varying between 12 and 14e b [18,19]) with little triaxiality. Therefore, the UC and the CRMF calculations do not support a TSD nuclear shape for the reported SD bands in the heavier $^{170-175}$Hf isotopes. Pronounced triaxial minima exist in the calculations, but they would result in even smaller calculated quadrupole moments. Three candidate TSD bands were also reported in ^{168}Hf [20], which is closer to the UC predicted neutron shell gap; however, none of the bands were linked to the known structures. The motivation for the present work was to search for possible decay pathways of these bands and investigate their properties to understand their nature.

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II. EXPERIMENTAL PROCEDURE AND RESULTS

High spin states in ^{168}Hf were populated through the reaction ^{96}Zr(^{76}Ge, 4*n*) at a beam energy of 310 MeV by using the ATLAS facility at Argonne National Laboratory. The target consisted of a self-supporting thin foil (∼0.67 mg/cm^2) of isotopically enriched ^{96}Zr. Coincident γ rays were measured by using the Gammasphere array [21], which consisted of 101 Compton-suppressed Ge detectors at the time of the experiments. A total of 2.2×10^9 five- and higher fold prompt coincidence events were collected. More details of the experiment can be found in a previous publication [20]. A detailed description of the procedures for the data analysis can be found in Ref. [17] regarding the γ-ray coincidence relationships and the γ-ray multipolarity measurements based on the γ-ray directional correlation from oriented states (DCO ratios) [22]. The extracted DCO ratios from $E2$-gated spectra fall into two distinct groups centered around 1.0 and 0.6 for stretched quadrupole and dipole transitions, respectively.

A partial level scheme of ^{168}Hf from this study is presented in Fig. 1. The two weaker bands reported in the previous publication [20], TSD2 and TSD3, were renamed ED and TSD2, respectively. The reason for the change will be discussed in the following. Other new results about ND structures will be published in an upcoming paper [23]. The strongest depopulating transition from band ED to lower spin ND structures is the 808-keV γ ray, which feeds the $I = 27\hbar$ level in the intermediate structure, band X1, before decaying to the yrast band AB. The intensities of the 808-keV decay-out and the 770-keV in-band transitions from the 29^- state are almost equal. Band X1 decays mainly to the band AB between the 26^+ and 22^+ levels with the 1371-keV γ ray being the strongest linking transition. Band X1 also feeds bands G and AE. However, the linking transitions to band AE could not be established. Several depopulating transitions from bands ED and X1 can be seen in the spectrum of Fig. 2.

DCO ratios were measured for all transitions in the three candidate TSD bands, except for those very weak transitions at the highest spins, and the results were consistent with expectations for $E2$ cascades. The 1371- and 1366-keV transitions depopulating band X1 have DCO ratios of 0.49(9) and 0.69(10), respectively, consistent with a stretched dipole character. Furthermore, we suggest negative parity for band X1 because the linking transitions most likely have an $E1$ multipolarity. An $M1$ transition of such high energy would be expected to exhibit an $E2$ admixture resulting in a larger DCO ratio. The 808-keV decay-out transition from ED has a DCO ratio of 0.91(11), indicating either a stretched $E2$ or a $\Delta I = 1 M1/E2$ character. The large error in the DCO ratio, caused by the low statistics, does not allow us to make further distinction between the two scenarios. However, the latter possibility can be ruled out since it would require the 702-keV γ ray, deexciting the 27^- level in X1 to a lower level in ED, to be an $M3$ transition, which is highly unlikely. Therefore, both the 808- and 702-keV γ rays are $E2$ transitions. The mixing of the two 27^- states in bands X1 and ED, which are 37.4 keV apart, causes the decay from ED to X1 and vice versa. This provides additional support for the spin-parity assignments for ED. Therefore, the band ED has a parity and signature $(\pi, \alpha) = (-, 1)$ (i.e., it is associated with odd spins).

