$\bar{K}\bar{K}N$ molecular state in a three-body calculation

Yoshiko Kanada-En'yo and Daisuke Jido

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

A $\bar{K}\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$ is investigated with nonrelativistic three-body calculations by using effective $\bar{K}N$ and $\bar{K}\bar{K}$ interactions. The present investigation suggests that a weakly bound state for the $\bar{K}\bar{K}N$ system can be formed below the two-body threshold of $\bar{K}$ and quasibound $\bar{K}N$ with a $40 \sim 60$ MeV decay width of $\bar{K}\bar{K}N \to \pi Y \bar{K}$. This corresponds to an excited Ξ baryon with $J^P = 1/2^+$ located around 1.9 GeV. Studying the wave function of the $\bar{K}\bar{K}N$ system obtained in this formulation, we find that the three-body bound system has a characteristic structure of $\bar{K}N + \bar{K}$ cluster with spatial extent.

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I. INTRODUCTION

The study of hadron structure is one of the most important issues in hadron physics. Recent interest in this line is developed in exploring quasibound systems of mesons and baryons governed by strong interactions among the hadrons. One of the long-standing candidates is the $\Lambda(1405)$ resonance considered as a quasibound state of the $\bar{K}N$ system [1]. It has been also suggested that the $f_0(980)$ scalar meson is a molecular state of $\bar{K}K$ [2]. For nuclear systems, bound states of an η meson in nuclei and an antikaon in light nuclei were predicted in Refs. [3–5]. Recently a multihadron state was proposed for the Θ^+ baryon resonance to explain its narrow width [6].

In such multihadron systems, antikaon plays a unique and important role due to its heavy mass and Nambu-Goldstone boson nature. The heavier kaon mass indicates stronger s-wave interactions around the threshold than those for pion according to the chiral effective theory. In addition, because typical kaon kinetic energy in the system estimated by range of hadronic interaction is small in comparison with the kaon mass, we may treat kaons in multihadron systems in many-body formulations.

The strong attraction in the $\bar{K}N$ system led to the idea of deeply bound kaonic states in light nuclei, such as K^-pp and K^-ppn, by Akaishi and Yamazaki [5,7–10]. Later, many theoretical studies on the structure of the K^-pp system have been done, for example, in Faddeev calculations [11,12] and in variational calculations [13,14], having turned out that the K^-pp system is bound with a large width. Experimental search for these states has been reported [15–18], whereas clear evidences are not obtained yet and interpretations of the experimental data are controversial [19,20]. Motivated by the strong $\bar{K}N(I = 0)$ attractions, quests of multi-$\bar{K}$ nuclei are challenging issues [8,21–25]. This is also one of the key subjects related to kaon condensation in dense nuclear matter [26–28].

The key issue for the study of the $\bar{K}N$ interaction is the resonance position of the $\Lambda(1405)$ in the $\bar{K}N$ scattering amplitude. The $\Lambda(1405)$ is observed around 1405 MeV in the $\pi\Sigma$ final-state interaction, as summarized in the particle data group [29]. Based on this fact, a phenomenological effective $\bar{K}N$ potential (AY potential) was derived in Refs. [5,13],

having relatively strong attraction in the $I = 0$ channel to provide the K^-p bound state at 1405 MeV. Recent theoretical studies of the $\Lambda(1405)$ based on chiral unitary approach have indicated that the $\Lambda(1405)$ is described as a superposition of two pole states and one of the state is considered to be a $\bar{K}N$ quasibound state embedded in the strongly interacting $\pi\Sigma$ continuum [30–35]. This double-pole conjecture suggests that the resonance position in the $\bar{K}N$ scattering amplitudes with $I = 0$ is around 1420 MeV, which is higher than the energy position of the nominal $\Lambda(1405)$ resonance. Based on this chiral SU(3) coupled-channels dynamics, Hyodo and Weise have derived another effective $\bar{K}N$ potential (HW potential) [36]. The HW potential provides a $\bar{K}N$ quasibound state at ~ 1420 MeV instead of 1405 MeV, and it is not as strong as the AY potential.

The common achievement of these studies on the s-wave $\bar{K}N$ effective potential is that the $\bar{K}N$ interaction with $I = 0$ is strongly attractive and describes the $\Lambda(1405)$ resonance as a $\bar{K}N$ quasibound state. Basing on these strong $\bar{K}N$ interactions, we examine possible bound states of a lightest two-anti-kaon nuclear system, namely $\bar{K}\bar{K}N$ with $I = 1/2$ and $J^P = 1/2^+$, in the hadronic molecule picture. We expect that the three-body $\bar{K}\bar{K}N$ system will form a bound state due to the strong attraction in the two-body $\bar{K}N$ subsystem producing the $10 \sim 30$ MeV binding energy. The questions raised here are whether the strong attraction is enough for a three-body bound state below the threshold of the $\bar{K}$ and the $\bar{K}N$ quasibound state and what structure the bound state has, if it is formed.

The $\bar{K}\bar{K}N$ molecule state may have characteristic decay patterns depending on the binding energy. If the three-body state is above the threshold of the $\bar{K}$ and the $\bar{K}N$ quasibound state, it can decay into $\bar{K}$ and $\Lambda(1405)$, and, consequently, the bound state has a large width. If the $\bar{K}\bar{K}N$ system is bound below the threshold, the bound state has a comparable width to the $\Lambda(1405)$, and the main decay mode is a three-body $\pi\Sigma\bar{K}$ and two-hadron decays are strongly suppressed in contrast to usual excited baryons. For deeply bound $\bar{K}\bar{K}N$ systems, the molecular picture may be broken down and two-hadron decays may be favored. Presently experimental data for $S = -2$ channel are very poor. But we expect that such an interesting molecule state could be observed as an exited state of Ξ baryon, for example, in double strangeness exchange

(K^-, K^+) processes at the Japan Proton Accelerator Research Complex (J-PARC).

In this article, we investigate the $\bar{K}\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$ with nonrelativistic three-body calculations by using the HW and AY potentials as effective $\bar{K}N$ interactions. In Sec. II, we describe the framework of the present calculations. We apply a variational approach with a Gaussian expansion method [37] to solve the Schrödinger equation of the three-body system. By treating the imaginary potentials perturbatively, we find the $\bar{K}\bar{K}N$ quasibound state. In Sec. III, we present our results of the three-body calculation. In analysis of the wave functions, we discuss the structure and the binding mechanism of the $\bar{K}\bar{K}N$ state. The effects of $\bar{K}\bar{K}$ interactions on the $\bar{K}\bar{K}N$ system are also discussed. Section IV is devoted to summary and concluding remarks.

II. FRAMEWORK

We consider a nonrelativistic three-body potential model for the $\bar{K}\bar{K}N$ system. We calculate the $\bar{K}\bar{K}N$ wave function by using effective two-body interactions [13,36] in local potential forms. We apply a variational approach with a Gaussian expansion method [37] in solving the Schrödinger equation for the three-body system. In this section we explain the details of the framework and interactions used in this work.

