

Kaon condensation in neutron stars in relativistic mean field theory with isovector-scalar channel interaction

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We study the properties of neutron stars involving antikaon condensation with three different models in the framework of relativistic mean field (RMF) approximation theory: the Glendenning-Moszkowski (GM) model, the Zimanyi-Moszkowski (ZM) model, and the hybrid derivative coupling (HD) model. We take the isovector-scalar (δ -meson) channel interaction into account in our calculations and find that large mass neutron stars with kaon condensation can exist if the kaon optical potential is appropriately weak. The δ -meson channel interaction has a significant influence on the property of neutron stars and the effects are a little different for the three models.

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I. INTRODUCTION

A neutron star (NS) is a natural laboratory to investigate the composition, the phase structure, and its transition of dense and cold/hot strongly interacting matter. Theoretical investigations have shown that, with the increase of density, some new degree of freedom other than nucleons such as hyperons [1–4], pion and kaon condensation [5–27], even quarks (see, for example, Refs. [28–36]) may appear in the interior of neutron stars. All of these exotic components in the core of NS soften the equation of state (EOS) [1,33,37,38] and lower the maximum mass and the gravitational redshift of neutron stars. On the other hand, a recent radio observation of the object PSR J0751+1807 yielded a neutron star mass (2.1 ± 0.2) M_{\odot} with 1σ error bars [39], and, 95% confidence, one of the pulsars Ter 5 I and J has a mass larger than $1.68 M_{\odot}$ [40]. In addition, recent observed gravitational redshift of the X-ray burster EXO 0748-676 implies $z \simeq 0.35$ [41] even though it is still in doubt (see for example Ref. [42]) and further confirmations are necessary. Since the observed mass and the redshift have been implemented for years to constrain the theoretical predictions of the EOS [31,32,34,37,38,43–45], the composition of large mass neutron stars has become then a hot topic for debate (see, for example, Refs. [46,47]). The existence of hyperons or quark matter in the core of neutron stars under the constraint of recent observations has been reinvestigated in various approaches (see, for example, Refs. [48–56]). The kaon condensation has also been revisited in the modified quark-meson-coupling model [57], chiral Lagrangian approach [58], and the Glendenning-Moszkowski (GM) model [27] in the framework of the relativistic mean field approximation.

It has been shown that the relativistic mean field (RMF) approximation, as a hadronic effective field theory, is able to reproduce not only the saturation properties of (astro)nuclear matter but also the ground state properties of finite nuclei (for

reviews see Refs. [1,59–64]), and even the nuclear shape phase transition [65]. Concerning the interactions among hadrons, one usually considers isoscalar-scalar, isoscalar-vector, and isovector-vector channels. In the mid-1990s, the contribution of the isovector-scalar [by exchanging the δ meson, or $a_0(980)$] channel was taken into account [66], and soon was expected to be important in neutron stars [67–73]. Technically, when one implements the RMF approximation to study the properties of neutron star matter, one has the Glendenning-Moszkowski (GM) model [8], the Zimanyi-Moszkowski (ZM) model [74], and the hybrid derivative coupling (HD) model [75], which handle the scalar meson field dependence of the effective mass of the nucleon in different ways. The GM model, even with the isovector-scalar channel being included, has been widely used to study not only asymmetric nuclear matter and neutron star property [2–4,7,8,66–73,76–78] but also heavy ion collisions [79–81]. However, the ZM model and the HD model [74,75,82–85] have not yet been implemented so popularly, especially in the case of including the interaction in the isovector-scalar (δ -meson) channel up to now. With the GM model, it has been shown that the δ -meson leads to a larger repulsion and a definite splitting of proton and neutron effective masses in dense neutron-rich matter and the involvement of the δ -meson mainly affects the behavior of the system at high density [70]. Nevertheless, the effects of the δ -meson on the observables such as the mass and redshift of neutron stars in the ZM and HD models have not yet been discussed. Since (anti)kaon condensation may appear in the high density range, we will then go deep into the topic of the property of neutron stars involving kaon condensation and manifest the effect of the interaction with δ -meson exchange in the three models in this work.

In order to illuminate the effect of the δ -meson channel (or the difference between the cases with and without the δ -meson) in the models, we have calculated the mass-radius relation, the gravitational redshifts and the relative population of the components of neutron stars in the GM, HD, and ZM models. This paper is organized as follows. We give in Sec. II a concise description of the GM model, the HD model, and the ZM model in the framework of RMF theory with kaon

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condensation in the case of the isovector-scalar δ -meson. In Sec. III we describe our calculations of neutron stars' structure and compare the results with and without the δ -meson and those among the three models. Finally, we give a summary in Sec. IV.