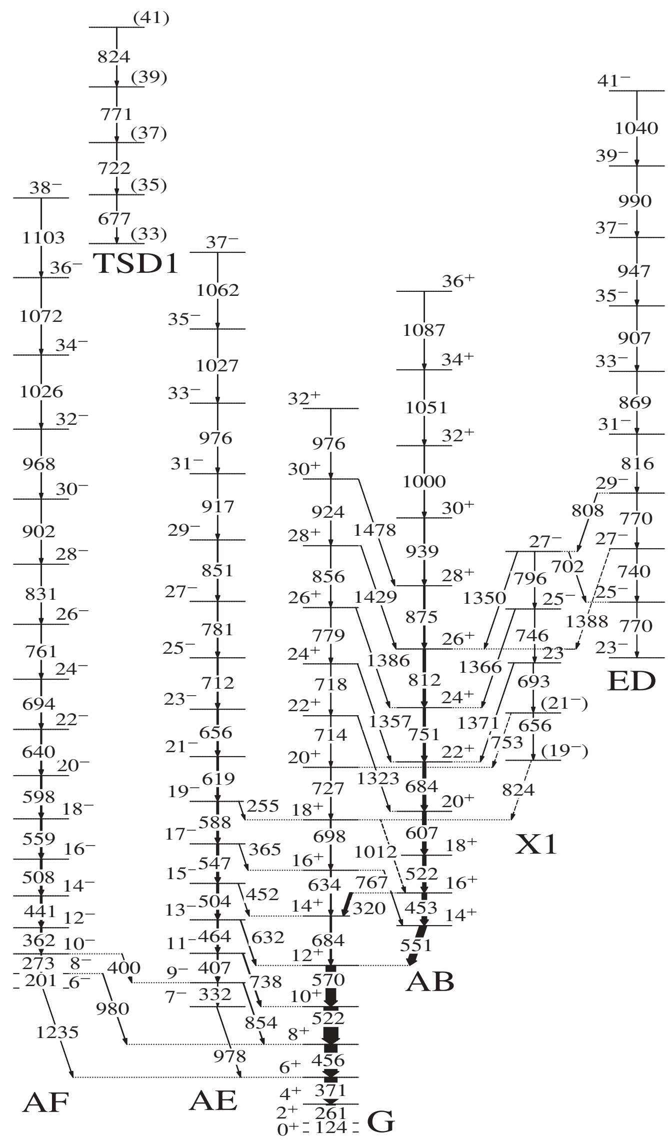

FIG. 1. Partial level scheme of ^{168}Hf from this work. Gamma-ray energies are in keV. Dashed lines represent tentative transitions. Gamma-ray energies of higher spin transitions in bands TSD1 (up to tentative $61\hbar$) and ED (former TSD2, up to $49\hbar$) are given on the spectra shown in Fig. 2. The spin, parity, and excitation energy of TSD1 are uncertain; see text for a detailed discussion. Band TSD2 (former TSD3) is not shown in the figure; its transition energies can be found in Ref. [20].

The band TSD1 decays mainly to the negative-parity bands AE and AF, whose members can be seen in the coincidence γ-ray spectrum, doubly-gated on TSD1 transitions, as shown in Fig. 2. The intensity of band TSD1 decreases in the two lowest transitions, 722 and 677 keV (see inset of Fig. 2). Therefore, the decay-out from TSD1 occurs over the lowest three levels of the band. This fact, together with the observation that TSD1 decays to at least two different bands, must result in a highly fragmented decay pattern. The exact decay pathways from TSD1 to these two bands could not be established. However, all γ rays in TSD1, including the lowest 677-keV transition, are in coincidence with transitions in band AE below

