# **Kaons and antikaons in asymmetric nuclear matter**

Amruta Mishra\*

*Department of Physics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi-110 016, India*

Stefan Schramm† and W. Greiner

*Frankfurt Institute for Advanced Studies, J. W. Goethe Universitat, Ruth-Moufang-Str. 1, D-60438 Frankfurt am Main, Germany ¨* (Received 6 February 2008; revised manuscript received 4 June 2008; published 5 August 2008)

The properties of kaons and antikaons and their modifications in isospin asymmetric nuclear matter are investigated using a chiral SU(3) model. These isospin-dependent medium effects are important for asymmetric heavy-ion collision experiments and will be especially relevant for the neutron-rich heavy-ion collision experiments in the future accelerator facility GSI Facility for Antiproton and Ion Research (GSI-FAIR). In the present work, the medium modifications of the energies of the kaons and antikaons, within the asymmetric nuclear matter, arise because of the interactions of kaons and antikaons with the nucleons and scalar mesons. The values of the parameters in the model are obtained by fitting the saturation properties of nuclear matter and kaon-nucleon scattering lengths for  $I = 0$  and  $I = 1$  channels. Furthermore, the isovector and isoscalar pion-nucleon scattering lengths are calculated within the chiral effective model and compared with earlier results from the literature. The kaon-nucleon and pion-nucleon  $\Sigma$  coefficients are also calculated within the present chiral SU(3) model.

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## **I. INTRODUCTION**

The study of the properties of hadrons at finite densities and temperatures is an important and challenging topic in strong interaction physics. This topic has direct implications for high-energy heavy-ion collision experiments in the study of astrophysical compact objects (like neutron stars) as well as in the early universe. The in-medium properties of kaons have been investigated particularly because of their relevance for neutron star phenomenology as well as for relativistic heavyion collisions. The drop in the mass of the antikaon in the nuclear medium arising from the attractive antikaon-nucleon interaction might lead to antikaon condensation in the interior of a neutron star as was first suggested by Kaplan and Nelson [\[1\]](#page-6-0). However, recent experimental observations on neutron star phenomenology impose constraints on the nuclear equation of state (EOS). The EOS for the nuclear matter obtained using an effective model should be consistent with the astrophysical bounds to be acceptable as an EOS for neutron star matter [\[2,3\]](#page-6-0). Recently, the nuclear matter EOS has been investigated consistent with the neutron star phenomenology as well as data for collective flow in heavy-ion collision experiments [\[4\]](#page-6-0). The in-medium modification of kaon/antikaon properties can be observed experimentally primarily in relativistic nuclear collisions. The experimental [\[5–9\]](#page-6-0) and theoretical studies [\[10–](#page-6-0) [19\]](#page-6-0) of  $K^{\pm}$  production in  $A + A$  collisions at GSI Schwerionen Synchrotron (SIS) energies of 1–2 *A* GeV show that the inmedium properties of kaons can be connected to the collective flow pattern of  $K^+$  mesons as well as to the abundance and spectra of antikaons.

was initiated by Brown and Rho [\[20\]](#page-6-0) who suggested that the modifications of hadron masses should scale with the scalar quark condensate  $\langle q\bar{q}\rangle$  at finite baryon density. The mass modifications of the hadrons like nucleons and vector mesons in dense nuclear matter have been extensively studied using an effective hadronic model within the framework of quantum hadrodynamics [\[21\]](#page-6-0). The first attempts to extract the antikaon-nucleus potential from the analysis of kaonic-atom data were in favor of very strong attractive potentials of the order of −150 to −200 MeV at normal nuclear matter density  $\rho_0$  [\[22,23\]](#page-6-0). However, more recent self-consistent calculations based on a chiral Lagrangian [\[24–27\]](#page-6-0) or coupled-channel *G* matrix theory (within meson-exchange potentials) [\[28\]](#page-6-0) only predicted moderate attraction with potential depths of −50 to  $-80$  MeV at density  $\rho_0$ .

The topic of medium modifications of hadron properties

The problem with the antikaon potential at finite baryon density is that the antikaon-nucleon amplitude in the isospin channel  $I = 0$  is dominated by the  $\Lambda(1405)$  resonant structure, which in free space is only 27 MeV below the  $\bar{K}N$  threshold. It is presently not clear if this physical resonance is an excited state of a "strange" baryon or some short-lived molecular state that, for instance, can be modeled in a coupled channel *T* -matrix scattering equation using a suitable meson-baryon potential. Additionally, the coupling between the  $\bar{K}N$  and  $\pi Y$  $(Y = \Lambda, \Sigma)$  channels is essential to get the proper dynamical behavior in free space. Correspondingly, the in-medium properties of the  $\Lambda(1405)$ , such as its pole position and its width, which in turn strongly influence the antikaon-nucleus optical potential, are very sensitive to the many-body treatment of the medium effects. Previous works have shown that a selfconsistent treatment of the  $\bar{K}$  self-energy has a strong impact on the scattering amplitudes [\[17,24,26–29\]](#page-6-0) and thus on the inmedium properties of the antikaons. Because of the complexity of this many-body problem, the actual kaon and antikaon

<sup>\*</sup>amruta@physics.iitd.ac.in

<sup>†</sup> schramm@th.physik.uni-frankfurt.de

self-energies (or potentials) are still a matter of debate. In Ref. [\[30\]](#page-6-0) it is pointed out that one has to take into account the effect of the Haar measure in mean-field approximations of nonlinear chiral models, which is especially relevant in the high-temperature regime. An extended discussion of the model presented here, including this contribution, should be performed in future studies.

