Two-component model for the axial form factor of the nucleon

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The axial form factor of the nucleon is studied in a two-component model consisting of a three-quark intrinsic structure surrounded by a meson cloud. The experimental data in the space-like region are well reproduced with a minimal number of parameters. The results are similar to those obtained from a dipole fit for $0 < Q^2 < 1 \, \mathrm{GeV}^2$, but outside this region there are important deviations from the dipole parametrization. Finally, the theoretical formula for the axial form factor is extrapolated by analytic continuation to the time-like region, thus providing the first predictions in this kinematical region which is of interest for present and future colliders.

I. INTRODUCTION

The electroweak structure of the nucleon is characterized by both electromagnetic and weak form factors and, in particular, by the weak axial form factor, $G_A(Q^2)$ (Q^2 is the four-momentum transfer squared), which is related to the nucleon axial current. The existing experimental information on the axial form factor in the space-like region can be obtained directly through the reaction $\nu_\mu + p \rightarrow \mu^+ + n$, or indirectly through charged pion electroproduction near-threshold experiments [1]. Axial form factors also play an important role in the analysis of parity violating electron scattering. Especially, in order to extract information on the strange form factors of the proton requires a good knowledge of the axial form factor [2].

Predictions on the axial for factor have been given in different models which describe the nucleon structure, such as the chiral constituent quark model [3,4], the chiral perturbation theory [5], the quark-soliton model [6], the light cone QCD sum rules [7]. Results from lattice QCD have become available recently [8].

The axial form factor is usually parametrized by means of a dipole form [1] which gives a reasonable description of the data up to $Q^2 = 1 \, \mathrm{GeV}^2$ covering the range of most of the available measurements. It is useful to have other parametrizations [9], even though it is difficult to discriminate among them on the basis of the existing data alone. Indeed for many years, the dipole parametrization was considered to provide a very good description of the proton and neutron magnetic form factors and the electric proton form factor, whereas the electric neutron form factor was assumed to be zero or very small and well described, for example, by the Galster parametrization [10].

However, it has recently been shown that the electric and magnetic form factors of the proton are actually very different, and that the ratio $\mu G_E^p / G_M^p$ drops almost linearly as a function of Q^2 [11], in contrast with the dipole description.

The Iachello, Jackson, and Landé model (IJL) [12] predicted this behavior by means of a two-component model for the electric proton form factor long before the data appeared. More recently, Bijker and Iachello (BI) [13] have shown that it is possible to refine the two-component model in oder to reproduce further details, in particular, concerning the electric and magnetic form factors of the neutron. The IJL and BI approaches are based on a two-component picture of the nucleon in terms of an intrinsic structure (qqq configuration) surrounded by a meson cloud ($q\bar{q}$ pairs). It has been shown to be rather successful in the description of the nucleon electromagnetic form factors both in the space- and in the time-like region [13–15]. Other applications of the two-component model include the deuteron [16] and the strange form factors of the proton [17].

The purpose of this paper is to apply the two-component model of nucleon form factors [12,13] to the axial form factor, and to study its analytic continuation to the time-like region for which the axial form factor has not yet been measured. Suggestions for its determination through the reaction $N\bar{p} \rightarrow \gamma^* N\pi$ and the crossed channels can be found in [18–20]. This problem can become very actual in connection with the physics planned with the antiproton beam which will be available at the FAIR accelerator complex.

II. AXIAL FORM FACTORS

The axial form factor has been measured directly in neutrino scattering, $\nu_\mu + p \rightarrow \mu^+ + n$, or indirectly, in near-threshold charged pion electroproduction in the space-like region. In both reactions the axial form factor is linked to weak charged currents. The available experimental information is usually

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parametrized in terms of a dipole [1]

$$G_A^D(Q^2) = \frac{G_A(0)}{\left(1 + Q^2/M_A^2\right)^2}. \tag{1}$$

At $Q^2 = 0$, the axial form factor can be determined from neutron β decay as $G_A(0) = 1.2695 \pm 0.0029$ [21]. The axial mass M_A is adjusted to the experimental data. From charged pion electroproduction one obtains $M_A = 1.069 \pm 0.018$ GeV, whereas in neutrino scattering experiments, M_A is extracted from a weighted average to be $M_A = 1.026 \pm 0.021$ GeV, which is somehow inconsistent with the best fit value obtained from the electroproduction experiments. Even if the neutrino data suffer from great uncertainties, the weighted average for the root mean square radius and thus also for M_A ($\langle r^2 \rangle_A = 12/M_A^2$) is considered to be quite reliable.