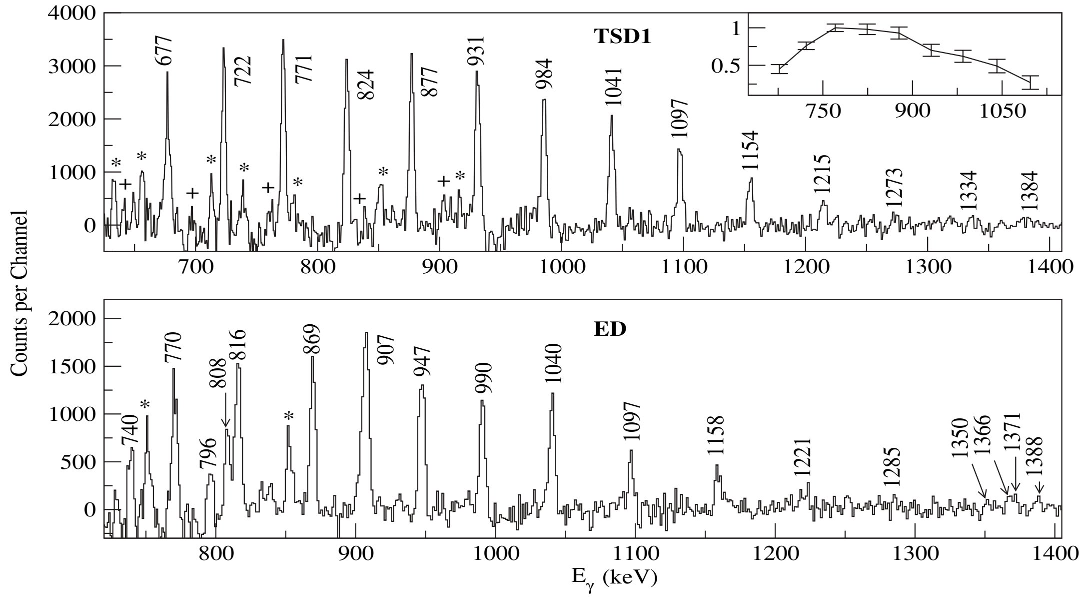

FIG. 2. Gamma-ray coincidence spectra of bands TSD1 and ED in ^{168}Hf, doubly gated by the band members that are labeled by γ-ray energies. In the TSD1 spectrum, the stars and plus signs indicate transitions in the normal deformed bands AE and AF, respectively. The transition with the highest spin marked in band AE is 917 keV ($31^- \rightarrow 29^-$), and that in band AF is 902 keV ($30^- \rightarrow 28^-$). The inset shows the intensity profile of band TSD1. In the ED spectrum, the decay-out transitions of bands ED and X1 are also labeled by energies, with the stars denoting the transitions in ND structures.

spin 31^- and in band AF below spin 30^-. The spin of the lowest TSD1 level would, most likely, be $32\hbar$ if this level decays to the 31^- state in band AE through a dipole transition, like the high-energy dipole γ rays in the statistical decay of the superdeformed bands in mass 150 and 190 regions [24,25]. Such a one-step direct link is not observed. Another, possibly more plausible, scenario would be a two-step link between the lowest TSD1 level and the states in bands AE and AF (e.g., a situation similar to the decay from band ED to X1 through level mixing, with subsequent decay to the yrast line). This would result in a spin of $33\hbar$ or $34\hbar$ for the lowest TSD1 level. Therefore, the spin of this level can only be determined approximately to be $\sim 33\hbar$, or higher. Consequently, the highest level in TSD1 is $61\hbar$, which is typical for the highest spins observed for nuclei in this region, or higher. The possibility of a three-step link cannot be ruled out but is less likely, since it would further raise the spin of band TSD1. The band decays to bands AE and AF, indicating its closer connection to the negative-parity structures, but the parity of TSD1 could not be determined. Band TSD2, consisting of nine transitions [20], is likely located at a higher excitation energy than TSD1 because it is more weakly populated. It feeds the yrast band, as well as another negative-parity ND band found in the present study (not shown in Fig. 1) [23], but its decay pathways could not be established.

III. DISCUSSION

A. Band ED

The kinematic ($J^{(1)}$) and dynamic ($J^{(2)}$) moments of inertia are presented in Fig. 3 for the three bands in ^{168}Hf, the ED bands in ^{171,175}Hf [17,19], and the SD band-2 in ^{175}Hf. The ED bands start from spins as low as $I \sim 15\hbar$–$20\hbar$. Their $J^{(2)}$ moments increase slightly with rotational frequency, excluding the low-spin region where the $J^{(2)}$ values are affected by paired band crossings and/or by interactions with ND bands. Over the