A. Hamiltonian

In the present formulation, we use the Hamiltonian for the $\bar{K}\bar{K}N$ system given by

$$H = T + V_{\bar{K}N}(r_1) + V_{\bar{K}N}(r_2) + V_{\bar{K}\bar{K}}(r_3), \quad (1)$$

with the kinetic energy T, the effective $\bar{K}N$ interaction $V_{\bar{K}N}$, and the $\bar{K}\bar{K}$ interaction $V_{\bar{K}\bar{K}}$. These interactions are given by local potentials as functions of $\bar{K}$-N distances, r_1, r_2, and the $\bar{K}$-$\bar{K}$ distance, r_3, which are defined by $r_1 = |\mathbf{x}_2 - \mathbf{x}_3|$, $r_2 = |\mathbf{x}_3 - \mathbf{x}_1|$, and $r_3 = |\mathbf{x}_2 - \mathbf{x}_1|$. The vectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ denote the spatial coordinates of the first antikaon ($\bar{K}_1$), the second antikaon ($\bar{K}_2$), and the nucleon, respectively. For convenience, we use Jacobian coordinates, $\mathbf{r}_c$ and $\mathbf{R}_c$, in three rearrangement channels $c = 1, 2, 3$ as shown in Fig. 1. In the present calculation, we assume isospin symmetry in the effective interactions, and we also neglect the mass differences between K^- and $\bar{K}^0$ and between proton and neutron by using the averaged masses. We do not consider three-body forces nor transitions to two-hadron decays.
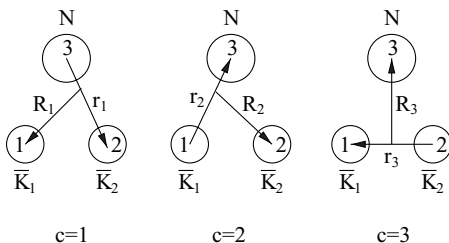

FIG. 1. Three Jacobian coordinates of the $\bar{K}\bar{K}N$ system.

The kinetic energy T is simply expressed by the Jacobian coordinates for a rearrangement channel as

$$T \equiv \frac{-1}{2\mu_{r_c}}\nabla^2_{r_c} + \frac{-1}{2\mu_{R_c}}\nabla^2_{R_c}, \quad (2)$$

with the reduced masses μ_{r_c} and μ_{R_c} for the corresponding configuration. For instance, $\mu_{r_1} = M_K M_N/(M_K + M_N)$ and $\mu_{R_1} = M_K(M_K + M_N)/(2M_K + M_N)$ for the rearrangement channel $c = 1$. Here M_N and M_K denote the averaged nucleon and antikaon masses, respectively, as $M_N = 938.9$ MeV and $M_K = 495.7$ MeV.

For the effective interactions, $V_{\bar{K}N}$ and $V_{\bar{K}\bar{K}}$, we use local potentials obtained by s-wave two-body scattering amplitudes with isospin symmetry. In the present three-body calculations, we take l-independent potentials for simplicity. The details of the local potentials will be discussed in Sec. II B. The coupled-channel effects of the $\bar{K}N$ to other relevant channels have been already implemented to the effective single-channel $\bar{K}N$ interaction $V_{\bar{K}N}$. Consequently $V_{\bar{K}N}$ has an imaginary part because of scattering states below the $\bar{K}N$ threshold, and the present Hamiltonian is not Hermitian. In the calculations of $\bar{K}\bar{K}N$ wave functions, we first use only the real part of $V_{\bar{K}N}$ in a variational approach, and then we calculate the energy E with the expectation value of the total Hamiltonian (1) with respect to the obtained wave functions. The width of the bound state is evaluated by the imaginary part of the energy as $\Gamma = -2\text{Im}E$. The details of the calculational procedure are described later.

B. Effective interactions

In this subsection, we explain the details of the effective interactions of the $\bar{K}N$ and $\bar{K}\bar{K}$ two-body subsystems in our formulation. We use two effective $\bar{K}N$ interactions that were derived in different ways and compare the results obtained by these interactions to estimate theoretical uncertainties. Both $\bar{K}N$ interactions have so strong attraction as to provide the $\Lambda(1405)$ as a quasibound state of the $\bar{K}N$ system. The important difference is the binding energy of the $\bar{K}N$ system, as already mentioned in introduction.

One of the $\bar{K}N$ interactions that we use here is given by Hyodo and Weise in Ref. [36]. This effective interaction was derived based on the chiral unitary approach [38] for s-wave scattering amplitude with strangeness $S = -1$ of the lowest-lying octet meson and baryon. A equivalent single-channel $\bar{K}N$ amplitude can be obtained by reducing the four (five)-channel problem for $I = 0$ ($I = 1$) to a $\bar{K}N$ single-channel problem, including dynamics of the rest channels. The local $\bar{K}N$ potential was constructed in coordinate space so as to reproduce the single-channel $\bar{K}N$ interaction as a solution of the Schrödinger equation for the $\bar{K}N$ system with the local potential.

The potential is written in a one-range Gaussian form as

$$V_{\bar{K}N} = U_{\bar{K}N}^{I=0} \exp[-(r/b)^2]P_{\bar{K}N}(I = 0)$$
$$+ U_{\bar{K}N}^{I=1} \exp[-(r/b)^2]P_{\bar{K}N}(I = 1), \quad (3)$$

with the isospin projection operator $P_{\bar{K}N}(I = 0, 1)$ and the range parameter $b = 0.47$ fm. The range parameter was optimized for the parametrization referred as HNJH [39] in

Ref. [36]. We refer this potential as "HW-HNJH potential." The strength $U_{\bar{K}N}(I = 0, 1)$ has energy dependence and is parametrized in terms of a third-order polynomial in the energy ω:

$$U_{\bar{K}N}^{I=0,1}(\omega) = K_0^{I=0,1} + K_1^{I=0,1}\omega + K_2^{I=0,1}\omega^2 + K_3^{I=0,1}\omega^3, \tag{4}$$

for 1300 MeV $\leqslant \omega \leqslant$ 1450 MeV. The coefficients $K_i^{I=0,1}$ for $I = 0$ and $I = 1$ are given in Table IV and V of Ref. [36], respectively. Here we use the corrected version of the local potentials [36].

In chiral unitary approaches for the meson-baryon interactions, two poles are generated at 1.4 GeV region, and the $\Lambda(1405)$ resonance is described as a $\bar{K}N$ quasibound state in the strongly interacting $\pi\Sigma$ continuum [30,32]. In this case, the peak position of the $\bar{K}N$ scattering amplitudes in the $I = 0$ channel appears at $\omega \sim 1420$ MeV, which is higher than the energy position of the $\Lambda(1405)$ resonance observed in the final-state interaction of the $\pi\Sigma$ channel. The HW potential reproduces this feature of $\bar{K}N$ scattering amplitudes, and therefore the position of the $\bar{K}N$ quasibound state is located at $\omega \sim 1420$ MeV. With the HW-HNJH potential, $\Lambda(1405)$ can be calculated as a $\bar{K}N$ quasibound state by solving the $\bar{K}N$ two-body Schrödinger equation in the s-wave $I = 0$ channel. We get a $\bar{K}N$ quasibound state at 1423 MeV with a width $\Gamma = 44$ MeV, treating the imaginary part of the potentials perturbatively. This is a weakly bound $\bar{K}N$ state located at 11 MeV below the $\bar{K}N$ threshold ($M_K + M_N = 1434$ MeV). The root-mean-square of the $\bar{K}$-N distance ($d_{\bar{K}N}$) is also calculated as 1.9 fm.

The HW-HNJH potential $V_{\bar{K}N}^{I=0,1}(\omega)$ depends on the energy of the subsystem $\bar{K}N$. We regard, however, ω as an interaction parameter and use fixed values of ω, because the energy dependence is small in the region of interest, $\omega = 1400 \sim 1440$ MeV. We take two values of $\omega = M_N + M_K - \delta\omega$ with $\delta\omega = 0$ and 11 MeV. The choice of $\delta\omega = 0$ MeV corresponds to $\bar{K}N$ at the threshold, whereas $\delta\omega = 11$ MeV is for the binding energy $B(\bar{K}N) = 11$ MeV of the $\bar{K}N$ bound state obtained by the HW-HNJH potential. In each of the fixed HW potential, we calculate the energy $E^{\text{Re}} = -B(\bar{K}\bar{K}N)$ of the $\bar{K}\bar{K}N$ system and also the energy $-B(\bar{K}N)$ of the $\bar{K}N$ system.

