Hadronization line in stringy matter

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The equation of state of the string model with linear strings comes close to describing the lattice quantum chromodynamics results and allows for the $E/N \approx 6T_0 = 1$ GeV relation found in phenomenological statistical model. The E/N value is derived from the zero pressure condition in quark matter and is a fairly general result. The baryochemical potential dependence of the hadron gas can be met if it is interpreted in the framework of an additive quark model. The conclusion is reached that stringy models explain the $E/N = 6T_0$ relation naturally and independently of the value of the string tension.

I. INTRODUCTION

The phenomenological success of the statistical hadronization model has been emphasized, questioned, and criticized repeatedly over the years. All experimental results of heavy-ion collisions on particle yields at energies ranging from GSI Schwerionen Synchrotron to the Relativistic Heavy Ion Collider are consistent with results falling in a narrow stripe in the parameter space of temperature and baryochemical potential. There are several descriptions of this so-called chemical freeze-out curve; for recent articles on this topic, see Refs. [1–9]. For a comprehensive review, see Ref. [10].

In this article we intend to explain the rather high value of the energy per particle, $E/N \approx 1$ GeV for a system of quarks and gluons with an equation of state containing a term depending on the color density. As a matter of fact the convex shape of this curve (cf. Fig. 3) is in agreement with several thermodynamic approaches incorporating fermions and bosons as ideal gases. The main puzzle is the quantitative value, because 1 GeV is six times higher than the associated hadronization temperature, $T_0 \approx 167$ MeV. We aim to understand how this value can be obtained starting from a massless quark-gluon plasma (QGP) by adding a color-density term.

On the hadronic side, considering massive matter, elementary nonrelativistic thermodynamics leads to $E/N = m + 3T/2$. With the known values cited above this requires an average mass in the range of the ρ-meson mass, $m \approx 750$ MeV. This value satisfies $m \gg T$, so the nonrelativistic approximation turns out to be acceptable.

However, such a high mass on the quark matter side cannot be a constituent mass; it can be the result only of strong interactions. A well-known example for treating this interaction as a mean field, the original MIT bag model, with $p = \kappa T^4 - B$, $e = 3\kappa T^4 + B$, and $n \approx \kappa T^3$ pressure, energy density, and particle density, respectively, would allow for $E/N = e/n \leqslant 4T$ only, on $p \geqslant 0$. Moreover the equation of state of the model quark matter has to be in accordance to results obtained from lattice quantum chromodynamics (QCD), the only model-independent nonperturbative approach from field theory to the equation of state at present. We present a simple thermodynamical model of massless quarks and gluons whose interactions are described by a free-energy contribution motivated by strings. This stringy matter will be studied at vanishing pressure for the hadronization curve and at high temperature for the lattice equation of state. It is an interesting conclusion that such stringy models do explain the $E/N = 6T_0$ relation easily and, remarkably, independent of the value of the string tension.

II. STRINGY THERMODYNAMICS

There are several ways to treat corrections to an ideal gas equation of state (EOS). Each model identifies a physical picture in which the leading-order interactions are calculated. The lattice QCD and pure Yang Mills EOS approach an effective massless ideal gas limit at high temperatures, but it deviates vastly from it around the color deconfinement temperature. In this article we test the string model picture for an interacting gluonic plasma, introduced in Refs. [11,12]. A quasiparticle model has been used successfully to extend the lattice QCD results to finite baryon chemical potential [12–16]. Our approach is different in that we use massless particles but introduce a density-dependent term in the free energy. There exist several attempts at obtaining the equation of state starting from perturbative QCD calculations, see e.g., Refs. [17–19], or from nonperturbative mass gap equations for gluons [20].

