Target mass corrections to the matrix elements in nucleon spin structure functions

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Target mass corrections to the twist-4 terms $\tilde{f}_2^{p,n,d}$ as well as to the leading-twist $\tilde{a}_2$ are discussed.

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We know that different approaches [1–7] have been employed to study higher-twist effect on nucleon structure functions. There have also been several phenomenological analyses of nucleon structure functions that studied quark-hadron duality and extracted the higher-twist contributions (like the ones of the twist-3 and twist-4 terms) from experimental measurements [8–11]. Those analyses are going to become more and more accurate because more and more precise measurements of the nucleon spin structure functions g_1 and g_2 are becoming available [11,12]. High precision data have been employed to study the validity of the quark-hadron duality for the nucleon structure function F_2 [13] and even for spin asymmetry A_1 by HERMES [14] recently. Several experiments to test the higher-twist effect on nucleon spin structure functions are being carried out in the Jefferson Laboratory [9,15].

It has been pointed out in the literature that target mass corrections (TMCs) should be considered in studies of nucleon structure functions [16] in a moderate Q^2 region and in studies of the Bloom-Gilman quark-hadron duality [17,18]. Therefore, only after important target mass corrections are removed from the experimental data can one reasonably extract the higher-twist effect [18]. There have been several papers about target mass corrections to $F_{1,2}(x, Q^2)$ and $g_{1,2}(x, Q^2)$ in the past [19]. Recently, target mass corrections to nucleon structure functions for polarized deep-inelastic scattering have been systematically studied [20,21]. In our previous work [22], TMCs to the twist-3 matrix element in nucleon structure functions are addressed. In this brief report, TMCs to the twist-4 terms $\tilde{f}_2^{p,n,d}$ as well as to the leading-twist $\tilde{a}_2$ are discussed.

Consider the Cornwall-Norton (CN) moments $g_{1,2}^{(n)}(Q^2) = \int_0^1 x^{n-1} g_{1,2}(x, Q^2)dx$, we know that the first CN moment of g_1 can be generally expanded in inverse powers of Q^2 in operator production expansion (OPE) [1,2] as

$$g_1^{(1)} = \int_0^1 dx g_1(x, Q^2) = \sum_{\tau=2,\text{even}}^{\infty} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}, \tag{1}$$

with the coefficients μ_τ relating to the nucleon matrix elements of operators of twist $\leqslant \tau$. In Eq. (1), the leading-twist (twist-2) component μ_2 is determined by the matrix elements of the axial vector operator $\bar{\psi}\gamma_\mu\gamma_5\psi$, summed over various quark flavors. The coefficient of the $1/Q^2$ term, $\mu_4 = \frac{1}{9}M^2(\tilde{a}_2 + 4\tilde{d}_2 + 4\tilde{f}_2)$, contains the contributions from the twist-2 $\tilde{a}_2$, twist-3 $\tilde{d}_2$, and twist-4 $\tilde{f}_2$, respectively. Usually, $\tilde{d}_2$ is extracted from the third moments of the measured $g_1(x, Q^2)$ and $g_2(x, Q^2)$ by using $\tilde{d}_2(Q^2) = \int_0^1 x^2(2g_1(x, Q^2) + 3g_2(x, Q^2))dx$. However, it is pointed out that this method for $\tilde{d}_2$ ignores the target mass corrections to the third moments of $g_{1,2}$, and the target mass corrections play a sizable role in $\tilde{d}_2$ [22] in a moderate Q^2 region.

To further estimate TMCs to the twist-4 of nucleon spin structure functions, one may assume that the contributions from a higher-twist term with $\tau > 6$ can be ignored [23] or assume this term to be a constant (neglecting any possible Q^2 dependence) [8]. Based on the first assumption, we have

$$\tfrac{4}{9}y^2\tilde{f}_2 + \tfrac{1}{2}\tilde{a}_0 = g_1^{(1)} - \tfrac{1}{9}y^2(\tilde{a}_2 + 4\tilde{d}_2). \tag{2}$$

When no TMCs are considered, $\tilde{a}_2$ and $\tilde{d}_2$ can be simply expressed by the CN moments of nucleon spin structure functions, and we get

$$\tfrac{4}{9}y^2\tilde{f}_2^{(0)} + \tfrac{1}{2}\tilde{a}_0 = g_1^{(1)} - \tfrac{2}{9}y^2\big(5g_1^{(3)} + 6g_2^{(3)}\big). \tag{3}$$