Similarly to the Rosenbluth separation for electromagnetic form factors, the axial (pseudoscalar) form factor is related to the slope (intercept) of the near threshold differential cross section as a function of the polarization of the virtual photon. By means of low energy theorems it is possible to calculate the electric dipole amplitude at the threshold in the case of soft pions. Model-dependent corrections, have to be introduced in order to take into account the finite pion mass. It has been shown in [1] that if one takes into account the corrections due to the finite pion mass in chiral perturbation theory which should be applied to the root mean square axial radius as extracted from charged pion electroproduction data, they do indeed correspond to an increase in the root mean square value. This leads to a lowering of the M_A value as extracted from electroproduction of the order of 5%, which makes it compatible with the neutrino value.

Additional experimental information on the axial form factor may be obtained from weak neutral current processes in parity violating electron scattering experiments. There is a proposal of the G0 collaboration for dedicated runs at backward angles in order to extract information on the axial coupling of the photon with the nucleon [22]. The SAMPLE experiment yielded values for the axial form factor by combining the results for proton and deuteron targets [23].

III. TWO-COMPONENT MODEL

In the two-component model [12,13], the axial nucleon form factor is described as

$$G_A(Q^2) = G_A(0)g(Q^2)\left[1 - \alpha + \alpha\frac{m_A^2}{m_A^2 + Q^2}\right],$$
$$g(Q^2) = \left(1 + \gamma Q^2\right)^{-2}, \tag{2}$$

with $Q^2 > 0$ in the space-like region. $g(Q^2)$ denotes the coupling to the intrinsic structure (three valence quarks) of the nucleon, and m_A is the mass of the lowest axial meson $a_1(1260)$ with quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$ and $m_A = 1.230$ GeV. We note that, unlike other studies, in which m_A is a parameter, here it corresponds to the mass of the axial meson $a_1(1260)$. In the present case, γ is taken from previous studies of the electromagnetic form factors of the nucleon [12,13]. Therefore, α is the only fitting parameter.

It is interesting to note, that this form of the axial form factor can give rise to a zero in the space-like region. If $\alpha > 1$, the axial form factor goes through zero at $Q^2 = m_A^2/(\alpha - 1)$. Since for large values of Q^2 the contribution of the axial meson cloud vanishes, the asymptotic behavior of the axial form factor of Eq. (2) is given by its intrinsic part only

$$\lim_{Q^2 \to \infty} G_A(Q^2) = \frac{G_A(0)(1 - \alpha)}{(\gamma Q^2)^2}, \tag{3}$$

which becomes negative if $\alpha > 1$.

The behavior of the axial form factor at low values of Q^2 can be used to determine the axial radius

$$\langle r^2 \rangle_A = -6 \left.\frac{dG_A(Q^2)}{dQ^2}\right|_{Q^2=0}$$

$$= \begin{cases} \frac{12}{M_A^2} & \text{dipole} \\ 6\left(2\gamma + \frac{\alpha}{m_A^2}\right) & \text{two-component}. \end{cases} \tag{4}$$

A comparison of the axial radius for the dipole and the two-component model may be used to express the coefficient α

$$\alpha = 2m_A^2\left(\frac{1}{M_A^2} - \gamma\right), \tag{5}$$

in terms of the mass of the lightest axial meson m_A, the fitted value of the axial mass M_A appearing in the dipole form and γ, which is proportional to the intrinsic radius.

IV. ANALYSIS OF THE DATA IN THE SPACE-LIKE REGION

In this section, we study the axial form factor of the proton in a two-component model. The experimental data are taken from a compilation of pion electroproduction experiments on the nucleon [1]. Since the present neutrino data suffer severe uncertainties [1], in the present analysis we only consider the pion electroproduction data.