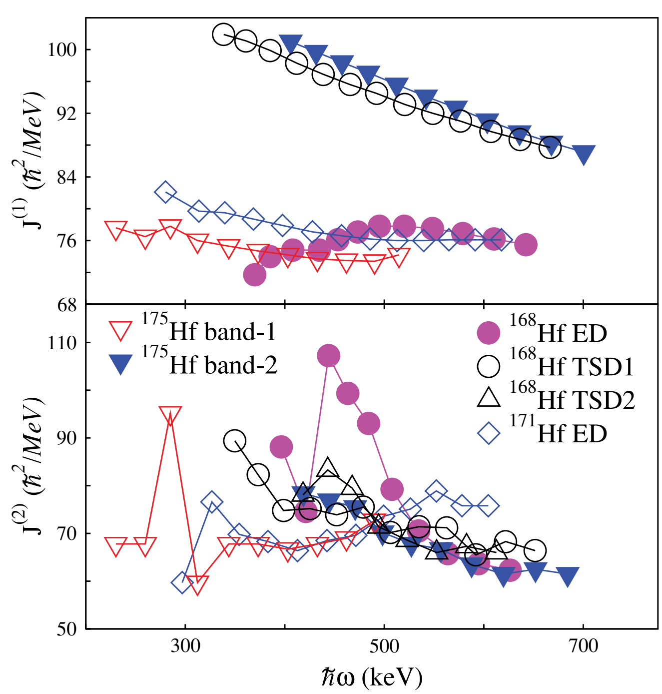

FIG. 3. (Color online) Kinematic ($J^{(1)}$) and dynamic ($J^{(2)}$) moments of inertia for highly deformed bands in ^{168,171,175}Hf. The values for the $J^{(1)}$ moment of band TSD1 in ^{168}Hf are plotted based on the adopted spin values; see text for details.

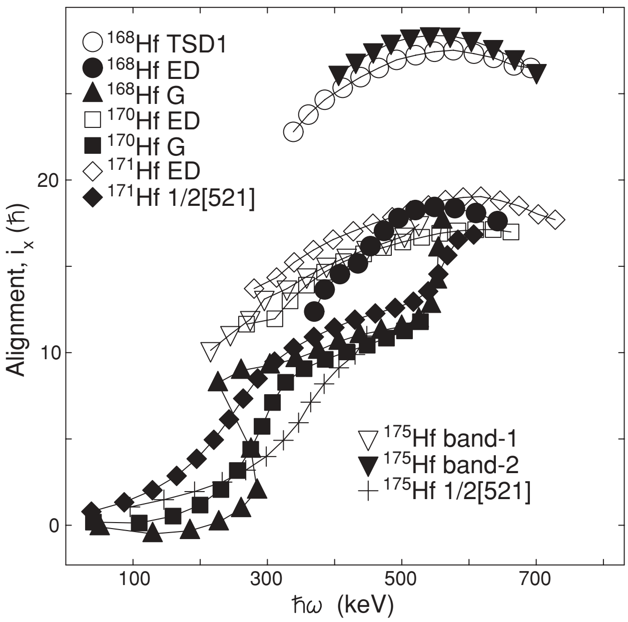

FIG. 4. Aligned angular momenta as a function of rotational frequency for the highly deformed bands and some ND bands in ^{168}Hf and ^{170,171,175}Hf. The band TSD1 in ^{168}Hf is plotted with the adopted spin values. Identical Harris parameters, $I_0 = 30\hbar^2$ MeV^{-1} and $I_1 = 40\hbar^4$ MeV^{-3}, were used for all bands.

entire frequency range, the $J^{(1)}$ moment of band ED in ^{168}Hf is similar to those of the ED bands in the heavier Hf isotopes, which are ∼20% larger than those of the ND bands in these nuclei. The $J^{(2)}$ moment of band ED in ^{168}Hf exhibits large irregularities, partly indicating a character change at lower spins, and partly reflecting the interaction with band X1. Its aligned angular momentum i_x (see Fig. 4) is comparable to those of the ED bands in ^{170}Hf [16] and ^{171,175}Hf and is clearly larger than that of the yrast band AB, which is an extension of ground-state band G after the first $i_{13/2}$ neutron band crossing at a rotational frequency $\hbar\omega \sim 0.28$ MeV. The large initial alignment at low frequencies is typical for structures with aligned high-j quasiparticles. Band ED in ^{168}Hf starts at a higher rotational frequency (∼0.38 MeV) than other ED bands, and its alignment increases gradually to 6.5ℏ above that of the yrast band, possibly because of a changed character at the lowest spins. The first proton alignment observed around $\hbar\omega \sim 0.55$ MeV in the ND bands is clearly missing in all ED bands.