II. RMF APPROXIMATION THEORY WITH KAON CONDENSATION

The RMF approximation theory is quite successful in understanding the properties of nuclear matter and finite nuclei (see for example Refs. [1,59–64]). In the standard RMF theory, the strong interaction between baryons is described by the exchange of isoscalar-scalar σ , isoscalar-vector ω , and isovector-vector ρ mesons through the Yukawa couplings. This picture has been consistently extended to include (anti)kaon condensation [12,13,17,27] and hyperons [2–4,8,45]. However, it does not include the contribution of the isovector-scalar δ -meson until the mid-1990s [66], which is soon expected to be important in neutron stars [67–73]. In this work we take the generalized RMF theory including the contribution of the δ -meson. Since the experiment has provided evidence that the interaction between Σ -hyperon and nucleon is repulsive [86], the Ξ -hyperon may emerge only in the very high density region of neutron stars [1], for simplicity, we are not concerned about the contribution of hyperons in the present work. The Lagrangian density of a nuclear system then reads

$$\begin{aligned} \mathcal{L} = & \sum_N \bar{\Psi}_N [i\gamma_\mu \partial^\mu - m_N^* - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma^\mu \vec{t} \cdot \vec{\rho}_\mu] \Psi_N \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) + \frac{1}{2} (\partial_\mu \vec{\delta} \cdot \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu} \\ & + \sum_\ell \bar{\Psi}_\ell (i\gamma_\mu \partial^\mu - m_\ell) \Psi_\ell, \end{aligned} \quad (1)$$

where m_N^* is the effective mass of nucleon, $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$ are the vector meson field tensors. The self-interactions of mesons are induced only in the σ -channel and read [87] $U(\sigma) = \frac{1}{3} b m (g_\sigma \sigma)^3 + \frac{1}{4} c (g_\sigma \sigma)^4$, in which m is the bare mass of the nucleon. The last line is the free Lagrangians for leptons which refer to e^- and μ^- here.

For the effective mass of the nucleon, m_N^* , which arises from the effect of the coupling with mesons, by extending that developed in Ref. [8], we have

$$m_N^* = m - g_{\sigma N} \sigma - t_{3N} g_{\delta N} \delta_3, \quad (2)$$

in which t_{3N} is the third-component of the isospin. Such a model has been used extensively [66–73,78] and is usually referred to as a GM model. We can also go along the way developed in Ref. [74] and define the effective mass of baryons as

$$m_N^* = \frac{m}{1 + \frac{g_{\sigma N} \sigma + t_{3N} g_{\delta N} \delta_3}{m}}. \quad (3)$$

Hereafter we denote this model as the ZM model (the difference between the present one and the original one is only that we include the interaction by exchanging the isovector-scalar δ -meson). Once more, extending the one developed in Ref. [75], we have the effective mass of the nucleon in the hybrid derivative coupling (HD) model as

$$m_N^* = m \frac{1 - \frac{g_\sigma \sigma + t_{3N} g_\delta \delta_3}{2m}}{1 + \frac{g_\sigma \sigma + t_{3N} g_\delta \delta_3}{2m}}. \quad (4)$$

From these expressions of the effective mass we can easily see that the isovector-scalar meson leads to a neutron-proton effective mass splitting due to the third-component of the isospin. It is also remarkable that the HD model is the intermediate between the ones in Eqs. (2) and (3).

Solving the Euler-Lagrange equations for meson fields in RMF approximation along the conventional way we get

$$m_\sigma^2 \sigma = g_\sigma \left[\sum_N C_N^i \rho_N^S - b m (g_\sigma \sigma)^2 - c (g_\sigma \sigma)^3 \right], \quad (5)$$

$$m_\delta^2 \delta_3 = g_\delta \sum_N t_{3N} C_N^i \rho_N^S, \quad (6)$$

$$m_\omega^2 \omega_0 = g_\omega (\rho_p + \rho_n), \quad (7)$$

$$m_\rho^2 \rho_{03} = g_\rho (\rho_p - \rho_n)/2, \quad (8)$$

in which ρ_N , ρ_N^S are the nucleon and scalar density, respectively. The coefficient C_N^i is different for the three models and the index $i = 1, 2, 3$ stands for the GM, HD, and ZM model, respectively. In detail, the coefficient is specialized as $C_N^1 = 1$, $C_N^2 = (1 + \frac{g_\sigma \sigma + t_{3N} g_\delta \delta_3}{2m})^{-2}$ and $C_N^3 = (\frac{m_N^*}{m})^2$. In addition, the field equations involve several parameters, for instance $\frac{g_\sigma}{m_\sigma}$, $\frac{g_\delta}{m_\delta}$, $\frac{g_\omega}{m_\omega}$, $\frac{g_\rho}{m_\rho}$, b , c , and so on.