The other $\bar{K}N$ interaction we use is given by Akaishi and Yamazaki (AY) in Refs. [5,13]. The AY potential was derived in a phenomenological way to start with the ansatz that the $\Lambda(1405)$ resonance is a K^-p bound state at 1405 MeV as reported by PDG [29]. The AY potential is energy independent and was parametrized to reproduce the $\bar{K}N$ quasibound state at the PDG values of the $\Lambda(1405)$:

$$V_{\bar{K}N}^{I=0}(r) = (-595 - 83i)\exp[-(r/0.66\,\text{fm})^2], \tag{5}$$

$$V_{\bar{K}N}^{I=1}(r) = (-175 - 105i)\exp[-(r/0.66\,\text{fm})^2]. \tag{6}$$

The AY potential has stronger attraction in the $I = 0$ channel because it was adjusted to generate a $\bar{K}N(I = 0)$ quasibound state at $\sim$1405 MeV. It corresponds to the $\bar{K}N$ binding energy $B(\bar{K}N) \sim 30$ MeV that is much larger than $B(\bar{K}N) = 11$ MeV for the HW-HNJH potential. By solving the $\bar{K}N$ two-body Schrödinger with a perturbative treatment of the imaginary potential, we get a $\bar{K}N$ ($I = 0$) state at 1403 MeV

with a width of 40 MeV, and the root-mean-square $\bar{K}$-N distance is $d_{\bar{K}N} = 1.4$ fm. Because of the strong attraction in the $I = 0$ channel of the AY potential, the $\bar{K}N(I = 0)$ quasibound state has the deeper binding and smaller radius than those given by the HW-HNJH potential. We comment that the size of the $\bar{K}$-N state is sensitive to the binding energy [40].

For the $\bar{K}\bar{K}$ interactions, there are few experimental data. The low energy theorem based on the current algebra suggests a repulsive interaction. For the s-wave interaction of $\bar{K}\bar{K}$, the $I = 0$ is forbidden due to Einstein-Bose statistics. Thus we assume $V_{\bar{K}\bar{K}}^{I=0} = 0$. We introduce the effective interaction of the subsystem $\bar{K}\bar{K}$ with $I = 1$, $V_{\bar{K}\bar{K}}^{I=1}$, in a Gaussian form

$$V_{\bar{K}\bar{K}}^{I=1}(r) = U_{\bar{K}\bar{K}}^{I=1}\exp[-(r/b)^2]P_{\bar{K}\bar{K}}(I = 1), \tag{7}$$

where the range parameter b is chosen to be the same value as that of the $\bar{K}N$ interaction. The interaction $V_{\bar{K}\bar{K}}^{I=1}$ is a real function, because there are no decay channels open for the $\bar{K}\bar{K}$ system.

The strength $U_{\bar{K}\bar{K}}^{I=1}$ is estimated by theoretical calculations of the scattering length of K^+K^+, which is equivalent to that of $\bar{K}\bar{K}$ with $I = 1$. Recently, the K^+K^+ scattering length has been obtained in lattice quantum chromodynamics (QCD) calculation as $a_{K^+K^+} = -0.141 \pm 0.006$ fm [41]. This value is consistent with the leading-order calculation of the chiral perturbation theory, $a_{K^+K^+} = -0.147$ fm, which is obtained by

$$a_{K^+K^+} = -\frac{m_K}{16\pi f_K^2}, \tag{8}$$

with $f_K = 115$ MeV. In the present calculation, the strength $U_{\bar{K}\bar{K}}^{I=1}$ is adjusted to reproduce the scattering length $a_{K\bar{K}}^{I=1} = -0.14$ fm. For the HW-HNJH potential, with the interaction range $b = 0.47$ fm, we obtain $U_{\bar{K}\bar{K}}^{I=1} = 313$ MeV, and we find $U_{\bar{K}\bar{K}}^{I=1} = 104$ MeV with $b = 0.66$ fm for the AY potential. We will also try weaker repulsion with $a_{K\bar{K}}^{I=1} = -0.10$ using $U_{\bar{K}\bar{K}}^{I=1} = 205$ MeV and 70 MeV for the HW-HNJH and the AY potentials, respectively. To examine how strong the $\bar{K}N$ interaction for the three-body system, at first we neglect the $\bar{K}\bar{K}$ repulsion, then we see the effect of the repulsive interaction with including the effective $\bar{K}\bar{K}$ interaction given above.

C. Three-body wave function

The three-body $\bar{K}\bar{K}N$ wave function Ψ is described as a linear combination of amplitudes $\Phi_{l_{KK}}^{(c)}(\mathbf{r}_c, \mathbf{R}_c)$ of two rearrangement channels $c = 1, 3$ (Fig. 1), because the channel $c = 2$ is included by symmetrization of the two antikaon in the wave function. In the present calculation, we take the model space limited to $l_c = 0$ and $L_c = 0$ of the orbital-angular momenta for the Jacobian coordinates $\mathbf{r}_c$ and $\mathbf{R}_c$ in the channel c. Then the wave function of the $\bar{K}\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$ is written as

$$\Psi = \frac{1 + P_{12}}{\sqrt{2}}\Phi, \tag{9}$$

$$\Phi = \Phi^{(c=1)}_{I_{KK}=0}(\mathbf{r}_1, \mathbf{R}_1)[[\bar{K}\bar{K}]_{I_{KK}=0}N]_{I=1/2}$$
$$+ \Phi^{(c=1)}_{I_{KK}=1}(\mathbf{r}_1, \mathbf{R}_1)[[\bar{K}\bar{K}]_{I_{KK}=1}N]_{I=1/2}$$
$$+ \Phi^{(c=3)}_{I_{KK}=1}(\mathbf{r}_3, \mathbf{R}_3)[[\bar{K}\bar{K}]_{I_{KK}=1}N]_{I=1/2}, \quad (10)$$

where P_{12} is the exchange operator between the two antikaons, $\bar{K}_1$ and $\bar{K}_2$ for two bosons. The $[[\bar{K}\bar{K}]_{I_{KK}}N]_{I=1/2}$ specifies the isospin configuration of the wave function $\Phi^{(c)}_{I_{KK}}(\mathbf{r}_c, \mathbf{R}_c)$, meaning that the total isospin $1/2$ for the $\bar{K}\bar{K}N$ system is given by combination of total isospin I_{KK} for the $\bar{K}\bar{K}$ subsystem and isospin $1/2$ for the nucleon. The isospin configuration $[[\bar{K}\bar{K}]_{I_{KK}=0}N]_{I=1/2}$ in the $c = 3$ is not necessary, because it vanishes after the symmetrization in the case of $l_3 = 0$.

As mentioned above, we omit basis wave functions with $l_c \geqslant 1$ and $L_c \geqslant 1$ in each rearrangement channel. This is consistent with the fact that the effective local potentials used in the present calculations are derived to reproduce the s-wave two-body dynamics. We comment that components with nonzero angular momenta of two-body subsystems are contained in the model wave function through rearrangement of three-body configurations, although the s-wave component is expected to be dominant.

