New measurements of the astrophysical S factor for ^{12}C$(p, \gamma)^{13}$N reaction at low energies and the asymptotic normalization coefficient (nuclear vertex constant) for the $p + {}^{12}$C $\rightarrow {}^{13}$N reaction

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New measurements of differential and total cross sections for the ^{12}C$(p, \gamma)^{13}$N reaction have been made at beam energies of $E_p = 354$, 390, 460, 463, 565, 750, and 1061 keV. Analysis of the astrophysical S factor $S(E)$ for the ^{12}C$(p, \gamma)^{13}$N reaction at low energies and of the reaction rates has been carried out within the R-matrix approach by using the previously measured nuclear vertex constant (or the respective asymptotic normalization coefficient) for the virtual decay ^{13}N $\rightarrow p + {}^{12}$C to fix the direct capture part of the amplitude in $S(E)$. It is demonstrated that the R-matrix approach, using the measured asymptotic normalization coefficient, can be employed as an ideal tool, minimizing the uncertainties associated with a calculation of the direct capture cross section of the ^{12}C$(p, \gamma)^{13}$N reaction at extremely low energies. New information on the proton and γ width for the first excited state of ^{13}N is obtained.

I. INTRODUCTION

The ^{12}C$(p, \gamma)^{13}$N reaction is the first in the CNO cycle, where it plays an important role both for nuclear energy generation in massive stars [1–3] and as a source of low-energy solar neutrinos for the GALLEX experiment [4–7]. Knowledge of the astrophysical S factor $S(E)$ [or cross section $\sigma(E)$] for the ^{12}C$(p, \gamma)^{13}$N reaction at astrophysically relevant energies is thus of interest in nuclear astrophysics, but uncertainties in this quantity are difficult to determine [8–11].

The ^{12}C$(p, \gamma)^{13}$N reaction proceeds by γ emission to the ^{13}N bound state via two strong resonant ($E^* = 2.365$ MeV; $1/2^+$ and $E^* = 3.502$ MeV; $3/2^-$) states and direct capture. Both of the excited resonance states decay partially by γ emission to the ground state of ^{13}N. Early experimental data for the ^{12}C$(p, \gamma)^{13}$N cross sections at rather low energies have been obtained in Refs. [12–19]. In Refs. [14,19], absolute values of the ^{12}C$(p, \gamma)^{13}$N cross section at the first resonance peak ($E = E_1^{(r)} = 421$ keV) have been given as $\sigma^{\exp}(E_1^{(r)}) = 127$ and 92 ± 10 μb, respectively. Further experiments observing the capture γ rays over a wide range of beam energies ($E_p = 150$–2500 keV) have been performed by the authors of Ref. [20] and they gave $\sigma^{\exp}(E_1^{(r)}) = 125 \pm 15$ μb, which has been deduced from the excitation functions measured at $\theta_{\gamma, \text{lab}} = 0°$ and $90°$. Therefore, in Ref. [9], by taking into account the scatter among the values of $\sigma^{\exp}(E_1^{(r)})$ measured in Refs. [19,20], the averaged value of 102 ± 8 μb deduced from the aforementioned data was used for a renormalization of the experimental data of Ref. [17]. Then the renormalized astrophysical S factor for the the ^{12}C$(p, \gamma)^{13}$N reaction for energies $E \leqslant 800$ keV was obtained within the R-matrix method. As a result, the value of Γ_1^γ changing within the limit of 0.45–0.50 eV was recommended by the authors of Refs. [9,19,21] for the resonance parameter for the γ width for the first excited state of ^{13}N, which differs noticeably from that of 0.67 eV (see,

e.g., Ref. [22] and references there). Therefore, to test both the accuracy of the experimental cross sections measured in Refs. [14,19,20] and the reliability of the aforementioned value for Γ_1^γ, new measurements of the ^{12}C$(p, \gamma)^{13}$N excitation functions performed at more than two detected angles $\theta_{\gamma, \text{lab}}$ are highly desirable.

Also, in Refs. [9,20] it was found that a direct capture (nonresonance) contribution to the ^{12}C$(p, \gamma)^{13}$N reaction had to be incorporated in the analysis. However, an ambiguity in the contribution of a direct capture component associated with both reliable values of the spectroscopic factor [20] and the proton reduced width [9] occurs there. It should be noted that at the lowest proton energies, direct capture is important but its contributions to $S(E)$ have a notable uncertainty, including a sensitivity to the theoretical methods used (see Refs. [9–11] and [20]). For example, in Ref. [20] the calculation of the direct radiative capture part performed within the two-body potential method shows that the empirical value of the spectroscopic factor for the ($p + {}^{12}$C) configuration in ^{13}N, $Z_{p^{12}\text{C};l_b}$ (where l_b is the angular orbital momentum of the proton in ^{13}N), obtained from the analysis of the experimental ^{12}C$(p, \gamma)^{13}$N S factors, depends noticeably on the choice of values of geometrical parameters (the radius r_0 and diffuseness a) of the adopted Woods-Saxon potential. One notes that an analogous problem occurs also for the empirical value of $Z_{p^{12}\text{C};l_b}$ obtained from the analysis of the ^{12}C$(d, n)^{13}$N and ^{12}C$(^3\text{He}, d)^{13}$N reactions [23–25]. Shell-model calculations of $Z_{p^{12}\text{C};l_b}$ performed by Cohen and Kurath [26] and by Vorma and Goldhammer [27] also gave different results. In Refs. [9] and [10] calculations of the $S(E)$ have been carried out within the framework of the R-matrix approach and the semimicroscopic two-cluster model, respectively. An important dependence of the obtained values of the parameters of the internal (resonance) amplitude, the reduced proton width, and the spectroscopic factor $Z_{p^{12}\text{C};l_b}$ of the external (direct capture) amplitude on the value of the channel radius r_c has been observed [9]. Therefore

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one may expect strong uncertainties in the choice of the
fitting parameter values obtained in Ref. [9]. This in its turn
makes calculations of $S(E)$ at extremely low energies model
dependent. One notes also the importance of a microscopic
calculation of the cross section $\sigma(E)$ performed in Ref. [10],
with inclusion of the antisymmetrization between the incident
proton and the ^{12}C nucleons. In Ref. [11] the astrophysical
S factor of the ^{12}C$(p, \gamma)^{13}$N reaction has been calculated
within the cluster generator coordinate method (CGCM) in
which the ^{12}C wave function is formed as three α particles in
a regular triangle of size R_c, with R_c being a free parameter of
the method. The calculation carried out in Ref. [11] has shown
that ^{13}N spectroscopic properties are sensitive to the ^{12}C wave
function, in particular to the value of R_c. Also, the calculated
value of $S(0)$ in Ref. [11] is rather sensitive to the form of the
adopted NN potential.

However, the peripheral nature of the direct radiative
capture of the ^{12}C$(p, \gamma)^{13}$N reaction at rather low energies
has been demonstrated [20] because of the strong Coulomb
repulsion of the colliding particles and the low binding energy
ϵ_p of ^{13}N in the $(p + {}^{12}$C$)$ channel $(\epsilon_p = 1.9435$ MeV).
Therefore, to calculate the direct capture part of the cross
section in the correct way, the radial part of the overlap function
for the bound state wave functions for ^{12}C and ^{13}N in the
direct capture amplitude can be approximated by a well-known
asymptotic expression [28] in which the asymptotic normal-
ization coefficient (ANC) $C_{p^{12}\text{C};l_b}$ determines the amplitude of
the tail of the ^{13}N bound state wave function in the $(^{12}$C$ + p)$
channel. Consequently, the overall normalization of the direct
capture part of the cross section (or astrophysical S factor)
of the ^{12}C$(p, \gamma)^{13}$N reaction should be expressed in terms of
the factor $C^2_{p^{12}\text{C};l_b} = Z_{p^{12}\text{C};l_b} b^2_{p^{12}\text{C};l_b}$ [28], where $b_{p^{12}\text{C};l_b}$ is the
single-particle (proton) ANC, which determines the amplitude
of the tail of the single-particle radial wave function of
the bound ^{13}N $= (^{12}$C$ + p)$ state. Thus the direct radiative
capture amplitude of the ^{12}C$(p, \gamma)^{13}$N reaction at rather
low energies is determined solely by the ANC $C_{p^{12}\text{C};l_b}$ (or
equivalently the nuclear vertex constant (NVC) $G_{p^{12}\text{C};l_b}$ for the
virtual decay ^{13}N $\rightarrow p + {}^{12}$C [28]) [29–32]. This important
circumstance as not used in Refs. [9,20].