With the increase of the density, kaon condensation may appear in the interior of the neutron stars. We take the Lagrangian of kaon condensation as the same as that in Refs. [12,13,15,17,18,27] which reads

$$\mathcal{L}_K = \mathcal{D}_\mu^* K^* \mathcal{D}^\mu K - m_K^* K^* K, \quad (9)$$

where $\mathcal{D}_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \vec{t}_K \cdot \vec{\rho}_\mu$ is the covariant derivative and the (anti)kaon effective mass should be extended from $m_K^* = m_K - g_{\sigma K} \sigma$ to $m_K^* = m_K - g_{\sigma K} \sigma - \frac{1}{2} g_{\delta K} \delta_3$ due to the involvement of the δ -meson. At the same time, the dispersion relation should also be modified as $\omega_K = m_K - g_{\sigma K} \sigma - \frac{1}{2} g_{\delta K} \delta_3 - g_{\omega K} \omega_0 - \frac{1}{2} g_{\rho K} \rho_{03}$. The $g_{\sigma K}$, $g_{\omega K}$, $g_{\rho K}$, and $g_{\delta K}$ are coupling constants between the kaon and the corresponding meson, respectively.

Now the meson field equations with kaon condensation and δ -meson can be written as

$$m_\sigma^2 \sigma = g_\sigma \left[\sum_N C_N^i \rho_N^S - b m (g_\sigma \sigma)^2 - c (g_\sigma \sigma)^3 \right] + g_{\sigma K} \rho_K, \quad (10)$$

$$m_\delta^2 \delta_3 = g_\delta \sum_N t_{3N} C_N^i \rho_N^S + g_{\delta K} \rho_K, \quad (11)$$

$$m_\omega^2 \omega_0 = g_\omega (\rho_p + \rho_n) - g_{\omega K} \rho_K, \quad (12)$$

$$m_\rho^2 \rho_{03} = g_\rho (\rho_p - \rho_n)/2 - g_{\rho K} \rho_K, \quad (13)$$

in which $\rho_K = 2m_K^* K^* K = 2(\omega_K + g_{\omega K}\omega_0 + \frac{1}{2}g_{\rho K}\rho_0)$ $K^* K$ is the kaon density. The expressions of the total energy density and the pressure including the contribution of the δ meson can be written as

$$\begin{aligned} \varepsilon = & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{b}{3}m(g_\sigma\sigma)^3 + \frac{c}{4}(g_\sigma\sigma)^4 + \frac{1}{2}m_\delta^2\delta_3^2 \\ & + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_0^2 + m_K^*\rho_K \\ & + \sum_{i=N,\lambda} \frac{v_i}{(2\pi)^3} \int_0^{k_F^i} d^3k \sqrt{k^2 + m_i^{*2}}, \end{aligned} \quad (14)$$

and

$$\begin{aligned} p = & -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{b}{3}m(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4 - \frac{1}{2}m_\delta^2\delta_3^2 \\ & + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_0^2 \\ & + \sum_{i=N,\lambda} \frac{1}{3} \frac{v_i}{(2\pi)^3} \int_0^{k_F^i} d^3k \frac{k^2}{\sqrt{k^2 + m_i^{*2}}}, \end{aligned} \quad (15)$$

in which k_F^i is the fermi momentum of baryons or leptons, $v_i = 2$ is the degeneracy of nucleons and leptons (electron and muon, here) in spin space. Note that, for the leptons, the effective mass m_i^* is just taken as the same as that in free space.

It has been known that, besides the pure normal phase and the pure kaon condensation phase, there exists a mixed phase with both the normal phase and the kaon phase [1, 12, 13, 27]. To deal with the mixed phase we implement the Gibbs condition in the same way as used in Ref. [27]. Here, for the sake of convenience later, we only show the formulism of the total density as

$$\rho_{\text{mix}} = (1 - \chi)\rho_N(\mu_n, \mu_e) + \chi\rho_K(\mu_n, \mu_e), \quad (16)$$

in which χ is the fraction of the kaon condensation. The discussion concerning the mixed phase has been accomplished in the GM and ZM models for the case without the δ meson in Ref. [27]. In this paper we will analyze the effect of the δ -meson on the mixed phase besides the pure kaon condensation phase not only in the GM and ZM models but also in the HD model. Up to now, we obtain the EOS of the neutron star matter in the RMF approximation theory with the δ -meson in three models for the effective mass of the nucleon.