In solving the Schrödinger equation for the $\bar{K}\bar{K}N$ system, we adopt the Gaussian expansion method for three-body systems given in Ref. [37]. The spatial wave functions $\Phi^{(c)}_{I_{KK}}(\mathbf{r}_c, \mathbf{R}_c)$ of the subcomponent of Eq. (10) are expanded in terms of the Gaussian basis functions, $\phi^G_n(\mathbf{r})$ and $\psi^G_n(\mathbf{R})$:

$$\Phi^{(c)}_{I_{KK}}(\mathbf{r}_c, \mathbf{R}_c) = \sum_{n_c, N_c}^{n_{max}, N_{max}} A^{c, I_{KK}}_{n_c, N_c} \phi^G_{n_c}(\mathbf{r}_c)\psi^G_{N_c}(\mathbf{R}_c). \quad (11)$$

The coefficients $A^{c, I_{KK}}_{n_c, N_c}$ are determined by variational principle when we solve the Schrödinger equation. In Eq. (11), n_{max} and N_{max} are the numbers of the Gaussian basis, and the basis functions are defined by

$$\phi^G_n(\mathbf{r}) = \mathcal{N}_n e^{-\nu_n r^2}, \quad (12)$$

$$\psi^G_n(\mathbf{R}) = \mathcal{N}_N e^{-\lambda_N R^2}, \quad (13)$$

where the normalization constants are given by $\mathcal{N}_n = 2(2\nu_n)^{3/4}\pi^{-1/4}$ and $\mathcal{N}_N = 2(2\lambda_N)^{3/4}\pi^{-1/4}$, and the Gaussian ranges, ν_n and λ_N, are given by

$$\nu_n = 1/r_n^2, \quad r_n = r_{min}\left(\frac{r_{max}}{r_{min}}\right)^{\frac{n-1}{n_{max}-1}}, \quad (14)$$

$$\lambda_N = 1/R_N^2, \quad R_N = R_{min}\left(\frac{R_{max}}{R_{min}}\right)^{\frac{N-1}{N_{max}-1}}. \quad (15)$$

We take enough bases for the present system by using the values given in Table I for the basis numbers and range parameters in the channel $c = 1$ and $c = 3$. We find that the mixing effect of the rearrangement channel $c = 3$ is very small in the present results of the $\bar{K}\bar{K}N$ system. This is because the $\bar{K}\bar{K}$ interactions are not attractive and therefore the $\bar{K}$-$\bar{K}$ correlation is not strong in the $\bar{K}\bar{K}N$ system. This is different from the case of the K^-pp system where the p-p correlation is significant because of the attractive NN interaction.

TABLE I. The numbers and range parameters of the basis functions for the rearrangement channels, $c = 1$ and $c = 3$.

Channel	n_{max}	r_{min} (fm)	r_{max} (fm)	N_{max}	R_{min} (fm)	R_{max} (fm)
$c = 1$	15	0.2	20	25	0.2	200
$c = 3$	15	0.2	20	15	0.2	20

D. Procedure of calculations

The wave function of the $\bar{K}\bar{K}N$ system is obtained by solving the Schrödinger equation:

$$[T + V_{\bar{K}N}(r_1) + V_{\bar{K}N}(r_2) + V_{\bar{K}\bar{K}}(r_3) - E]\Psi = 0. \quad (16)$$

The effective interaction $V_{\bar{K}N}$ is complex due to the presence of the decay channels below the threshold, whereas $V_{\bar{K}\bar{K}}$ is expressed by real numbers.

To solve this equation with variational principle, we treat the imaginary part of the potentials perturbatively. Separating the real part of the Hamiltonian, we write

$$H^{Re} = T + \text{Re}V_{\bar{K}N}(r_1) + \text{Re}V_{\bar{K}N}(r_2) + V_{\bar{K}\bar{K}}(r_3). \quad (17)$$

We first calculate the wave function for the real part of the Hamiltonian, H^{Re}, with variational principle in the model space of the Gaussian expansion described in Sec. II C. This is equivalent to determine the eigenenergy E^{Re} and the coefficients $A^{c, I_{KK}}_{n_c, N_c}$ of the Gaussian wave functions $\phi^G_{n_c}(\mathbf{r}_c)\psi^G_{N_c}(\mathbf{R}_c)[[\bar{K}\bar{K}]_{I_{KK}}N]_{I=1/2}$ by diagonalizing the norm matrix and Hamiltonian matrix

$$\langle \phi^G_{n'_c}(\mathbf{r}_c)\psi^G_{N'_c}(\mathbf{R}_c)[[\bar{K}\bar{K}]_{I'_{KK}}N]_{I=1/2}|H^{Re}|\phi^G_{n_c}(\mathbf{r}_c)\psi^G_{N_c}(\mathbf{R}_c)$$
$$\times [[\bar{K}\bar{K}]_{I_{KK}}N]_{I=1/2}\rangle. \quad (18)$$

After this variational calculation, we take the lowest-energy solution for H^{Re}. The binding energy $B(\bar{K}\bar{K}N)$ of the three-body system is given as $B(\bar{K}\bar{K}N) = -E^{Re}$. It should be checked if the bound state of the $\bar{K}\bar{K}N$ is lower than the threshold of the $\bar{K}$ and the quasibound state of $\bar{K}N$, because the $\bar{K}\bar{K}N$ solution obtained above the $\bar{K}N + \bar{K}$ threshold is not a three-body bound state but a two-body continuum state with $\bar{K}$ and $\bar{K}N$ quasibound state.

Next we estimate the imaginary part of the energy E for the total Hamiltonian H by calculating the expectation value with the wave function Ψ obtained by the Hamiltonian H^{Re}:

$$E^{Im} = \langle \Psi|\text{Im}V_{\bar{K}N}|\Psi\rangle. \quad (19)$$

The total decay width for $\bar{K}\bar{K}N$ is estimated as $\Gamma = -2E^{Im}$. In the present calculation, we have only three-body decays to $\pi\Sigma\bar{K}$ and $\pi\Lambda\bar{K}$ by the model setting.

We also calculate several quantities characterizing the structure of the three-body system, such as spatial configurations of the constituent particles and probabilities to have specific isospin configurations. These values are calculated as expectation values of the wave functions. The root-mean-square radius of the $\bar{K}$ distribution is defined as the average of the distribution of each anti-kaon by

$$r_{\bar{K}} \equiv \sqrt{\langle \Psi|\tfrac{1}{2}(\mathbf{x}_1^2 + \mathbf{x}_2^2)|\Psi\rangle}, \quad (20)$$

which is measured from the center-of-mass of the three-body system. For the two-body $\bar{K}N$ system, $r_{\bar{K}}$ is given by the spatial coordinate of $\bar{K}$, $\mathbf{x}_{\bar{K}}$, measured from the center-of-mass of the two-body system as $r_{\bar{K}} = \sqrt{\langle \mathbf{x}_{\bar{K}}^2 \rangle}$. We also calculate the root-mean-square value of the relative $\bar{K}$-$\bar{K}$ distance defined by

$$d_{\bar{K}\bar{K}} \equiv \sqrt{\langle \Psi | \mathbf{r}_3^2 | \Psi \rangle}. \qquad (21)$$

The probabilities for the three-body system to have the isospin $I_{\bar{K}\bar{K}}$ states are introduced by

$$\Pi([\bar{K}\bar{K}]_{I_{KK}}) \equiv \langle \Psi | P_{\bar{K}\bar{K}}(I_{KK}) | \Psi \rangle, \qquad (22)$$

where $P_{\bar{K}\bar{K}}(I_{KK})$ is the projection operator for the isospin configuration $[[\bar{K}\bar{K}]_{I_{KK}} N]_{I=1/2}$, as given before. We also calculate the root-mean-square radius of the $\bar{K}$ distribution for the each isospin state

$$r_{\bar{K}}|_{I_{KK}} \equiv \sqrt{\frac{\langle \Psi | \frac{1}{2}(\mathbf{x}_1^2 + \mathbf{x}_2^2) | P_{\bar{K}\bar{K}}(I_{KK}) \Psi \rangle}{\langle \Psi | P_{\bar{K}\bar{K}}(I_{KK}) | \Psi \rangle}}, \qquad (23)$$

which is normalized by Eq. (22).