An "indirectly measured" value of the ANC $C_{p^{12}\text{C};l_b}$ has
been obtained in Refs. [25,33,34] by the analysis of the
experimental differential cross sections for the peripheral
proton transfer ^{12}C$(^3$He$, d)^{13}$N reaction at two beam energies,
where the ambiguity arising from both the model dependence
of the extracted value of the ANC (or NVC) on the geometric
parameters of the adopted Woods-Saxon potential used for the
calculation of the bound $(^{12}$C$ + p)$ state and on the choice
of the optical-model parameters in the initial and final states
is reduced to a minimum, within the stated uncertainties for
the experimental data. One notes that the indirectly measured
value of the ANC (or NVC) obtained in Refs. [33,34] differs
noticeably from that which can be obtained from the result
of Ref. [9] (see Sec. IV). Therefore, it is of interest also to
perform an accurate extrapolation of the ^{12}C$(p, \gamma)^{13}$N cross
section $\sigma(E)$ [or the astrophysical S factor $S(E)$] at stellar
energies ($\approx$25–50 keV) by taking into account information
about the value of the ANC obtained by the proton-stripping
experiments [25,33,34].

In this study, the results of new measurements of the
^{12}C$(p, \gamma)^{13}$N cross sections near the resonant energy of
421 keV with rather high precision ($\approx$10%) and reanalysis of
the experimental astrophysical S factor of the ^{12}C$(p, \gamma)^{13}$N
reaction obtained by us and previously in Ref. [20] are
presented. The analysis is performed within the framework
of the one-channel R-matrix approach by taking into account
independent information about the ANC (or NVC) previously
obtained in Refs. [33,34]. One notes that introduction of
additional information about the ANC (or NVC) into the
R-matrix method leads to a minimum in the uncertainties
arising in a calculation of the direct capture (external) part of
the amplitude for the ^{12}C$(p, \gamma)^{13}$N reaction since hard-sphere
scattering phase shifts are used in taking into account the
p^{12}C-scattering in the initial state.

The contents of the paper are as follows. In Sec. II details
and results of the new experiment are presented. In Sec. III
a brief description of the calculation of the astrophysical S
factor $S(E)$ within the framework of the R-matrix method is
given. In Secs. IV and V the results of an analysis of $S(E)$
for the ^{12}C$(p, \gamma)^{13}$N reaction, using the previously obtained
ANC value as input, and the calculation of the reaction rates,
respectively, are given. The conclusion is given in Sec. VI.

II. THE EXPERIMENT

In this section the result of the measurement of differential
cross sections for the reaction ^{12}C$(p, \gamma)^{13}$N is presented for
the most important (with respect to astrophysics) transition to
the ground state in ^{13}N at angles of $0°$, $45°$, $90°$, and $135°$ with
respect to the direction of the incident proton beam.

The electrostatic tandem accelerator UKP-2-1 at the In-
stitute of Nuclear Physics in Kazakhstan was used for our
studies over the energy range of $E_{p,\text{lab}} = 356–1065$ keV
with proton beam currents of 5–15 μA. We measured γ-ray
yields starting at the highest beam energy, with other lower
energies in sequence. The characteristics of the accelerator
have been described in Ref. [35]. The energy calibration of the
accelerator was tested during the course of the experiment by
using the well-known resonances [36,37] in ^{27}Al$(p, \gamma)^{28}$Si at
$E_{p,\text{lab}} = 632, 773, 992$, and 1089 keV and in ^{19}F$(p, \alpha\gamma)^{16}$O
at $E_{p,\text{lab}} = 340$ keV and is known to a precision of ± 1 keV.

The proton energy spread of the beam was found to be below
1.2 keV. A high-resolution HpGe detector (ORTEC GEM20P
of volume 111 cm^3) was used to observe the reaction γ rays.
The resolution of the detector was typically 0.9, 1.8, and
2.5 keV at $E_\gamma = 122, 1408$, and 2614 keV. The absolute
detector γ-ray efficiency for $E_\gamma = 661–3253.6$ keV was
determined by using calibrated ^{137}Cs, ^{60}Co, and ^{56}Co sources,
with intensities known to better than 1%. Uncertainties in the
source position, dead time, and counting statistics lead to an
overall uncertainty of 6% for the detector photopeak efficiency.

A schematic diagram of the experimental setup for our
γ-ray detection is shown in Fig. 1. The proton beam passed
through two beam-defining apertures, collimator 1 (14 mm
in diameter) and collimator 2 (10 mm in diameter) 60 cm
downstream focused onto the target (80 cm distance from
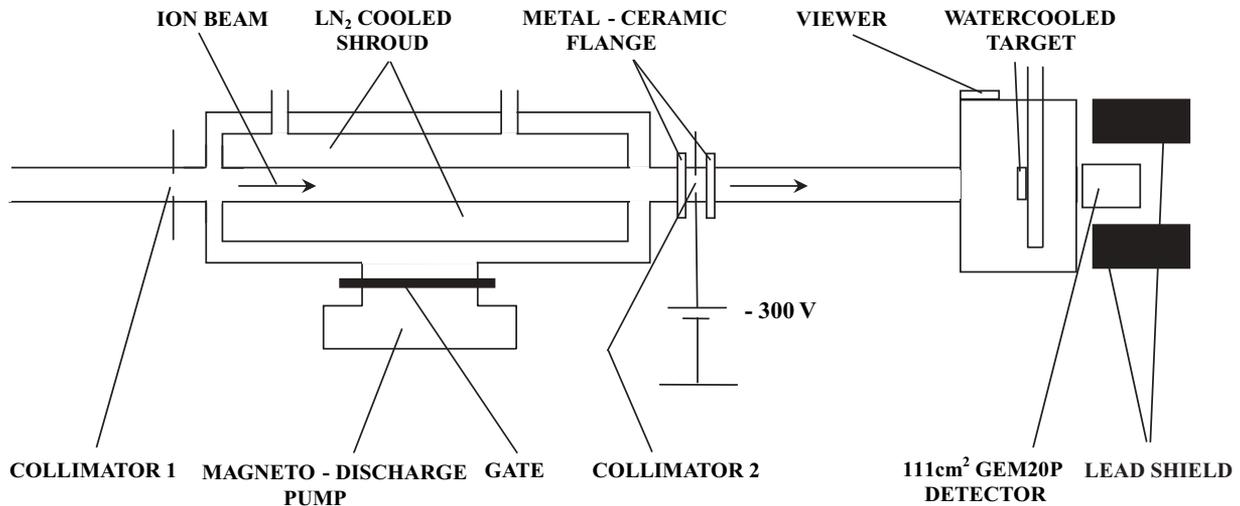collimator 2) into a spot of about 5 mm in diameter. A

FIG. 1. Schematic diagram of the experimental setup used in the ^{12}C$(p, \gamma)^{13}$N reaction study.

liquid-nitrogen (LN$_2$) cooled in-line stainless steel shroud (50 cm in length) and a magneto-discharge pump were installed between the apertures. With this pump and shroud and with metal-sealed UHV components (of annealed copper and indium) in the vacuum system, carbon buildup on the targets was found to be negligible. To minimize γ-ray absorption and Compton scattering in the setup, the target chamber (a vertical cylinder of 8 cm in diameter and 10 cm in height) was made of stainless steel with a wall thickness of 1.5 mm. The beam line was hermetically sealed to the water-cooled target holder and the viewport; a quartz glass for obtaining a luminous image of the beam shape in front of the target was mounted at the upper face of the target chamber. By an external handle, the quartz glass could be placed in the course of the beam in front of the target for alignment. The cylindrical form of the target chamber allowed measurement of γ-ray angular distributions at angles from 0° to 135°. A voltage of -300 V was applied to collimator 2 between the metal-ceramic flanges to suppress electrons ejected from the target. The long beam pipe together with the target chamber was isolated from ground and other parts of the setup so that the beam current could be integrated, to within an uncertainty of 3%.

