## Unified formula of half-lives for $\alpha$ decay and cluster radioactivity

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In view of the fact that  $\alpha$  decay and cluster radioactivity are physically analogical processes, we propose a general formula of half-lives and decay energies for  $\alpha$  decay and cluster radioactivity. This new formula is directly deduced from the WKB barrier penetration probability with some approximations. It is not only simple in form and easy to see the physical meanings but also shows excellent agreement with the experimental values. Moreover, the difference between two sets of parameters to separately describe  $\alpha$  decay and cluster radioactivity is small. Therefore, we use only one set of adjustable parameters to simultaneously describe the  $\alpha$  decay and cluster radioactivity data for even-even nuclei. The results are also satisfactory. This indicates that this formula successfully combines the phenomenological laws of  $\alpha$  decay and cluster radioactivity. We expect it to be a significant step toward a unified phenomenological law of  $\alpha$  decay and cluster radioactivity.

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## I. INTRODUCTION

The spontaneous emission of a charged particle heavier than an  $\alpha$  particle but lighter than a fission fragment is known as cluster radioactivity. There is a whole family of such a disintegration mode: <sup>14</sup>C radioactivity, <sup>24</sup>Ne radioactivity, <sup>28</sup>Mg radioactivity, and so on. This new nuclear radioactivity decay mode was first predicted in 1980 by Săndulescu, Poenaru, and Greiner [1]. Subsequently in 1984 Rose and Jones experimentally observed this new kind of radioactivity, <sup>14</sup>C from <sup>223</sup>Ra [2]. Later primary studies of this new radioactivity were widely carried out [3-7]. As a result of the special interest in the studies of heavy and superheavy nuclei, in particular the heaviest ones, the interest in  $\alpha$ -decay systematics is continuing [8-10], and the interest in systematic analysis of cluster radioactivity is increasing. Many semiempirical relationships for  $\alpha$  decay have been developed [11–14]. We also notice that there are some formulas to determine the halflives of cluster radioactivity [15–19]. All of these formulas should be considered as effective methods to calculate the half-lives of  $\alpha$  decay and cluster radioactivity because they are based on different models or different variations of Gamow's theory. In addition, a very simple formula for proton emission systematics was proposed by Delion et al. [20]. It takes into account the centrifugal barrier, the structure of the decaying nucleus, and the corresponding preformation probability so well that the experimental data of proton emitters with Z > 50lie along two straight lines.

Our group have implemented some work for the description of  $\alpha$  decay and cluster radioactivity. New calculations of  $\alpha$ -decay half-lives by the Viola-Seaborg formula were carried out [21]. The formula with new parameters can reproduce the experimental half-lives of even-even nuclei within a factor of 1.3 and predict the half-lives of some superheavy nuclei well. Besides, a new formula between half-lives and decay energies of cluster radioactivity was proposed [17], which is

$$\log_{10} T_{1/2} = a Z_c Z_d Q^{-1/2} + c Z_c Z_d + d + h, \qquad (1)$$

where  $Z_c$  and  $Z_d$  are the atomic number of the cluster and daughter nuclei, respectively, and h accounts for a blocking factor associated with unpaired nucleons for odd-A nuclei. This formula can be considered as a natural extension of both the Geiger-Nuttall law and the Viola-Seaborg formula from simple  $\alpha$  decay to complex cluster radioactivity. Nowadays, considering many resemblances between  $\alpha$  decay and cluster radioactivity, we hope to give a universal formula that can simultaneously describe them. On the one hand,  $\alpha$  decay and cluster radioactivity are physically similar processes. Both are fundamentally quantum tunneling processes through the potential barrier. On the other hand, in recent years many new data of  $\alpha$  decay, especially the data for the superheavy nuclei, have been observed experimentally. During the same time the data of cluster radioactivity from <sup>14</sup>C to <sup>34</sup>Si have been accumulated. They provide an excellent opportunity to unify the phenomenological laws of  $\alpha$  decay and cluster radioactivity. In this article, we start from the quantum tunneling effect and hope to find a unified formula of half-lives for  $\alpha$  decay and cluster radioactivity.