However, we assume that a decreasing proportion of color charges are still connected by strings at high temperature, above the crossover value to deconfinement $T > T_c$. This assumption is supported by some simple and general properties of the thermal distribution of relative momentum squared between pairs of massless particles,

$$P(Q^2) = \langle \delta(Q^2 - 2E_1 E_2 (1 - \cos \vartheta)) \rangle$$
$$= \frac{\int_0^\infty \int_{Q^2/4E_1}^\infty E_1 E_2 f(E_1) f(E_2)}{4 \int_0^\infty \int_0^\infty E_1^2 E_2^2 f(E_1) f(E_2)}. \quad (1)$$

This distribution has a nonperturbative contribution to the equation of state stemming from relative momenta below a

fixed value ($Q^2 < \Lambda^2$ with say $\Lambda \approx 1$ GeV):

$$F = \int_0^{\Lambda^2} P(Q^2) \, dQ^2. \tag{2}$$

This quantity, not having another energy scale than the temperature in case of massless particles, is given by a scaled integral

$$F = \int_0^{\Lambda^2/T^2} \mathcal{P}(x) \, dx. \tag{3}$$

For high temperatures $T \gg \Lambda$ this proportion is approximately given by $F \approx \mathcal{P}(0) \, \Lambda^2/T^2$, for low temperature, $T \ll \Lambda$ by $F \approx 1$ due to normalization. Because

$$\mathcal{P}(0) = \left\langle \frac{T}{2E} \right\rangle^2 \tag{4}$$

is nonzero [cf. Eq. (1)], there is always a nonperturbative contribution to the equation of state at any temperature; it just becomes subleading order in the pressure. Nevertheless, in the interaction measure ($e - 3p$) such an $\mathcal{O}(T^2\Lambda^2)$ term mixes to the leading order.

A. Equation of state with strings

A general class of EOS is obtained by taking into account a term in the free energy proportional to a fractional power of the density. Here we consider a "color density," a weighted sum of the number of particles:

$$c = \sum n_i c_i. \tag{5}$$

Colorless objects do not pull strings, nor do they take part in screening (ending) them. Furthermore, different color charges like quarks or gluons may have different effective string constants [21]; this effect is taken into account in the factors c_i.

The contribution to strings of an average length of

$$\langle \ell \rangle \propto c^{-\gamma} \tag{6}$$

shall be multiplied by the density, with γ being a fractional power between zero and 1. For straight strings in three dimensions, $\gamma = 1/3$.

Because we consider density-dependent modifications of the EOS at a given temperature, we use the free-energy density, denoted f, as the fundamental potential. We consider

$$f = \tilde{f}(n_i, T) + \frac{A}{1-\gamma} c^{1-\gamma}. \tag{7}$$

Here the coefficient A comprises eventual average geometrical shape factors in addition to the string tension, so it cannot be taken from the Regge slope of meson resonances directly. Instead, we fit it to the lattice EOS later. The $\tilde{f}$ is a general free-energy density for describing a quark-gluon plasma to be specified later.

The chemical potentials associated to the component i are given by the derivatives with respect to n_i:

$$\mu_i = \tilde{\mu}_i + A c^{-\gamma} c_i, \tag{8}$$

where $\tilde{\mu}_i = \partial \tilde{f}/\partial n_i$ follows from $\tilde{f}$. We use so far the additive form of Eq. (7). The chemical equilibrium establishes, if possible, at constant values of this chemical potential. These values are determined by the conserved charges of the components. For the sake of simplicity we consider here the baryon charge only (zero for gluons, $1/3$ and $-1/3$ for quarks and antiquarks, respectively), but further quantities may be introduced into this scheme easily. From the equations

$$\mu_i = q_i \mu_B \tag{9}$$

the corresponding number densities can be expressed

$$n_i^{\text{eq}} = \nu_i(T, \mu_B; q_i, c_i). \tag{10}$$

In this chemical equilibrium situation the color density defined in Eq. (5) takes its equilibrium value expressed by the ν_i-s.