The targets were produced by evaporation of natural carbon onto 1.5-mm-thick Cu backings. The developed technology of the target production consisted of the following: A Ta layer of about 20 mg/cm^2 was evaporated onto one of two equal backings and a fine Al layer of about 20 μg/cm^2 was evaporated onto the other backing. Since Cu contains low-mass impurities, the purpose of the Ta was to reduce the possibility of the production of background γ rays by reducing the energy of the protons before they reached the Cu. The backings with the Ta layers were used in all measurements of absolute cross sections. The other backing with the Al layer was necessary for determination of the evaporated carbon thickness (see the following). Then thin layers of natural carbon were simultaneously (during a single exposure) evaporated onto the surfaces of both backings, so that the thicknesses of the layers at both backings were equal. After measurement of the target yield curves over the ^{27}Al$(p, \gamma)^{28}$Si resonance at $E_{p, \text{lab}} =$

992 keV, which has a total width of less than of 0.10 keV, for an aluminum foil and the target with the Al layer was made, target thicknesses were taken from the shift of the resonance energy obtained by comparing these yield curves. Then stopping power tables [38] were used to obtain the atomic density of the carbon layer. This method allowed us to determine thicknesses of layers in the range of this experiment with an uncertainty about 5%. The difference in the thicknesses of carbon layers obtained by simultaneous evaporation onto two backings was found to be less than 5% from experimental determination during the development of their production technology. For this purpose, an aluminum foil with dimensions exceeding the total dimension of the two evaporated targets was used as the backing. The foil was covered by a thin carbon layer (under the same conditions as in the target production) and then was cut by two equal parts. After that, the carbon layer thickness of both parts was determined. In the present work six targets with thicknesses of 9.0 ± 0.6 and 11.0 ± 0.7 μg/cm^2 (for measurements at $E_p = 354$, 390, 460, and 463 keV), 13.5 ± 0.4 and 23.5 ± 1.2 μg/cm^2 (for $E_p = 565$ and 750 keV), and 40 ± 2 and 48.0 ± 2.4 μg/cm^2 (for $E_p = 1061$ keV) have been used. At $E_p = 1061$, 750, and 565 keV energy losses of protons in the targets, $\Delta(E_p)$, were no more than 10.5, 6.5, and 7.8 keV, respectively. In other cases energy losses did not exceed 5 keV. By using different targets at the same energies E_p it was found that the effective beam energies within all targets at the corresponding energies $E_{p, \text{lab}}$ were well described by $E_{\text{eff}} \equiv E_p = E_{p, \text{lab}} - 0.5\Delta(E_p)$ [3] and so no procedure concerning target-thickness correction has been made. The targets were able to withstand beam currents of $\leqslant 15$ μA for periods greater than several days without noticeable deterioration (within the statistical uncertainty of the measurements, which varied from 3% to 5.8%). No change in γ-ray yield during the exposures was noted. For the angle of 135° the HpGe detector was about 5 cm from the beam spot on the target whereas at angles of 0°, 45°, and 90° the distance was about 4 cm. In all measurements of γ spectra the HpGe detector was surrounded by a 5-cm-thick lead shield to reduce the room background. The accumulated charges on the target
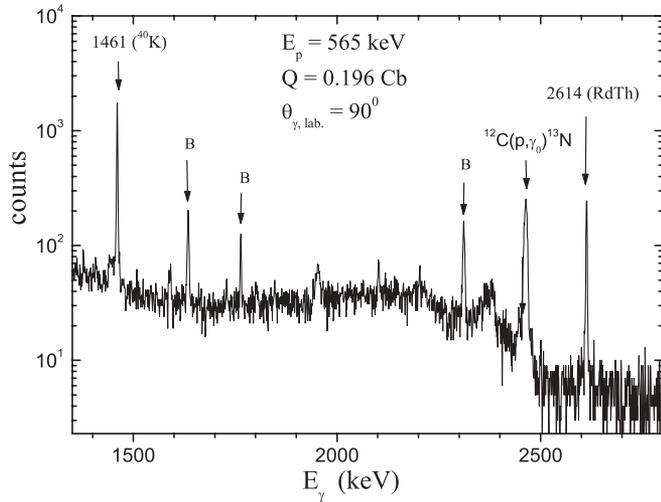
FIG. 2. γ-ray spectrum of the $^{12}C(p, \gamma)^{13}N$ reaction for the radiative capture transition to the ground state in ^{13}N as obtained at the effective proton energy of 565 keV ($\theta_{\gamma,\text{lab}} = 90°$) by the GEM20P detector of volume 111 cm^3, located 4 cm from the reaction region. The "B" denotes a background line.

were from $Q = 3$ mC at $E_p = 460$ keV to $Q = 300$ mC at $E_p = 1061$ keV. Dead-time effects were kept below 1.5% at all beam energies.

Figure 2 shows the γ-ray spectrum obtained at $E_p = 565$ keV and $\theta_{\gamma,\text{lab}} = 90°$. The room background lines at 1461 keV (^{40}K) and at 2614 keV (RdTh) are observable. The well-known energies of these γ-ray lines allowed the energy calibration of each spectrum to be independently determined. The primary transition to the ground state in ^{13}N is the dominant line in the spectrum. The differential cross sections for the transition to the ground state shown in Fig. 3 were determined at effective beam energies of $E_p = 354, 390, 750, 1061, 460, 463$, and 565 keV, by using the relation

$$N_\gamma = N_p N_C \varepsilon_\gamma d\sigma/d\Omega,$$

where N_p is the number of incident protons (3% uncertainty), N_C is the areal density of ^{12}C atoms, ε_γ is the absolute detector γ-ray efficiency (6% uncertainty), and N_γ is the number of counts observed for the capture transition (3%–5.8% uncertainty). For the first four energies, the measurements were performed at angles of $0°, 45°, 90°, 135°$ and for the latter energies at angles of $0°, 90°,$ and $135°$. At angles of $45°$ and $135°$ data were obtained for the first time. The overall uncertainty in the differential cross sections reported here is 10.2%.

The angular distributions of the $^{12}C(p, \gamma)^{13}N$ reaction were fitted at seven fixed energies from the energy region of $E = 326.8$–979.4 keV by polynomials

$$\frac{d\sigma}{d\Omega} = a_0(E)\left[1 + \sum_k a_k(E)Q_k P_k(\cos\theta)\right], \quad (1)$$

where a_k are expansion coefficients and Q_k are the attenuation coefficients (independently obtained by calculation and experimental measurement for the detector: $Q_1 = 0.95$ and $Q_2 = $
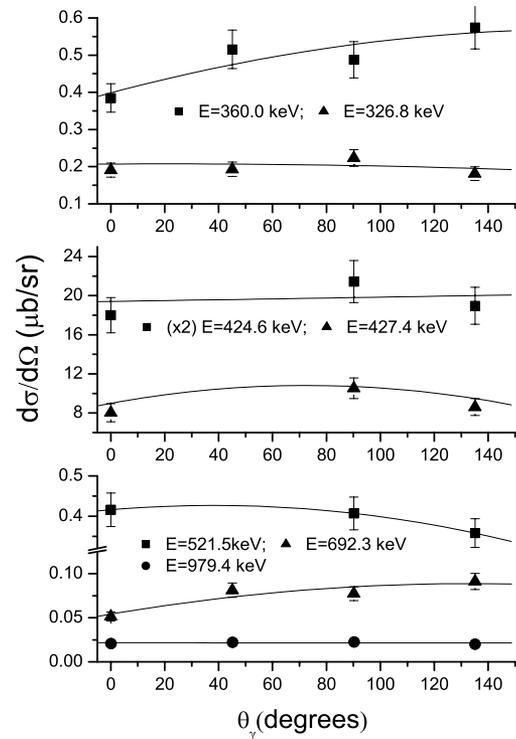

FIG. 3. The differential cross sections of the $^{12}C(p, \gamma)^{13}N$ reaction for the γ-ray transition to the ground state in ^{13}N, in the form of the γ-ray angular distributions. The curves are the results of polynomial fits.