To understand the intrinsic configurations of the ^{168}Hf bands, we performed cranking calculations using the UC code. Pairing is taken into account in the code, and the standard parameters [26] were used for the Nilsson potential. The calculated configurations are labeled as $P(\pi_1, \alpha_1)N(\pi_2, \alpha_2)$. For protons, $\pi_1 = 0$ (or 1) represents the positive (or negative) parity, and $\alpha_1 =$ (signature $\times 2$). The π_2 and α_2 symbols are defined similarly for neutrons. There are four theoretical bands with an aligned angular momentum close to that of band ED. They all have the proton configuration P(1, 2) (see Fig. 5). Of the four neutron configurations, N(0, 0) is energetically favored, lying about 0.5 MeV below the

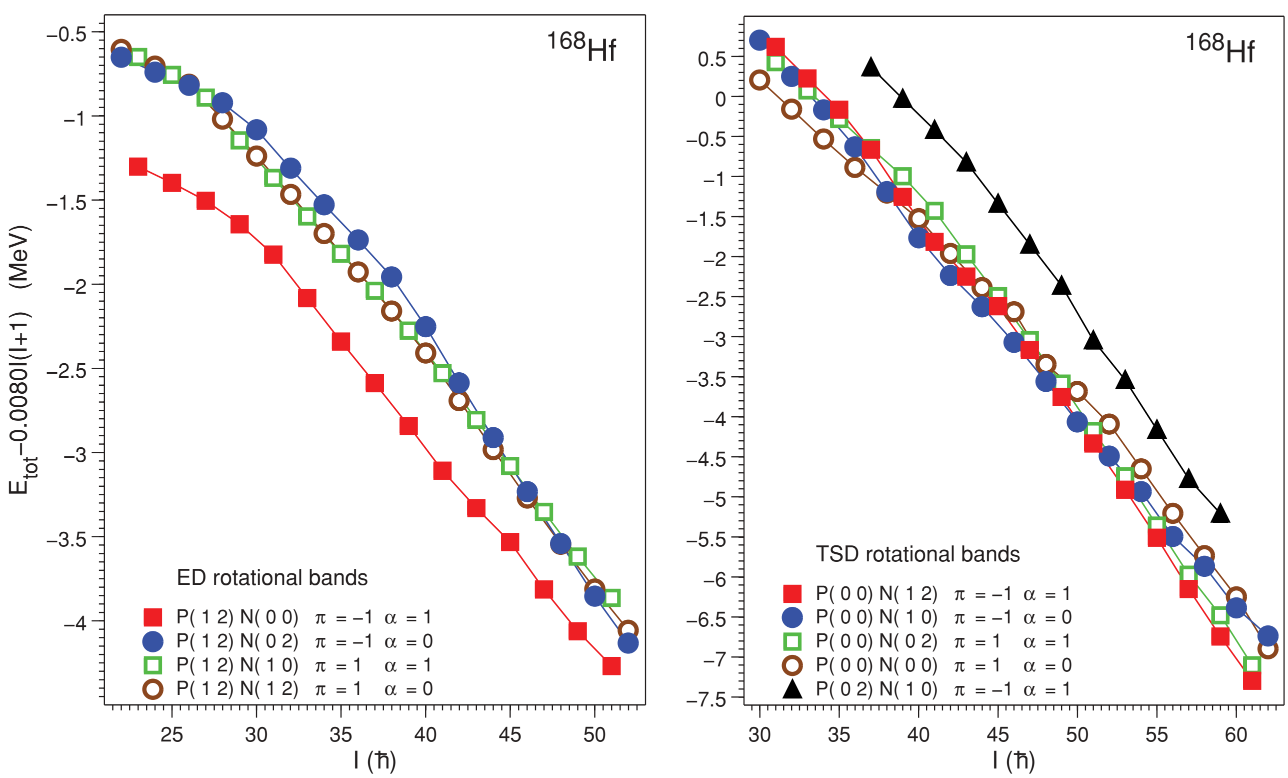

FIG. 5. (Color online) Theoretically calculated energies of ED bands (left) and TSD bands (right). A rigid-rotor reference energy has been subtracted.

