PHYSICAL REVIEW C **78**, 034612 (2008)

Semiclassical calculations of observable cross sections in breakup reactions

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(Received 8 July 2008; published 29 September 2008)

We develop a semiclassical procedure to calculate breakup reaction products' angular and energy distributions in the laboratory frame of reference. The effects of the Coulomb and nuclear interaction potentials on the classical trajectories, as well as bound-bound, bound-continuum, and continuum-continuum couplings, are included. As an example we consider the $^{8}B + ^{58}Ni$ system at $E_{\rm lab} = 26$ MeV and find very good agreement with the available experimental data.

DOI: 10.1103/PhysRevC.78.034612 PACS number(s): 25.60.Gc, 24.10.−i

I. INTRODUCTION

The study of breakup reactions of loosely bound nuclei has received much attention from the nuclear physics community, specially because of the interest in the coupling with this channel in fusion reactions induced by those projectiles [1]. The earliest studies of the breakup process in collisions of weakly bound nuclei [2] were based on the polarization potential approach, in which the effects of breakup coupling are expressed as a dynamic polarization potential in the Schrödinger equation for the elastic channel. The breakup cross section then corresponds to the absorption resulting from the imaginary part of this potential. These models could only predict inclusive quantities like the total breakup cross section. Predictions of other observable quantities, as angular and energy distributions of the breakup fragments, required more powerful theories, taking into account the three-body kinematics. Nowadays, the standard procedure adopted to theoretically describe breakup reactions is to employ the Continuum Discretized Coupled-Channel (CDCC) method [3]. In a previous article [4] we developed a semiclassical approximation to this method, which greatly simplifies the numerical calculations involved in a quantum mechanical approach. There, it was demonstrated that the results of the semiclassical method compare favorably with the more sophisticated full quantum mechanical treatments. The main limitation of the calculations presented in Ref. [4] is that only the angular distribution of the center of mass of the projectile fragments was obtained. To calculate angular and energy distributions of the individual fragments in the laboratory frame, which are the measured quantities, it is necessary to consider a full three-body kinematics. This type of calculation was done by Tostevin, Nunes, and Thompson [5] for a full quantum coupled-channels calculation. In the present work we do it within the framework of the semiclassical approximation, with the intention of extending these calculations in the future to treat fusion reactions induced by weakly bound nuclei.

The article is organized as follows: in Sec. II we summarize the semiclassical model employed in our treatment of breakup reactions. In Sec. III we calculate the momentum distribution of the fragments in the projectile frame, which we employ in Secs. IV and V to calculate their angular and energy distributions, respectively. The calculated fragment energy distributions at several laboratory angles in the case of a $^{8}B + ^{58}Ni$ collision at $E_{\rm lab} = 25.8$ MeV are presented and compared to experimental data in Sec. VI. Our conclusions and suggestions for further work are given in Sec. VII.

II. DESCRIPTION OF THE SEMICLASSICAL METHOD

As in our previous article [4] we consider a weakly bound projectile, but in the present treatment we allow any number of bound states of this nucleus, instead of the only one considered in that work. This projectile is described in the two-cluster model as a particle (c_1) orbiting around a core (c_2), and we consider that the target nucleus is not excited as a result of the collision with the projectile. The scattering process is described in terms of the vector joining the centers of the projectile and the target, $\mathbf{R}$, and the intrinsic vector, $\mathbf{r}$, joining the centers of c_1 and c_2, as illustrated in Fig. 1. The initial ground state of the projectile is coupled to its excited bound states, if any, and to the continuum of $c_1 + c_2$. For a given impact parameter b the projectile-target relative motion is given by a classical trajectory $\mathbf{R}(t)$, and the intrinsic dynamics is treated as a time-dependent quantum mechanics problem. Analogously to Refs. [6] and [7], the interaction is

$$V(\mathbf{R}, \mathbf{r}) = V_1(\mathbf{r}_1) + V_2(\mathbf{r}_2)\,, \qquad (1)$$

where V_1 is the interaction between c_1 and the target and V_2 is the interaction of the target with c_2. From Fig. 1, we obtain the relations

$$\mathbf{r}_1 = \mathbf{R} + \frac{A_2}{A_P}\,\mathbf{r}\,, \quad \mathbf{r}_2 = \mathbf{R} - \frac{A_1}{A_P}\mathbf{r},$$

so that we can write

$$V(\mathbf{R}, \mathbf{r}) = V_1\left(\mathbf{R} + \frac{A_2}{A_P}\mathbf{r}\right) + V_2\left(\mathbf{R} - \frac{A_1}{A_P}\mathbf{r}\right). \qquad (2)$$

Above, A_1 and A_2 are the mass numbers of c_1 and c_2, and $A_P = A_1 + A_2$ is the mass number of the projectile. The potentials V_1 and V_2 contain nuclear and Coulomb parts. For the semiclassical calculation, the interaction is split into an "optical potential," V_0, and a coupling interaction, $U(\mathbf{R}, \mathbf{r})$, which leads to breakup. The real part of V_0 only affects the

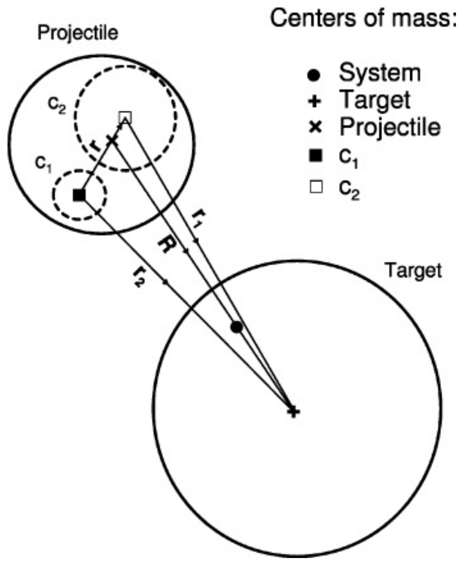

FIG. 1. Coordinates used in the main text to describe the breakup reaction of a weakly bound projectile by a heavy target.

classical trajectory of the projectile-target system, while its imaginary part produces absorption along the trajectory. We take

$$V_0(\mathbf{R}) = V(\mathbf{R}, \mathbf{r}=0) = V_1(\mathbf{R}) + V_2(\mathbf{R}), \tag{3}$$

and

$$U(\mathbf{R}, \mathbf{r}) = V(\mathbf{R}, \mathbf{r}) - \mathrm{Re}\,\{V_0(\mathbf{R})\}. \tag{4}$$