The Q^2 dependence of the nucleon axial form factor $G_A(Q^2)$, has been measured in several pion electroproduction experiments at threshold over the last few decades. The slope of the total unpolarized differential cross section at threshold contains information on $G_A(Q^2)$, but the numerical value of this form factor is highly model dependent. In general, four different approaches have been used to extract the values of the axial form factor of the nucleon: the soft pion approximation (SP) [24], the partially conserved axial current approximation (PCAC) [25], the Furlan approximation (FPV) [26] (enhanced soft pion production), and the Dombey and Read approximation (DR) [27]. As a consequence of these competing approaches, up to four experimental values may be extracted from a single measurement (at fixed Q^2). A total of 67 experimental points are available, corresponding to 32 measurements. Data from Ref. [28] were considered separately, as they correspond to Δ excitation in the final state. In order to evaluate the systematic error, the data were therefore separated into five groups according to the approach used and the processes measured. The data from [29] were not considered in the fit, following Ref. [1], as they are

TABLE I. Fitted α parameter and corresponding $\chi^2/$n.d.f. for the different model-dependent extractions of the axial data.

Model	DR	FPV	SP	PCAC	Δ	Global	Ref.
α	1.38 ± 0.08	1.90 ± 0.14	1.15 ± 0.06	1.80 ± 0.06	1.21 ± 0.07	1.57 ± 0.04	[12]
$\chi^2/$n.d.f.	0.19	0.81	3.9	0.79	0.43	1.78	
α	0.75 ± 0.10	1.23 ± 0.15	0.52 ± 0.09	1.17 ± 0.07	0.53 ± 0.09	0.95 ± 0.05	[13]
$\chi^2/$n.d.f.	0.40	0.75	3.45	0.67	0.49	1.45	

systematically larger, nor were data from [24]. The data, normalized to one, are plotted in Fig. 1. Different symbols correspond to different models used for the extraction of the data but may correspond to the same experiment.

The form factor $g(Q^2)$ in the two-component model describes the coupling to the intrinsic structure (three valence quarks) of the nucleon, where γ was determined from a fit of nucleon electromagnetic form factors to be $\gamma = 0.25$ GeV^{-2} [12] or $\gamma = 0.515$ GeV^{-2} in a more recent fit [13]. We note however that the former value is not good from a t channel point of view, because it gives a pole in the physical region at $t_0 = 1/\gamma = 4$ GeV^2 ($> 4m^2 = 3.52$ GeV^2, the corresponding threshold). In the latter case, the pole is shifted to the unphysical region. In our calculations of the axial form factors we keep γ as a fixed parameter, and consider both values mentioned above.

Individual one–parameter fits to the five data sets were performed, as well as a global fit, according to Eq. (2). The
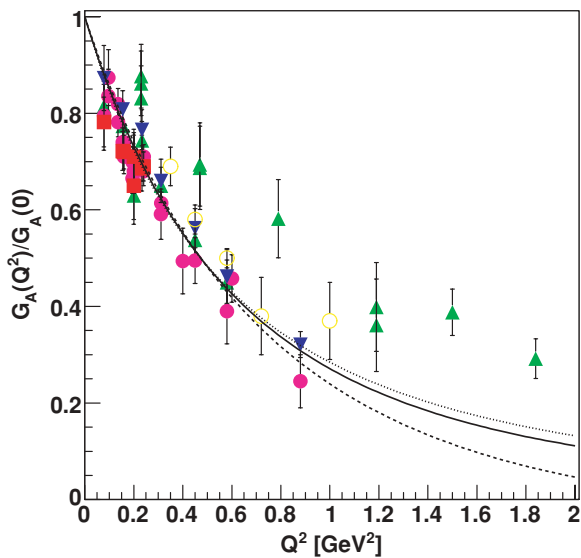

FIG. 1. (Color online) Comparison between theoretical and experimental values of the axial form factor of the nucleon $G_A(Q^2)$ as a function of Q^2. The theoretical values are calculated in the two-component model using Eq. (7) with $\alpha = 1.57$ and $\gamma = 0.25$ GeV^{-2} [12] (dashed line), and $\alpha = 0.95$ and $\gamma = 0.515$ GeV^{-2} [13] (solid line), and the dipole form of Eq. (1) with $M_A = 1.069$ GeV (dotted line). The experimental values were extracted according different models: PCAC [25] (pink, solid circles), FPV (red, solid squares) [26], SP (green, solid triangles) [24], DR (blue, trianglesdown) [27], Δ (yellow, open circles) [28].

results are shown in Table I and in Fig. 1. The global fit gives $\alpha = 1.57 \pm 0.04$ with $\chi^2/$n.d.f. $= 85.36/48 = 1.78$ for $\gamma = 0.25$ GeV^{-2} [12], and $\alpha = 0.95 \pm 0.05$ with $\chi^2/$n.d.f. $= 69.60/48 = 1.45$ for $\gamma = 0.515$ GeV^{-2} [13]. In both cases, the χ^2 for individual fits may be smaller than the global χ^2, owing to the dispersion of the data, but the errors associated to the parameters of the global fits are smaller, owing to the larger number of points. These values of α can be considered as an average of the different corrections. The associated systematic error, which takes into account the dispersion of the model analysis, can be evaluated from the results of the individual fits to be $< |0.35|$.