III. NUMERICAL RESULTS AND DISCUSSIONS

Substituting the EOS we obtained in the last section into the Tolman-Oppenheimer-Volkoff (TOV) equations [88, 89] for the structure of a static, spherically symmetric, relativistic star, we can obtain the mass-radius (M-R) relation of neutron star. The TOV equations read

$$\frac{dp}{dr} = \frac{[p(r) + \epsilon(r)][M(r) + 4\pi r^3 p(r)]}{r[r - 2M(r)]}, \quad (17)$$

$$M = 4\pi \int_0^R \epsilon(r)r^2 dr. \quad (18)$$

It has been known that the general relativity predicts not only a maximal mass for a star stable to radial perturbations for a given EOS but also a redshift for photons leaving the surface of a star with a strong gravitational field. The gravitational redshift obeys the relation as

$$z = \left(1 - \frac{2GM}{c^2 R}\right)^{-1/2} - 1, \quad (19)$$

which can be easily determined by the M-R relation.

As mentioned in the last section, to determine the EOS in a practical calculation, we need parameters $\frac{g_\sigma}{m_\sigma}$, $\frac{g_\delta}{m_\delta}$, $\frac{g_\omega}{m_\omega}$, $\frac{g_\rho}{m_\rho}$, b , c , and so forth. In our present calculation, for the parameter $\frac{g_\delta}{m_\delta}$, we take a value by rescaling the one given in Ref. [67]. For the others, we fix them with the bulk properties of nuclear matter at the saturation density ρ_0 . The saturation nuclear matter properties being fitted are $\rho_0 = 0.153 \text{ fm}^{-3}$, $E/A = -16.3 \text{ MeV}$, $a_{\text{sym}} = 32.5 \text{ MeV}$, $K = 265 \text{ MeV}$, and $m^* = 0.8m$, where the compression modulus is selected the same as mentioned in Ref. [75]. The obtained results are listed in Table I in which the notation $f_i \equiv (\frac{g_i}{m_i})^2$ with $i = \sigma, \delta, \omega, \rho$ is adopted. Moreover, the coupling constants between the vector meson and kaon $g_{\omega K}$, $g_{\rho K}$ are determined by the meson SU(3) symmetry as $g_{\omega K} = \frac{1}{3}g_\omega$, $g_{\rho K} = g_\rho$. The scalar coupling constant $g_{\sigma K}$ is fixed to the optical potential of the K^- at ρ_0 with $U_K(\rho_0) = -g_{\sigma K}\sigma(\rho_0) - g_{\omega K}\omega_0(\rho_0) = -90 \text{ MeV}$. While the additional parameter $g_{\delta K}$ is chosen as $g_{\delta K} = -0.1g_\delta$ for the case of the δ meson.

Since the constraint on the EOS of neutron stars by the recent observed maximal mass and the gravitational redshift has been discussed in our previous paper [27], we did not emphasize it here. In the following, we discuss then the results obtained in the three different RMF models and show the effects of the involvement of the δ -meson.

TABLE I. Parameters used in our calculations ($f_i \equiv (\frac{g_i}{m_i})^2$ with $i = \sigma, \delta, \omega, \rho$ are fitted by the saturation nuclear matter properties except for f_δ).

Parameter	$f_\delta(\text{fm}^2)$	$f_\sigma(\text{fm}^2)$	$f_\omega(\text{fm}^2)$	$f_\rho(\text{fm}^2)$	b	c
GM δ	10.0	8.91425	4.24085	13.75890	0.00695	0.01294
GM	0.0	8.91425	4.24085	4.87904	0.00695	0.01294
HD δ	10.0	8.97982	4.24085	10.40608	0.00035	0.00013
HD	0.0	8.97982	4.24085	4.87904	0.00035	0.00013
ZM δ	10.0	9.02106	4.24085	8.26147	-0.00660	0.00428
ZM	0.0	9.02106	4.24085	4.87904	-0.00660	0.00428

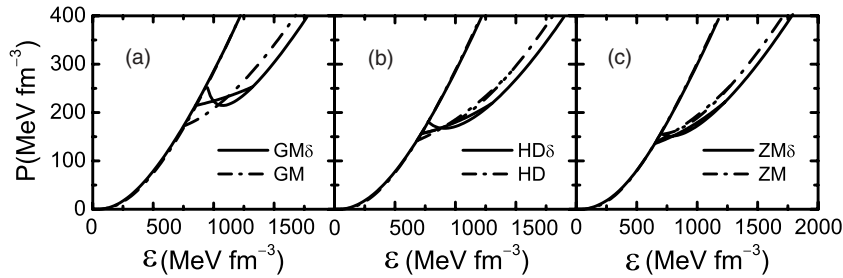


FIG. 1. Calculated equation of state (EOS) in the three models for the cases with (solid line) and without (dotted-dash line) the δ -meson channel interaction [(a), (b), (c) denote the results in the GM model, HD model, ZM model, respectively].