As a first step to derive the semiclassical coupled-channel equations, we introduce the set of intrinsic eigenstates of the projectile. We call s_i and ν_i the spin of particle c_i $(i = 1, 2)$ and its z projection, respectively, and denote $\chi_{s_i\nu_i}$ as their internal states. In the cluster model considered, the continuum eigenfunctions of total angular momentum J_α and z projection M_α can be written as

$$\Psi_\alpha \equiv \Psi_{\varepsilon_\alpha l_\alpha j_\alpha J_\alpha M_\alpha} = \sum_{m\,\nu_2} \langle j_\alpha\, m\, s_2\, \nu_2 \mid J_\alpha\, M_\alpha\rangle\, \varphi_{\varepsilon_\alpha l_\alpha j_\alpha m}\; \chi_{s_2 \nu_2}, \tag{5}$$

where $\varphi_{\varepsilon_\alpha l_\alpha j_\alpha m}$ describes the motion of the particle c_1 around the particle c_2, with orbital angular momentum l_α,

$$\varphi_{\varepsilon_\alpha l_\alpha j_\alpha m}(\mathbf{r}) = \mathcal{R}_{\varepsilon_\alpha l_\alpha j_\alpha}(r) \sum_{m_l \nu_1} \langle l_\alpha\, m_l s_1 \nu_1 | j_\alpha m\rangle\; Y_{l_\alpha\, m_l}(\hat{\mathbf{r}})\chi_{s_1\nu_1}, \tag{6}$$

with $\mathcal{R}_{\varepsilon_\alpha l_\alpha j_\alpha}(r)$ standing for the radial wave function of the relative c_1-c_2 motion. The label α represents the set of quantum numbers $\{\varepsilon_\alpha, l_\alpha, j_\alpha, J_\alpha, M_\alpha\}$. Because the spins of c_1 and c_2 have fixed values, they are not explicitly included in this set. The continuum states are normalized so as to satisfy the relation

$$\langle \Psi_\alpha | \Psi_\beta\rangle = \delta(\varepsilon_\alpha - \varepsilon_\beta)\, \delta(l_\alpha, l_\beta)\, \delta(j_\alpha, j_\beta)\, \delta(J_\alpha, J_\beta)\, \delta(M_\alpha, M_\beta). \tag{7}$$

The radial wave functions $\mathcal{R}_{\varepsilon_\alpha l_\alpha j_\alpha}(r)$ are solutions of the angular momentum-projected Schrödinger equation for the c_1-c_2 relative motion.

We now derive the semiclassical coupled-channel equations. The time-dependent wave function describing the c_1-c_2 relative motion in the projectile frame is expanded as

$$\Psi(b, t) = \sum_i c_i(b, t)\psi_i e^{-i\varepsilon_i t/\hbar} + \Psi_C(b, t), \tag{8}$$

where ψ_i represents the bound states and $\Psi_C(b, t)$ is the component of $\Psi(b, t)$ in the continuum,

$$\Psi_C(b, t) = \sum_{l_\alpha j_\alpha J_\alpha M_\alpha} \int d\varepsilon_\alpha\; c_\alpha(b, t)\; e^{-i\varepsilon_\alpha t/\hbar}\; \Psi_\alpha. \tag{9}$$

The amplitudes $c_i(b, t)$ and $c_\alpha(b, t)$ are functions of time, and also of the classical trajectory, which is specified by the collision energy and the impact parameter b. As the dependence of these coefficients on impact parameter is important to evaluate cross sections, we include it explicitly in the argument.

Because the collision is initiated with the projectile in its ground state $(i = 0)$, the amplitudes have the initial values $c_0(b, t \to -\infty) = 1$, $c_{i\neq 0}(b, t \to -\infty) = 0$, and $c_\alpha(b, t \to -\infty) = 0$. These amplitudes evolve as the collision proceeds and their final values contain the relevant information on the breakup cross section. The time evolution of these coefficients is obtained from the Schrödinger equation with the Hamiltonian

$$H = h(\mathbf{p}_r, \mathbf{r}) + U(t, \mathbf{r}), \tag{10}$$

where h is the Hamiltonian operator associated with the $c_1 - c_2$ relative motion. The time-dependent interaction $U(t, \mathbf{r}) \equiv U(\mathbf{R}(t), \mathbf{r})$ corresponds to the coupling potential of Eq. (4), with the replacement of the projectile-target separation vector $\mathbf{R}$ by its value along the classical trajectory, $\mathbf{R}(t)$. Note that, in contrast to Refs. [8–10], the classical trajectories of the present calculation take into account the effects of both the Coulomb and the nuclear parts of the optical potential. Using the expansion of Eqs. (8) and (9) in the time-dependent Schrödinger equation,

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = [h + U(t)]\; \Psi(t), \tag{11}$$

we get the semiclassical coupled-channel equations

$$i\hbar\, \dot{c}_i(b, t) = \sum_j U_{ij}(t)\, e^{i(\varepsilon_i - \varepsilon_j)t/\hbar}\, c_j(b, t) + \sum_{l_\beta j_\beta J_\beta M_\beta} \int d\varepsilon_\beta\, U_{i\beta}(t)\, e^{i(\varepsilon_i - \varepsilon_\beta)t/\hbar}\, c_\beta(b, t) \tag{12}$$

and

$$i\hbar\, \dot{c}_\alpha(b, t) = \sum_j U_{\alpha j}(t)\, e^{i(\varepsilon_\alpha - \varepsilon_j)t/\hbar}\, c_j(b, t) + \sum_{l_\beta j_\beta J_\beta M_\beta} \int d\varepsilon_\beta\, U_{\alpha\beta}(t)\, e^{i(\varepsilon_\alpha - \varepsilon_\beta)t/\hbar}\, c_\beta(b, t). \tag{13}$$