To investigate the structure of the $\bar{K}\bar{K}N$ system further, we calculate the expectation values with the unsymmetrized wave function Φ given in Eq. (10). Although these expectation values are not observable in the real $\bar{K}\bar{K}N$ system having two identical bosons, they are helpful to understand the structure of the three-body system and to investigate the symmetrization effect. We calculate norms of the wave functions for the isospin $[[\bar{K}_2 N]_{I_{KN}} \bar{K}_1]_{I=1/2}$ states, where the total isospin $1/2$ for the $\bar{K}\bar{K}N$ system is given by combination of total isospin I_{KN} for the $\bar{K}_2 N$ subsystem and isospin $1/2$ for the $\bar{K}_1$,

$$\Pi([\bar{K}_2 N]_{I_{KN}})_{\Phi} \equiv \langle \Phi | P_{\bar{K}_2 N}(I_{KN}) | \Phi \rangle, \qquad (24)$$

where $P_{\bar{K}_2 N}(I_{KN})$ is again the isospin projection operator. Note that this *cannot* be interpreted as a probability for the $[[\bar{K}_2 N]_{I_{KN}} \bar{K}_1]_{I=1/2}$ states in the physical $\bar{K}\bar{K}N$ system, because the norm is not normalized to be unity without the symmetrization of the antikaon wave functions. The root-mean-square values of the $\bar{K}_2$-N distance and the $\bar{K}_2 N$-$\bar{K}_1$ distance for Φ are evaluated as

$$d_{\bar{K}_2 N} \equiv \sqrt{\frac{\langle \Phi | \mathbf{r}_1^2 | \Phi \rangle}{\langle \Phi | \Phi \rangle}}, \qquad (25a)$$

$$d_{(\bar{K}_2 N) - \bar{K}_1} \equiv \sqrt{\frac{\langle \Phi | \mathbf{R}_1^2 | \Phi \rangle}{\langle \Phi | \Phi \rangle}}, \qquad (25b)$$

respectively.

The perturbative treatment performed above is justified qualitatively in the case of $|\langle \Psi | \mathrm{Im} V | \Psi \rangle| \ll |\langle \Psi | \mathrm{Re} V | \Psi \rangle|$. For the two-body system $\bar{K}N$, we find the perturbative treatment good, observing that $|\langle \mathrm{Im} V_{\bar{K}N} \rangle| = 22$ MeV is much smaller than $|\langle \mathrm{Re} V_{\bar{K}N} \rangle| \sim 100$ MeV in the HW-HNJH potential case, for instance. This is responsible for that we get the reasonable energy $E = 1423 - 22i$ MeV for the $\bar{K}N$ with the perturbative treatment. (In a full calculation with the $\bar{K}N$ effective interaction, the scattering amplitude reproduces the $\Lambda(1405)$ resonance at 1420 MeV with 40 MeV width.) Also in the case of the $\bar{K}\bar{K}N$ system, it is found that the

absolute values of the perturbative energy $|\langle \Psi | \mathrm{Im} V_{\bar{K}N} | \Psi \rangle| = 20 \sim 30$ MeV is much smaller than the real potential energy $|\langle \Psi | \mathrm{Re} V_{\bar{K}N} + \mathrm{Re} V_{\bar{K}\bar{K}} | \Psi \rangle| = 100 \sim 200$ MeV in the present calculations. This is because the dominant contribution of $\langle V_{\bar{K}N} \rangle$ comes from the $I = 0$ channel that has the strong attractive potential with the weak imaginary part compared with the real part.

III. RESULTS

In this section, we show the results of calculation of the $\bar{K}\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$. For the $\bar{K}N$ interactions, we compare two potentials, the HW-HNJH potential and the AY potential, as discussed in the previous sections. We first show the calculation without the $\bar{K}\bar{K}$ interactions $V_{\bar{K}\bar{K}}$, to see if the $\bar{K}N$ interaction is strong enough for binding the three-body system. Later in subsection III B, we discuss the effect of $V_{\bar{K}\bar{K}}$ by introducing possible repulsive $\bar{K}\bar{K}$ interactions in the $I = 1$ channel.

A. Bound $\bar{K}\bar{K}N$ state without $\bar{K}\bar{K}$ interaction

In this subsection, we present the results of the $\bar{K}\bar{K}N$ state calculated without the $V_{\bar{K}\bar{K}}$ interactions.

1. Energy, width, and decay modes of $\bar{K}\bar{K}N$ state

First, it is very interesting that the $\bar{K}\bar{K}N$ bound state is obtained below the threshold of the $\bar{K}$ and the quasibound $\bar{K}N$ state in both calculations with the HW-HNJH potential and with the AY potential, as seen in Fig. 2, where we show the level structure of the $\bar{K}\bar{K}N$ system. In Table II, we show the results for the energies and radius of the $\bar{K}\bar{K}N$ state as well as those for the $\bar{K}N$ state. For the energy-dependent HW-HNJH potential, we take two energies $\delta\omega = 0$ and 11 MeV with $\omega = M_N + M_K - \delta\omega$. In the table, $B(\bar{K}N)$ and $B(\bar{K}\bar{K}N)$ denote the binding energies for the $\bar{K}N$ and $\bar{K}\bar{K}N$ systems measured by the energies of the two-body and three-body break-up states, respectively.

The $\bar{K}\bar{K}N$ bound state appears as small as 1 MeV below the threshold of the $\bar{K}$ and quasibound $\bar{K}N$ state$[-B(\bar{K}N)]$
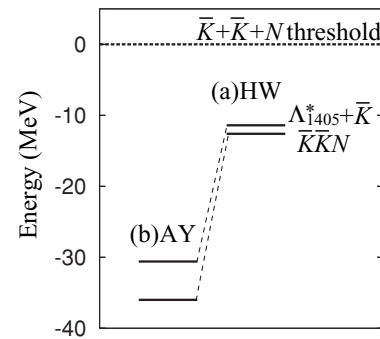

FIG. 2. Energies of the $\bar{K}\bar{K}N$ calculated with (a) the HW-HNJH ($\delta\omega = 11$ MeV) potential and (b) the AY potential without the $\bar{K}\bar{K}$ interaction. The calculated thresholds of $\bar{K}$ and the quasibound $\bar{K}N$ state are also shown. The $\bar{K} + \bar{K} + N$ threshold is located at 1930 MeV.

TABLE II. Energies and root-mean-square radii and distances of the $\bar{K}N$ and the $\bar{K}\bar{K}N$ states calculated without the $\bar{K}\bar{K}$ interaction. For the $\bar{K}N$ interactions, the AY potential and the HW-HNJH potential with the energy parameter $\omega = M_N + M_K - \delta\omega$ (MeV) are used. For the $\bar{K}N$ state, the real energy $E = -B(\bar{K}N)$, the imaginary energy E^{Im}, the root-mean-square $\bar{K}$-N distances$(d_{\bar{K}N})$, $r_{\bar{K}}$ are shown. For the $\bar{K}\bar{K}N$ state, the real energy $E = -B(\bar{K}\bar{K}N)$, the energy relative to the $\bar{K}N + \bar{K}$ threshold, the imaginary energy E^{Im}, $r_{\bar{K}}$ are shown. We also list the expectation values of $\mathrm{Im}V_{\bar{K}N}^{I=0}$ and $\mathrm{Im}V_{\bar{K}N}^{I=1}$ separately.

| | AY | HW-HNJH | |
		$\delta\omega = 0$	$\delta\omega = 11$
$\bar{K}N(I = 0)$ state			
$-B(\bar{K}N)$ (MeV)	-30.6	-10.4	-11.4
E^{Im} (MeV)	-19.9	-20.9	-21.8
$d_{\bar{K}N}$ (fm)	1.4	2.0	1.9
$r_{\bar{K}}$ (fm)	0.9	1.3	1.2
$\bar{K}\bar{K}N(I = 1/2)$ state			
$-B(\bar{K}\bar{K}N)$ (MeV)	-36.0	-11.3	-12.6
$-B(\bar{K}\bar{K}N) + B(\bar{K}N)$ (MeV)	-5.4	-1.0	-1.2
E^{Im} (MeV)	-28.3	-25.4	-26.8
$\langle\mathrm{Im}V^{I=0}\rangle$ (MeV)	-24.8	-24.2	-25.5
$\langle\mathrm{Im}V^{I=1}\rangle$ (MeV)	-3.5	-1.2	-1.3
$r_{\bar{K}}$ (fm)	2.0	4.2	3.8

in both cases of the HW-HNJH potential. The calculated value for $r_{\bar{K}}$ is about 4 fm, which is much larger than that of the two-body $\bar{K}N$ bound state. It indicates that the $\bar{K}\bar{K}N$ state is loosely bound with a large radius. In the case of the AY potential, the $\bar{K}\bar{K}N$ energy is -36 MeV, which is 5 MeV below the $\bar{K}N + \bar{K}$ threshold (-31 MeV). The AY potential gives a deeper binding and a smaller radius, $r_{\bar{K}} = 2$ fm than the HW-HNJH potential, reflecting the stronger $\bar{K}N(I = 0)$ attraction in the AY potential.