In Fig. 1 we show a comparison between the experimental and theoretical values of the axial form factor for the dipole fit, the global fit with $\alpha = 1.57$ and $\gamma = 0.25$ GeV^{-2} [12], and $\alpha = 0.95$ and $\gamma = 0.515$ GeV^{-2} [13]. It is possible to give a reasonable description of the data, if we consider the average value, since we average not only statistical errors, but also the systematic errors related to the model-dependence extraction of the data. In the range up to $Q^2 = 1$ GeV^2 the description of the data is comparable to the quality of a dipole fit, though it is clear that already around $Q^2 = 1$ GeV^2 the three parametrizations start to show a different behavior. According to Eq. (3), the two fitted values of the α parameter imply a different asymptotic behavior with a change of sign at $Q^2 = 2.65$ GeV^2 for the IJL parametrization [12], but not for BI [13], nor for the dipole (see also Fig. 2).
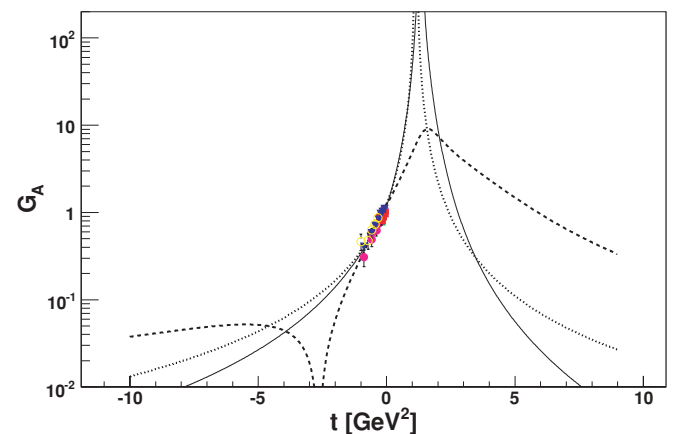

FIG. 2. (Color online) As Fig. 1, but for the absolute value of the axial form factor $|G_A(t)|$ in the space-like ($t < 0$) and time-like ($t > 0$) regions. In the time-like region, $\delta = 0.925$ for IJL [14] and $\delta = 0.397$ for BI [13].

The values of α obtained in the global fits are close to the values that can be derived from Eq. (5) with $m_A = 1.230$ GeV and $M_A = 1.069$ GeV in which it is assumed that the axial radius for the dipole and the two-component model is the same: $\alpha = 1.89$ for $\gamma = 0.25$ GeV^{-2} [12] and $\alpha = 1.09$ for $\gamma = 0.515$ GeV^{-2} [13].

The axial radius $\sqrt{\langle r^2 \rangle_A}$ can be obtained from Eq. (4): 0.60 fm for IJL, 0.62 fm for BI and 0.64 fm for the dipole. In the two-component model the contributions of the quark core and the axial meson cloud to the axial radius are given by

$$\langle r^2 \rangle_A = \begin{cases} 12\gamma(1-\alpha) & \text{quark core} \\ 6\alpha\left(2\gamma + \frac{1}{m_A^2}\right) & \text{axial meson cloud.} \end{cases} \quad (6)$$

The difference between the two parametrizations of the two-component model (IJL and BI) is in the values of γ and α. The value of γ corresponds to the spatial extent of the intrinsic dipole form factor $\langle r^2 \rangle^{1/2} \simeq 0.34$ fm [12] and $\simeq 0.49$ fm [13], whereas α is related to the coupling of the axial meson. Finally, the contributions of the core and the meson cloud to $\langle r^2 \rangle_A$ are -1.71 and 10.94 GeV^{-2} for IJL, and 0.31 and 9.64 GeV^{-2} for BI. Therefore, both for IJL and BI the dominant contribution to the axial radius of the nucleon comes from the meson cloud.