Above, the *form factors*

$$U_{\alpha\beta}(t) \equiv U_{\alpha\beta}(\mathbf{R}(t)) = \langle \Psi_\alpha \mid U(\mathbf{R}(t), \mathbf{r}) \mid \Psi_\beta\rangle \tag{14}$$

are the matrix elements of the coupling potential U of Eq. (4), with similar expressions for U_{ij}, $U_{i\alpha}$, and $U_{\alpha j}$. Using Eq. (5), and assuming that the coupling operator is independent of the spin of the core, they can be put in the form

$$U_{\alpha\beta}(t) = \sum_{mm'\nu_2} \langle j_\beta\, m' s_2\, \nu_2 \mid J_\beta\, M_\beta \rangle \times \langle J_\alpha\, M_\alpha \mid j_\alpha\, m\, s_2 \nu_2 \rangle\; f_{\alpha\beta,\nu_2}(t) \tag{15}$$

$$f_{\alpha\beta,\nu_2}(t) = f^{c_1}_{\alpha\beta,\nu_2}(t) + f^{c_2}_{\alpha\beta,\nu_2}(t), \tag{16}$$

with

$$f^{s}_{\alpha\beta,\nu_2}(t) = \langle \varphi_{\varepsilon_\alpha l_\alpha j_\alpha (M_\alpha-\nu_2)} | U_s(|\,\mathbf{R}(t) - \kappa_s \mathbf{r}\,|)|\, \varphi_{\varepsilon_\beta l_\beta j_\beta (M_\beta-\nu_2)} \rangle, \tag{17}$$

where $\kappa_s = -m_2/(m_1+m_2)$ when $s = c_1$ and $\kappa_s = m_1/(m_1+m_2)$ when $s = c_2$. Carrying out the usual multipole expansion, these matrix elements can be written as [11]

$$f^{s}_{\alpha\beta,\nu_2}(\mathbf{R}) = \sqrt{\pi} \sum_\lambda (-1)^{-M_\beta+\nu_2-\frac{1}{2}} \left[\frac{1+(-1)^{l_\alpha+l_\beta+\lambda}}{2}\right] \times \sqrt{\frac{(2j_\alpha+1)(2j_\beta+1)}{2\lambda+1}} \times \langle j_\alpha\ (\nu_2 - M_\alpha)\ j_\beta\ (M_\beta-\nu_2) \mid \lambda\ (M_\beta - M_\alpha)\rangle \times \left\langle j_\alpha\ \frac{1}{2}\ j_\beta\ \frac{-1}{2} \middle| \lambda\ 0 \right\rangle\ Y^{*}_{\lambda\,(M_\alpha-M_\beta)}(\hat{\mathbf{R}})\quad I^{\lambda s}_{\alpha\beta}(R), \tag{18}$$

where

$$I^{\lambda s}_{\alpha\beta}(R) = \frac{2}{2\lambda+1}\int_0^\infty dr\, r^2\, \mathcal{R}^*_\alpha(r)\, \mathcal{F}^s_\lambda(r,R) \mathcal{R}_\beta(r) \tag{19}$$

and

$$\mathcal{F}^s_\lambda(r,R) = \frac{2\lambda+1}{2}\int_0^\pi d\theta\, \sin\theta\, P_\lambda(\cos\theta) \times U(\sqrt{\kappa_s{}^2 r^2 + R^2 - 2\kappa_s r R\, \cos\theta})\,. \tag{20}$$

In these relations, $\mathcal{R}_\alpha(r)$ is the radial part of the $\varphi_\alpha(\mathbf{r})$ wave function, and $Y_{\lambda\mu}$ and P_λ are the usual spherical harmonics and Legendre polynomials, respectively. For simplicity of notation, we omit the argument in $\mathbf{R}(t)$ (that is, $\hat{\mathbf{R}}(t) \to \hat{\mathbf{R}}, R(t) \to R$).

For the practical solution of the semiclassical coupled-channel equations, we need to discretize the continuum. For this purpose, we follow the method of Ref. [3], dividing the continuum into energy bins of variable widths $\Delta_n\ (\equiv \Delta_1, \Delta_2, \Delta_3, \ldots)$ centered at $\varepsilon_n\ (\equiv \varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots)$. We construct the set q as the combination of a bin n and the angular momentum quantum numbers $q \equiv \{n, l, j, J, M\}$ and define that a state α belongs to the set if $\varepsilon_n - \Delta_n/2 < \varepsilon_\alpha < \varepsilon_n + \Delta_n/2$ and $l_\alpha = l$, $j_\alpha = j$, $J_\alpha = J$, and $M_\alpha = M$. We then approximate all the wave functions within an energy bin by the one at its center. This implies that the form factors are the same for each pair of states belonging to the same pair of sets: $U_{\alpha\alpha'} = U_{\beta\beta'}$ if $\alpha, \beta \in q$ and $\alpha', \beta' \in q'$. Consequently, these states are equally populated along the collision. If Ψ_α is normalized to one state with angular momenta $\{l, j, J, M\}$ per energy unit, the population of states within a set q is given by new amplitudes, $a_i = c_i$ and $a_q = \sqrt{\Delta_n} c_q$. It can be easily checked that these amplitudes satisfy the equations

$$i\hbar\, \dot{a}_i(b,t) = \sum_j U_{ij}(t)\, e^{i(\varepsilon_i-\varepsilon_j)t/\hbar}\, a_j(b,t) + \sum_{q'} U_{iq'}(t)\sqrt{\Delta_{n'}}\, e^{i(\varepsilon_i-\varepsilon_{n'})t/\hbar} a_{q'}(b,t) \tag{21}$$

and

$$i\hbar\, \dot{a}_q(b,t) = \sum_i U_{qi}(t)\sqrt{\Delta_n}\, e^{i(\varepsilon_n-\varepsilon_i)t/\hbar} a_i(b,t) + \sum_{q'} U_{qq'}(t)\sqrt{\Delta_n}\sqrt{\Delta_{n'}} e^{i(\varepsilon_n-\varepsilon_{n'})t/\hbar} a_{q'}(b,t). \tag{22}$$