This article is organized in the following way. In Sec. II, we deduce a new formula of half-lives and decay energies for  $\alpha$  decay and cluster radioactivity, directly from the WKB barrier penetration probability with some approximations. In Sec. III, we use the new formula to make a systematic study of the  $\alpha$  decay and cluster radioactivity data, respectively. The experimental data are well reproduced by the formula. Furthermore, we use the formula to investigate the total experimental data of  $\alpha$  decay and cluster radioactivity for even-even nuclei. The results are also satisfactory. A summary is given in Sec. IV.

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### II. THE UNIFIED FORMULA OF HALF-LIVES FOR α DECAY AND CLUSTER RADIOACTIVITY

 $\alpha$  decay was first described in 1928 as a quantum tunneling through the potential barrier [22]. The cluster radioactivity observed from 1984 is fundamentally also the quantum tunneling process through the decay barrier. In terms of the semiclassical approximation, the decay width (or decay constant) can be expressed by the equation [23–26]

$$\Gamma \equiv \hbar \ln 2/T_{1/2} = P_0 F P, \qquad (2)$$

where  $P_0$  is the preformation probability of the cluster in the parent nucleus, which differs from one decay mode to another but does not change very much for a given radioactivity [27]. *F* is the frequency of the cluster inside the barrier, and *P* is the probability of transmission through the barrier, which is given in the WKB approximation by

$$P = \exp\left(-\frac{2}{\hbar}\int_{R_t}^{R_c}\sqrt{2\mu[V(r)-Q]}dr\right).$$
 (3)

Here  $R_t$  is the touching radius,  $R_t = R_c + R_d$ , where  $R_c$  and  $R_d$  are the hard-sphere radii for the cluster and daughter nuclei, respectively. The potential is given by  $V(r) = Z_c Z_d e^2/r$ , and  $R_C$  is the classical turning point,  $R_C = Z_c Z_d e^2/Q$ .  $\mu$  is the reduced mass of the cluster-daughter system measured in unit of the nucleon mass,  $\mu = A_c A_d/(A_c + A_d)$ . Combining the above results, one obtains

$$\log_{10} T_{1/2} = \log_{10}(\hbar \ln 2/P_0 F) + \frac{2}{\ln 10} \frac{\sqrt{2\mu}e^2}{\hbar} Z_c Z_d Q^{-1/2} \times [\arccos(x) - x\sqrt{1 - x^2}], \qquad (4)$$

where  $x = \sqrt{R_t/R_c}$ . As the first approximation of the last part of Eq. (4), we obtain

$$\log_{10} T_{1/2} = \log_{10}(\hbar \ln 2/P_0 F) + \frac{\sqrt{2\mu\pi}e^2}{\hbar \ln 10} Z_c Z_d Q^{-1/2} - \frac{4e\sqrt{2\mu}R_t}{\hbar \ln 10} (Z_c Z_d)^{1/2}.$$
 (5)

Due to the rough assumption of  $R_t$  and the little change of  $\sqrt{R_t}$  for  $\alpha$  decay and cluster radioactivity of heavy and superheavy nuclei, we treat the factor  $\sqrt{R_t}$  as a constant here. In the case of  $\alpha$  decay the reduced mass changes 0.3% as the daughter( $A_d$ ) changes from 200 to 300; that is to say, it approaches to a constant for  $\alpha$  decay of heavy and superheavy nuclei. Therefore, the factor  $\sqrt{\mu}$  was usually omitted as a constant in the prevenient  $\alpha$ -decay systematics. However, for heavier clusters the reduced mass is sensitive to the mass of the fragments. With this feature in mind, without loss of generality we have

$$\log_{10} T_{1/2} = \log_{10}(\hbar \ln 2/P_0 F) + c_1 \sqrt{\mu} Z_c Z_d Q^{-1/2} + c_2 \sqrt{\mu} (Z_c Z_d)^{1/2}.$$
(6)