Although the real solution of this system of equations can be involved in the general case, in some particular systems, e.g., for a massless ideal gas with strings in the Boltzmann approximation, it can be given in analytic form (see later). A remarkable general property, however, can be derived without the explicit form of this solution. This property related to the hadronization of the QGP is the energy per particle at the edge of the stability: when the pressure vanishes. For this purpose we obtain the entropy density,

$$s = -\frac{\partial f}{\partial T} = \tilde{s}, \tag{11}$$

the pressure

$$p = \sum \mu_i n_i - f = \tilde{p} - \gamma \frac{A}{1-\gamma} c^{1-\gamma}, \tag{12}$$

and the energy density

$$e = f + Ts = \tilde{e} + \frac{A}{1-\gamma} c^{1-\gamma}. \tag{13}$$

We note here that in stringy models the mean-field correction is density dependent and therefore the energy density and pressure corrections do not compensate each other. As it can be seen from the above equations [(12) and (13)], the attarctive pressure correction is $-\gamma$ times the energy correction. A bag model-like contribution, i.e., a term proportional to the metric tensor in the energy-momentum tensor, would arise for $\gamma = 1$ only, whereas the lattice equation of state seems to support a value around $\gamma = 1/3$.

At the end point of the mechanical stability $p = p_0 = 0$ and therefore

$$\tilde{p}_0 = \frac{\gamma A}{1-\gamma} c^{1-\gamma}. \tag{14}$$

From this the color-weighted density can be obtained at this point,

$$c_0 = \left(\frac{1-\gamma}{A\gamma} \tilde{p}_0 \right)^{\frac{1}{1-\gamma}}, \tag{15}$$

and the energy density is expressed as

$$e_0 = \tilde{e}_0 + \tilde{p}_0/\gamma. \tag{16}$$

This is a remarkable result. For massless constituents only $\tilde{p} = T^4 \phi(\mu_B/T)$ and therefore $\tilde{e} = 3\tilde{p}$. For straight strings

$\gamma = 1/3$, so we arrive at $e_0 = 6\tilde{p}_0$. As long as the Boltzmann approximation is applicable, for an ideal gas mixture $\tilde{p} = nT$ with $n = \sum n_i$, and one concludes that

$$\frac{E}{N} = e_0/n = 6T_0. \tag{17}$$

With $T_0 = 167$ MeV fitted to hadronization data (and predicted by lattice QCD as the crossover temperature) one obtains $E/N = 1$ GeV at this point. This derivation applies to the quark-gluon side of the hadronization curve.

B. Chemical equilibrium with strings

From Eqs. (8) and (9) it follows that the equilibrium number densities in general satisfy

$$n_i^{\text{eq}} = n_{i,A=0}^{\text{eq}}(T, q_i\mu_B - Ac^{-\gamma}c_i). \tag{18}$$

Summing with the color weight factors, c_i we arrive at an implicit equation for the effective color (string source) density:

$$c = \sum c_i \, n_{i,A=0}^{\text{eq}}(T, q_i\mu_B - Ac^{-\gamma}c_i). \tag{19}$$

Its solution in general depends on the temperature T, baryochemical potential μ_B, and the specific color and baryon charges c_i and q_i.

Because this implicit equation contains the unknown, c, on its right-hand side as a given correction to the chemical potential only, we are able to deduce that there is no solution beyond a critical point. This point is characterized by the fact that the derivative of Eq. (19) with respect to c is also satisfied,

$$1 = \gamma Ac^{-\gamma-1} \sum c_i^2 \, \frac{\partial n^{\text{eq}}}{\partial\mu}(T, q_i\mu_B - Ac^{-\gamma}c_i). \tag{20}$$

We denote the color density in this end point where the chemical equilibrium ceases to be possible by c_E. A further question is how this end point of possible chemical equilibrium solutions is related to the zero pressure boundary.

In the particular case of Boltzmann-like dependence on the chemical potential $\partial n_i/\partial\mu = n_i/T$ (i.e., the exponential of μ/T is a factor in the number density). We note that for a Boltzmann system with all c_i's equal to either 1 or zero $c_i^2 = c_i$ and therefore the ratio of Eqs. (19) and (20) gives a simple condition for the critical color density $c = c_E$:

$$\gamma \frac{A}{T} c_E^{-\gamma} = 1. \tag{21}$$

In the same approximation one can obtain an analytic solution to the chemical equilibrium problem. We get

$$c = \left[\sum_{c_i=1} \chi_i(T)e^{q_i\mu_B/T} \right] e^{-Ac^{-\gamma}/T} \tag{22}$$