The $\bar{K}\bar{K}N \rightarrow \pi Y\bar{K}$ decay width of the $\bar{K}\bar{K}N$ state is evaluated by the imaginary $\bar{K}N$ potentials as $\Gamma(\bar{K}\bar{K}N \rightarrow \pi Y\bar{K}) = -2E^{\mathrm{Im}}$. In the present results, we obtain the width in the range of 51 to 57 MeV. This implies that the $\bar{K}\bar{K}N$ state has a comparable width to that of the $\bar{K}N$ state for $\Lambda(1405)$. It is interesting to note that the dominant contribution in the decay width of the $\bar{K}\bar{K}N$ state comes from the $I = 0$ channel for the $\bar{K}N$ subsystem, whereas the $I = 1$ channel gives a contribution of only a few MeV. This means that the $\bar{K}\bar{K}N$ state has dominant $\pi\Sigma\bar{K}$ and relatively small $\pi\Lambda\bar{K}$ decay modes. This characteristic decay pattern comes from the strong attraction in the $\bar{K}N$ channel with $I = 0$ reproducing the $\Lambda(1405)$ as a quasibound state.

2. Structure of $\bar{K}\bar{K}N$ state

Let us discuss the binding mechanism and the structure of the $\bar{K}\bar{K}N$ system with $I = 1/2$ in detail. For this purpose, we analyze the wave functions obtained in the present few-body calculation in terms of the isospin configuration and the spatial structure of the $\bar{K}\bar{K}N$ system, because the potential energy is determined by both of the isospin and spatial structure.

We first discuss the binding mechanism based on the isospin configurations. For convenience, we introduce the following notations; $[\bar{K}\bar{K}]_I$ denotes the isospin configuration $[[\bar{K}\bar{K}]_{I_{KK}=I}N]_{I=1/2}$ for the two-body $\bar{K}\bar{K}$ subsystem with $I = 0$ or 1 in the three-body system and $[\bar{K}N]_0$ stands for $[[\bar{K}N]_{I_{KN}=0}\bar{K}]_{I=1/2}$ for the two-body $\bar{K}N$ subsystem with $I = 0$, which is $\Lambda(1405) + \bar{K}$-like isospin configuration.

According to the following argument, it is easy to understand that the $[\bar{K}\bar{K}]_1$ and $[\bar{K}N]_0$ configurations are energetically favored to gain potential energy of the $\bar{K}N$ subsystem. The effective $\bar{K}N$ interactions $V_{\bar{K}N}^I$ have strong attraction in $I = 0$ and weak attraction in the $I = 1$ channel. From group-theoretical arguments it is easily found that, on one hand, the former configuration $[\bar{K}\bar{K}]_1$ consists of $[\bar{K}N]_0$ and $[\bar{K}N]_1$ components with a ratio 3:1. As a result, both of two antikaons effectively feel the potential $\frac{3}{4}V_{\bar{K}N}^{I=0} + \frac{1}{4}V_{\bar{K}N}^{I=1}$, which is moderate attraction. However, in the latter configuration $[\bar{K}N]_0$, one of the antikaons couples with the nucleon in $I_{KN} = 0$ and gains the strong attraction of $V_{\bar{K}N}^{I=0}$, and, at the same time, the other antikaon feels much weaker attraction as $\frac{1}{4}V_{\bar{K}N}^{I=0} + \frac{3}{4}V_{\bar{K}N}^{I=1}$.

In the present calculation, it is found that the probability $\Pi([\bar{K}\bar{K}]_1)$ for the three-body system to have the $[\bar{K}\bar{K}]_1$ configuration is dominant in the $\bar{K}\bar{K}N$ wave functions as shown in Table III, in which $\Pi([\bar{K}\bar{K}]_1)$ is 0.87 for the result of the HW-HNJH ($\delta\omega = 11$ MeV) potential and it is 0.91 in the case of the AY potential. These values are in between two limits, $[\bar{K}N]_0$ and $[\bar{K}\bar{K}]_1$. In the $[\bar{K}N]_0$ limit, the probability $\Pi([\bar{K}\bar{K}]_1)$ should be 0.75 when the symmetrization of two antikaons is ignored, whereas $\Pi([\bar{K}\bar{K}]_1) = 1$ for the pure

TABLE III. Properties such as the isospin configurations and radii in the three-body $\bar{K}\bar{K}N$ system calculated without the $\bar{K}\bar{K}$ interaction. In the upper part, we show the values in the total wave function Ψ obtained after the symmetrization. The probabilities $P([\bar{K}\bar{K}]_{0,1})$ to have the $[\bar{K}\bar{K}]_{0,1}$ configurations are listed. The $r_{\bar{K}}$ in the $[\bar{K}\bar{K}]_0$ and $[\bar{K}\bar{K}]_1$ components are shown separately. The calculated values for $d_{\bar{K}\bar{K}}$ are also shown. We also present the expectation values for the unsymmetrized wave function Φ. The norms of the wave functions for the isospin configurations $[\bar{K}_2N]_0$ and $[\bar{K}_2N]_1$, and distances $d_{\bar{K}_2N}$ and $d_{(\bar{K}_2N)-\bar{K}_1}$ calculated with the wave function Φ are shown.

| | AY | HW-HNJH | |
		$\delta\omega = 0$	$\delta\omega = 11$
Expectation values for Ψ			
$\Pi([\bar{K}\bar{K}]_0)$	0.09	0.13	0.13
$\Pi([\bar{K}\bar{K}]_1)$	0.91	0.87	0.87
$r_{\bar{K}}|_{I_{KK}=0}$ (fm)	2.7	5.2	4.8
$r_{\bar{K}}|_{I_{KK}=1}$ (fm)	1.9	4.0	3.7
$d_{\bar{K}\bar{K}}$ (fm)	3.1	6.4	5.9
Expectation values for Φ			
$\Pi([\bar{K}_2N]_0)_\Phi$	0.80	0.81	0.80
$\Pi([\bar{K}_2N]_1)_\Phi$	0.00	0.00	0.00
$d_{\bar{K}_2N}$ (fm)	1.2	1.7	1.6
$d_{(\bar{K}_2N)-\bar{K}_1}$ (fm)	3.2	6.8	6.2

$[\bar{K}\bar{K}]_1$ state. The present calculation implies that the $\bar{K}\bar{K}N$ state is regarded as an admixture of the isospin configurations $[\bar{K}\bar{K}]_1$ and $[\bar{K}N]_0$. Therefore, the rearrangement of the isospin configurations is essential in the $\bar{K}\bar{K}N$ bound state.