III. MOMENTUM DISTRIBUTION

From the semiclassical calculation described in the previous section we obtain the amplitudes $a_i(b,t)$ and $a_q(b,t)$. However, to calculate cross sections and energy distributions in the laboratory frame of reference we need to determine the relative momentum distribution. For a collision with impact parameter b at time t in which the spins of c_1 and c_2 have projections ν_1 and ν_2, respectively, the relative momentum distribution after breakup is given by

$$A_{\nu_1\nu_2}(\mathbf{k},t,b) = \left\langle \Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t) \middle| \Psi_C(b,t)\right\rangle, \tag{23}$$

where $\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t)$ is the wave function describing the scattering of c_1 from c_2, with incoming boundary conditions. In Appendix A, we calculate this wave function and evaluate the relative momentum distribution of Eq. (23). The result is

$$A_{\nu_1\nu_2}(\mathbf{k},t,b) = \frac{-\hbar}{\sqrt{\mu k}} \sum_{ljJM} \langle JM | j(M+\nu_2) j_2(-\nu_2)\rangle \times \langle j(M+\nu_2)| l(M+\nu_1+\nu_2) s(-\nu_1)\rangle \times Y^*_{l(M+\nu_1+\nu_2)}(\hat{\mathbf{k}})\, e^{i\sigma_l}\, i^l\, (-1)^{J-M} \times c_{\varepsilon_k l j J(-M)}(b,t), \tag{24}$$

where μ is the reduced mass associated with the c_1-c_2 relative motion. This distribution is the starting point for the derivation of the differential cross sections presented in the following two sections.

IV. INCLUSIVE ANGULAR DISTRIBUTIONS

The interpretation of the momentum distribution is that $|A_{\nu_1\nu_2}(\mathbf{k}, t\to\infty, b)|^2$ is the probability per unit of volume in $\mathbf{k}$ space that the projectile breaks into two fragments with relative momentum $\hbar\mathbf{k}$ in a collision with impact parameter b. The triple differential cross section for this process can then be calculated from

$$\frac{d\sigma}{d^3\mathbf{k}} = \frac{\pi}{K^2}\sum_{L\nu_1\nu_2}(2L+1)\left|A^L_{\nu_1\nu_2}(\mathbf{k})\right|^2, \tag{25}$$

where $A^L_{\nu_1\nu_2}(\mathbf{k}) = A_{\nu_1\nu_2}(\mathbf{k}, t \to +\infty, b)$, and $\hbar K$ and $L = Kb$ are the relative momentum and the orbital angular momentum in units of $\hbar$, respectively, of the projectile-target relative motion. Because $d^3\mathbf{k} = k^2\,dk\,d\omega$ and $\varepsilon = \hbar^2 k^2/2\mu$, we can write

$$\frac{d\sigma}{d\omega} = \frac{\pi}{K^2}\sum_{L\nu_1\nu_2}(2L+1)\int \left|A^L_{\nu_1\nu_2}(\mathbf{k})\right|^2 \frac{\mu k}{\hbar^2}d\varepsilon. \quad (26)$$

Owing to the discretization of the continuum, the integral over $d\varepsilon$ becomes a sum over the index n and the above equation takes the form

$$\frac{d\sigma}{d\omega} = \frac{\pi}{K^2}\sum_{L\nu_1\nu_2 n}(2L+1)\left|A^L_{\nu_1\nu_2}(\mathbf{k}_n)\right|^2 \frac{\mu k_n}{\hbar^2}\Delta\varepsilon_n. \quad (27)$$

The next step is to calculate the angular distribution for one of the breakup fragments, say c_1, in the laboratory frame. Because we are dealing with a three-body system (c_1, c_2, and the target), the relations between angles in the laboratory and in the projectile frames depend on the breakup energy, ε_n. We adopt the following notation: quantities in the projectile frame are represented by lowercase letters; the relative velocity associated with $\mathbf{k}$ is denoted by $\mathbf{u}$ ($\mathbf{u} = \hbar\mathbf{k}/\mu$) and its angular coordinates are θ and ϕ; variables in the laboratory frame are represented by capital letters; the velocity of c_1 is $\mathbf{V}_1$ and its angular coordinates are Θ_1 and Φ_1. The angular distribution of fragment c_1 is given by

$$\frac{d\sigma(\Theta_1,\Phi_1)}{d\Omega_1} = \frac{\pi}{K^2}\sum_{L\nu_1\nu_2 n}\frac{\mu k_n}{\hbar^2}(2L+1) \times \left|A^L_{\nu_1\nu_2}(\mathbf{k}_n)\right|^2 \frac{\sin\theta}{\sin\Theta_1}J_n^{-1}\,\Delta\varepsilon_n. \quad (28)$$

We have introduced the Jacobian

$$J_n = \left|\frac{\partial(\Theta_1,\Phi_1)}{\partial(\theta,\phi)}\right| = \frac{\sin\theta}{\sin\Theta_1}\cdot\frac{u\;|\mathbf{u}\cdot\mathbf{V}_1|}{V_1^3}, \quad (29)$$

where $u = |\mathbf{u}|$. We use the subscript n in the Jacobian to stress the fact that it depends on the energy bin. The derivation of Eq. (29) is presented in Appendix B. To use Eq. (28), it is necessary to obtain the angles θ and ϕ associated with Θ_1 and Φ_1. From kinematics, more than a solution may exist. In such cases one must sum their respective contributions.

Because in the experiment the reaction plane is not determined, one should take the average over the azimuthal angle. That is,

$$\frac{d\sigma(\Theta_1)}{d\Omega_1} = \frac{1}{2\pi}\int \frac{d\sigma(\Theta_1,\Phi_1)}{d\Omega_1}d\Phi_1. \quad (30)$$

V. INCLUSIVE ENERGY DISTRIBUTIONS

To calculate energy distributions we begin with the triple differential cross section for breakup, Eq. (25), and the relation

$$\frac{d\sigma}{d\varepsilon\,d\Omega_k} = \frac{\mu k}{\hbar^2}\;\frac{d\sigma}{d^3\mathbf{k}}. \quad (31)$$

In the laboratory frame, this cross section is

$$\left[\frac{d\sigma(\Theta_1,\Phi_1,\varepsilon)}{d\varepsilon\,d\Omega_1}\right]_{\varepsilon=\varepsilon_n} = \frac{\pi\mu k_n}{\hbar^2K^2}\sum_{L\nu_1\nu_2}(2L+1) \times \frac{\sin\theta}{\sin\Theta_1}J_n^{-1}\left|A^L_{\nu_1\nu_2}(\mathbf{k}_n)\right|^2. \quad (32)$$