0.83). In view of the limited number of angles, the fits were carried out by including only $k = 1$ and 2 [39]. Fitted angular distributions and the deduced $a_0, a_1,$ and a_2 coefficients are shown in Fig. 3 (solid curves) and Table I, respectively. It is seen from Fig. 3 that the angular distributions measured in this work are isotropic within uncertainties, as expected for an S-wave resonance reaction. The experimental total cross sections, defined by the equation $\sigma^{\text{exp}}(E) = 4\pi a_0(E)$, are presented in the fourth column of Table I, and the corresponding experimental astrophysical S factor values, $S^{\text{exp}}(E)$, are presented in the fifth column of Table I and Fig. 4(a). The presented uncertainties are the standard deviations of the mean resulting from the fits to the angular distribution data, and thus they include implicitly all the data uncertainties discussed so far, as well as the geometrical and normalization uncertainties associated with measurement of the angular distributions. This observation is in good agreement with previous results obtained in Ref. [20] at $E \leqslant 980$ keV.

III. R-MATRIX APPROACH TO THE $^{12}C(p, \gamma)^{13}N$ REACTION

To calculate the astrophysical S factor for the radiative capture $^{12}C(p, \gamma)^{13}N$ reaction, here we use the R-matrix approach developed in Refs. [40–42]. The astrophysical S factor $S(E)$ is defined by the relation

$$S(E) = Ee^{2\pi\eta}\sigma(E), \quad (2)$$

TABLE I. Coefficients of Legendre polynomial (a_1 and a_2) fitting, experimental total cross sections [$\sigma^{\exp}(E) = 4\pi a_0(E)$], and experimental astrophysical S factors [$S^{\exp}(E)$].

E (keV)	$a_1(E)$	$a_2(E)$	$\sigma^{\exp}(E)$ (μb)	$S^{\exp}(E)$ (MeV b)
326.8	0.043	-0.019	2.5 ± 0.3	$(0.18 \pm 0.02) \times 10^{-1}$
360.0	-0.157	-0.041	6.5 ± 0.8	$(0.31 \pm 0.03) \times 10^{-1}$
424.6	-0.027	-0.012	$(1.24 \pm 0.12) \times 10^2$	0.33 ± 0.03
427.4	-0.040	-0.032	$(1.16 \pm 0.12) \times 10^2$	0.31 ± 0.03
521.5	0.118	-0.072	5.0 ± 0.5	$(0.7 \pm 0.07) \times 10^{-2}$
692.3	-0.188	-0.113	1.0 ± 0.1	$(0.67 \pm 0.07) \times 10^{-3}$
979.4	0.065	-0.001	0.27 ± 0.03	$(0.85 \pm 0.08) \times 10^{-4}$

where $\sigma(E)$ is the radiative capture total cross section, E is the relative kinetic energy of the $p + {}^{12}$C system, and η is the Coulomb parameter corresponding to $p{}^{12}$C scattering. We use the system of units in which $\hbar = c = 1$. The cross section $\sigma(E)$ for the ^{12}C$(p, \gamma){}^{13}$N reaction to the ground state of ^{13}N is given by [42]

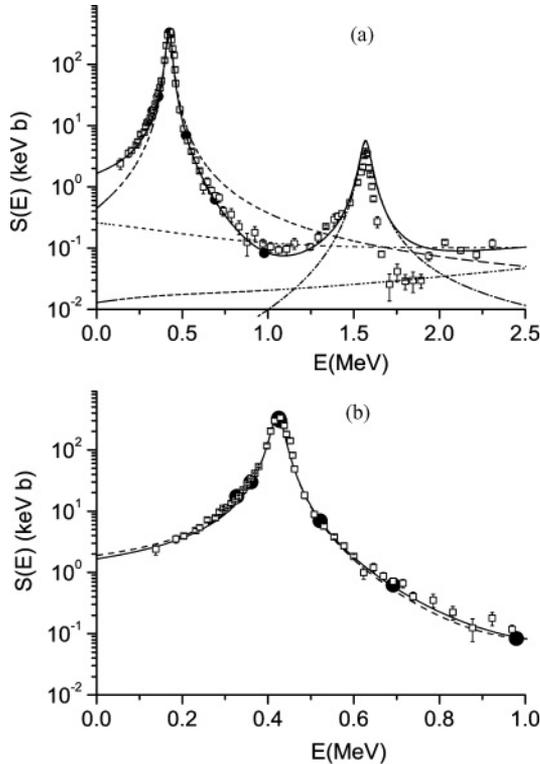$$\sigma(E) = \frac{\pi}{2k^2} \sum_{JlI} (2J + 1) |M_{JlI}(E)|^2, \qquad (3)$$

FIG. 4. The astrophysical S factor for the ^{12}C$(p, \gamma){}^{13}$N reaction. The experimental data are from Ref. [47]($\square$) and the present work ($\bullet$). (a) The solid line is our fit, the dotted line is our calculated contribution for the direct radiative capture, the dashed (dashed-dotted) line shows our calculated contribution for the first (second) resonance and the dashed-dotted-dotted line presents our calculated contribution for the third ($E^* = 10.250$ MeV, $J^\pi = 1/2^+$) γ-resonance tail. (b) The solid line is our fit; the dashed line is our calculation performed with the same values for the parameters, except the ANC, which is taken equal to $C_1 = 1.72$ fm$^{-1/2}$ [56].

where k is the relative momentum of the colliding particles and $J(l)$ is the total (relative orbital) angular momentum of the colliding particles. The R-matrix expression for the amplitude $M_{JlI}(E)$ can be derived from Refs. [40–42] by taking into account the contributions from the two resonance states of ^{13}N [$E^* = 2.365$ MeV; $J^\pi = 1/2^+(E1$ transition) and $E^* = 3.502$ MeV; $J^\pi = 3/2^-(M1$ transition)] with amplitudes $M_{JlI}^{(R\lambda)}(E)(\lambda = 1$ and 2 for the first and second resonances of ^{13}N, respectively) and direct $E1$ and $M1$ captures with amplitudes $M_{JlI}^{(D, E1)}(E)$ and $M_{JlI}^{(D, M1)}(E)$, respectively. In the single-level R-matrix approximation the resonance amplitude $M_{JlI}^{(R\lambda)}(E)$, which corresponds to the $p + {}^{12}$C $\rightarrow {}^{13}$N$^* \rightarrow \gamma + {}^{13}$N mechanism with spin J of the resonance level λ, is given by

$$M_{JlI}^{(R\lambda)}(E) = -i e^{i\delta_l} \frac{\left[\Gamma_{\lambda;lJ}^p(E)\right]^{1/2}\left[\Gamma_{\lambda;IJ}^\gamma(E)\right]^{1/2}}{E_\lambda^{(r)} - i \frac{\Gamma_{\lambda;J}}{2} - E}. \qquad (4)$$

Here δ_l is the sum of the Coulomb and hard-sphere phase shifts for $p{}^{12}$C scattering, $E_\lambda^{(r)}$ is the resonance energy of level λ, $\Gamma_{\lambda;J}$ is the total width of the resonant state of the nucleus ^{13}N*, $\Gamma_{\lambda;J} \approx \sum_l \Gamma_{\lambda;lJ}^p(E)$ and $\Gamma_{\lambda;lJ}^p(E)$, and $\Gamma_{\lambda;IJ}^\gamma(E)$ for the radiative transition of Ith order are given by

$$\Gamma_{\lambda;lJ}^p(E) = \frac{2P_l(E)\left(\gamma_{\lambda;lJ}^p\right)^2}{1 + \left(\gamma_{\lambda;lJ}^p\right)^2 \left(\frac{dS_{\lambda;l}(E)}{dE}\right)_{E=E_\lambda^{(r)}}}, \qquad (5)$$

$$\Gamma_{\lambda;IJ}^\gamma(E) = \frac{2k_\gamma^{2I+1}\left(\gamma_{\lambda;IJ}^\gamma\right)^2}{1 + \left(\gamma_{\lambda;lJ}^p\right)^2 \left(\frac{dS_{\lambda;l}(E)}{dE}\right)_{E=E_\lambda^{(r)}}}. \qquad (6)$$

Here $\gamma_{\lambda;lJ}^p$ and $\gamma_{\lambda;IJ}^\gamma$ are the reduced proton and γ widths, respectively, $S_{\lambda;l}(E)$ is the energy part of the Thomas shift $\Delta_{\lambda;l}(E)$ [$\Delta_{\lambda;l}(E) = -\gamma_{\lambda;lJ}^p S_{\lambda;l}(E)$] [40], $P_l(E)$ is the penetrability, and k_γ is the γ-ray momentum. One notes that in Eqs. (4)–(6) the linear approximation for the Thomas shift $\Delta_{\lambda;l}(E)$ in the vicinity of $E = E_\lambda^{(r)}$ and the boundary condition $B_{\lambda;l} = S_{\lambda;l}(E_\lambda^{(r)})$ for the constant $B_{\lambda;l}$ are used [40,42]. The reduced-width amplitude $\gamma_{\lambda;IJ}^\gamma$ is given by the sum of the internal and external (or channel) reduced width amplitudes [41,42]

$$\gamma_{\lambda;IJ}^\gamma = \gamma_{\lambda;IJ}^\gamma(\text{int}) + \gamma_{\lambda;IJ}^\gamma(\text{ch}). \qquad (7)$$

One notes also that the internal (channel) reduced width is a real (complex) number and the channel reduced width

$\gamma^{\gamma}_{\lambda;IJ}$(ch) contains the ANC $C_{p^{12}C;l_b}$ for ^{13}N in the ($p + ^{12}$C) configuration and $\gamma^{p}_{\lambda;lJ}$ [43] as parameters.