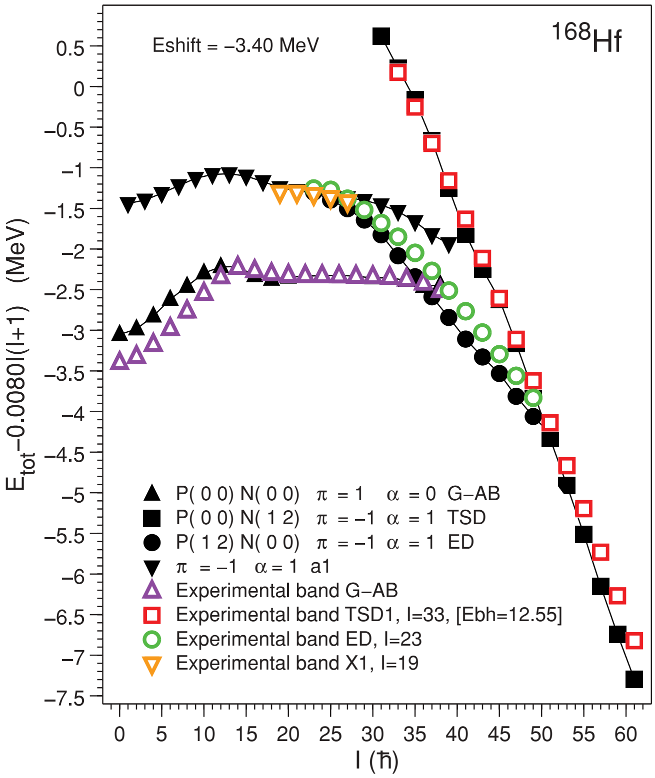

FIG. 6. (Color online) Experimental and UC calculated excitation energies minus a rigid-rotor reference for bands in ^{168}Hf. The experimental bands are shifted downward by 3.4 MeV so that the average energy of levels between $14\hbar$ and $36\hbar$ in the yrast band overlaps with that of the calculated band. The zero point of the energy scale corresponds to the spherical nonrotating liquid drop energy. Band TSD1 is plotted with assumed bandhead spin and energy values, as discussed in the text. The calculated band a1 is a prime candidate for the ND band X1.

nearly degenerate configurations N(1, 0) and N(1, 2). The N(0, 2) configuration has the highest energy, lying about 0.7 MeV above the N(0, 0) configuration. The P(1, 2)N(0, 0) configuration has $(\pi, \alpha) = (-, 1)$ (i.e., negative parity and odd spins), in agreement with the values established for band ED. The configurations P(1, 2)N(1, 2) and P(1, 2)N(1, 0) are signature partners with practically no signature splitting. Such bands are not observed experimentally. Therefore, band ED is likely associated with the configuration P(1, 2)N(0, 0), or $\pi(i_{13/2}h_{9/2}) \otimes \nu(i_{13/2})^2$. The UC calculated excitation energies for such a band, minus a rigid-rotor reference, are compared with the experimental values in Fig. 6. The experimental bands are shifted downward by 3.4 MeV so that the average energies of levels between $14\hbar$ and $36\hbar$ in the yrast band overlap with those of the calculated band. The calculated ED band fits the observed band ED well, but with a slightly lower excitation energy. In addition, the calculated aligned angular momenta are $5.6\hbar$, $2.5\hbar$, and $6.1\hbar$ for the $\pi i_{13/2}$, $\pi h_{9/2}$, and $\nu(i_{13/2})^2$ orbitals, respectively, with a total alignment of $14.2\hbar$. This amount is slightly higher than the $12.3\hbar$ initial alignment of band ED, which actually approaches this value with increasing spin, as shown in Fig. 4. The potential energy