It is known from available experimental cases that the larger the cluster the smaller the preformation probability [27]. So the preformation probability of a cluster depends practically upon the size of the cluster. Because the same cluster is emitted by different nuclei, the preformation probability should associate with the size of the parent or the daughter. Based on these simple experimental facts, we assume that the preformation probability of a cluster is an exponential function  $P_0 = 10^{-c_3\sqrt{\mu}(Z_cZ_d)^{1/2}+c_4}$ , where  $c_3$  and  $c_4$  are constants. For a given cluster, in particular an  $\alpha$  particle, in heavy nuclei  $\sqrt{\mu}$  and  $(Z_cZ_d)^{1/2}$  change with the parent nuclei very smoothly so that the preformation probability of the cluster does not change very much, as shown by Iriondo, Jerrestam, and Liotta [27]. Moreover, the probability decreases quickly with the increase of the charge number of the cluster, as one would expect. Then we write the right side of Eq. (6) as the sum of the  $\sqrt{\mu}Z_cZ_dQ^{-1/2}$  term, the  $\sqrt{\mu}(Z_cZ_d)^{1/2}$  term and a constant, where the term related to the preformation probability is also included. The equation of half-lives and decay energies for  $\alpha$  decay and cluster radioactivity can be written as

$$\log_{10} T_{1/2} = a \sqrt{\mu} Z_c Z_d Q^{-1/2} + b \sqrt{\mu} (Z_c Z_d)^{1/2} + c, \quad (7)$$

where a, b, and c are the constants to be determined.

Furthermore, for the case of odd-A and odd-odd nuclei, the structures of the ground states of the parent and the daughter are in general different. This causes the hindrance of the transition between these states. The hindrance is characterized by the cluster preformation probability and is independent of the decay energy [28]. Generally speaking, the larger the hindrance, the smaller the preformation probability. Looking at the form of the cluster preformation probability above, the first term of the exponential can describe the changes with the cluster and the parent nuclei well, but for a given cluster and some parent nuclei close to each other, it approaches to a constant; i.e., it is not able to reflect the hindrance for each kind of nuclei. In view of this fact, to describe the difference among the hindrances for different kinds of nuclei, we assume that the second term of the exponential  $c_4$  varies with the kinds of parent nuclei, which corresponds to the parameter c of the formula which has different values for even-even, even-odd, odd-even, and odd-odd nuclei.

This new formula is not only simple in form and easy to see the physical meanings but also at the same time describes the complicated processes of  $\alpha$  decay and cluster radioactivity effectively.

# **III. NUMERICAL RESULTS AND DISCUSSIONS**

In our analysis of  $\alpha$ -decay half-lives, we concentrate on heavier nuclei with  $Z \ge 84$  and  $N \ge 128$ . We take the latest values of experimental half-lives from Ref. [29]. We also use some other sources [30–34]. The parameter *c* of the formula has different values for even-even, even-odd, odd-even, and odd-odd nuclei while the other parameters have the same values. This is similar to the technique of the famous Viola-Seaborg formula. Before bringing forth our numerical results for  $\alpha$  decay, we compare the new formula of any cluster emission with the Royer formula of  $\alpha$  decay [14], which is  $\log_{10} T_{1/2} = aZQ^{-1/2} + bZ^{1/2}A^{1/6} + c$ , where Z and A are the proton number and the mass number of a parent nucleus, respectively. When we treat  $\sqrt{\mu}$  rather than  $\sqrt{R_t}$  as a constant and fix  $Z_c = 2$  for  $\alpha$  decay, the new formula comes naturally back to the Royer formula. Based on this, the new formula can be considered as a natural generalization of the Royer formula



FIG. 1. Deviations between the logarithms of the experimental half-lives and of the calculated values for  $\alpha$  decay of even-even nuclei with proton number Z = 84-118.

from simple  $\alpha$  decay to any cluster emission, including  $\alpha$  decay.