We stress the fact that a given pair of angles (Θ_1, Φ_1) may correspond to more than one pair of angles in the projectile frame. In this case, their contributions should be summed. Because the collision has axial symmetry, the reaction plane is not defined. The cross section should then be averaged over the azimuthal angle as

$$\frac{d\sigma(\Theta_1,\varepsilon)}{d\varepsilon\,d\Omega_1} = \frac{1}{2\pi}\int \frac{d\sigma(\Theta_1,\Phi_1,\varepsilon)}{d\varepsilon\,d\Omega_1}d\Phi_1. \quad (33)$$

To obtain the energy distribution in the laboratory frame, one follows the above procedure. However, one should also transform the fragment energy to the laboratory frame. To do so we start from the expression

$$\frac{d\sigma(\Theta_1,\Phi_1,E_1)}{dE_1\,d\Omega_1} = \frac{\pi}{K^2}\sum_{L\nu_1\nu_2 n}(2L+1) \times \frac{\mu k_n}{\hbar^2}\;\frac{\sin\theta}{\sin\Theta_1}\mathcal{J}_{nL}^{-1}\left|A^L_{\nu_1\nu_2}(\mathbf{k}_n)\right|^2, \quad (34)$$

where $\mathcal{J}_{nL}$ is the Jacobian

$$\mathcal{J}_{nL} = \left|\frac{\partial(E_1,\Theta_1,\Phi_1)}{\partial(\varepsilon,\theta,\phi)}\right| = \left(\frac{A_P - A_1}{A_P}\right)\frac{\sin\theta}{\sin\Theta_1}\;\frac{u}{V_1}. \quad (35)$$

Note that the above Jacobian depends on L and n. Equation (35) is derived in Appendix B. Finally, the desired cross section is obtained evaluating the average over Φ_1,

$$\frac{d\sigma(\Theta_1,E_1)}{dE_1\,d\Omega_1} = \frac{1}{2\pi}\int \frac{d\sigma(\Theta_1,\Phi_1,E_1)}{dE_1\,d\Omega_1}d\Phi_1. \quad (36)$$

VI. BREAKUP OF ^{8}B

In this section we consider the ^{8}B breakup on a ^{58}Ni target, for which there are experimental data of energy distributions at $E_{\rm lab} = 25.8$ MeV [12]. This system has already been analyzed by Tostevin, Nunes, and Thompson [5] by means of a full quantum calculation using the coupled-channels continuum discretization method. We take the same potentials used in our previous work [4]. The continuum is discretized into 19 bins that cover the energies between 0 and 8 MeV. In all our calculations continuum-continuum coupling is included. The calculations have been done taking the values of L from 0 up to 260 in steps of 5. Following Ref. [5] the ^{7}Be intrinsic spin is neglected, but the proton spin is considered.

It is to be noted that in semiclassical calculations the energy is not conserved as the projectile follows a classical trajectory determined by initial conditions corresponding to the elastic channel. To correct for this we use the standard prescription of Alder and Winther [13] for inelastic scattering; that is, we consider a trajectory associated with a center of mass energy, $E_{\rm c.m.}$, given by the geometrical mean between the

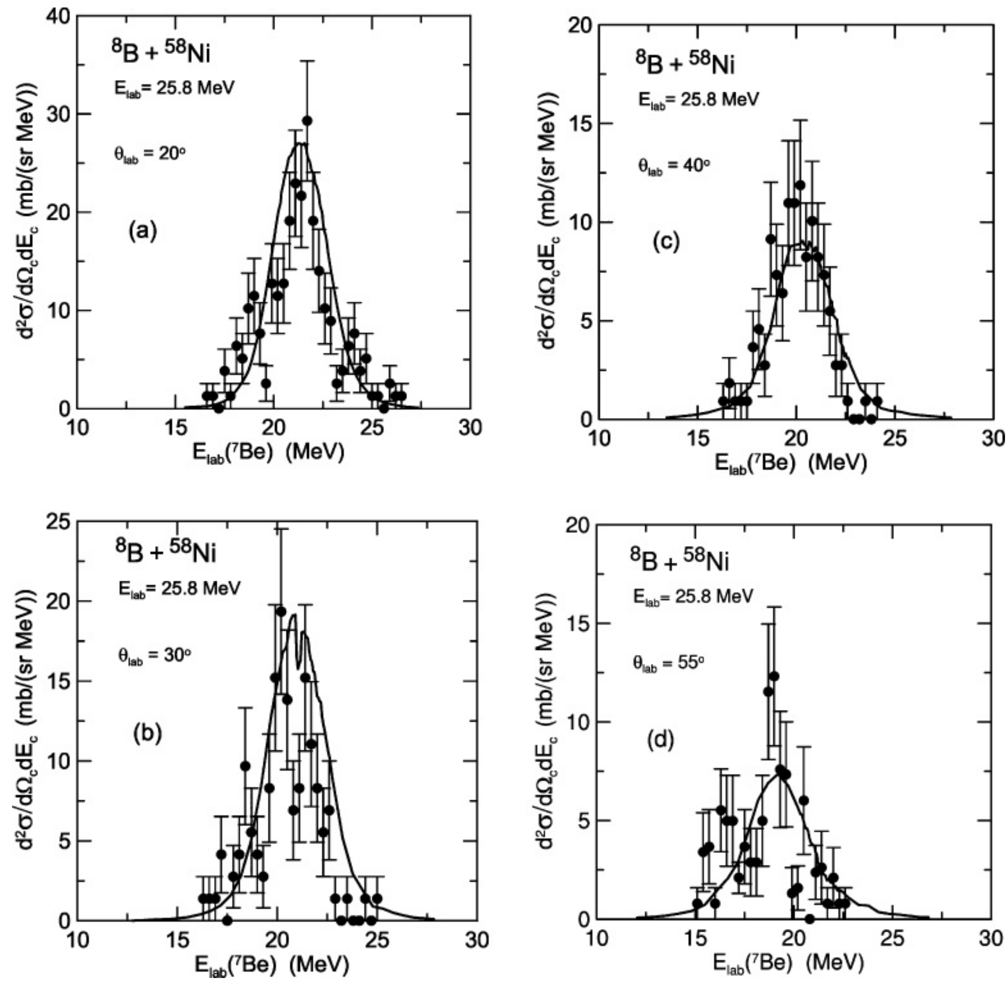

FIG. 2. Inclusive energy distribution of the ^{7}Be fragment detected at the laboratory angles: (a) 20°, (b) 30°, (c) 40°, and (d) 55° degrees, with respect to the incident beam direction. The energy of the incident ^{8}B projectiles is $E_{\rm lab} = 25.8$ MeV. The solid circles are the experimental data of Ref. [12] and the solid lines are the results of the semiclassical calculations of the present work.

one corresponding to the elastic channel, $E^{(o)}_{\rm c.m.}$, and $E^{(o)}_{\rm c.m.} - \epsilon$, where ϵ is the average value of the breakup energy.