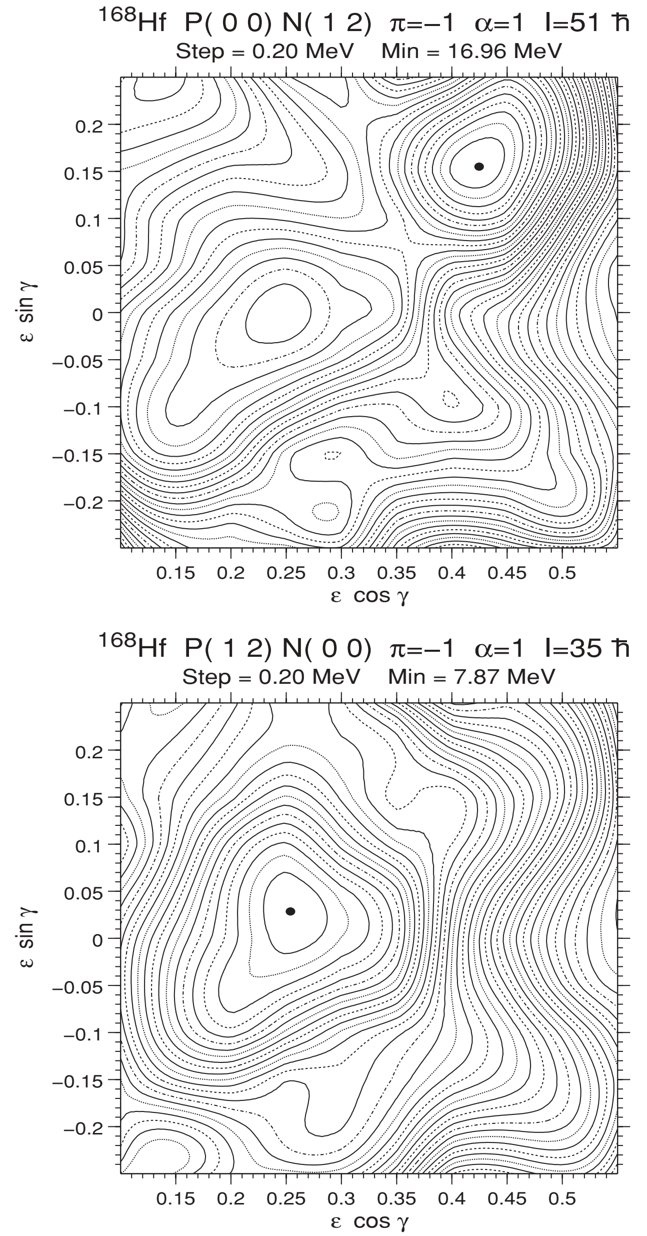

FIG. 7. Total energy surfaces for the P(0, 0)N(1, 2) (upper) and P(1, 2)N(0, 0) (lower) configurations, which contain the lowest TSD and ED minima, respectively. The contour line separation is 0.2 MeV.

surface for the P(1, 2)N(0, 0) configuration is presented in Fig. 7 for $I = 35\hbar$. The ED minimum is located at $(\varepsilon_2, \gamma) = (0.26, 9.3^\circ)$. This deformation is slightly enhanced compared to the ND bands but is similar to deformations calculated for the ED bands observed systematically in ^{170,171,175}Hf [17]. These bands are all built on the proton $i_{13/2}h_{9/2}$ configuration, but they are coupled to different neutron configurations.

B. Band TSD1

Band TSD1 and the SD band-2 in ^{175}Hf [19] are located at higher spins than the ED bands. Their $J^{(1)}$ moments are considerably larger than the $J^{(2)}$ values and both $J^{(1)}$ and $J^{(2)}$ moments decrease smoothly with increasing rotational frequency. The large aligned angular momentum of TSD1 in ^{168}Hf can be achieved in configurations involving one or two $i_{13/2}$ protons and at least one $j_{15/2}$ neutron. In UC calculations, all 16 configurations, which represent the lowest energy for all possible combinations of proton and neutron parity and signature, fulfill this requirement at a TSD shape for at least a part of the experimentally observed spin range. The calculations, as depicted in Fig. 5, show that the proton configuration P(0, 0) is energetically favored by 1 MeV or more, compared to other proton configurations. The four neutron configurations, however, lie much closer in energy; over most of the experimental spin range they are within 0.3 MeV of each other. It is, therefore, likely that the observed band TSD1 will have the P(0, 0) proton configuration, combined with a neutron configuration that cannot be specified further in view of the small energy differences just noted. Thus, four configurations are possible for band TSD1. If the band has negative parity, the configuration is either P(0, 0)N(1, 2) (odd spin) or P(0, 0)N(1, 0) (even spin). If the band has positive parity, the configuration is either P(0, 0)N(0, 0) (even spin) or P(0, 0)N(0, 2) (odd spin). The proton configuration is always the same, namely $\pi(i_{13/2})^2$, and all four neutron configurations contain one neutron in the $j_{15/2}$ subshell. The negative-parity bands have a slightly lower energy than the positive-parity ones, suggesting that the observed band TSD1 may have negative parity. The most probable intrinsic configuration is then $\pi(i_{13/2})^2 \otimes \nu(j_{15/2}i_{13/2})$. The excitation energies of the P(0, 0)N(1, 2) configuration are plotted in Fig. 6 together with that of band TSD1 with a bandhead spin $I = 33\hbar$ and an assumed energy of 12.55 MeV, which fits the calculated band the best. The uncertainties in the assumed energy of the lowest TSD1 level will change the vertical position of the plotted band. With the rigid-rotor reference energies chosen in the plot, the band TSD1 appears like a straight line. The slope of the line is closely related to the moment of inertia and to the spin assumed for the bandhead. If the adopted spin of TSD1 is changed by $1\hbar$, the slope of the line changes by ~5.3%. The calculated quadrupole moment of such a band is $\sim 10.5\,e$ b, in agreement with the experimentally measured value of $Q_t = 11.4^{+1.1}_{-1.2}\,e$ b [20], and is substantially larger than $Q_t \approx 6.4(0.5)\,e$ b for the yrast band [23]. As seen in the potential energy surface in Fig. 7, the calculated band is associated with a TSD minimum at $(\varepsilon_2, \gamma) \sim (0.43, 20^\circ)$. A neutron shell gap at large triaxiality for $N = 97$ is essential for this TSD minimum [6]. When going above $N \sim 97$, rotational bands with similar moments of inertia and aligned angular momentum are predicted to appear in several Hf isotopes, but not necessarily at the same triaxial deformation. For example, similarities are seen in Figs. 3 and 4 when the band TSD1 in ^{168}Hf$_{96}$ and band-2 in ^{175}Hf$_{103}$ are compared. However, our UC calculations, as well as previous CRMF calculations [17], show that the rotational properties of band-2 are reproduced by a band built on a prolate minimum. The calculated quadrupole moment, $Q_t \sim 12\,e$ b, agrees reasonably well with the preliminary values of $Q_t \sim 13\,e$ b measured for the band [19]. The same characteristic high-j orbitals as in TSD1 of ^{168}Hf, $\pi i_{13/2}$ and $\nu j_{15/2}$, are occupied. Details of the UC calculations will be published elsewhere [27].