Next we discuss the spatial structure of the $\bar{K}\bar{K}N$ state. Because two antikaons are identical bosonic particles, $\bar{K}_1$ and $\bar{K}_2$ cannot be identified in the symmetrized wave function Ψ. For intuitive understanding, it is helpful to analyze the wave function Φ obtained before the symmetrization. For the wave function Φ, we can definitely calculate the expectation values for the $\bar{K}_2$-N and $(\bar{K}_2 N)$-$\bar{K}_1$ distances in the rearrangement channel $c = 1$ as given by Eq. (25). In the calculated results, it is found that $d_{\bar{K}_2 N}$ is almost the same as the $\bar{K}$-N distance in the $\bar{K}N$ quasibound state, whereas $d_{(\bar{K}_2 N)-\bar{K}_1}$ is remarkably large. Actually, $d_{(\bar{K}_2 N)-\bar{K}_1}$ is more than three times larger than $d_{\bar{K}_2 N}$ in all choices of the interactions as shown in Table III. For example, we obtain $d_{\bar{K}_2 N} = 1.6$ fm and $d_{(\bar{K}_2 N)-\bar{K}_1} = 6.2$ fm with the HW-HNJH ($\delta\omega = 11$ MeV) potential. It indicates that one of the antikaons widely distributes around the nucleon with very loosely binding and the other antikaon is moving in the vicinity of the nucleon. In addition, the wave function Φ contains dominantly the $[\bar{K}_2 N]_0$ component and little $[\bar{K}_2 N]_1$ component, which are shown as the norms of the corresponding isospin wave functions in Table III. This means that the $\bar{K}_2 N$ subsystem has more likely the $I = 0$ component and, thus, the Φ has a $\Lambda(1405) + \bar{K}_1$ cluster structure.[1] It is worth noting that, even though the group theoretical argument suggests that the probability $\Pi([\bar{K}\bar{K}]_1)$ for the normalized wave function should be 0.75 without the symmetrization, the $[\bar{K}\bar{K}]_1$ component is obtained as $\Pi([\bar{K}\bar{K}]_1) \sim 0.9$ for the symmetrized wave function Ψ, as discussed above. This is because of the symmetrization effect.

Now let us combine the analysis of the isospin configurations and the spatial structure. The $\bar{K}\bar{K}N$ bound state can be interpreted as a hybrid of the two configurations: the $[\bar{K}\bar{K}]_1$ in the inner region (I) and the $\Lambda(1405) + \bar{K}$ cluster in the asymptotic region (II) as shown in the schematic figure (Fig. 3). As mentioned above, one of the antikaons distributes in a spatially wide region and the other antikaon distributes near the nucleon. In the inner region (I), two antikaons couple in the isospin symmetric configuration $[\bar{K}\bar{K}]_1$ because they occupy the same orbit and are spatially symmetric. In the outer region (II), the nucleon and one of the antikaons form a $\Lambda(1405)$ state and the other antikaon is moving around the $\Lambda(1405)$. This kaon halolike structure is reminiscent of the neutron halo observed in unstable nuclei.

B. Effect of repulsive $\bar{K}\bar{K}$ interaction

In the previous subsection, we presented the results calculated without $\bar{K}\bar{K}$ interactions. We discuss possible effects of the $\bar{K}\bar{K}$ interactions. As already mentioned, the $I = 0$ is forbidden for s-wave $\bar{K}\bar{K}$ states due to Einstein-Bose statistics. We here investigate how the repulsive $\bar{K}\bar{K}$ interactions may affect the $\bar{K}\bar{K}N$ state by introducing $V_{\bar{K}\bar{K}}^I$ for the $I = 1$ channel.

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[1]The idea of $\Lambda(1405)$ cluster in kaonic nuclei was proposed in the $K^- pp$ system in Ref. [22].
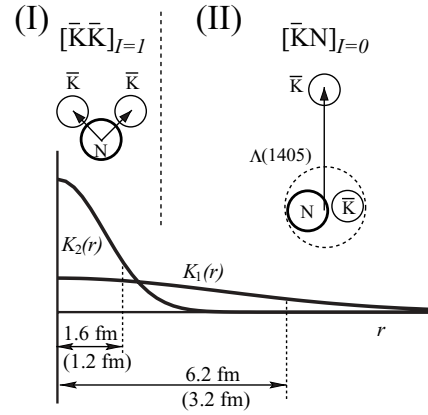

FIG. 3. Schematic figure for the $\bar{K}\bar{K}N$ state. $K_1(r)$ and $K_2(r)$ indicate the antikaon wave functions. One of the antikaons distributes in a wide region and the other antikaon distributes near the nucleon. The values shown in the figure without the parenthesis are $d_{\bar{K}_2 N}$ and $d_{(\bar{K}_2 N)-\bar{K}_1}$, calculated with the HW-HNJH potential ($\delta\omega = 11$ MeV), whereas the values in the parentheses are those for the AY potential. In the inner region (I), two antikaons couple in isospin symmetric configuration $[[\bar{K}\bar{K}]_{I_{KK}=1}N]_{I=1/2}$ because they occupy the same orbit and are spatial symmetric. In the outer region (II), the nucleon and one of the antikaons form a $\Lambda(1405)$ state and the other antikaon is moving around the $\Lambda(1405)$.

We use the $V_{\bar{K}\bar{K}}^{I=1}$ interactions adjusted to reproduce the scattering length $a_{KK}^{I=1} = -0.14$ fm of lattice QCD calculation. We also use weaker repulsion with $a_{KK}^{I=1} = -0.10$ fm. The calculated results with $V_{\bar{K}\bar{K}}^{I=1}$ are shown in Table IV. It is found that the $\bar{K}\bar{K}N$ bound state is obtained even with the possible repulsive $\bar{K}\bar{K}$ interactions. Because of the repulsive $\bar{K}\bar{K}$ interactions, the antikaon is further loosely bound, and the $\Lambda(1405) + \bar{K}$ cluster spatially develops more with a large intercluster distance. As a result, the absolute value of the imaginary energy ImE decreases as shown in Table IV. The value of $d_{(\bar{K}_2 N)-\bar{K}_1}$ is extremely large as $d_{(\bar{K}_2 N)-\bar{K}_1} \geqslant 16$ fm in the HW-HNJH potential, and the $\Lambda(1405) + \bar{K}$ cluster feature becomes more remarkable.

There still remains ambiguity of the strengths of the $\bar{K}\bar{K}$ interactions due to few experimental data. Here we estimate possible boundaries of the repulsive interaction for the formation of the bound $\bar{K}\bar{K}N$ state. For the AY potential, the $\bar{K}\bar{K}N$ state is still bound even with a further strong $\bar{K}\bar{K}$ interaction as $a_{KK}^{I=1} = -0.20$ fm. In the case of the HW-HNJH potential, the $\bar{K}\bar{K}N$ energy is found above the $\bar{K}N + \bar{K}$ threshold with more repulsive $\bar{K}\bar{K}$ interactions than $a_{KK}^{I=1} = -0.15$ fm. In such a case, a resonance state or a virtual state might appear near the threshold. This might be a broad state. For the detailed structure of such states, we need to use formulations beyond the present framework, because the continuum states are not taken into account in the present calculations.

IV. SUMMARY AND CONCLUDING REMARKS

We have investigated the $\bar{K}\bar{K}N$ system with $I = 1/2$ and $J^P = 1/2^+$ as an example of multihadron systems with

TABLE IV. Theoretical results of the $\bar{K}\bar{K}N$ state calculated with the $\bar{K}\bar{K}$ interactions. The range parameters of the $\bar{K}\bar{K}$ interactions are taken to be $b = 0.47$ fm and $b = 0.66$ fm for the HW-HNJH and the AY potentials, respectively. Energies and root-mean-square radii and distances are listed. The expectation values for the various isospin configurations, and those for the unsymmetrized wave function Φ are also shown. The detailed descriptions are given in captions of Tables II and III. In the calculations for HW-HNJH($\delta\omega = 0$ MeV) with $U_{KK}^{I=1} = 70$ MeV, we take the extended basis space as $(n_{\max}, N_{\max}) = (15, 28)$ and $(r_{\min}, r_{\max}, R_{\min}, R_{\max}) = (0.2, 20, 0.2, 400)$ (fm) for the configuration $c = 1$ to get the convergent solution.

