The (π^-, K^+) reaction on ^{28}Si and the Σ-nucleus potential

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We calculate in the impulse approximation the kaon spectrum from the (π^-, K^+) reaction on ^{28}Si. The strength V_Σ of the real part of the single particle potential of the Σ hyperons produced in the reaction is obtained from the Nijmegen model F of the baryon-baryon interaction, and the strength W_Σ of the imaginary (absorptive) part is determined by the ΣN cross sections for the $\Sigma\Lambda$ conversion and also for the elastic ΣN scattering (this elastic scattering introduces a strong dependence of W_Σ on the Σ momentum). The momentum dependence of V_Σ is also considered. Our calculated inclusive kaon spectrum agrees reasonably with the spectrum measured at KEK. The strengths of the real and absorptive Σ potentials used in the present calculation are compatible with these strengths determined in the analyses of Σ atoms and of the strangeness exchange reactions.

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Our knowledge of the single particle (s.p.) potential $U_\Sigma = V_\Sigma - iW_\Sigma$ of the Σ hyperon in nuclear matter comes from the analyses of Σ atoms [1,2], of the strangeness exchange (K^-, π) reactions [3,4], and of the hyperon-nucleon scattering data, to which the hyperon-nucleon interaction potential may be fitted [5–8] (and with this potential one may calculate U_Σ [9–11]). All these analyses lead to the conclusion that the ΣN interaction is well represented by the Nijmegen model F of the baryon-baryon interaction [6], which leads to a repulsive V_Σ with the strength of about 25 MeV at the equilibrium density of nuclear matter [11,12].

Recently, a new source of information on U_Σ became available: the final state interaction of Σ hyperons in the associated production reaction (π, K^+). The first measurement of the inclusive K^+ spectrum from the (π^-, K^+) reaction on the ^{28}Si target was performed in KEK at a pion momentum of 1.2 GeV/c [13–15]. The existing impulse approximation analyses [13,15,16] of this KEK experiment imply a repulsive V_Σ with a surprisingly great strength of about 100 MeV, which is inconsistent with the previous estimates.

In the present article we show that the inconsistency may disappear if we consider (i) the contribution of the elastic ΣN scattering to W_Σ (and not only of the $\Sigma\Lambda$ conversion) and (ii) the dependence of V_Σ on the Σ momentum.

In our discussion we apply the simple impulse approximation described in Ref. [16] and described in more detail in Ref. [17].[1] We apply the local density approximation and treat each point of the nuclear core as a piece of nuclear matter with the local nucleon density ρ. For W_Σ we use the expression in terms of the total cross sections for the $\Sigma\Lambda$ conversion process $\Sigma N \to \Lambda N'$ and for the ΣN total elastic (including charge exchange) scattering:

$$W_\Sigma = W_c + W_e, \tag{1}$$

$$W_c = \rho \frac{\hbar^2}{4\mu_{\Sigma N}} \langle k_{\Sigma N} Q_\Lambda \sigma(\Sigma^- p \to \Lambda n)\rangle, \tag{2}$$

$$W_e = \rho \frac{\hbar^2}{4\mu_{\Sigma N}} \langle k_{\Sigma N} Q_\Sigma [\sigma(\Sigma^- n \to \Sigma^- n) + \sigma(\Sigma^- p \to \Sigma^- p) + \sigma(\Sigma^- p \to \Sigma^0 n)]\rangle, \tag{3}$$

where $\langle\ \rangle$ denotes the average value in the Fermi sea, $\hbar k_{YN}$ is the relative YN momentum $(Y = \Sigma, \Lambda)$, μ_{YN} is the YN reduced mass, and Q_Y is the exclusion principle operator in the YN channel (a projection operator onto nucleon states above the Fermi sea). Relations (1)–(3) were derived in Ref. [18] (see also Ref. [9]) within the low order Brueckner (LOB) theory with two coupled channels YN, by applying the optical theorem, in which terms with squares of the reaction matrix were approximated by the corresponding cross sections ($|\mathcal{K}_{\Sigma N,\Sigma N}|^2$ by ΣN elastic cross section, $|\mathcal{K}_{\Lambda N,\Sigma N}|^2$ by $\Sigma\Lambda$ conversion cross section).

Notice that in the case of the nucleon optical potential $V_N + iW_N$ only the elastic NN scattering contributes to W_N, and the situation is similar to that in the case of the contribution W_e to W_Σ. A semiclassical expression for W_N, analogous to expression (3), was first proposed by Goldberger [19] for high incident energies and was shown later by Lane and Wandel [20] to be applicable also at lower energies. (The problem of the accuracy of expression (2)—discussed by Gal [21]—appears similar.)