The configuration of band TSD1 is very different from that of the wobbling bands observed in neighboring Lu isotopes, where only one aligned high-j intruder orbital, the $i_{13/2}$ proton, is involved. The zero-phonon bands in Lu isotopes start from spins as low as $6.5\hbar$–$12.5\hbar$ (rotational frequency $\hbar\omega \sim$ 100–200 keV), with excitation energies less than 100 keV above the yrast line at low spins. These bands are strongly populated (e.g., ~10% and ~8% relative to yrast bands in ^{163}Lu [8] and ^{167}Lu [11], respectively). The wobbling excitations built on these zero-phonon bands are more favored in energy than the TSD bands based on quasiparticle excitations in the TSD minimum, which are located at higher energies (e.g., about 1 MeV higher than the zero-phonon band in ^{163}Lu [28]). The band TSD1 in ^{168}Hf is located at much higher spin and excitation energy than the zero-phonon bands in the Lu isotopes. Consequently, its intensity, 0.26(10)% relative to the yrast band, is very weak as compared to those of the Lu bands. Furthermore, the calculations suggest that there are several quasiparticle configurations located very close to TSD1, including those shown in Fig. 5, which may compete with wobbling excitations more favorably. Therefore, it will be difficult to observe collective wobbling excitations built on band TSD1. A similar situation was suggested to be present for the TSD bands of ^{163}Tm [29]. We note that band TSD2 could correspond to one of the four low-lying TSD bands predicted by the UC calculations, considering the fact that the $J^{(2)}$ moments of TSD2 and TSD1 are similar (see Fig. 3), and that TSD2 is likely located at a higher excitation energy than TSD1 because it is more weakly populated. If band TSD2 was plotted in Fig. 6 with an assumed bandhead spin of $40\hbar$ and energy about 2 MeV higher than that of the suggested $33\hbar$ bandhead of the band TSD1, band TSD2 would follow the behavior of TSD1 very closely.

IV. CONCLUSION

Decay pathways of previously reported candidates for TSD structures in ^{168}Hf were analyzed. Discrete links were firmly established for band ED. The spin of the lowest level in TSD1 was determined to be $\sim 33\hbar$ or higher, based on the observed γ-ray coincidence relationships. Detailed rotational properties of the bands were investigated. The results of cranking calculations using the UC code reproduce all experimental observables rather well. The measured properties and the overall agreement with a theoretical analysis provide strong support for an interpretation where band TSD1 is associated with a TSD minimum with $(\varepsilon_2, \gamma) \sim (0.43, 20^\circ)$, involving the $\pi(i_{13/2})^2$ and the $\nu(j_{15/2})$ high-j orbitals. This constitutes a confirmation of the existence of a first TSD band, long-predicted in Hf isotopes. Band ED is likely associated with a near-prolate shape and a deformation slightly enhanced with respect to the normal deformed bands. It is proposed to be built on the $\pi(i_{13/2}h_{9/2}) \otimes \nu(i_{13/2})^2$ configuration. Such ED bands have recently been observed systematically in several heavier Hf isotopes.

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