with

$$\chi_i(T) = n_{i,A=0}^{\text{eq}}(T, 0). \tag{23}$$

The sum in the brackets is a c-independent factor; we denote it by $\varphi(T, \mu_B) = T^3 a^3(\mu_B, T)$. The solution of Eq. (22) is given

by

$$z = \frac{\gamma Ac^{-\gamma}}{T} = -W\left(-\frac{\gamma A}{T}\varphi^{-\gamma}\right) \tag{24}$$

with $W(x)$ being Lambert's W function defining the solution of the transcendental equation $We^W = x$. The form occurring in Eq. (24), $-W(-x)$ has a real solution for positive x values only if $x > 1/e$. The argument $x = e^{-1}$ is the end point of stringy thermodynamics, the end-point temperature is connected to the effective string tension A as

$$\gamma \frac{A}{T_E} \varphi(T_E, \mu_B)^{-\gamma} = e^{-1}. \tag{25}$$

For considering massless particles as sources and endings of strings the factor $\varphi = a^3 T^3$ has a special form: $a(\mu_B, T)$ can be a function of the ratio μ_B/T only. At vanishing baryochemical potential, $\mu_B = 0$, a is a constant. In this case Eq. (25) can be resolved for the end-point temperature:

$$T_E = (\gamma Aea^{-3\gamma})^{\frac{1}{3\gamma+1}}. \tag{26}$$

In particular for $\gamma = 1/3$ it is $T_E = \sqrt{Ae/3a}$ and $a = a(\mu_B/T_E)$. This implicitly determines the end line $\mu_B(T_E)$.

III. MASSLESS BOLTZMANN GAS WITH STRINGS AT $\mu_B = 0$

In the followings we investigate the one-component massless Boltzmann gas at vanishing net baryon density. In this case a is constant and the pressure and energy density are expressed easily with help of explicit chemical equilibrium solution

$$c = a^3 T^3 \left[\frac{-W(-x)}{x} \right]^{-1/\gamma} \tag{27}$$

with

$$x = e^{-1} \left(\frac{T_E}{T} \right)^{3\gamma+1}. \tag{28}$$

Because for ideal Boltzmann gases the Boyle-Mariotte law holds, $\tilde{p} = cT$, and for vanishing mass $\tilde{e} = 3\tilde{p}$, the total pressure is reconstructed as [cf. Eq. (12)]:

$$p = cT\left(1 - \frac{z}{1-\gamma}\right), \tag{29}$$

where z is taken from the solution (24). The energy density is given by

$$e = cT\left[3 + \frac{z}{\gamma(1-\gamma)}\right]. \tag{30}$$

A. High-temperature asymptotics

When comparing with lattice results, the high-temperature asymptotics of the stringy massless Boltzmann EOS described implicitly by Eqs. (24), (29), and (30) is particularly important. In our model $T \to \infty$ is realized by the $x \to 0$ limit [we consider $\gamma \in (0, 1)$ only]. The Lambert W function has the series expansion $z = -W(-x) = x + x^2 + \cdots$ and hence the

high-temperature (low-x) color density behaves like

$$c = a^3 T^3 \left(1 - \frac{1}{\gamma} x + \cdots\right), \tag{31}$$

the pressure like

$$p = a^3 T^4 \left[1 - \frac{1}{\gamma(1-\gamma)} x + \cdots\right], \tag{32}$$

and the energy density like

$$e = a^3 T^4 \left[3 + \frac{3\gamma - 2}{\gamma(1-\gamma)} x + \cdots\right]. \tag{33}$$

The interaction measure is given by

$$e - 3p = a^3 T^4 \left[\frac{3\gamma + 1}{\gamma(1-\gamma)} x + \cdots\right]. \tag{34}$$

This analysis reveals that the high-temperature asymptotics of the pressure is T^4 as it should be; this fixes the value of a^3 by comparing to lattice EOSs [5.21 for 2+1 flavor QCD, 1.7 for pure SU(3)].