Figures 1 and 2 illustrate the EOS and the M-R relation with kaon condensation in the cases with and without the δ meson in the GM, HD, ZM models, respectively. From these figures, we can notice generally that the kaon condensation softens the EOS and would lower the maximal mass of NS. There exists the mixed phase which connects the normal phase and the pure kaon phase in all three models except the case without the δ -meson in the GM model. In more detail, Fig. 1(a) shows that the range for the mixed phase to appear in the case with the δ -meson is larger than that of without the δ -meson in the GM model, which in fact disappears. Figure 1(b) (for the HD model) and (c) (for the ZM model) manifests that the energy density range of the mixed phase is nearly the same for the cases including and not including the δ -meson channel interaction. The figures also show that the pressure of the mixed phase is higher in the case with the δ -meson than that without the δ -meson in the GM and HD models. As a consequence, the maximal mass of the neutron star in the case with the δ -meson is larger than that without the δ -meson in these two models, while nearly the same in the ZM model. The exact values for the cases with and without the δ -meson are $1.962 M_{\odot}$ and $1.875 M_{\odot}$ in the GM model, $1.905 M_{\odot}$ and $1.841 M_{\odot}$ in the HD model, and $1.908 M_{\odot}$ and $1.885 M_{\odot}$ in the ZM model, respectively, which are listed in Table II. The radii corresponding to the maximal masses are a little different but all about $12 \sim 14$ km in the three models and the difference between those with and without the δ -meson is not as obvious (the concrete values are also listed in Table II). However, the radii of the intermediate mass neutron stars in the cases with the δ -meson are more different than those in the cases without the δ -meson and the ones with the δ -meson are larger than those without the δ -meson if they hold the same intermediate mass.

Comparing the results shown in the figures, we find that the range of the mixed phase in the HD and ZM models is much wider than that in the GM model and the maximal mass of the neutron stars is all larger than $1.9 M_{\odot}$ in the case with the δ -meson, whereas a little smaller in the case without the δ -meson.

For the relation between the gravitational redshift and the neutron star mass, we display the calculated results in the GM, HD, ZM models in Fig. 3. From the figure we notice that, for the low mass neutron stars, the difference between those with and without the δ -meson is very small in the GM model, while a little larger in the HD and ZM models. However, the gravitational redshift corresponding to the maximal mass neutron stars with the δ -meson is slightly larger than that without the δ -meson in the GM model, while a little smaller in the HD and ZM models. Comparing the concrete values of the redshift (listed in Table II), we know that it is the largest in the case with the δ -meson in the GM model, and the smallest in the ZM model with the δ -meson.

It is remarkable that, although the maximal mass of neutron stars calculated in the case without the δ -meson in all three models is a little smaller than $1.9 M_{\odot}$ and the gravitational redshift is lower than 0.35 for some cases, we cannot exclude the existence of the large mass and high redshift neutron stars, since the kaon optical potential we take is rather strong [with $U_K(\rho_0) = -90$ MeV] and the effective nucleon mass is quite large [$m^*(\rho_0) = 0.8 m$]. Our previous work [27] shows that the smaller the effective nucleon mass or the smaller the absolute value of the kaon optical potential at saturation nuclear density, the larger the maximal mass and the gravitational redshift of the neutron star, and some investigations indicate that the kaon optical potential can be quite weak [with $U_K(\rho_0) = -80$ MeV even -50 MeV] [90–92]. Then we can infer that, as we decrease one of the two factors, we could obtain a much

TABLE II. Calculated mass (M_{\max}), radius ($R(M_{\max})$), redshift ($z(M_{\max})$), baryon density at the center (ρ_{cent}), baryon density range for the mixed phase to appear ($\rho_{\text{mix}} = (\rho_1, \rho_2)$), the fraction of the kaon condensation at the center (χ_{cent}), and the baryon density for kaon condensation to appear if no mixed phase exists (ρ_{pkc}) of the maximal mass neutron star in the models.

Model	M_{\max}/M_{\odot}	$R(M_{\max})/\text{km}$	$z(M_{\max})$	$\rho_{\text{cent}}/\rho_0$	ρ_{mix}/ρ_0	χ_{cent}	ρ_{pkc}/ρ_0
GM δ	1.962	12.26	0.377	6.04	(4.85, 6.97)	0.51	5.25
GM	1.875	12.01	0.362	6.00	(4.49, 4.49)	1.00	4.49
HD δ	1.905	12.71	0.339	5.56	(4.21, 6.99)	0.41	4.49
HD	1.841	12.25	0.341	5.68	(4.10, 7.76)	0.32	4.27
ZM δ	1.908	12.83	0.335	5.34	(3.88, 6.65)	0.46	4.11
ZM	1.885	12.32	0.350	5.53	(3.95, 6.97)	0.45	4.12

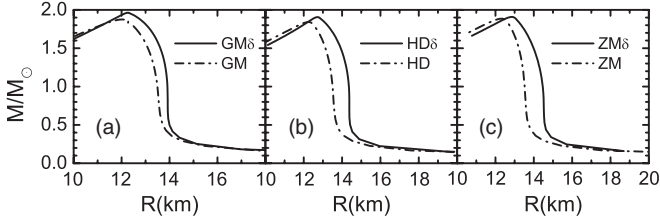


FIG. 2. Calculated mass-radius relation of the neutron star with kaon condensation in the three models with and without the δ -meson channel interaction (marked as GM δ , GM, HD δ , HD, ZM δ , ZM, respectively).

larger maximal mass and gravitational redshift for neutron stars. Furthermore, from the analysis above we know that the δ -meson channel interaction makes the EOS stiffer and the maximal mass of neutron stars gets larger.