Experimental proton and γ widths are

$$\Gamma_p^{exp} = \left|\Gamma_{\lambda;lJ}^{p}\left(E_\lambda^{(r)}\right)\right|, \quad \Gamma_\gamma^{exp} = \left|\Gamma_{\lambda;IJ}^{\gamma}\left(E_\lambda^{(r)}\right)\right|. \quad (8)$$

In the long-wavelength approximation the direct capture part of the amplitude corresponding to EI and $M1$ captures is given by [41–44]

$$M_{JlI}^{(D,EI)}(E) = i^{l+I-l_b}\sqrt{\frac{2\mu}{k}}k_\gamma^{I+1/2}e\left(\frac{\mu}{m_p}\right)^I\left[1+(-1)^I\frac{6}{12^I}\right]$$

$$\times\sqrt{\frac{(2I+1)(I+1)}{I}}\frac{1}{(2I+1)!!}$$

$$\times\sqrt{2(2l+1)}C_{l0I0}^{l_b0}W\left(Il_bJ\frac{1}{2};l\frac{1}{2}\right)I_{\alpha;ll_bI}, \quad (9)$$

$$M_{Jl1}^{(D,M1)}(E) = i^{l+1-l_b}\sqrt{\frac{\mu}{3k}}k_\gamma^{3/2}\frac{e}{m_p}$$

$$\times\left[\frac{25}{13}\sqrt{(2l_b+1)(2J+1)l(l+1)}\right.$$

$$\times W\left(1lJ\frac{1}{2};l\frac{1}{2}\right) + 2\mu_p(-1)^{\frac{1}{2}-J}\sqrt{6}\hat{J}$$

$$\left.\times W\left(1\frac{1}{2}Jl;\frac{1}{2}\frac{1}{2}\right)\right]\delta_{ll_b}I_{0;ll_bI}, \quad (10)$$

$$I_{\alpha;ll_bI} = C_{p^{12}C;l_b}\int_{r_c}^{\infty}drr^{I-\alpha}W_{-\eta_b,l_b+1/2}(2\kappa r)$$

$$\times[I_l(r) - e^{2i\delta_l}O_l(r)]. \quad (11)$$

Here μ is the reduced mass of the p and ^{12}C, m_j is the mass of particle j, μ_p is the magnetic moment of the proton in nuclear magnetons, $\kappa = \sqrt{2\mu\epsilon_p}$, $I_l(r)$ [$O_l(r)$] is the incoming (outgoing) solution of the radial Schrödinger equation for p^{12}C scattering, $W_{-\eta_b,l_b+1/2}(2\kappa r)$ is the Whittaker function determining the behavior of the radial overlap function $I_{p^{12}C;l_b}(r)$ at $r \geqslant r_c$ for the ^{13}N nucleus in the ($p + ^{12}$C) channel, η_b is the Coulomb parameter for the ($p + ^{12}$C) bound state, $C_{j_1m_1j_2m_2}^{jm}[W(abcd;ef)]$ is the Clebsh-Gordon (Racah) coefficient, and $\alpha = 0(1)$ for an $EI(M1)$ transition. One notes that the ANC $C_{p^{12}C;l_b}$ is related to the NVC $G_{p^{12}C;l_b}$ for the virtual decay ^{13}N $\rightarrow p + ^{12}$C as [28]

$$G_{p^{12}C;l_b} = -i^{l_b+\eta_b}\frac{\sqrt{\pi}}{\mu}C_{p^{12}C;l_b}, \quad (12)$$

where the antisymmetrization factor arising from the nucleon identity has been absorbed into the ANC $C_{p^{12}C;l_b}$. The ANC $C_{p^{12}C;l_b}$ is related to the parameters of the R-matrix method—the reduced width $\gamma^{(p)}_{f;l_b1/2}$, the spectroscopic factor $(Z_{p^{12}C;l_b})$ for ^{13}N in the ($p + ^{12}$C) configuration, and the channel radius r_c—as [31,42,43,45]

$$C_{p^{12}C;l_b} = \sqrt{\frac{2}{r_c}}\left(Z_{f;l_b1/2}\theta^{(p)}_{f;l_b1/2}\right)^{1/2}N_f^{1/2}$$

$$\times[W_{-\eta_b;l_b+1/2}(2\kappa r_c)]^{-1}, \quad (13)$$

where $\theta^{(p)}_{f;l_b1/2}$ and the factor N_f are determined by Eqs. (15) and (23) of Ref. [42], respectively.

Thus as seen from Eqs. (9)–(11), use of the indirectly measured value of the ANC for the ground state of ^{13}N for the ($p + ^{12}$C) channel allows us to fix the absolute normalization of the nonresonance (direct capture) amplitude and the channel radiative width.

One should outline the main differences of the R-matrix approach adopted here from the calculation methods used in Refs. [9,20]. First, in Ref. [9] the direct radiative capture amplitude found by taking into account a contribution from the external region ($r \geqslant r_c$) involves both the resonance phase shifts $\delta_l^{(res)}$ (the channel contribution) and the nonresonance (Coulomb and nuclear) phase shifts. Indeed, according to Ref. [41], the channel contribution ($r \geqslant r_c$) connected with the resonance phase shifts $\delta_l^{(res)}$, which is given by the terms of $\gamma^{\gamma}_{\lambda;IJ}$(ch) in Eqs. (6) and (8) and is separated from the direct capture (external) part of the amplitude $M_{JlI}(E)$, must be included in the resonance part of the amplitude $M_{JlI}(E)$ [41,42]. Also in Ref. [9], the channel radiative amplitude and nonresonant (direct capture) amplitude are expressed in terms of the reduced-width amplitude, which is not directly observable and depends strongly on the channel radius r_c.

Second, in Ref. [20] the direct capture amplitude containing both the resonance phase shifts and the nonresonance phase shifts calculated for the Woods-Saxon (or square) potential at fixed geometrical parameters is factorized in terms of the spectroscopic factor $Z_{p^{12}C;l_b}$. Here the direct capture amplitudes $M_{JlI}^{(D,EI)}(E)$ and $M_{JlI}^{(D,M1)}(E)$ given by Eqs. (9)–(11) are determined by means of values of the ANC $C_{p^{12}C;l_b}$ and the nonresonance hard-sphere phase shifts for p^{12}C scattering. Consequently, taking into account the fact that a value of the ANC $C_{p^{12}C;l_b}$ was determined in a correct manner in Refs. [25,33] and that p^{12}C scattering is given by the hard-sphere phase shifts, the uncertainty related to an accurate calculation from the external contribution to the ^{12}C(p, γ)^{13}N reaction at rather low energies does not occur here.

It should be noted that the parametrization of the direct capture part of the amplitude $M_{JlI}(E)$ in terms of the ANC in the R-matrix method were used by the authors of Refs. [31] and [46] for the analysis of the ^{9}Be(p, γ)^{10}B and ^{13}C(p, γ)^{14}N reactions, respectively, at extremely low energies, but the channel contribution, given by $\gamma^{\gamma}_{\lambda;IJ}$(ch), was not included.