The power γ can be fixed from the subleading terms (and leading term in the interaction measure), $x T^4$. Using Eq. (28) such terms scale like $T^{3-3\gamma}$. To agree with lattice EOS results, i.e., $x T^4 \sim T^2$, the only possible choice we are left with is $\gamma = 1/3$. This agrees with our statistical argument for leading nonperturbative effects on the EOS at $T > T_c$, and in particular favors the straight string geometry in quark matter.

B. The zero pressure point as reference

Because on the lattice only positive-pressure states can be achieved by Monte Carlo techniques, the zero pressure point, T_0, is a more physical reference than the end point of chemical equilibrium. This occurs at $z_0 = 1 - \gamma = 2/3$ and due to $z = x e^z$ at $x_0 = \frac{2}{3} e^{-2/3} \approx 0.34$. However, $x_0 = e^{-1}(T_E/T_0)^2$ due to Eq. (28), so we arrive at $T_0/T_E = (x_0 e)^{-1/2} \approx 1.04$. In the stringy massless Boltzmann quark matter approximation these two temperatures are quite close to each other. Reference to the zero pressure point can be taken by substituting $x = x_0(T_0/T)^2 \approx 0.34(T_0/T)^2$ in general.

With respect to T_0 the scaled pressure and interaction measure are given as

$$\frac{p}{T^4} = \frac{x^3 a^3}{z^3} \left(1 - \frac{3z}{2}\right)$$
$$\frac{e - 3p}{T_0^2 T^2} = \frac{9 a^3 x^2}{z^2} x_0. \tag{35}$$

This reveals a factor of $9 x_0 \approx 3.04$ between the high-temperature asymptotic values of the scaled interaction measure $(e - 3p)/T^2 T_0^2$ and scaled pressure p/T^4. Although this gives a qualitatively correct picture of the quark matter EOS, unfortunately, for a quantitative comparison, Monte Carlo data are still spread over a remarkable range (cf. Fig. 1).

The whole temperature dependence of the stringy EOS in Boltzmann approximation works surprisingly well for the pressure but is only qualitatively correct for the interaction measure. This we plot in Fig. 1. The lattice data show the

most sudden jump in the T^2-scaled interaction measure to a nearly constant value. For comparison, the stringy model has a somewhat milder rise. Quantitatively, the constant value of the scaled interaction measure differs from the lattice result in the pure SU(3) case [22,23] as well as probably also in the full QCD case [24–27]. The massless Boltzmann approximation is clearly an oversimplification.

C. Degenerate Fermi gas at $T = 0$

Another analytically solvable case is represented by the degenerate Fermi gas at zero temperature with stringy contributions to thermodynamics. In this case, considering only one type of fermions with color charge $c_i = 1$ the quark chemical potential fixes the Fermi energy. Let us consider the case of massless fermions; the Fermi momentum is also $\mu = \mu_Q - A c^{-1/3}$, reduced by the stringy interaction. The chemical equilibrium relates the color density to this potential via

$$c = \frac{d}{6\pi^2} (\mu_Q - A c^{-1/3})^3, \tag{36}$$

with a degeneracy factor $d = (2S + 1) N_c N_f = 12$ for light quark matter and $\mu_Q = q \mu_B$, a value related to the baryocemical potential.

This value can be expressed by the effective quark chemical potential (Fermi energy) with introducing the parameter $B = (6\pi^2/d)^{1/3}$:

$$\mu_Q = \mu + \frac{AB}{\mu}. \tag{37}$$

Figure 2 plots $\mu_B = 3\mu_Q$ as a function of the quark μ and $\mu_B = 3\mu$ used in the additive quark model. The stable solution of Eq. (37) for μ at a given μ_Q belongs to the higher value of c:

$$c = \left(\frac{\mu + \sqrt{\mu^2 - 4AB}}{2B}\right)^3. \tag{38}$$

It is easy to see that the critical end-point value of the chemical potential in the stringy model is given by the minimum of the expression Eq. (37):

$$\mu_Q^{(E)} = 2\sqrt{AB}. \tag{39}$$

For smaller μ_Q there is no equilibrium, the strings pull the matter infinitely. Because the end point and the zero pressure points are close to each other both at zero baryochemical potential and at zero temperature, we conjecture that this is the case all over the hadronization line.