We note, that the negative sign of the contribution of the quark core to the nucleon axial radius for the IJL parametrization is related to the change in sign of the axial form factor at $Q^2 = m_A^2/(\alpha - 1) = 2.65$ GeV^2 and the occurrence of a pole in the physical region at $t_0 = 1/\gamma = 4$ GeV^2, which indicates that BI is the preferred parametrization. This is not surprising, since the IJL and BI parameters were determined in a fit to experimental data available in 1973 and 2004, respectively.

V. TIME-LIKE REGION

The extension of the axial form factor of the nucleon in the two-component model to the time-like region can be done by analytic continuation, just as for the case of the electromagnetic form factors [13,14]: (i) the kinematical variable Q^2 is changed into $Q^2 \to -t$, (ii) a complex phase $e^{i\delta}$ is introduced into the intrinsic form factor of Eq. (2), similar to Refs. [13,14], and (iii) the vector-meson dominance term corresponding to the exchange of an axial meson has to be modified in order to take into account the considerable width of the axial meson. Here it has been substituted by a Breit-Wigner formula with $\Gamma_A = 400$ MeV. These modifications lead to the following expression for the axial form factor in the time-like region

$$G_A(t) = G_A(0)g(t)\left[1 - \alpha + \alpha\frac{m_A^2\left(m_A^2 - t + im_A\Gamma_A\right)}{\left(m_A^2 - t\right)^2 + (m_A\Gamma_A)^2}\right], \quad (7)$$

with

$$g(t) = \left(1 - e^{i\delta}\gamma t\right)^{-2}. \quad (8)$$

Once the parameter α has been determined from the space-like data, the time-like behavior of nucleon axial form factor can be calculated using Eqs. (7), (8). In Fig. 2, we show the axial form factor in the space-like ($t < 0$) and time-like

($t > 0$) regions for the two-component model obtained from Eq. (7) with $\alpha = 1.57$, $\gamma = 0.25$ GeV^{-2} and $\delta = 0.925$ [12,14] (dashed line), and $\alpha = 0.95$, $\gamma = 0.515$ GeV^{-2} and $\delta = 0.397$ [13] (solid line), the dipole form of Eq. (1) with $M_A = 1.069$ GeV (dotted line) and the experimental data used in the fit of the axial form factor in the space-like region.

Even though the different parametrizations of the axial form factor coincide in the range of $0 < Q^2 < 1$ GeV^2, outside this range they show large and important differences. The position and the shape of the peak in the time-like region is determined by the values of γ and δ in the intrinsic form factor. It is interesting to note that, outside the region of the peak, the magnitude of the axial form factor is significantly higher in the time-like region than in the space-like region. Moreover, contrary to the other calculations, the IJL parametrization [12] predicts a zero at $Q^2 = 2.65$ GeV^2 in the space-like region.

VI. CONCLUSIONS

In conclusion, we have proposed a new parametrization of the existing space-like data for the axial nucleon form factor by means of a two-component model of the nucleon. The physical interpretation of this model corresponds to a compact core surrounded by a meson cloud. This parametrization satisfies the analytical properties of the form factors and can be extended to the whole kinematical region. The axial form factor of the two-component model displays a behavior similar to that of the dipole parametrization in the space-like region up to $Q^2 = 1$ GeV^2, whereas outside this region the behavior is quite different: IJL predicts a zero around $Q^2 = 2.65$ GeV^2, whereas the dipole and BI do not show a change of sign.

It is important to note that the values of the axial form factor extracted from the experimental data in the space-like region are model-dependent, whereas in the time-like region there is no experimental information available. A possible way to access the axial form factor in the time-like region and in the unphysical region (below the reaction threshold) has been suggested through the reactions $N\bar{p} \to \gamma^* N\pi$ and the crossed channels [18–20]. The cross section related to these processes is large and such experiments may be performed in future colliders, such as FAIR (Germany), BES3 (China), DANAE (Italy). Such experiments also seem to be encouraged by our finding of a non-negligible time-like axial form factor, at least up to a few GeV^2, as shown in Fig. 2.

We have also discussed the importance of accurate knowledge of the axial form factor in order to be able to extract good data on the strange form factors in parity-violating experiments. Possible improvements of the present analysis, which will be required in the event of new and more precise data, can be foreseen in two directions. First, since the axial meson a_1 has a large decay width, even larger than that of the ρ meson, the corresponding propagator has to be modified to a more complicated form, similar to what was done for the ρ meson [12]. Secondly, one may consider the contribution of other axial mesons with higher masses. A similar study can be applied to the pseudoscalar nucleon form factors.

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