To show the effect of the δ -meson in neutron stars further, we have also calculated the distribution of the relative population of the constituents in neutron stars in the three models. The obtained results of density distribution are illustrated in Fig. 4, while that of the radius distribution are illustrated in Fig. 5. In Fig. 4, we manifest also the center density ρ_c of the neutron star with the maximal mass and the baryon density range (ρ_1, ρ_2) for the mixed phase to exist. Their concrete values are listed in Table II. From the left panels (a) and (d) of Fig. 4 and Table II, we notice that, in the GM model, $\rho_c \sim 6.0\rho_0$ for both with and without the δ -meson. The center density of the neutron star with a δ -meson is in the middle of the baryon density range of the mixed phase, so that the pure kaon condensation cannot appear in this case. While, in the case without the δ -meson in the GM model, the mixed phase shrinks to one point (i.e., $\rho_1 = \rho_2$) and its density is smaller than the center density. It indicates apparently that the pure kaon phase exists. As the middle panels [(b) and (e)] and right panels [(c) and (f)] of Fig. 4 show for the HD and ZM models, the center density is evident between the density range of the mixed phase, i.e., $\rho_c \in (\rho_1, \rho_2)$, so that the pure kaon condensation phase does not exist in either the HD model or the ZM model for both cases with and without the δ -meson. Table II also manifests evidently the difference between the center densities and the density range of mixed phase. We find that the center density of the GM model is larger than that of the HD and ZM models, and the density ranges of the mixed phase in HD and ZM models are larger than that in the GM model. Concerning more concretely the density range for the mixed phase to exist, we learn that the HD model without the δ -meson is the widest

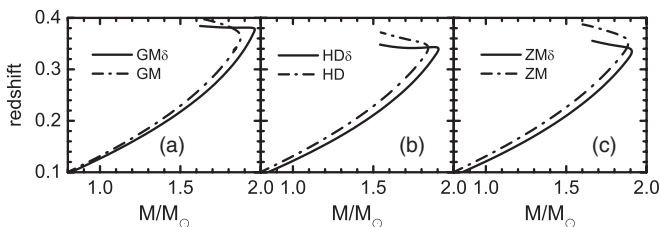


FIG. 3. Calculated gravitational redshift vs the mass of the neutron star with kaon condensation [(a), (b), (c) show the result of the GM model, HD model, ZM model, respectively].

and that the GM model without δ -meson is the narrowest, which is one point in fact. Furthermore, from Fig. 4, one can recognize that the δ -meson channel interaction enhances the relative population of charged particles [proton with positive charge, electron, muon, and (anti)kaon with negative charge] and descend that of neutrons. Such an effect on the relative population of the components may influence the observation properties (for instance, the magnetic field, the rotation, and so on) of neutron stars. In addition, if we do not take the mixed phase into account, we obtain that the baryon density for the kaon condensation to appear is larger than the ρ_1 mentioned above (the details can be seen from the last column of Table II, i.e., ρ_{pkc}/ρ_0). It is obvious that the mixed phase induces the kaon condensate to emerge in the lower density region.

Figure 5 shows the radial distribution of the relative population of the components in the NS with the maximal mass in the cases with and without the δ -meson, respectively. We find that the radius range for kaon condensation to exist gets larger and larger going from the GM model to the HD model, and then to the ZM model in both cases and the differences are more apparent for the case with the δ -meson. Comparing the same panels (a) and (d), (b) and (e), (c) and (f) of Fig. 5, we find that the range for the kaon condensation to exist is much larger in the case without the δ -meson than that with the δ -meson in the GM and HD models, while those in the ZM model are only slightly different.

From Figs. 4 and 5, we also notice that the relative population of the proton is larger than that of the neutron in some range close to the center of NS. Combining the expression of ω_K and the equation of motion for δ -meson, we know that once the population of the proton becomes higher than that of the neutron, the kaon energy ω_K gets larger than that without the δ -meson. It means that, when considering the existence of the δ -meson, the kaon condensation may appear later than that without the δ -meson, as the data listed in Table II and characteristics displayed in Fig. 4 show.