Now we determine the three parameters a, b, and  $c_{e-e}$  for even-even nuclei. Through a least-square fit to the experimental data of 71 even-even (with Z = 84-118, N = 128-176) nuclei, we obtain a set of parameters whose values are

$$a = 0.39961,$$
  
 $b = -1.31008,$   
 $c_{e-e} = -17.00698.$ 

To evaluate the proposed relation, the calculated favored  $\alpha$  decay half-lives of even-even nuclei are compared with the experimental data as shown in Fig. 1. To aid the eye, consecutive isotopes of a given element are connected with a line segment. It can be seen that the values of  $\log_{10}(T_{\text{exp.}}/T_{\text{cal.}})$  are generally within the range of about  $\pm 0.3$ , which corresponds to the values of the ratio  $T_{exp.}/T_{cal.}$  within the range of about 0.5–2.0, except the values of the nuclei <sup>212</sup>Po, <sup>290</sup>116 and <sup>292</sup>116. This means that the calculated  $\alpha$ -decay half-lives are in good agreement with the experimental data for even-even nuclei. With the parameters a and bfixed, the parameter c for 68 e-o (with Z = 84-116, N =129–175), 52 o-e (with Z = 85-115, N = 128-172), and 53 o-o (with Z = 85-115, N = 129-173) nuclei are determined to be  $c_{e-o} = -16.26029, c_{o-e} = -16.40484$ , and  $c_{o-o} = -16.40484$ -15.85337. The results are illustrated in Fig. 2, where the x axis is  $\mu^{1/2} Z_c Z_d Q^{-1/2}$  and the y axis is the other parts of the formula. We also show the results when the parameter c has the same value for the four cases. When the parameter c has different values, we can obviously see that the experimental points lie approximatively in a single straight line as predicted by our formula in a better way. This is an active response to our assumption that the parameter  $c_4$  of the cluster preformation probability varies with the kinds of parent nuclei.



FIG. 2. The comparison of the logarithm of the calculated halflives with the logarithm of the experimental data for  $\alpha$  decay. The line represents the calculated values and the points represent the experimental ones. For the case of even-even, even-odd, odd-even, and odd-odd nuclei, (a) the parameter *c* has the same value  $c_{e-e}$ ; (b) the parameter *c* has different values  $c_{e-e}$ ,  $c_{e-o}$ ,  $c_{o-e}$ , and  $c_{o-o}$ .

The standard deviation is defined as

$$\sqrt{\langle \sigma^2 \rangle} = \sqrt{\sum_{i=1}^{N} \left[ \log_{10} \left( T_{\text{exp.}}^i / T_{\text{cal.}}^i \right) \right]^2 / N}, \qquad (8)$$

and the mean deviation is given by

$$\langle \sigma \rangle = \sum_{i=1}^{N} \left| \log_{10} \left( T_{\text{exp.}}^{i} / T_{\text{cal.}}^{i} \right) \right| / N.$$
(9)

In Table I, the first column denotes the type of nuclei and the second column denotes the number of nuclei. The standard deviations and the mean deviations are listed in columns 3 and 4, respectively. One can see that the experimental data of  $\alpha$  decay are well reproduced by the new formula.

Now we describe the cluster radioactivity data with the new formula. The parameter c has different values for even-even and odd-A nuclei while the other parameters have the same values, which is similar to the description of  $\alpha$  decay. We determine the three parameters a, b, and  $c_{e-e}$  through a least-square fit to the available data of 11 even-even emitters. With the values of a and b kept fixed, the parameter  $c_{o-A}$  is obtained

TABLE I. Results for  $\alpha$  decay obtained with the new formula. The deviations 0.1, 0.2, 0.6, 0.7, 0.8 of the logarithms correspond to the deviations between the experimental half-lives and the theoretical ones by factors of 1.3, 1.6, 4.0, 5.0, 6.3, respectively.