Figure 2 shows the double differential cross sections $d^2\sigma(\Theta_c, E_c)/d\Omega_c dE_c$ as a function of E_c for given values of Θ_c. The suffix c indicates the ^{7}Be fragment. Symbols are the experimental data of Ref. [12]; solid lines are the cross sections obtained from Eqs. (28) and (30). We note that the calculations presented in this work reproduce the experimental data as satisfactorily as the full quantal calculations of Ref. [5]. Thus the semiclassical approximation to the CDCC presented in this work leads to reliable results.

VII. CONCLUSIONS

In a previous publication [4] we have calculated the angular distribution of the center of mass of the two particles in which the projectile breaks, using a semiclassical approach. There we have demonstrated that such a calculation was in very good agreement with a full quantum one [7]. In this work we have extended the method, developing the necessary three-body formalism to obtain laboratory angular and energy distributions of the ejectiles.

We have exemplified with the ^{8}B + ^{58}Ni system at $E_{\rm lab} =$ 25.8 MeV, for which both data and quantum mechanical calculations are available. We have considered the effect on the trajectory of the Coulomb and nuclear interactions and taken into account all couplings in the projectile dynamics.

The comparison with experimental data is very good and gives support to this type of calculation, which has the advantage of being both simpler and easier to visualize than full quantum coupled-channels calculations. We believe that this type of calculation can be important not only to analyze experimental results of breakup reactions but also to serve as a basis to describe more complex reaction dynamics, such as the incomplete or sequential complete fusion reaction processes that take place in collisions involving weakly bound nuclei. In fact, as a byproduct of the calculations, one has detailed information of the probability amplitudes associated with the position and momenta of the breakup fragments produced in the collision process. From that knowledge the eventual fusion of one or both of the fragments with the target nucleus can be determined employing any available fusion computational procedure. Work along these lines is already in process.

ACKNOWLEDGMENTS

We are indebted to Dr. V. Guimarães for providing us with the experimental data in tabular form. We have benefited from very fruitful discussions with Drs. A. J. Pacheco, J. O. Fernández Niello, and D. Heimann. This work was supported in part by the FAPERJ-CNPq-PRONEX Program (Brazil) and by the PEDECIBA (Uruguay).

APPENDIX A: SEMICLASSICAL MOMENTUM DISTRIBUTION

In this appendix we calculate the momentum distribution of Eq. (23),

$$A_{\nu_1\nu_2}(\mathbf{k},t,b) = \langle \Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t)|\Psi_C(b,t)\rangle.$$

Here, $\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t)$ is the scattering wave function with incoming boundary conditions given by

$$\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t) = \mathcal{T}\Psi^{(+)}_{-\nu_1-\nu_2}(-\mathbf{k},-t), \tag{A1}$$

where $\Psi^{(+)}_{\nu_1\nu_2}(\mathbf{k},t) = \Psi^{(+)}_{\nu_1\nu_2}(\mathbf{k})\exp(-i\varepsilon_k t/\hbar)$ is the scattering wave function with outgoing boundary conditions and $\mathcal{T}$ is the time reversal operator.

We introduce the spin-angle basis

$$\mathcal{Y}_{ls_1js_2JM}(\hat{\mathbf{r}}) = i^l \sum_{mm_l\nu_1\nu_2} \langle jms_2\nu_2|JM\rangle \times \langle lm_l s_1\nu_1|jm\rangle\, Y_{lm_l}(\hat{\mathbf{r}})\chi_{s_1\nu_1}\chi_{s_2\nu_2} \tag{A2}$$

and express the continuum wave functions of Eq. (5) in this set of states. That is,

$$\Psi_{\varepsilon ljJM} = i^{-l}\mathcal{Y}_{ls_1js_2JM}(\hat{\mathbf{r}})\;\; \mathcal{R}_{\varepsilon lj}(r). \tag{A3}$$

In Eq. (A3),

$$\mathcal{R}_{\varepsilon lj}(r) = -\sqrt{\frac{2\mu k}{\pi\hbar}}\frac{u_{lj}(kr)}{kr}, \tag{A4}$$

where the radial wave function $u_{lj}(kr)$ is defined such that outside the range of the nuclear potential it has the form

$$u_{lj}(kr) = \frac{i}{2}e^{-i\delta_{lj}}\left[H^{(-)}_l(kr) - \bar{S}_{lj}H^{(+)}_l(kr)\right]. \tag{A5}$$

Above, $H^{(+)}_l(kr)(H^{(-)}_l(kr))$ is the Coulomb wave function with outgoing (ingoing) boundary condition and $\bar{S}_{lj} = \exp(2i\delta_{lj})$ are the components of the nuclear S matrix. With the normalization of Eqs. (A4) and (A5), the continuum states are normalized as in Eq. (7). The incident wave also can be expanded in the spin-angle basis,

$$\begin{aligned}\Phi_{\nu_1\nu_2}(\mathbf{k},\mathbf{r}) &= \Phi_{\mathrm{Cou}}(\mathbf{k},\mathbf{r})\,\chi_{s_1\nu_1}\chi_{s_2\nu_2}\\ &= \frac{2}{(2\pi)^{1/2}}\sum_{ljJM} Y_{lm_l}(\hat{\mathbf{k}})\langle JM|jms_2\nu_2\rangle\\ &\quad\times\langle jm|lm_l s_1\nu_1\rangle e^{i\sigma_l}\mathcal{Y}_{ls_1js_2JM}(\hat{\mathbf{r}})\frac{\hat{F}_l(kr)}{kr},\end{aligned} \tag{A6}$$