For obtaining the pressure and energy density we use the corresponding expressions for a degenerate Fermi gas:

$$\tilde{p} = \frac{d}{24\pi^2} (\mu - A c^{-1/3})^4$$
$$\tilde{e} = 3\tilde{p}. \tag{40}$$

On using Eq. (36) this leads to the total pressure

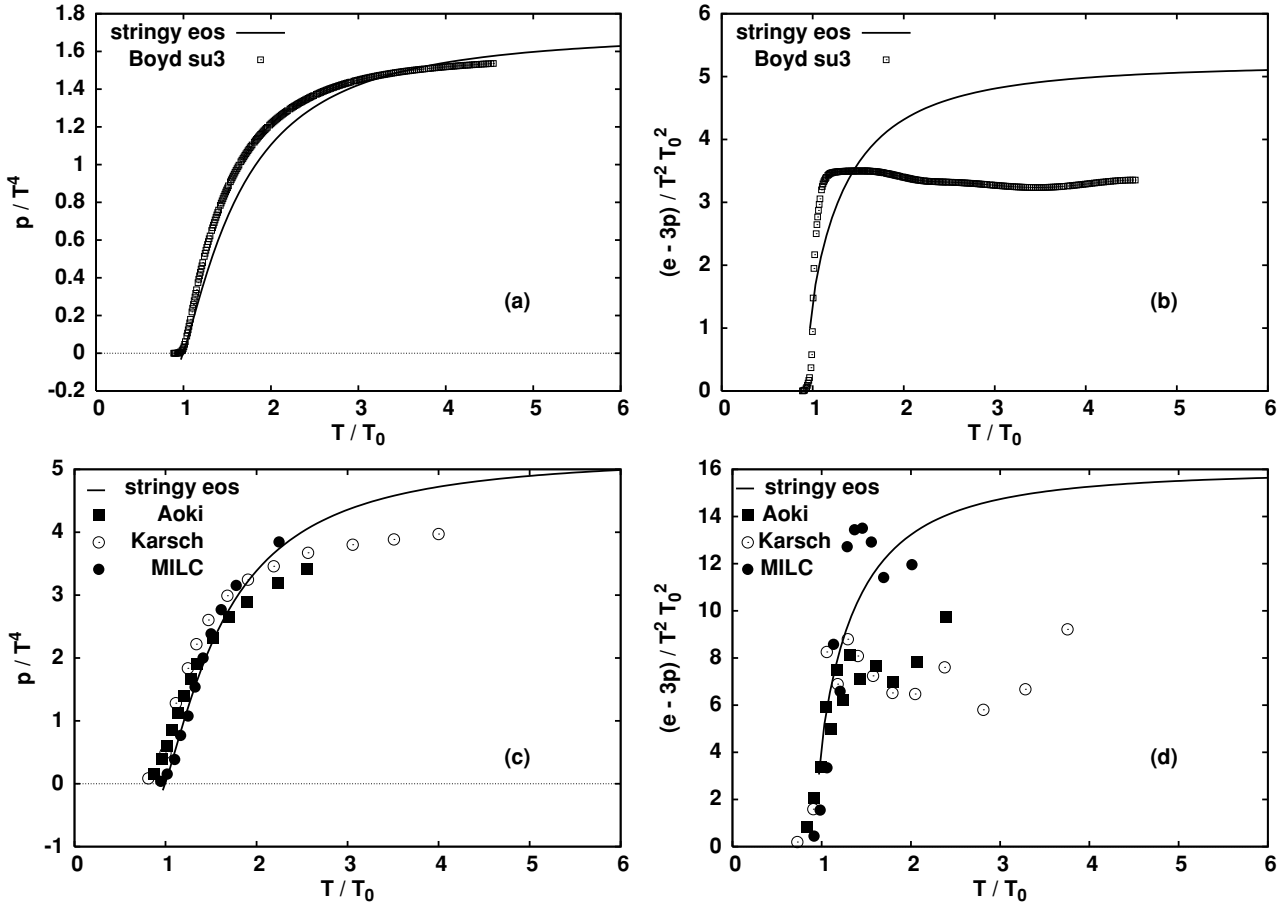$$p = \tfrac{1}{4}\mu c - \tfrac{3}{4} A c^{2/3}, \tag{41}$$