In applying expressions (2) and (3), we used for the $\Sigma\Lambda$ conversion cross section the parametrization of Gal *et al.* [22], and for the cross sections appearing in expression (3) we used the cross sections tabulated by Rijken [23]. We consider the case of the (π^-, K^+) reaction on the ^{28}Si target: $\pi^- + ^{28}\text{Si} \to \text{K}^+ + ^{27}\text{Al} \otimes \Sigma^-$. In our simple model the 27 nucleons in the final state are distributed uniformly within the sphere of radius $R = 3.7559$ fm, with the corresponding density $\rho = 0.122$ fm^{-3}.

The exclusion principle operators Q_Y, approximated in expressions (2) and (3) by their averages over the angle between the (conserved) total and relative YN momenta, depend on the YN momenta in the final state of the $\Sigma\Lambda$ conversion and ΣN elastic scattering. These relative YN momenta are determined by the energy conservation equation,

[1] Notice that the conclusions of Ref. [17] are not correct because they were reached by a comparison with the KEK results presented in Ref. [13] in a figure that contained an error (corrected in Ref. [14]).

which in the case of the $\Sigma\Lambda$ conversion has the form

$$e_\Sigma(k_\Sigma) + e_N(k_N) + (M_\Sigma - M_N)c^2 = e_\Lambda(k'_\Lambda) + e_N(k'_N), \quad (4)$$

where $e_\Sigma(k_\Sigma)$, $e_N(k_N)$, and $e_\Lambda(k'_\Lambda)$, $e_N(k'_N)$ are the s.p. energies of the particles, respectively, in the initial and final state of the $\Sigma\Lambda$ conversion process $\Sigma N \rightarrow \Lambda N'$. In the case of the ΣN elastic scattering the energy conservation has a similar form (without, of course, the mass difference term). For the s.p. energies of the hyperons, we use the form

$$e_Y(k_Y) = \epsilon_Y(k_Y) + D_Y, \quad (5)$$

where ϵ_Y denotes the hyperon kinetic energy and D_Y denotes its s.p. potential. At the equilibrium density $\rho_0 = 0.166$ fm^{-3} of nuclear matter, for D_Y implied by the Nijmegen model F of the baryon-baryon interaction [6], the following values were obtained in Refs. [12] and [24]: $D^0_\Sigma = 23.5$ MeV, $D^0_\Lambda = -31.4$ MeV. At the density $\rho = 0.122$ fm^{-3}, we use the values $D_\Sigma = 17.25$ MeV and $D_\Lambda = -23.05$ MeV, obtained by linear interpolation of the values at ρ_0. For the nucleon s.p. energy $e_N(k'_N)$ for $k'_N > k_F$ (k_F is the Fermi momentum of nuclear matter), we use the form

$$e_N(k'_N) = \epsilon_N(k'_N) - \epsilon_N(k_F) + e_N(k_F) \quad (6)$$

and adjust $e_N(k_N < k_F)$ to $\rho_0 = 0.166$ fm^{-3} and to the energy -15.8 MeV per nucleon of nuclear matter at the equilibrium density ρ_0.

Our procedure of determining the absorptive potential of Σ in nuclear matter is described in detail in Appendix A of Ref. [2]. Our results obtained for W_c, W_e, and W_Σ for nuclear matter at the density $\rho = 0.122$ fm^{-3} are shown in Fig. 1. With increasing momentum k_Σ the $\Sigma\Lambda$ conversion cross section decreases, on the other hand the suppression of W_c by the exclusion principle weakens. As the net result W_c does not change very much with k_Σ. The same two mechanisms act in the case of W_e. Here, however, the action of the exclusion principle is much more pronounced: at $k_\Sigma = 0$ the suppression of W_e is complete. At higher momenta, where the Pauli blocking is not important, the total elastic cross section is much bigger than the conversion cross section, and we have $W_e \gg W_c$, and consequently $W_\Sigma \gg W_c$. Notice that the action of the absorptive potential W_Σ on the Σ wave function (decrease of this wave function) is similar to the action of a repulsive V_Σ.

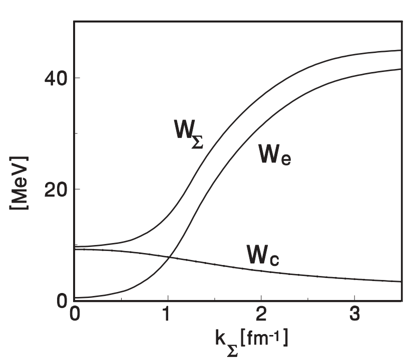

FIG. 1. The absorptive potential W_Σ and its components W_c and W_e in nuclear matter of density $\rho = 0.122$ fm^{-3} (in the case of $\nu = 1$).