| Nuclei type         | Number | $\sqrt{\langle \sigma^2  angle}$ | $\langle \sigma \rangle$ |
|---------------------|--------|----------------------------------|--------------------------|
| Even $Z$ , even $N$ | 71     | 0.159                            | 0.125                    |
| Odd $Z$ , odd $N$   | 53     | 0.805                            | 0.680                    |
| Even $Z$ , odd $N$  | 68     | 0.692                            | 0.548                    |
| Odd $Z$ , even $N$  | 52     | 0.616                            | 0.474                    |

| Decay   | Q (MeV) | $\log_{10} T_{1/2}^{\text{exp.}}(s)$ | $\log_{10} T_{1/2}^{\text{formula}}(s)$ | Ref. |
|---|---------|--------------------------------------|---|------|
| $^{221}$ Fr $\rightarrow ^{207}$ Tl + $^{14}$ C                   | 31.29   | 14.52                                | 14.63                                   | [29] |
| $^{221}$ Ra $\rightarrow ^{207}$ Pb + $^{14}$ C                   | 32.40   | 13.39                                | 13.47                                   | [29] |
| $^{222}$ Ra $\rightarrow ^{208}$ Pb + $^{14}$ C                   | 33.05   | 11.02                                | 11.02                                   | [35] |
| $^{223}$ Ra $\rightarrow ^{209}$ Pb + $^{14}$ C                   | 31.84   | 15.21                                | 14.54                                   | [36] |
| $^{224}$ Ra $\rightarrow ^{210}$ Pb + $^{14}$ C                   | 30.53   | 15.87                                | 15.87                                   | [35] |
| $^{226}$ Ra $\rightarrow ^{212}$ Pb + $^{14}$ C                   | 28.21   | 21.34                                | 20.91                                   | [36] |
| $^{225}\text{Ac} \rightarrow ^{211}\text{Bi} + {}^{14}\text{C}$   | 30.48   | 17.16                                | 18.23                                   | [37] |
| $^{228}$ Th $\rightarrow ^{208}$ Pb + $^{20}$ O                   | 44.73   | 20.72                                | 21.53                                   | [29] |
| $^{230}$ Th $\rightarrow $ $^{206}$ Hg + $^{24}$ Ne               | 57.77   | 24.64                                | 24.57                                   | [29] |
| $^{231}$ Pa $\rightarrow ^{208}$ Pb $+ ^{23}$ F                   | 51.84   | 26.02                                | 25.67                                   | [38] |
| $^{231}$ Pa $\rightarrow ^{207}$ Tl + $^{24}$ Ne                  | 60.41   | 22.89                                | 23.09                                   | [38] |
| $^{230}\text{U} \rightarrow {}^{208}\text{Pb} + {}^{22}\text{Ne}$ | 61.39   | 19.60                                | 20.09                                   | [39] |
| $^{232}\text{U} \rightarrow ^{208}\text{Pb} + ^{24}\text{Ne}$     | 62.31   | 20.39                                | 20.36                                   | [29] |
| $^{233}\text{U} \rightarrow ^{209}\text{Pb} + {}^{24}\text{Ne}$   | 60.49   | 24.84                                | 24.41                                   | [29] |
| $^{234}\text{U} \rightarrow {}^{206}\text{Hg} + {}^{28}\text{Mg}$ | 74.12   | 25.75                                | 25.24                                   | [40] |
| $^{236}$ Pu $\rightarrow$ $^{208}$ Pb + $^{28}$ Mg                | 79.68   | 21.65                                | 20.75                                   | [41] |
| $^{238}$ Pu $\rightarrow$ $^{206}$ Hg + $^{32}$ Si                | 91.20   | 25.30                                | 25.57                                   | [42] |
| $^{242}\text{Cm} \rightarrow ^{208}\text{Pb} + ^{34}\text{Si}$    | 96.52   | 23.15                                | 23.51                                   | [43] |

TABLE II. The comparison of the logarithm of calculated half-lives from our new formula with the logarithm of experimental data for cluster radioactivity.

through the least-square fit to the experimental data of seven odd-*A* nuclei. Their values are as follows:

$$\begin{cases} a = 0.38617, \\ b = -1.08676, \\ c_{e-e} = -21.37195, \\ c_{o-A} = -20.11223. \end{cases}$$

The standard deviation and the mean deviation of both eveneven and odd-A nuclei are 0.489 and 0.378, respectively. Looking at the present data of cluster radioactivity, both the decay energies and the half-lives of many nuclei need to be measured with improved accuracy. Therefore, the standard deviation and the mean deviation are larger than they should be. The numerical results are shown in Table II and Fig. 3.