When TMCs are considered, we must employ the Nachtmann moments

$$\begin{aligned} M_1^{(n)}(Q^2) &= \int_0^1 dx \frac{\xi^{n+1}}{x^2}\left\{\left[\frac{x}{\xi} - \frac{n^2}{(n+2)^2}y^2x\xi\right]g_1(x, Q^2) - y^2x^2\frac{4n}{n+2}g_2(x, Q^2)\right\}, \\ M_2^{(n)}(Q^2) &= \int_0^1 dx \frac{\xi^{n+1}}{x^2}\left\{\frac{x}{\xi}g_1(x, Q^2) + \left[\frac{n}{n-1}\frac{x^2}{\xi^2} - \frac{n}{n+1}y^2x^2\right]g_2(x, Q^2)\right\}, \end{aligned} \tag{4}$$

where the Nachtmann variable $\xi = \frac{2x}{1+r}$ (with $r = \sqrt{1+4y^2x^2}$), $y^2 = M^2/Q^2$, and x is the Bjorken variable. The two Nachtmann moments are simultaneously constructed by the two spin structure functions $g_{1,2}$. If $g_{1,2}(x, Q^2)$ are replaced by the ones with TMCs (see Refs. [20–22]), one can easily expand the two Nachtmann moments with respect to y^2. The results are $M_1^{(n)} = \frac{1}{2}\tilde{a}_{n-1} + \mathcal{O}(y^8)$ and $M_2^{(n)} = \frac{1}{2}\tilde{d}_{n-1} + \mathcal{O}(y^8)$. The two expressions explicitly tell that, different from the CN moments, one can get the contributions of a pure twist-2 with spin-n and a pure twist-3 with spin-$(n-1)$ operators from the Nachtmann moments. The advantage of the Nachtmann moments means that they contain only dynamical

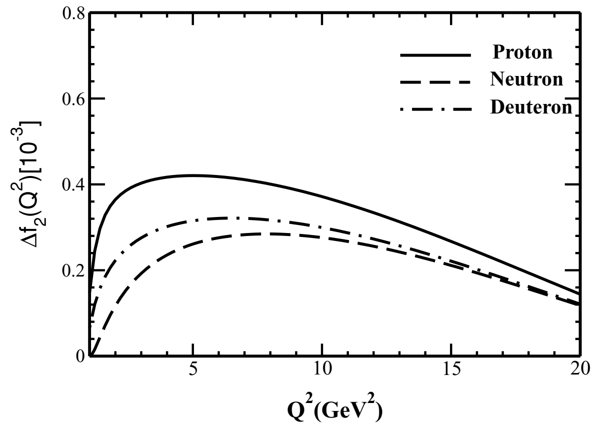

FIG. 1. Difference Δf_2. The solid, dashed, and dotted-dashed curves are the results of the proton, neutron, and deuteron, respectively.

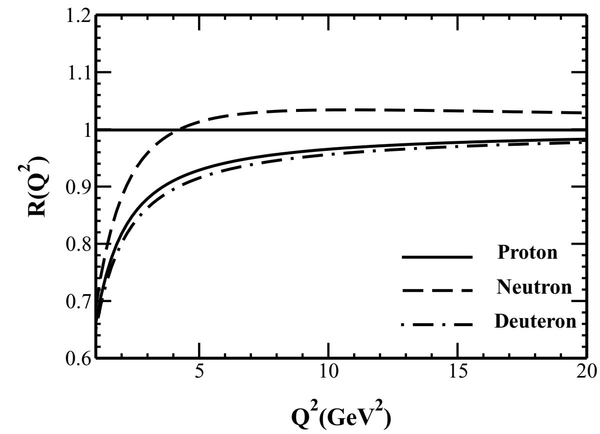

FIG. 2. Ratio R_{a_3}. The solid, dashed, and dotted-dashed curves are the results of the proton, neutron, and deuteron, respectively.

higher-twists, which are the ones related to the correlations among the partons. As a result, they are constructed to protect the moments of nucleon spin structure functions from the target mass corrections. Consequently, to extract the higher-twist effect, say twist-3 or twist-4 contribution, one is required to consider the Nachtmann moments instead of the CN moments.

We use the Nachtmann moments to express $\tilde{a}_n$ and $\tilde{d}_n$ and obtain

$$\frac{4}{9}y^2\tilde{f}_2+\frac{1}{2}\tilde{a}_0 = g_1^{(1)}-\frac{2}{9}y^2\int_0^1\frac{\xi^4}{x^2}dx\left[\left(\frac{5x}{\xi}-\frac{9}{25}y^2x\xi\right)g_1(x,Q^2) + \left(6\frac{x^2}{\xi^2}-\frac{27}{5}y^2x^2\right)g_2(x,Q^2)\right]. \tag{5}$$

Thus, the TMC to the twist-4 contribution, due to the two different moments, is $\Delta f_2=\tilde{f}_2-\tilde{f}_2^0$. Here, we employ the parametrization forms of the spin structure functions of the proton, neutron, and deuteron [11,12] to estimate Δf_2. Note that the well-known Wandzura and Wilczek (WW) relation [24] $g_2(x,Q^2)=g_2^{WW}(x,Q^2)=-g_1(x,Q^2)+\int_x^1\frac{g_1(y,Q^2)}{y}dy$ is valid if only the leading-twist is considered, and TMCs to the twist-2 contribution do not break the WW relation. However, if the higher-twist operators, like twist-3 and twist-4, are considered, the WW relation $g_2(x,Q^2)=g_2^{WW}(x,Q^2)$ no longer preserves. Thus, one may write $g_2(x,Q^2)=g_2^{WW}(x,Q^2)+\bar{g}_2(x,Q^2)$ [8,9], where $\bar{g}_2$ represents the violation of the WW relation. The nonvanishing value of $\bar{g}_2$ just results from the higher-twist effect.