IV. ANALYSIS OF THE EXPERIMENTAL DATA OF THE ^{12}C(p, γ)^{13}N REACTION

We have reanalyzed the astrophysical S factor $S(E)$ for the ^{12}C(p, γ)^{13}N reaction at energies $E \leqslant 2.5$ MeV by taking into account the contributions of the two resonances with energies $E_1^{(r)} = 421$ keV ($J^\pi = 1/2^+$) and $E_2^{(r)} = 1556$ keV ($J^\pi = 3/2^-$), direct capture, and their interference. The ^{12}C(p, γ)^{13}N experimental data for the ground-state ^{13}N are taken from the latest works [20,47] together with our data, which are presented in the fifth column of Table I and Fig. 4(a). For the reaction under consideration, the value of l_b is taken to be equal to 1, $l = 0$ (the $E1$ transition) and $l = 1$ (the $M1$ transition) are

taken for the first and second resonances ($E^* = 2.365$ MeV; $J^\pi = 1/2^+$ and $E^* = 3.512$ MeV; $J^\pi = 3/2^-$), respectively, and the value of l is taken to be equal to 1 for the $E2$ and $M1$ transitions of a direct capture.

In Refs. [33,34] the indirectly measured ANC (NVC) value $C_{l_b} \equiv C_{p^{12}\mathrm{C};l_b}$ ($G_{l_b} \equiv G_{p^{12}\mathrm{C};l_b}$) for $p + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N}$ was found to be equal to $C_1 = 1.43 \pm 0.09$ fm$^{-1/2}$ ($|G_1|^2 = 0.34 \pm 0.04$ fm). One notes that the latter has been obtained from the analysis of the experimental differential cross sections for the peripheral proton transfer ${}^{12}\mathrm{C}({}^3\mathrm{He}, d){}^{13}\mathrm{N}$ reaction at two ${}^3\mathrm{He}$ energies by using two theoretical methods [33,34]. The analysis was performed within both the "post" approximation of the modified distorted wave Born approximation (DWBA) [25,48,49] and the distorted wave pole approximation [33] in which the contribution of the three-body Coulomb dynamics of the proton transfer mechanism is taken into account correctly by an approach combining the dispersion method and the DWBA approach (see Refs. [33,50] and references therein). Both approximations gave the same result for C_1^2 (or $|G_1|^2$) [33,34]. The uncertainty quoted in the value of $|G_1|^2$ (or C_1^2) is the averaged square error, which includes the experimental overall errors ($\approx 7\%$) [25,34] and the uncertainty of the method used ($\approx 9\%$); the latter involves the uncertainty of the modified single-particle cross section caused by the choice of the single-proton bound-state potential ($\leqslant 4.5\%$) and the uncertainty of optical-model parameters in the initial and final states (2%) [25]. It should be emphasized that this value of the ANC for $p + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N}$ obtained in Refs. [33,34] is in good agreement with the value $C_1 = Z_1^{1/2} b_1 = 1.46 \pm 0.22$ fm$^{-1/2}$ deduced by us independently from the results of Ref. [20], but it has noticeably larger uncertainty than the result of Refs. [33,34]. One notes that the value of $C_1 = 1.46 \pm 0.22$ fm$^{-1/2}$ obtained by using the recommended [20] empirical value of $Z_1 = 0.49 \pm 0.15$ and the value of $b_1 = 2.08$ fm$^{-1/2}$ corresponds to the values of $r_0 = 1.22$ fm and $a = 0.60$ fm recommended in Ref. [20], where $Z_{l_b} \equiv Z_{p^{12}\mathrm{C};l_b}$ and $b_{l_b} \equiv b_{p^{12}\mathrm{C};l_b}$. However, one would like to note here that the geometrical parameters r_0 and a are not directly measured and, consequently, cannot be fixed unambiguously. Hence, the spectroscopic factor Z_1 obtained in Ref. [20] depends strongly on these model parameters, which arises because of a strong dependence of b_1 on the parameters r_0 and a (see, e.g., Ref. [51] and references there), whereas the value of C_1^2 extracted from the peripheral proton transfer reaction does not depend significantly on a variation of b_1 (or the parameters r_0 and a) [25]. Because of this, in Ref. [20] use of the parametrization of the direct capture amplitude of the ${}^{12}\mathrm{C}(p, \gamma){}^{13}\mathrm{N}$ reaction in terms of the spectroscopic factor Z_1 is not justified [51]. Besides, one can obtain another estimate for C_1 by use of the Eq. (13) and recommended [9] values of the parameters for the proton reduced width $\theta^{(p)}$, spectroscopic factor $Z_p \equiv Z_1$ for ${}^{13}\mathrm{N}$ in the ($p + {}^{12}\mathrm{C}$) configuration, and the channel radius r_c. In Ref. [9], these parameters have been obtained by a best fit of the calculated cross sections $\sigma(E)$ for the ${}^{12}\mathrm{C}(p, \gamma){}^{13}\mathrm{N}$ reaction in the energy range $150 \leqslant E \leqslant 800$ keV to the experimental one, which was reached at $r_c = 5.0$ fm ($\theta^{(p)} = 0.41$ and $Z_1 = 0.78$). One notes that the value of $\theta^{(p)}$ has been obtained by us from Eq. (32) of Ref. [9] by using the values for the geometric parameters given there

($r_0 = 1.25$ fm and $a = 0.65$ fm [9]), whereas the value of Z_1 is taken from data plotted in Fig. 5 of Ref. [9]. As a result, from Eq. (13) one obtains $C_1 = 1.84$ fm$^{-1/2}$ ($|G_1|^2 = 0.56$ fm). It is seen that the ANC obtained by us from the data of Ref. [9] differs noticeably from those obtained in Refs. [33,34] and deduced by us from the results of Ref. [20]. Therefore, in our calculation an ANC value $C_1 = 1.43 \pm 0.09$ fm$^{-1/2}$ for $p + {}^{12}\mathrm{C} \to {}^{13}\mathrm{N}$ is taken from Refs. [33,34] since it has been determined in the correct way and its uncertainty is less than that from Ref. [20].

Because of the fact that the experimental cross sections (or the experimental astrophysical S factors) measured at higher energies ($E \geqslant 1.0$ MeV) have about 40% uncertainty [47], we fix values of the resonance parameters corresponding to the second excited ($E^* = 3.512$ MeV) state, which are taken equal to $\Gamma_2^\gamma = 0.64$ eV [52] and $\Gamma_2^p = 62$ keV [52,53]. The resonance parameters for the first excited ($E^* = 2.365$ MeV) state found by different authors prove to be rather different (see Sec. I and Refs. [52,53]). For example, as mentioned in Sec. I, the value of Γ_1^γ obtained by different authors is in the ranges from 0.45 to 0.67 eV. Therefore, in our analysis the width parameters of Γ_1^γ and Γ_1^p, the resonant energies, and the channel radius r_c are varied by means of fitting to prior experimental data of Ref. [20] and the present work to minimize χ^2 only in the energy range of $E \leqslant 979.4$ keV. The fitted $S(E)$ is plotted in Fig. 4(a) (the solid line). There the dotted line corresponds to the contribution for direct radiative capture, and the dashed (dashed-dotted) line shows the calculated contribution for the first (second) resonance. In Fig. 4(a), the dashed-dotted-dotted line is our calculated contribution for the tail of the third ($E^* = 10.250$ MeV, $J^\pi = 1/2^+$) γ resonance. As the energy dependencies for the proton and γ widths are determined by Eqs. (5) and (6), without this contribution being taken into account, good agreement can hardly be obtained with the experimental data, $S^{\exp}(E)$, in the energy region $E \geqslant 0.55$ MeV, by including the position of the minimum ($E \approx 1.13$ MeV). For this resonance state, only a value of the resonance parameter Γ_3^γ has been fitted as the value of the proton width Γ_3^p is known [52]. The recommended value of Γ_3^γ is given also in Table II. There the uncertainties quoted for the parameters (Γ_1^p and Γ_1^γ) obtained by us are evaluated by using standard statistical methods [54,55], which involve the experimental uncertainties of $S^{\exp}(E)$. The solid line in Fig. 4(a) corresponds to the channel radius $r_c = 5.0$ fm, providing a minimum χ^2 equal to 11.8 for 47 degrees of freedom from the energy region with $E \leqslant 979.4$ keV for which the fitting is done. As an illustration of this fact, Fig. 5 shows that the asymptotic behaviors of the $p^{12}\mathrm{C}$ scattering and bound ($p + {}^{12}\mathrm{C}$) state wave functions, calculated by using the Wood-Saxon potential with the recommended [20] values of $r_0 = 1.22$ fm and $a = 0.60$ fm, are reached simultaneously at $r_c \geqslant 5.0$ fm, and so at $r_c \geqslant 5.0$ fm their substitution for these wave functions in the external part of the R-matrix amplitude is correct. Also the sensitivity of the calculated $S(E)$ to a variation of the channel radius r_c has been verified. We find that a change of r_c within the accepted range has little influence upon calculated values of $S(E)$.