FIG. 1. Equation of state from lattice and from the massless Boltzmann stringy model. Pure SU(3) gauge theory results [22] to the top; 2+1 flavor QCD results [24,25,27] to the bottom. The scaled pressure (left sides) and interaction measure (right sides) curves are compared.

and to the energy density
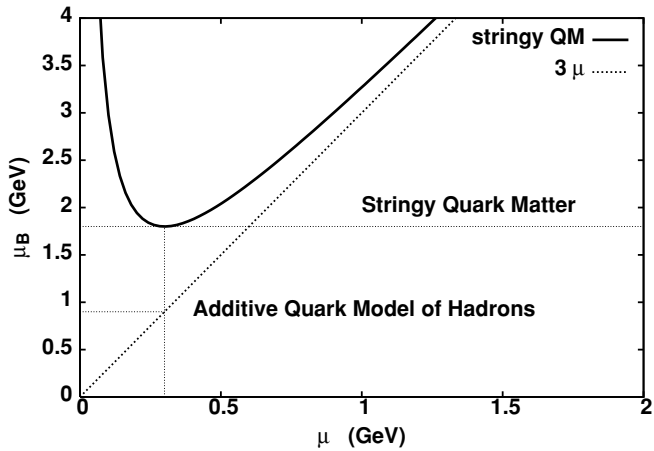
$$e = \tfrac{3}{4}\mu c + \tfrac{3}{4}Ac^{2/3}. \tag{42}$$

FIG. 2. The baryochemical potential, μ_B, is plotted as a function of the Fermi energy of quarks, μ, at $T = 0$ in the stringy QGP ($\mu_B = 3\mu_Q$) and in the additive model ($\mu_B = 3\mu$). The minimum point represents the end point of the stringy model; the corresponding additive quark model value is the half of it.

At zero pressure, $p = 0$, the energy per particle becomes $e/n = \mu$ and the value of the chemical potential at this point can be obtained from Eq. (41) as satisfying $\mu_0 c_0 = 3Ac_0^{2/3}$. Together with Eq. (36) this leads to

$$\mu_0 = 3\sqrt{AB/2} = \frac{3}{2\sqrt{2}}\mu_E \approx 1.06\,\mu_E \tag{43}$$

at the color density $c_0 = (2A/B)^{3/2}$. If this meets the $E/N = 1$ GeV line, then at $T = 0$ $\mu_0 = 1$ GeV.

Relating μ_Q to the hadronic baryochemical potential depends on the hadronization scenario. First, regarding the hadrons fitted in the statistical model as objects made in the constituent (additive) quark model, one considers that the quark number density is equal to the ideal one at $c = n_{id}(0, \mu_B/3)$. In this case at the end point of the stringy equation of state $\mu_Q^{(E)} = 2\mu^{(E)} = 2\mu_B/3$ is taken.