One can calculate Δf_2 with the parametrizations of $g_{1,2}$. The results are plotted in Fig. 1. We see that the typical values of the differences are in the order of $10^{-3}\sim 10^{-4}$. There are several theoretical estimated values for the twist-4 term $\tilde{f}_2$ in the literature (see Table I), like the ones of the bag model [4], of the QCD sum rule [5,6], of the empirical analyses of the experimental measurements [8,23], and of the instanton model [25]. Comparing the estimated differences in Fig. 1 to those estimated values displayed in Table I, we conclude that TMCs to the twist-4 term $\tilde{f}_2$ are negligible (less than 2%). We also find that Δf_2 of the proton and deuteron are always larger than that of the neutron.

In addition, we check TMCs to the leading twist term (with spin-3) $\tilde{a}_2$. If no TMCs are considered, $\tilde{a}_2^{(0)}=2g_1^{(3)}$. When TMCs are taken into account, we get, from the Nachtmann moments,

$$\tilde{a}_2=\int_0^1 2\frac{\xi^4}{x^2}dx\left\{\left[\frac{x}{\xi}-\frac{9}{25}y^2x\xi\right]g_1(x,Q^2) - \frac{12}{5}y^2x^2g_2(x,Q^2)\right\}. \tag{6}$$

Figure 2 displays the Q^2 dependence of the ratio $R=\tilde{a}_2/\tilde{a}_2^{(0)}$ for the proton, neutron, and deuteron targets. The sizable effect of TMCs is clearly seen, because the ratios all diverge from unity obviously. When $Q^2\sim 5$ GeV^2, the effect of TMCs is still about 10% for the proton and deuteron targets. In addition, the effect on the proton and deuteron targets is much larger than that on the neutron. Here the Q^2 dependences of the three ratios are similar to those of the twist-3 terms [22]. The

TABLE I. The estimated values for $\tilde{f}_2$ in different approaches in the literature.

Reference	$\tilde{f}_2^p$	$\tilde{f}_2^n$	Reference	$\tilde{f}_2^p$	$\tilde{f}_2^n$
Ref. [4]	0.050 ± 0.034	-0.018 ± 0.017	Ref. [5]	-0.028	0
Ref. [6]	0.037 ± 0.006	0.013 ± 0.006	Ref. [8]	–	0.034 ± 0.043
Ref. [23]	-0.10 ± 0.05	-0.07 ± 0.08	Ref. [25]	-0.046	0.038

sizable effect tells us that TMCs should be taken into account. Therefore, to estimate the matrix element of $\tilde{a}_2$, the Nachtmann moments must be employed.

In summary, we have explicitly shown the target mass corrections to the twist-4 $\tilde{f}_2$ term and to the leading-twist one (spin-3) $\tilde{a}_2$. It is reiterated that to precisely and consistently extract the contributions of the leading-twist $\tilde{a}_2$, of the twist-3 $\tilde{d}_2$, and of the twist-4 $\tilde{f}_2$ with a definite spin and with a moderate Q^2 value, one is required to employ the Nachtmann moments $M_{1,2}$ instead of the CN moments. Our results show that TMCs evidently play a role in $\tilde{a}_2$ when Q^2 is small. The above conclusion does not change if different parametrizations of the structure functions are employed. We also show that TMCs to the twist-4 term are much smaller than those to the twist-3 term and to the leading-twist term.

Finally, the expressions of the differences Δf_2 and Δa_2 between the CN and Nachtmann moments are

$$\Delta f_2 = \tilde{f}_2 - \tilde{f}_2^{(0)} = \frac{y^2}{10}\left\{\left[\frac{384}{5}g_1^{(5)} - 234y^2 g_1^{(7)} + 736y^4 g_1^{(9)}\right] + \left[87g_2^{(5)} - 258y^2 g_2^{(7)} + 798y^4 g_2^{(9)}\right]\right\} + \mathcal{O}(y^8),$$

$$\begin{aligned}\Delta a_2 &= \tilde{a}_2 - \tilde{a}_2^0 = 2M_1^{(3)} - 2g_1^{(3)} \\ &= y^2\left\{\left[-\frac{168}{25}g_1^{(5)} + \frac{108}{5}y^2 g_1^{(7)} - \frac{352}{5}y^4 g_1^{(9)}\right] + \left[-\frac{24}{5}g_2^{(5)} + \frac{96}{5}y^2 g_2^{(7)} - \frac{336}{5}y^4 g_2^{(9)}\right]\right\} + \mathcal{O}(y^8).\end{aligned} \quad (7)$$

One sees that the two expressions mainly depend on the higher-moment of the nucleon spin structure function and, therefore, on the spin structure function in the large-x region. In most of the empirical analyses of the Ellis-Jaffe sum rule (the first moment of g_1), the contribution from the spin structure function in the large-x region is assumed to be trivial, because it behaves like $(1-x)^3$. When the higher-moment of the spin structure function is considered, the effect of the spin structure function in the large-x region becomes important. Consequently, the measurement of the nucleon spin structure function in the large-x region with high precision is required.

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