In Fig. 4(b) the sensitivities of the fitted $S(E)$ (dashed line) to the ANC value are displayed, where the theoretical value of

TABLE II. Resonance parameters for the ^{12}C$(p, \gamma)^{13}$N reaction.

Resonance $J_i^\pi, E_\lambda^{(r)}$ (MeV)	Transition	Γ_λ^p (keV)	Γ_λ^γ (eV)
$\frac{1}{2}^+$, 0.421 $\lambda = 1$	$E1$	36.5 ± 0.9 [22], 33.7 ± 1.8 [22] 33.9 ± 0.9 [19], 36.0 ± 1.8 [19] 31.7 ± 0.8 [53], 33 [9] 39 ± 2 [20], 35.0 ± 1.0 (this work)	0.67 [22], 0.50 ± 0.04 [9,53] 0.45 ± 0.05 [19] 0.65 ± 0.07 (this work)
$\frac{3}{2}^-$, 1.570 $\lambda = 2$	$M1$	62.0 ± 4.0 [52,53], 65 ± 2 [20] 65 ± 5 [9], 62 (this work)	0.64 ± 0.07 [52] 0.64 [53]
$\frac{1}{2}^-$, 8.371 $\lambda = 3$	$E1$	280 [52]	6000 (this work)

the ANC (NVC) $C_1 = 1.72$ fm$^{-1/2}$ ($|G_1|^2 = 0.48$ fm) [56] is used. The quantitative comparison between the curves plotted in Fig. 4(b) shows that the ratio of the $S(E)$ calculated with the theoretical ANC value [56] to that calculated with the indirectly measured ANC value [34] changes from 1.14 to 1.06 under the range of E from 2 to 270 keV. It follows from here that extrapolated values of the $S(E)$ are noticeably sensitive to the value of the ANC at extremely low energies. However, it should be noted that in Ref. [56] a violation of a self-consistency occurred since the shell-model wave function for the ^{13}N nucleus used there and the NVC were calculated

FIG. 5. The radial behavior of the p^{12}C- scattering wave function and the bound state ($p + ^{12}$C) radial wave function (regular in the origin) calculated for the Wood-Saxon potential with the geometric parameters of $r_0 = 1.22$ fm and $a = 0.60$ fm (solid lines). The dashed lines are the asymptotic behavior for the corresponding radial wave functions. The asymptotic behavior of the radial bound state wave function is the tail of the function $bW_{-\eta_B, 3/2}(2\kappa r)$ for $b = 2.081$ fm$^{-1/2}$.

by using different forms of NN potential. Therefore, the value of NVC calculated in Ref. [56] could have an additional uncertainty associated with the aforementioned violation of the self-consistency.

Our analysis reproduces the value of $\Gamma_1^\gamma = 0.65 \pm 0.07$ eV, which differs notably from the values of $\Gamma_1^\gamma = 0.50 \pm 0.04$ [9] and $\Gamma_1^\gamma = 0.45 \pm 0.05$ eV [21]. One notes that the resulting Γ_1^γ value obtained by us is in excellent agreement with the value of $\Gamma_1^\gamma = 0.67$ eV [22]. The same agreement occurs for the Γ_1^p value, which is deduced in the present work and obtained by other authors (see Table II also). Besides, the calculation shows that, although near each of the resonance peaks the corresponding resonance term dominates, the relative weight of the first resonance term to the direct capture part of the amplitude is rather noticeable as the value of E goes to 0, whereas the contribution of the second and third resonances can be disregarded at extremely low energies.

Thus, it follows that an allowance for additional information about the previously indirectly measured value of the NVC (or ANC) for the virtual decay ^{13}N $\rightarrow p + ^{12}$C [33,34] in the analysis of the experimental ^{12}C$(p, \gamma)^{13}$N astrophysical S factors obtained in Ref. [20] and in the present work allows us to strongly restrict the existing spread for γ width, corresponding to the first resonance level of ^{13}N, and to obtain a new estimate for γ width to this level.

The results of our calculations of the astrophysical S factors at the astrophysically most important energies $E = 0$, 25, and 50 keV are $S(0) = 1.62 \pm 0.20$ keV b, $S(25 \text{ keV}) = 1.75 \pm 0.22$ keV b, and $S(50 \text{ keV}) = 1.88 \pm 0.24$ keV b, respectively. The uncertainties quoted for these astrophysical S factors are associated with those for the parameters of the proton and γ widths and the ANC given earlier. Our central value for $S(25\text{keV})$ is consistent at the 1σ level with that of $S(25 \text{ keV}) = 1.54 \pm 0.08$ keV b obtained by Barker and Ferdous [9] and is about $1.4\sigma(2\sigma)$ larger than the central value of $S(25 \text{ keV}) = 1.33 \pm 0.15$ keV b (1.45 ± 0.20 keV b) obtained by Hebbard and Vogl [16,17] (by Rolfs and Azuma [20]). It is seen that our result for $S(25 \text{ keV})$ gives reasonable agreement with those obtained independently by other authors. However, our result for $S(0)$ is noticeably larger than that of $S(0) = 1.0$ and 1.3 keV b obtained by the authors of Ref. [11] within CGCM using the Minnesota and V2 forms of the NN potential, respectively, as well as the value of $S(0) = 1.4$ keV b recommended in Ref. [57].
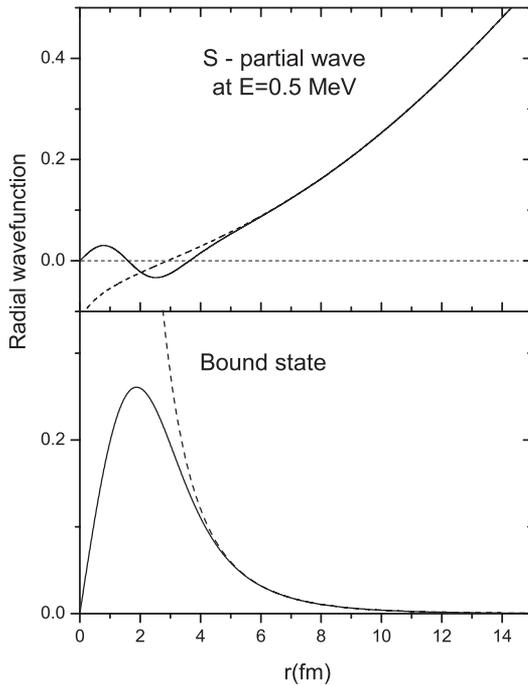
It should be emphasized that a value of $S(25 \text{ keV}) = 1.54 \pm 0.08$ keV b [9] has been also obtained within the R-matrix approach without taking into account the information about the ANC (or NVC). One notes once more that the free parameters used in Ref. [9] are the resonance amplitude $M_{i,f}^{(r)}$, the proton reduced width $\gamma^{(p)}$, spectroscopic factor Z_p for ^{13}N in the $(p + ^{12}$C$)$ configuration, and the channel radius r_c, which were determined by a best fit of the calculated cross section $\sigma(E)$ to the experimental one by minimizing χ^2. But the observed difference in values of the ANC C_1 (or Z_1) used by us and obtained from the results of Ref. [9] for the fitted parameters may lead to overestimated values of the direct capture part the cross sections as calculated in Ref. [9] as compared with the resonance part of $\sigma(E)$. In contrast with this, the ANC value for $p + ^{12}$C $\rightarrow ^{13}$N derived in Refs. [33,34] independently from the peripheral proton transfer reaction defines the nonresonance (direct) capture cross section in a correct way.