We expect, therefore, to achieve with strong absorption the same final effect with a relatively weaker repulsion.

To describe the associated production reaction on the ^{28}Si target we consider the (π^-, K^+) reaction in which the pion π^- with momentum $\mathbf{k}_\pi$ hits a proton in the ^{28}Si target in the state ψ_P and emerges in the final state as kaon K^+ moving in the direction $\hat{k}_K$ with energy E_K, whereas the hit proton emerges in the final state as a Σ^- hyperon with momentum $\mathbf{k}_\Sigma$. We apply the simple impulse approximation described in Ref. [17], with K^+ and π^- plane waves, assume a zero range spin independent interaction for the elementary process $\pi^- P \rightarrow K^+\Sigma^-$ (with transition matrix t adjusted to the total cross section for this process measured in Ref. [25]), and obtain

$$\frac{d^3\sigma}{d\hat{k}_\Sigma d\hat{k}_K dE_K} = \frac{E_K E_\pi M_\Sigma c^2 k_K k_\Sigma}{(2\pi)^5(\hbar c)^6 k_\pi} \times \left| t \int d\mathbf{r} \exp(-i\mathbf{q}\mathbf{r}) \psi_{\Sigma,\mathbf{k}_\Sigma}(\mathbf{r})^{(-)*} \psi_P(\mathbf{r}) \right|^2, \quad (7)$$

where the momentum transfer $\mathbf{q} = \mathbf{k}_K - \mathbf{k}_\pi$ and $\psi_{\Sigma,\mathbf{k}_\Sigma}(\mathbf{r})^{(-)}$ is the Σ scattering wave function that is the solution of the s.p. Schrödinger equation with the s.p. potential

$$U_\Sigma(r) = (V_0 - iW_0)\theta(R - r), \quad (8)$$

where for W_0 we use the nuclear matter results for W_Σ presented in Fig. 1 and for V_0 the value $D_\Sigma = 17.25$ MeV. The Coulomb interaction of Σ^- may be taken into account by a proper change in V_0. For the proton s.p. potential $V_P(r)$, which determines ψ_P, we also assume the square well form with the radius R (and with a spin-orbit term). The parameters of $V_P(r)$ are adjusted to the proton separation energies (in particular $R = 3.7559$ fm). The Coulomb interaction of the target proton is not taken into account explicitly. Its average value inside the nucleus is 6 MeV, and we assume that it is included in the depth of $V_P(r)$.

In the inclusive KEK experiments [13–15] only the energy spectrum of kaons at fixed $\hat{k}_\Sigma$ was measured. To obtain this energy spectrum, we have to integrate the cross section (7) over $\hat{k}_\Sigma$.

We present our results for the inclusive cross section as a function of B_Σ, the separation (binding) energy of Σ from the hypernuclear system produced. B_Σ is related to Σ momentum k_Σ.[2] Thus the value of k_Σ that we need to calculate W_0 is determined by B_Σ.

So far, the real s.p. Σ potential $V_0 = D_\Sigma$ has been assumed to be independent of the Σ momentum k_Σ (whereas our imaginary Σ potential W_Σ depends strongly on k_Σ—see Fig. 1). In the case of nucleon s.p. potential in nuclear matter, with increasing nucleon momentum, the potential becomes less attractive. This is so because at high momenta the nucleon-nucleon interaction is dominated by the hard core repulsion. This dependence of the nucleon s.p. potential on the nucleon momentum may be described by replacing in Eq. (6) the nucleon mass M_N by the effective nucleon

[2]In the simplest case when the kaon hits the least bound proton in the silicon target (proton in the $d_{5/2}$ state), we have $-B_\Sigma = \hbar^2 k_\Sigma^2/2M_\Sigma$.

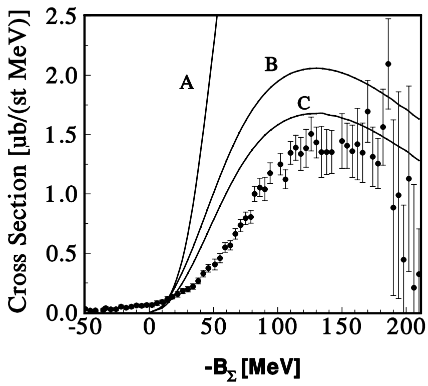

FIG. 2. Kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6^\circ$ at $p_\pi = 1.2$ GeV/c. See text for explanation.