A second, more traditional, hadronization scenario assumes phase equilibrium between quark and hadronic matter resulting in $\mu_Q^{(E)} = \mu_B/3$. The physical difference between the two scenarios—beyond the rapidity of the hadronization process—lies in the treatment of energy stored in the strings: whether they built in the baryon rest mass constructing one baryon of each quark triple ($\mu_Q = 2\mu_B$/three-scenario), or
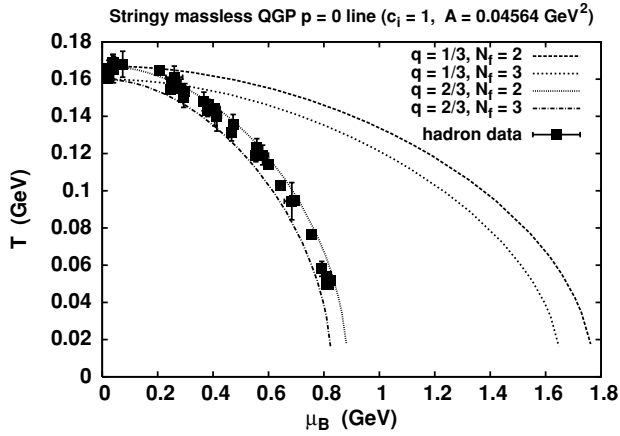
FIG. 3. The zero pressure line of the stringy, massless QGP (labeled by $q = 1/3$) with two and three quark flavors. We also plotted the stringy QGP fast hadronization lines with halved baryochemical potentials corresponding to an additive quark model interpretation of this process (labeled by $q = 2/3$). For comparison the statistical model results (boxes), fit by $T = 0.167 - 0.139 \mu_B^2 - 0.053 \mu_B^4$, Ref. [8] are indicated.

they just disappear and there will be as many baryons as the phase equilibrium dictates ($\mu_Q = \mu_B$/three-scenario).

The corresponding hadronization curves are plotted in Fig. 3. The $p = 0$ line (which is close to the end line of the stringy thermodynamics within a few percentages) is indicated for a quark-gluon plasma with massless gluons and $N_f = 2$ and $N_f = 3$ massless quark and antiquark flavors and assuming $\mu_B = 3\mu_Q = 6\mu$, respectively. For comparison the statistical model results are indicated by filled boxes. Following our previous comment about the additive quark model interpretation of the stringy QGP results, the same hadronization lines are also plotted as functions of three times the quark Fermi level, $\mu_B = 3\mu$. In this second case the coverage with the hadronic fit results is intriguing. In Fig. 4 we plot the energy per particle along the hadronization line; it is roughly constant for moderately low values of the baryochemical potential but then decreases somewhat.

Summarizing, the stringy EOS with $\gamma = 1/3$ comes close to describing the $T > T_0$ lattice QCD results and at the same time offers an explanation for the $E/N \approx 6T_0 = 1$ GeV hadronization condition found in the phenomenological statistical model. It differs from the simple bag model, where the pressure and energy density corrections are opposite and equal in magnitude; the present density-dependent mean interaction term represents a more general approach.
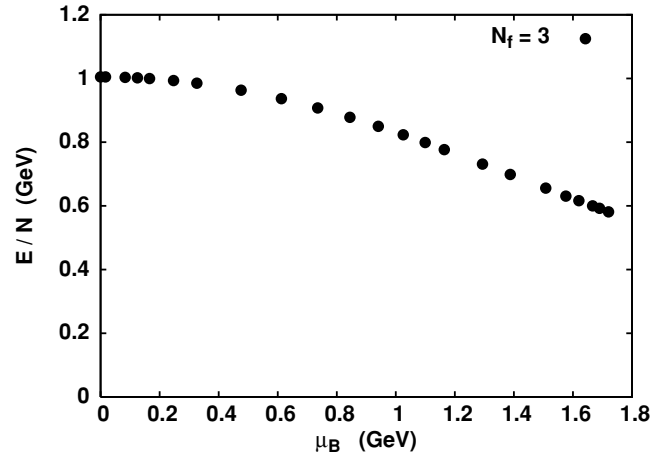

FIG. 4. The energy per particle for a stringy, massless QGP with three light flavors. Until the value $\mu_B \approx 0.8$ GeV the E/N ratio does not change more than 10%.

Although the massless Boltzmann approximation should be dropped in the view of lattice EOS data, the E/N value at zero pressure is a much more general result; it hopefully survives as a possible physical picture for the quark matter side at hadronization in relativistic heavy-ion collisions. Admittedly, there is no reason for the E/N ratios being equal in the hadronic and quark matter, but it is not easy to imagine how this value could be doubled from $3T_0$ to $6T_0$ at the hadronization moment.

The baryochemical potential dependence of the hadronic side fit by the statistical model can be met if the chemical potential is interpreted in the framework of an additive quark model (parameterized by the half of the stringy QGP value at the end point of the stringy model at $T = 0$).

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