One notes once more that in Ref. [9] the absolute renormalized value of the ^{12}C$(p, \gamma)^{13}$N peak cross section at $E = E_1^{(r)} = 421$ keV has been underestimated, which led to the underestimated value of $\Gamma_1^{\gamma} = 0.50 \pm 0.05$ eV. But the result of the present work for the cross section near this resonance peak is equal to 124 ± 12 μb, which is in excellent agreement with that obtained in Refs. [20,58]. It follows from here that, in reality, the evaluation of $\Gamma_1^{\gamma} = 0.50 \pm 0.05$ eV recommended in Ref. [9] has been obtained with the underestimated value of $\sigma(421 \text{ keV})$ and the overestimated value of the ANC for $p + ^{12}$C $\rightarrow ^{13}$N.

Therefore, good agreement between the experimental and the calculated [9] reactions cross sections at rather low energies can apparently be explained by the fact that, owing to fitting of $\theta^{(p)}$ and $Z_p \equiv Z_1$(or C_1) as well as of $M_{i,f}^{(r)}$, the overestimated contribution of the pure direct radiative capture amplitude compensates for the underestimated values of $M_{i,f}^{(r)}$ because each of them depends noticeably on the value of the channel radius parameter r_c. Also, the resonance cross section $\sigma(E_1^{(r)})$ in Ref. [9] is underestimated with respect to those obtained in the present work and in Refs. [20,58]. Perhaps that is one of the possible reasons for the observed discrepancy between the values recommended in Refs. [9,19] and our value of the γ width Γ_1^{γ} for the first resonance level of ^{13}N. In connection with this, it is stressed that in the R-matrix approach, as the absolute normalization of the direct capture amplitude of the ^{12}C$(p, \gamma)^{13}$N reaction is expressed in terms of the previously indirectly measured ANC (or NVC), the resonance amplitude may be determined in a correct way by fitting experimental data for minimum χ^2. In this case the astrophysical S factor for the ^{12}C$(p, \gamma)^{13}$N reaction at rather low energies can be used as an independent source of additional information on the values of the proton and γ widths for the resonance states of ^{13}N.

V. THERMONUCLEAR REACTION RATE

The calculated astrophysical S factors $S(E)$ [or the cross sections $\sigma(E)$] given by Eq. (2) can be used for calculating the reaction rates as a function of stellar temperature T_9 within the
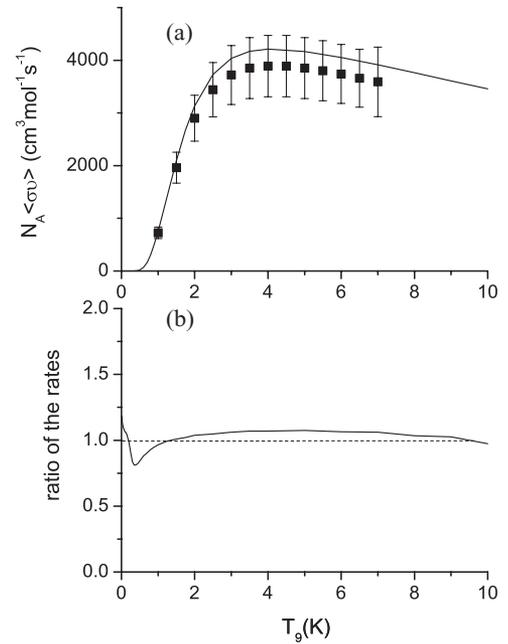

FIG. 6. (a) The reaction rate calculated in the present work (solid line) and points taken from Ref. [60] performed for the geometry of our experiment. (b) The ratio of the ^{12}C$(p, \gamma)^{13}$N reaction rates $N_A(\sigma\vartheta)$ of the present work to those from Ref. [47] (solid line).

range of $10^{-3} \leqslant T_9 \leqslant 10$, where T_9 is the temperature T in units of 10^9 K. The Maxwellian-averaged reaction rates $N_A(\sigma v)$ as a function of the temperature are defined by [57,59]

$$N_A(\sigma v) = N_A \left(\frac{8}{\pi \mu}\right)^{1/2} (k_B T)^{-3/2}$$
$$\times \int_0^\infty \sigma(E) \exp(-E/k_B T) E \, dE, \quad (14)$$

where N_A is Avogadro's number, k_B is the Boltzmann constant, and $v = \sqrt{2E/\mu}$. In Fig. 6(a), we present the reaction rates of our calculation (solid line) and its comparison with the data (points) of Ref. [60]. It is seen that the result of our calculation is in good agreement with that recommended in Ref. [60]. The ratio of our calculation of reaction rates $N_A(\sigma v)$ to the result recommended in Ref. [47] (solid line) is also displayed in Fig. 6(b). As is seen from Fig. 6(b) there is a noticeable difference (up to $\approx$20%) between our recommended results and those presented in Ref. [47] within a wide interval of stellar temperature, including solar temperatures. The probable reason for the observed difference between the adopted reaction rates in Ref. [47] and our recommended results is that different approaches for calculating the astrophysical S factors were used. One notes that in Ref. [47] the calculation of the reaction rates included all the experimental astrophysical S factors obtained in Refs. [16,17,20,47], some of which have uncertainties up to 40%, by a smooth spline fit. However, our astrophysical S-factor calculation over the wide energy range has been performed within the analytic R-matrix method in which the additional new more precise experimental astrophysical S factors (or the cross sections) and previously measured values of the ANC [33,34] are used. Also one notes

once more that our result for the reaction rates coincides well with that of Ref. [60] obtained by quite a different technique over a wide temperature range.

VI. CONCLUSION

We have measured the differential and total cross sections for the ^{12}C$(p, \gamma)^{13}$N reaction with absolute experimental uncertainties of about 10% at several beam energies ($E_p = 354$–1061 keV or $E = 326.8$–979.4 keV) leading to γ emission to the bound state and the strong resonance ($E^* = 2.366$ MeV; $1/2^+$) state. Our data are in good agreement with those previously obtained in Ref. [20].

We have analyzed the experimental astrophysical S factors $S(E)$ for the ^{12}C$(p, \gamma)^{13}$N reaction at extremely low energies within the one-level R-matrix approach where the direct part of the amplitude is expressed in terms of the ANC (C_1) for ^{13}N in the ($p + {}^{12}$C) channel. Such a parametrization allowed us to calculate the direct capture part of the amplitude in a correct manner using the indirectly measured value of C_1 found previously in Refs. [33,34] from the analysis of the peripheral ^{12}C$(^3$He, $d)^{13}$N reaction. It is demonstrated that using information about the ANC C_1 provides good fitting of the astrophysical S factor for the ^{12}C$(p, \gamma)^{13}$N reaction populating the first and second excited states of the ^{13}N and reduces to a minimum the model dependence of the calculated direct capture part of the astrophysical S factor on the parameters of the R-matrix approach.

A new estimate for the resonance γ width corresponding to the first resonance level ($E^* = 2.365$ MeV; $J^\pi = 1/2^+$) of ^{13}N has been obtained. Also, new estimates for $S(E)$ at astrophysically relevant energies of $E = 0$, 25, and 50 keV and reaction rates within the stellar temperature range of $10^{-3} \leqslant T_9 \leqslant 10$ have been obtained. Our recommended value for $S(25$ keV) is up to 2σ larger than previous estimates obtained by the authors of Refs. [9,16,17,20]. The observed mutual agreement

between the experimental data for $S^{\exp}(E)$ measured in the present work and those obtained previously in Ref. [20] and a comparison of the value of the ANC C_1 obtained by us from Ref. [9] ($C_1 = 1.84$ fm$^{-1/2}$) and recommended in Refs. [33,34] ($C_1 = 1.43 \pm 0.06$ fm$^{-1/2}$) allow us to conclude the following: Apparently in Ref. [9], where an analysis of the cross sections at extremely low energies for the same radiative capture reaction has been carried out within the R-matrix approach without taking into account the information about the ANC C_1, there is a noticeable overestimation (underestimation) of the contribution from the pure direct radiative capture amplitude (the resonance cross section at the first resonance peak).

It has been shown that the present reaction rates are in good agreement with those recommended in Ref. [60] by using a very different technique from that described in the present work, whereas a notable difference (up to $\approx$20%) occurs between our result and that presented in Ref. [47] within the wide interval of stellar temperatures, including solar temperatures.

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