In Table II, the first column denotes the mode of cluster radioactivity and the second column is the experimental decay energy. The logarithms of the experimental half-lives and of the calculated ones are listed in columns 3 and 4, respectively. We can see that in many cases the deviation between the

experimental value and the calculated one is less than 0.5. This means that the experimental half-lives of cluster radioactivity are reproduced by our formula within a factor of 3.

This new formula makes it easy to see the physical meanings as it can be approximately derived. The signs and values of the constants in the formula also agree with our expectation. After we use the formula to separately describe  $\alpha$  decay and cluster radioactivity for even-even nuclei, we find that the difference of the three parameters *a*, *b*, and  $c_{e-e}$  in the two cases is small, as we expected in the course of deducing the new formula. With more and more accumulation of data for cluster radioactivity and better and better precision of data, it will be interesting to see whether the difference is visibly smaller. If that happens, then we can use one set of parameters to simultaneously describe the  $\alpha$  decay and cluster radioactivity data with high accuracy.

Nowadays we use three parameters a, b, and  $c_{e-e}$  to describe the total experimental data for even-even nuclei. Through a linear least-square fit to 82 experimental values,



FIG. 3. The comparison of the logarithm of the calculated half-lives with the logarithm of the experimental data for cluster radioactivity. The parameter *c* has different values for even-even and odd-*A* nuclei. The small figures on the right are for the radioactivity of <sup>14</sup>C and <sup>24</sup>Ne, respectively. The line represents the calculated values and the points represent the experimental ones.



FIG. 4. Deviations between the logarithms of the experimental data and of the calculated values for even-even nuclei (a) when we use two sets of parameters to describe  $\alpha$  decay and cluster radioactivity respectively and (b) when we use one set of parameters to describe both  $\alpha$  decay and cluster radioactivity at the same time.

consisting of the  $\alpha$  decay and cluster radioactivity data for even-even nuclei, we obtain

$$\begin{cases} a = 0.39980, \\ b = -1.13263, \\ c_{e-e} = -21.85863 \end{cases}$$

The standard deviation and the mean deviation are 0.310 and 0.202, respectively. The deviations between the logarithms of the experimental half-lives and of the calculated values for even-even nuclei are shown in Fig. 4. We also show the deviations when using different sets of parameters to separately

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describe  $\alpha$  decay and cluster radioactivity for even-even nuclei, in which  $T_{1/2}^{\text{exp.}}$  are better reproduced. This is not unexpected because of the larger number of parameters in the latter case. Nevertheless, in the former case, we can see that the values of the deviations are within the range of about  $\pm 0.5$ in many cases, which means that the calculated half-lives from the formula agree with the experimental data within a factor of three. This indicates that the formula successfully combines the phenomenological laws of  $\alpha$  decay and cluster radioactivity. We expect it to be an important step toward a unified phenomenological law of  $\alpha$  decay and cluster radioactivity.

## **IV. SUMMARY**

In summary, a general formula of half-lives and decay energies for  $\alpha$  decay and cluster radioactivity is deduced from the WKB barrier penetration probability with some approximations. We systematically investigate the experimental data of  $\alpha$  decay and cluster radioactivity with this new formula, respectively. The results show excellent agreement between the experimental data and the calculated values. Furthermore, it is only three adjustable parameters that we use to describe the  $\alpha$  decay and cluster radioactivity data for even-even nuclei at the same time. The results are also satisfactory.

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