where $\Phi_{\mathrm{Cou}}(\mathbf{k},\mathbf{r})$ is the incident Coulomb wave with momentum $\hbar\mathbf{k}$, σ_l is the Coulomb phase shift at the lth partial wave, and $F_l(kr)$ is the corresponding regular Coulomb function. The scattering wave function has a similar expansion. It can be written as

$$\begin{aligned}\Psi^{(+)}_{\nu_1\nu_2}(\mathbf{k}) = \frac{-\hbar}{\sqrt{\mu k}}\sum_{ljJM} &Y^*_{l(M-\nu_1-\nu_2)}(\hat{\mathbf{k}})\,\langle JM|j(M-\nu_2)s_2\nu_2\rangle\\ &\times\langle j(M-\nu_2)|l(M-\nu_1-\nu_2)s_1\nu_1\rangle\\ &\times e^{i\sigma_l}\,\mathcal{Y}_{lj_1jj_2JM}(\hat{\mathbf{r}})\mathcal{R}_{\varepsilon_k lj}(r),\end{aligned} \tag{A7}$$

or, with Eq. (A3),

$$\begin{aligned}\Psi^{(+)}_{\nu_1\nu_2}(\mathbf{k}) = \frac{-\hbar}{\sqrt{\mu k}}\sum_{ljJM}&\langle JM|j(M-\nu_2)s_2\nu_2\rangle\\ &\times\langle j(M-\nu_2)|l(M-\nu_1-\nu_2)s_1\nu_1\rangle\\ &\times Y^*_{l(M-\nu_1-\nu_2)}(\hat{\mathbf{k}})\,e^{i\sigma_l}i^l\Psi_{\varepsilon_k ljJM}.\end{aligned} \tag{A8}$$

Now we use the above equation and the time reversal relation, Eq. (A1), to calculate $\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k})$. We obtain

$$\begin{aligned}\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k}) = \frac{-\hbar}{\sqrt{\mu k}}\sum_{ljJM}&\langle JM|j(M+\nu_2)s_2(-\nu_2)\rangle\\ &\times\langle j(M+\nu_2)|l(M+\nu_1+\nu_2)s_1(-\nu_1)\rangle\\ &\times Y_{l(M+\nu_1+\nu_2)}(-\hat{\mathbf{k}})e^{-i\sigma_l}i^{-l}\mathcal{T}\Psi_{\varepsilon_k ljJM}.\end{aligned} \tag{A9}$$

Applying the time reversal operator to Eq. (5), we get

$$\begin{aligned}\mathcal{T}\Psi_{\varepsilon ljJM} = \sum_{\nu_1\nu_2}&\langle j(M-\nu_2)s_2\nu_2\mid JM\rangle\\ &\times\langle l(M-\nu_1-\nu_2)s_1\nu_1\mid j(M-\nu_2)\rangle\\ &\times\mathcal{R}_{\varepsilon lj}(r)\,Y^*_{l(M-\nu_1-\nu_2)}(\hat{\mathbf{r}})(-1)^{s_1+\nu_1}\\ &\times\chi_{s_1-\nu_1}(-1)^{s_2+\nu_2}\,\chi_{s_2-\nu_2},\end{aligned} \tag{A10}$$

To obtain the above result, we used the time reversal properties of the spin function states,

$$\mathcal{T}\chi_{s_i\nu_i} \;=\; (-1)^{s_i+\nu_i}\,\chi_{s_i-\nu_i}. \tag{A11}$$

Taking into account properties of the spherical harmonics and of the Clebsch-Gordan coefficients we can easily see that

$$\mathcal{T}\Psi_{\varepsilon ljJM} = (-1)^{J-l-M}\Psi_{\varepsilon ljJ(-M)}. \tag{A12}$$

We can now evaluate the momentum distribution. Using the above equations in Eq. (23), we have

$$\begin{aligned}A_{\nu_1\nu_2}(\mathbf{k},t,b) = \frac{-\hbar}{\sqrt{\mu k}}\sum_{ljJMl'j'J'M'}&\langle JM|j(M+\nu_2)s_2(-\nu_2)\rangle\\ &\times\langle j(M+\nu_2)|l(M+\nu_1+\nu_2)s_1(-\nu_1)\rangle\\ &\times Y^*_{l(M+\nu_1+\nu_2)}(-\hat{\mathbf{k}})\;\;e^{i\sigma_l}\;\;i^l\;\;e^{\frac{i}{\hbar}\varepsilon_k t}\;\;(-1)^{J-l-M}\\ &\times\int d\varepsilon e^{-\frac{i}{\hbar}\varepsilon t}c_{\varepsilon ljJM}(b,t)\\ &\times\langle\Psi_{\varepsilon_k ljJ(-M)}|\Psi_{\varepsilon l'j'J'M'}\rangle.\end{aligned} \tag{A13}$$

From the normalization conditions, Eq. (7), and the property $Y_{lm}(-\hat{\mathbf{k}}) = (-1)^l Y_{lm}(\hat{\mathbf{k}})$ we get the final expression

$$A_{\nu_1\nu_2}(\mathbf{k},t,b) = \frac{-\hbar}{\sqrt{\mu k}} \sum_{ljJM} \langle JM|j(M+\nu_2)s_2(-\nu_2)\rangle \times \langle j(M+\nu_2)|l\,(M+\nu_1+\nu_2)s_1(-\nu_1)\rangle \times Y^*_{l(M+\nu_1+\nu_2)}(\hat{\mathbf{k}})\, e^{i\sigma_l}\, i^l\, (-1)^{J-M} \times c_{\varepsilon_k ljJ(-M)}(b,t). \tag{A14}$$