From the above discussion, we notice that the mixed phase is quite important in the neutron star matter and there does not exist a pure kaon condensation phase in the neutron stars in five cases we considered. However, with the definition of the hadron density of the mixed phase in Eq. (16), we can extract a fraction of the kaon condensation phase. The obtained results of the fraction of the kaon condensation in the neutron star with the maximal mass in the GM, HD, ZM models with and without the δ -meson are illustrated in Fig. 6. Looking over Fig. 6 carefully, we notice that, except for the pure kaon condensation phase which can appear in the region close to the center of the NS with the maximal mass in the GM model without the δ -meson (as mentioned above), a fraction of the kaon condensation phase is not very large (even if at the center of the maximal mass NS, the fraction is about 51% in the GM model with the δ -meson, and only about 41% with the δ -meson and 32% without the δ -meson in the ZM model, 45% in the HD model with and without δ -meson). Comparing the results with and without the δ -meson, one can recognize that the existence of the interaction with δ -meson exchange decreases the fraction of the kaon condensation in the GM model while increases it in the HD model and maintains the same in the ZM model. Combining these results with those illustrated in Fig. 5,

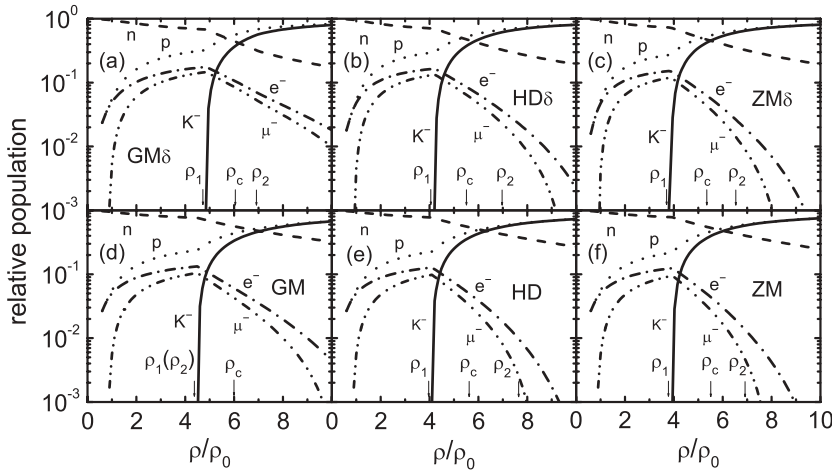


FIG. 4. Calculated relative population of the components in neutron star matter with kaon condensation vs the baryon density in the GM (a), HD (b), and ZM (c) models in the case including the δ -meson channel interaction and the results in the case not including the δ -meson channel interaction [(d), (e), (f) for that of the GM model, HD model, ZM model, respectively].

we find that the higher χ_{cent} and larger radius range involving the kaon condensation exist in the case without the δ -meson in the GM model, so that the maximal mass of the NS is smaller than that with the δ -meson because of the softening effect of kaon condensation. In the HD model, the circumstance is different, despite the higher χ_{cent} for the case with the δ -meson, the maximal mass is still larger than that without the δ -meson, since the range for the kaon condensation to exist is quite narrow. While a small difference of the maximal masses of NSs in the cases with and without the δ -meson in the ZM model comes from the small difference of χ_{cent} and nearly the same existence range of kaon condensation.

IV. SUMMARY

In this paper we have studied antikaon condensation in neutron stars with the isovector-scalar δ -meson and compared the results with those without the δ -meson in three models: the GM model, the ZM model, and the HD model which are all in the framework of the RMF approximation theory. In the calculations, we adopted the Gibbs conditions instead of the Maxwell construction to deal with the mixed phase with both the normal baryons and the kaon condensation.

In our calculations we make use of the parameters obtained by fitting the saturation nuclear matter properties: $\rho_0 = 0.153 \text{ fm}^{-3}$, $E/A = -16.3 \text{ MeV}$, $a_{\text{sym}} = 32.5 \text{ MeV}$, $K = 265 \text{ MeV}$, and $m^* = 0.8 m$. And we take the kaon-nucleon optical potential at the saturation density $U_K = -90 \text{ MeV}$ as a representative. With these parameters we obtain the equation of state, the mass-radius relation, the gravitational redshift, the distribution of the relative population of the particles, and kaon condensation in neutron star in the GM, HD, and ZM models with and without the δ -meson.