mass $M_N^* = \nu_N M_N$, or equivalently the kinetic energies ϵ_N by ϵ_N/ν_N. In the Nijmegen model F the YN interactions have repulsive hard cores similar to the cores in the NN interaction, and we expect a similar momentum dependence of the s.p. potential of the Y hyperon as in the case of the nucleon potential, and we describe this dependence by replacing in Eq. (5) the hyperon mass M_Y by the effective mass $M_Y^* = \nu_Y M_Y$, or equivalently the kinetic energy ϵ_Y by ϵ_Y/ν_Y. In this way the Y s.p. potential $V_Y = e_Y(k_Y) - \epsilon_Y(k_Y) = D_Y(k_Y) + (1/\nu_Y - 1)\epsilon_Y(k_Y)$ acquires a momentum dependent component that for $\nu < 1$ is repulsive. Now in Eq. (8), we use for V_0 the value $D_\Sigma + (1/\nu_\Sigma - 1)\epsilon_\Sigma(k_\Sigma)$. We assume that $\nu_N = \nu_\Sigma = \nu_\Lambda = \nu = 0.7$. Whereas our value of ν_N is compatible with the empirical energy dependence of V_N [26], values of ν_Σ and ν_Λ are not well known (although our value of ν_Λ is supported by early estimates [10,27]).[3] Consequently, our result obtained with $\nu = 0.7$ represents only a rough estimate of the effect of the momentum dependence of the s.p. potentials V_Σ, V_Λ, and V_N on the observed K^+ spectrum.

Our results for the kaon spectrum from (π^-, K^+) reaction on ^{28}Si at $\theta_K = 6^\circ$ at $p_\pi = 1.2$ GeV/c together with the experimental results of Ref. [15] are shown in Fig. 2. In the case of curves A and B, the momentum dependence of the s.p. potentials was not considered, *i.e*, we assumed $\nu = 1$. In calculating curves A and B, we used for W_0 the nuclear matter results for W_c and $W_\Sigma = W_c + W_e$, respectively [see Eqs. (1)–(3)]. The momentum dependence of the s.p. potentials was considered in the case of curve C, with $\nu = 0.7$ (and with $W_0 = W_\Sigma$).

The data points in Fig. 2 at positive values of B_Σ correspond to reactions with emission of Λ hyperons (except for the final Σ bound state possible only for attractive V_Σ). In the present article, we do not consider processes with emission of Λ hyperons, and consequently, our calculated curves in Fig. 2 start at $-B_\Sigma = 0$.

The distortion of the pion and kaon waves, neglected in our simple plane wave impulse approximation, obviously diminishes the cross section for the associated Σ production. As was noticed already in Ref. [13], the distortion effect reduces the magnitude of the inclusive spectrum but does not affect the spectrum shape very much. Consequently, the distortion effect is expected to push curves B and C down into the range of the experimental data.

We conclude that we are able to reproduce the kaon spectrum from the (π^-, K^+) reaction observed at KEK, assuming for V_Σ the strength implied by the model F of the Nijmegen baron-baryon interaction, and for W_Σ the strength determined from the ΣN scattering cross section.

Comparing curves A and B, we see that to obtain this result it is essential to include into W_Σ not only the contribution W_c of the $\Sigma\Lambda$ conversion cross section but also the contribution W_e of the elastic ΣN scattering, which introduces a strong dependence of W_Σ on Σ momentum and reduces the magnitude of our calculated kaon spectrum A to that of the spectrum B. Similar is the effect of the momentum dependence of the real Σ potential V_Σ, illustrated by the comparison of curves B and C.

Notice that at low k_Σ we have $W_\Sigma \simeq W_c$ and $V_\Sigma \simeq D_\Sigma$, and these strengths of the absorptive and real Σ potential are compatible with these strengths determined in the analyses of Σ atoms and of the strangeness exchange reactions, where mainly low values of k_Σ are relevant.

[3] The result $\nu_\Sigma(k_\Sigma = 0) = 1.14$ was obtained in Ref. [10] with model D of the Nijmegen interaction, rejected in all analyses mentioned at the beginning of this report. Furthermore ν_Σ obtained in Ref. [10] is varying with k_Σ, and $\nu_\Sigma(k_\Sigma \simeq 1.6\ \text{fm}^{-1}) = 1$. An extrapolation to $k_\Sigma \gtrsim 1.6\ \text{fm}^{-1}$ suggests that $\nu_\Sigma(k_\Sigma \gtrsim 1.6\ \text{fm}^{-1}) < 1$. In our analysis of KEK data $0 < k_\Sigma < 3.7\ \text{fm}^{-1}$, and the range of small Σ momenta turns out to be relatively unimportant. Consequently the assumption $\nu_\Sigma = 0.7$ appears a reasonable approximation, consistent with the dominant role of the hard core in the ΣN interaction at high momenta.

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