APPENDIX B: JACOBIANS FOR THE TRANSFORMATION FROM PROJECTILE TO LABORATORY FRAME

Let us consider that the motion of the projectile relative to the target is in the xz plane. The semiclassical calculations are performed in the projectile frame while we want the angular distribution of one breakup fragment in the laboratory frame. The starting point is the probabilities $P(b,\mathbf{k})$, where b is the impact parameter and $\mathbf{k}$ is the relative momentum of the fragments in the projectile frame. We adopt the following notation: velocities in the projectile frame are represented by $\mathbf{u}$; velocities in the projectile-target center of mass frame by $\mathbf{v}$, and velocities in the laboratory frame by $\mathbf{V}$. The velocity of fragment 1 in the projectile frame, $\mathbf{u}_1$, is related to the relative momentum as

$$\mathbf{u}_1 = \frac{A_2}{A_P}\frac{\hbar\mathbf{k}}{\mu}, \tag{B1}$$

and its components are given in terms of the spherical coordinates as

$$u_{1x} = u_1 \sin\theta\cos\phi, \tag{B2}$$

$$u_{1y} = u_1 \sin\theta\sin\phi, \tag{B3}$$

$$u_{1z} = u_1 \cos\theta. \tag{B4}$$

The velocity of fragment 1 in the laboratory frame is

$$\mathbf{V}_1 = \mathbf{u}_1 + \mathbf{V}_P = \mathbf{u}_1 + \mathbf{v}_{\text{c.m.}} + \mathbf{V}_0, \tag{B5}$$

where $\mathbf{V}_P$ is the final velocity of the projectile in the laboratory frame. It corresponds to the sum of the projectile-target relative velocity in their center of mass frame, $\mathbf{v}_{\text{c.m.}}$, with the velocity of this center of mass in the laboratory frame, $\mathbf{V}_0 = V_0\hat{\mathbf{z}}$. The projectile velocity can be expressed in terms of the deflexion angle, $\Theta_d \equiv \Theta_d(b)$, as

$$\mathbf{V}_P = v_{\text{c.m.}}(\sin\Theta_d\hat{\mathbf{x}} + \cos\Theta_d\hat{\mathbf{z}}) + V_0\hat{\mathbf{z}}.$$

Above, the velocities $v_{\text{c.m.}}$ and V_0 are given by

$$v_{\text{c.m.}} = \sqrt{\frac{2E_{\text{c.m.}}}{\mu}\frac{A_T + A_P}{A_T A_P}}, \tag{B6}$$

$$V_0 = \sqrt{\frac{2E_{\text{c.m.}}}{\mu}\frac{A_P}{(A_T + A_P)\,A_T}}. \tag{B7}$$

The components of the vector $\mathbf{V}_1$ are

$$V_{1x} \equiv V_1\sin\Theta_1\cos\Phi_1 = u_1\sin\theta\cos\phi + v_{\text{c.m.}}\sin\Theta_d \tag{B8}$$

$$V_{1y} \equiv V_1\sin\Theta_1\sin\Phi_1 = u_1\sin\theta\sin\phi, \tag{B9}$$

$$V_{1z} \equiv V_1\cos\Theta_1 = u_1\cos\theta + v_{\text{c.m.}}\cos\Theta_d + V_0, \tag{B10}$$

and the spherical coordinates of fragment 1 in the laboratory frame can be expressed in terms of V_{1x}, V_{1y}, and V_{1z} as

$$\Theta_1 = \tan^{-1}\left(\frac{V_{1z}}{\sqrt{V_{1x}^2 + V_{1y}^2}}\right);\quad \Phi_1 = \tan^{-1}\left(\frac{V_{1y}}{V_{1x}}\right). \tag{B11}$$

We begin evaluating the Jacobian defined in Eq. (29),

$$J_n = |J_{11}J_{22} - J_{12}J_{21}|, \tag{B12}$$

where

$$J_{11} = \frac{\partial\Theta_1}{\partial\theta},\quad J_{12} = \frac{\partial\Theta_1}{\partial\phi}, \quad J_{21} = \frac{\partial\Phi_1}{\partial\theta},\quad J_{22} = \frac{\partial\Phi_1}{\partial\phi}. \tag{B13}$$

We must express the components of $\mathbf{V}_1$ in terms of the angles θ and ϕ and use Eq. (B11) to evaluate the partial derivatives. Following this procedure and performing lengthy calculations we obtain

$$J_n = \frac{u|\mathbf{u}\cdot\mathbf{V}_1|\sin\theta}{V_1^3\sin\Theta}. \tag{B14}$$

The next stage is to calculate the Jacobian

$$\mathcal{J}_{nL} = \left|\frac{\partial(E_1,\Theta_1,\Phi_1)}{\partial(\varepsilon,\theta,\phi)}\right| = \left\|\begin{matrix} \frac{\partial E_1}{\partial\varepsilon} & \frac{\partial E_1}{\partial\theta} & \frac{\partial E_1}{\partial\phi} \\ \frac{\partial\Theta_1}{\partial\varepsilon} & \frac{\partial\Theta_1}{\partial\theta} & \frac{\partial\Theta_1}{\partial\phi} \\ \frac{\partial\Phi_1}{\partial\varepsilon} & \frac{\partial\Phi_1}{\partial\theta} & \frac{\partial\Phi_1}{\partial\phi} \end{matrix}\right\|. \tag{B15}$$

The derivatives $\frac{\partial\Theta_1}{\partial\theta}$, $\frac{\partial\Theta_1}{\partial\phi}$, $\frac{\partial\Phi_1}{\partial\theta}$, and $\frac{\partial\Phi_1}{\partial\phi}$ were calculated to obtain J_n . $\frac{\partial\Theta_1}{\partial\varepsilon}$ and $\frac{\partial\Phi_1}{\partial\varepsilon}$ can be obtained using Eq. (B11), and the partial derivatives of E_1 can be calculated from

$$E_1 = \tfrac{1}{2}m_1V_1^2 = \tfrac{1}{2}m_1\big[u_1^2 + v_{\text{c.m.}}^2 + V_0^2 + 2v_{\text{c.m.}}V_0\,\cos\theta_d + 2u_1v_{\text{c.m.}}\sin\theta\,\cos\phi\,\sin\theta_d + 2u_1\big(v_{\text{c.m.}}\cos\theta_d + V_0\big)\,\cos\theta\big], \tag{B16}$$

which can be obtained by squaring Eqs. (B8), (B9), and (B10) and summing, and where m_1 is the mass of fragment 1. The relation between u_1 and ε is

$$u_1^2 = 2\left(1 - \frac{A_1}{A_P}\right)^2\,\frac{\varepsilon}{\mu}. \tag{B17}$$

Again, after a lengthy calculation, we arrive at

$$\mathcal{J}_{nL} = \left|\frac{\partial(E_1,\Theta_1,\Phi_1)}{\partial(\varepsilon,\theta,\phi)}\right| = \left(\frac{A_P - A_1}{A_P}\right)\frac{\sin\theta}{\sin\Theta_1}\,\frac{u}{V_1}. \tag{B18}$$

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