Our calculations indicate that the existence of the interaction with δ -meson exchange enlarges the maximal mass and the radius of neutron stars in all three models. Meanwhile, even though the kaon optical taken here is quite strong ($U_K = -90 \text{ MeV}$), the maximal mass of the neutron can be larger than $1.68 M_\odot$ for both with and without the δ -meson in all models and larger than $1.9 M_\odot$ in the case with the δ -meson. One can then infer that large mass neutron stars with kaon condensation can exist if an appropriately weaker kaon optical potential is adopted. At the same time, the gravitational redshift of the maximal mass neutron star in the GM model with the δ -meson can be as large as 0.377, and all others are a little smaller than 0.35 (the smallest one is 0.335, obtained in the ZM model with the δ -meson). Comparing the results given in the models with the δ -meson and those without the δ -meson, one can learn that the interaction with δ -meson exchange makes the redshift a little larger in the GM model, while smaller in the ZM and HD models. Our calculation also gives the distribution of the relative population (particles and kaon condensation) in neutron stars. It shows that the interaction with a δ -meson enhances the relative population of charged particles (proton with positive charge, electron, muon, and (anti)kaon-condensation with negative charge) and descends that of neutrons. It also manifests that the mixed phase with both normal nuclear matter and kaon condensation matter can exist in neutron stars in five cases and the pure kaon condensation phase can only appear in the region close to the center of a neutron star in the GM model without the δ -meson channel interaction. However, the baryon density range for the mixed phase to exist is different in different models, where the HD model without a δ -meson holds the widest and the GM model without a δ -meson gives the narrowest which, in fact,

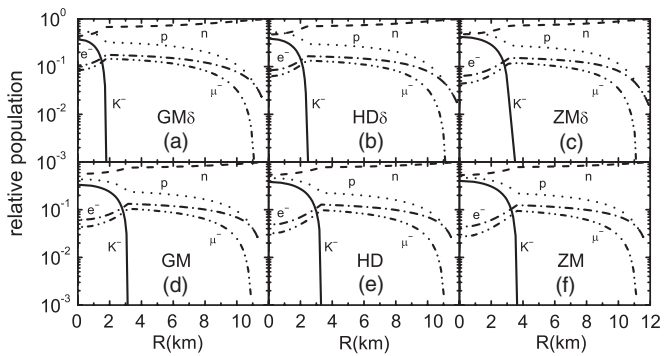


FIG. 5. Calculated relative population of the components in neutron star matter with kaon condensation vs the radius of the maximal mass neutron star in the GM (a), HD (b), and ZM (c) models in the case including the δ -meson channel interaction and the results in the case not including the δ -meson channel interaction [(d), (e), (f) for that of the GM model, HD model, ZM model, respectively].

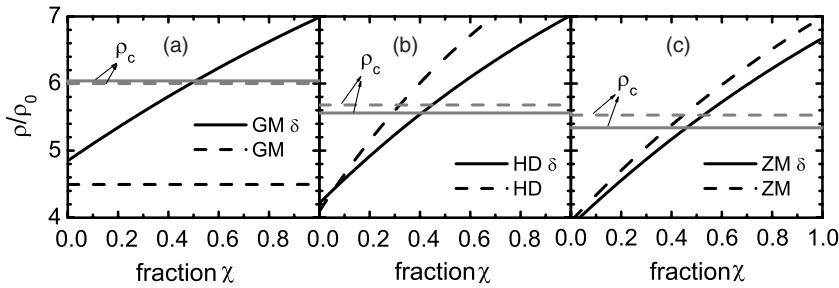


FIG. 6. Calculated density range of the mixed phase vs the fraction of the kaon condensation χ in the GM model (a), HD model (b), and ZM model (c) (the gray lines are the center densities of the NS).

shrinks to one point, i.e., no mixed phase exists. Furthermore, the interaction with δ -meson exchange influences the fraction of the kaon condensation in the mixed phase. In the GM model, the fraction gets decreased and, in turn, the maximal mass becomes larger. Meanwhile, the decrease of a fraction of the kaon condensation in the GM model induces an increase of the redshift. Wherever the cases are different in the ZM and HD models.

Finally, it should be mentioned that we have not taken into account the contribution of the constituents of hyperons in neutron stars in our present calculations. Since the experiment has provided evidence for the interaction between the Σ -hyperon and nucleon is repulsive [86], the Σ -hyperon may not then appear in neutron star matter. Previous investigations have shown that Ξ -hyperons may emerge only in the very high density region of neutron stars. Their contribution may not be significant, so that our results of the existence of (anti)kaon condensation and the effect of the interaction in the δ -meson channel would not change drastically as the Ξ -hyperon degree of freedom is considered. Moreover due to the neutral charge characteristic of the Λ -hyperon and

the fact that the attractive interaction between Λ -hyperons is quite weak [93], the Λ -hyperon may not influence our results obviously even though such a constituent may exist in neutron stars. In short, our present results about the existence of kaon condensation in large mass neutron stars and the effect the interaction in the δ -meson exchanging channel may also be available to neutron stars including hyperons (mainly with Λ -hyperon and fewer Ξ -hyperons). However a concrete study is required. A related investigation is under progress.

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