	AY+$V_{\bar{K}\bar{K}}$		HW-HNJH $+V_{\bar{K}\bar{K}}$			
			$\delta\omega = 0$		$\delta\omega = 11$	
$a_{KK}^{I=1}$ (fm)	1.0	1.4	1.0	1.4	1.0	1.4
$U_{KK}^{I=1}$ (MeV)	70	104	205	313	205	313
			$\bar{K}\bar{K}N(I = 1/2)$ state			
$-B(\bar{K}\bar{K}N)$ (MeV)	-33.2	-32.3	-10.5	-10.4	-11.5	-11.4
$-B(\bar{K}\bar{K}N) + B(\bar{K}N)$ (MeV)	-2.6	-1.7	-0.1	-0.005	-0.1	-0.01
E^{Im} (MeV)	-25.1	-23.8	-21.7	-21.0	-22.8	-22.0
$\langle \mathrm{Im}V^{I=0} \rangle$ (MeV)	-22.9	-22.1	-21.5	-21.0	-22.5	-22.0
$\langle \mathrm{Im}V^{I=1} \rangle$ (MeV)	-2.2	-1.7	-0.2	0.0	-0.3	-0.1
$r_{\bar{K}}$ (fm)	2.7	3.2	12	49	10	32
			Expectation values for Ψ			
$\Pi([\bar{K}\bar{K}]_0)$	0.13	0.14	0.20	0.24	0.20	0.23
$\Pi([\bar{K}\bar{K}]_1)$	0.87	0.86	0.80	0.76	0.80	0.77
$r_{\bar{K}}|_{I_{KK}=0}$ (fm)	3.4	3.9	13	50	11	33
$r_{\bar{K}}|_{I_{KK}=1}$ (fm)	2.6	3.0	12	48	9.8	31
$d_{\bar{K}\bar{K}}$ (fm)	4.1	4.9	18	76	16	49
			Expectation values for Φ			
$\Pi([\bar{K}_2N]_0)_\Phi$	0.85	0.87	0.93	0.99	0.92	0.98
$\Pi([\bar{K}_2N]_1)_\Phi$	0.00	0.00	0.00	0.00	0.00	0.00
$d_{\bar{K}_2N}$ (fm)	1.2	1.3	1.9	2.0	1.8	1.9
$d_{(\bar{K}_2N)-\bar{K}_1}$ (fm)	4.2	5.0	19	76	16	49

antikaons. We have performed a nonrelativistic three-body calculation by using the effective $\bar{K}N$ interactions proposed by Hyodo-Weise and Akaishi-Yamazaki. With the $\bar{K}N$ potentials and no $\bar{K}\bar{K}$ interaction, the present calculation suggests that a weakly bound $\bar{K}\bar{K}N$ state can be formed below the $\bar{K}N + \bar{K}$ threshold energy. The AY potential for the $\bar{K}N$ interactions provides a deeper bound state of the $\bar{K}\bar{K}N$ system because of the stronger attraction in the $\bar{K}N$ channel with $I = 0$ than the case of the HW potential.

Investigating the wave function obtained by the three-body calculation, we found that the $\bar{K}\bar{K}N$ bound state can be interpreted as a hybrid of two configurations; In the inner region, two antikaons are spatially symmetric and couple in $I_{KK} = 1$, whereas in the asymptotic region where one antikaon is far from the nucleon, the system is regarded as the $\Lambda(1405) + \bar{K}$ like cluster. The root-mean-square radius of the $\bar{K}$ distribution is found to have a large value due to the $\Lambda(1405) + \bar{K}$ like component with a loosely bound antikaon around the $\Lambda(1405)$. This kaon-halo like structure is reminiscent of the neutron-halo observed in unstable nuclei.

We have also evaluated the decay width of the $\bar{K}\bar{K}N$ system to $\pi Y \bar{K}$, obtaining $\Gamma = 40 \sim 60$ MeV, which is comparable to the width of $\Lambda(1405)$. The dominant mode is found to be $\bar{K}\bar{K}N \to \bar{K}(\pi\Sigma)_{I=0}$, reflecting the $\bar{K} + \Lambda(1405)$ cluster structure. In this estimation, we have not considered three-body forces nor transitions to two-hadron decays. Nevertheless, such effects are expected to be suppressed because the overlap of the wave functions of three particles in a compact region is very small in the present case that one of the $\bar{K}$s is loosely bound and distributes very widely around the $\bar{K}N$ subsystem.

To estimate the effect of unknown $\bar{K}\bar{K}$ interactions, we have introduced a repulsive interaction with $I = 1$ that reproduces the K^+K^+ scattering length obtained by lattice QCD calculation. It is interesting that a bound state of the $\bar{K}\bar{K}N$ system is possible even with the repulsive $\bar{K}\bar{K}$ interactions. The repulsive nature of the $\bar{K}\bar{K}$ interaction suggests that the $\Lambda(1405) + \bar{K}$ cluster develops more and the antikaon is further loosely bound.

The peculiar structure that the wave function of one of the antikaon spreads for long distance is a consequence that the $\bar{K}N$ interaction is strong enough to form a quasibound $\bar{K}N$ and $\bar{K}\bar{K}N$ states but is not so strong for the deeply bound $\bar{K}\bar{K}N$ state. This is very significant for the multikaon system. In such systems, some of the antikaon could be bound very loosely, so that many-body absorptions of antikaons are suppressed and the antikaons keep their identity.

So far the experimental data for the $S = -2$ channel have been very poor. In the near future, detailed experimental investigations could clarify the structure of excited Ξ baryons,

for instance, in double strangeness exchange reactions at J-PARC. The $\bar{K}\bar{K}N$ molecular state suggested in the present calculation is one of the excited Ξ baryon with $J^P = 1/2^+$ sitting around 1.9 GeV and having characteristic properties. The main decay mode is the three-body decay of the $\bar{K}\pi\Sigma$ with $I = 0$ for the final $\pi\Sigma$, because the $\Lambda(1405)$ component in the three-body system is a doorway for the decay. In addition, productions of such a multihadron system may be strongly dependent on transfered momentum, because the loosely bound state has a large spatial distribution, which leads a softer form factor. These would be good indications for identifying the molecular state in experiments.

In the present work, we have discussed the bound $\bar{K}\bar{K}N$ state obtained by the single-channel three-body calculation, where effects of coupled-channel meson-baryon interactions are taken into account as effective single-channel $\bar{K}N$ interactions. Such coupled-channels effects could debase clear resonance shape for the three-body quasibound state in the spectrum or could push the quasibound state up to the two-body continuum and the quasibound state could be a virtual state. Nevertheless, in principle, the spectra that will be observed in experiments have information of resonances and virtual states. For complete understanding of the $\bar{K}\bar{K}N$ molecule state, further detailed studies involving dynamics of three-body resonances are necessary. For instance, coupled-channels calculations of the three-body system, including $\bar{K}N \leftrightarrow \pi\Sigma$

[42], such as Faddeev-type calculations done for the ppK^- system in Refs. [11,12] and for two-meson and one-baryon system with $S = -1$ in Ref. [43], will be required to obtain qualitatively precise values of the energy and width.

Our suggestion of the $\bar{K}\bar{K}N$ quasibound state implies a possible existence of an excited baryon with a molecule structure of two mesons surrounding a baryon. The present investigation may be an important step to lead fundamental information on the physics of multistrange systems, such as antikaons in nuclear medium.

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