The isospin effects in hot and dense hadronic matter [\[31\]](#page-6-0) are important in isospin asymmetric heavy-ion collision experiments. Within the UrQMD model the density dependence of the symmetry potential has been studied by investigating observables like the  $\pi^{-}/\pi^{+}$  ratio, the *n/p* ratio [\[32\]](#page-7-0), and the  $\Delta^{-}/\Delta^{++}$  ratio as well as the effects on the production of  $K^0$ and  $K^+$  [\[33\]](#page-7-0) and on pion flow [\[34\]](#page-7-0) for neutron-rich heavy-ion collisions. Recently, the isospin dependence of the in-medium *NN* cross section [\[35\]](#page-7-0) has also been studied.

In the present investigation we use a chiral  $SU(3)$  model for the description of hadrons in the medium [\[36,37\]](#page-7-0). The properties of vector mesons [\[37,38\]](#page-7-0) in the nuclear medium within this model have also been examined and were seen to have significant contributions from the Dirac sea polarization effects. The chiral  $SU(3)_{\text{flavor}}$  model was generalized to  $SU(4)_{\text{flavor}}$  to study the mass modification of D mesons arising from their interactions with the light hadrons in hot hadronic matter in Ref. [\[39\]](#page-7-0). The energies of kaons (antikaons), as modified in the medium because of their interaction with nucleons, consistent with the low-energy *KN* scattering data [\[40,41\]](#page-7-0), were also studied within this framework [\[42,43\]](#page-7-0). In the present work, we investigate the effect of isospin asymmetry on the kaon and antikaon optical potentials in the asymmetric nuclear matter, consistent with the low-energy kaon-nucleon scattering lengths for channels  $I = 0$  and  $I = 1$ . Furthermore, the isovector and isoscalar pion-nucleon scattering lengths are also calculated and compared with the earlier literature. In addition, the kaon-nucleon and pion-nucleon  $\Sigma$  coefficients,  $\Sigma_{\pi N}$  and  $\Sigma_{KN}$ , are determined within the present effective chiral model.

The outline of the article is as follows: In Sec. II we briefly review the SU(3) model used in the present investigation. Section III describes the medium modification of the  $K(\bar{K})$ mesons in this effective model. In Sec. IV, we discuss the results obtained for the optical potentials of the kaons and antikaons and the isospin-dependent effects on these optical potentials in asymmetric nuclear matter. Section V summarizes our results and discusses possible extensions of the calculations.

#### **II. THE HADRONIC CHIRAL SU(3) × SU(3) MODEL**

In this section the various terms of the effective hadronic Lagrangian used,

$$
\mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB}, \quad (1)
$$

are discussed. Equation (1) corresponds to a relativistic quantum field theoretical model of baryons and mesons adopting a nonlinear realization of chiral symmetry [\[44–46\]](#page-7-0) and broken scale invariance (for details see Refs. [\[36–38\]](#page-7-0)) to describe strongly interacting nuclear matter. The model was used successfully to describe nuclear matter, finite nuclei, hypernuclei, and neutron stars. The Lagrangian contains the baryon octet, the spin-0, and spin-1 meson multiplets as the elementary degrees of freedom. In Eq. (1),  $\mathcal{L}_{kin}$  is the kinetic energy term,  $\mathcal{L}_{BW}$  contains the baryon-meson interactions in which the baryon-spin-0 meson interaction terms generate the baryon masses.  $\mathcal{L}_{\text{vec}}$  describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields.  $\mathcal{L}_0$  contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential.  $\mathcal{L}_{SB}$  describes the explicit chiral symmetry breaking.

The baryon-scalar meson interactions generate the baryon masses through coupling of the baryons to the nonstrange *σ*(∼  $\langle \bar{u}u + \bar{d}d \rangle$ ) and the strange  $\zeta$ (∼  $\langle \bar{s}s \rangle$ ) scalar quark condensates. The parameters corresponding to these interactions are adjusted to fix the baryon masses to their experimentally measured vacuum values. It should be emphasized that the nucleon mass also depends on the *strange condensate ζ* . For the special case of ideal mixing, however, the nucleon mass depends only on the nonstrange quark condensate.

In analogy to the baryon-scalar meson coupling there exist two independent baryon-vector meson interaction terms corresponding to the *F*-type (antisymmetric) and *D*-type (symmetric) couplings. Here we use the antisymmetric coupling [\[36,43\]](#page-7-0) because, following the universality principle [\[47\]](#page-7-0) and the vector meson dominance model, one can conclude that the symmetric coupling should be small. Additionally we choose the parameters [\[36,43\]](#page-7-0) so as to decouple the strange vector field  $\phi_{\mu} \sim \bar{s} \gamma_{\mu} s$  from the nucleon, corresponding to an ideal mixing between  $\omega$  and  $\phi$ . A small deviation of the mixing angle from the ideal mixing [\[48–50\]](#page-7-0) has not been taken into account in the present investigation.

The Lagrangian densities corresponding to the interaction for the vector meson,  $\mathcal{L}_{\text{vec}}$ , the meson-meson interaction  $\mathcal{L}_0$ , and that corresponding to the explicit chiral symmetry breaking  $\mathcal{L}_{SB}$  have been described in detail in Refs. [\[36,43\]](#page-7-0).

To investigate the hadronic properties in the medium, we write the Lagrangian density within the chiral SU(3) model in the mean-field approximation and determine the expectation values of the meson fields by minimizing the thermodynamical potential [\[37,38\]](#page-7-0).

## **III. KAON (ANTIKAON) INTERACTIONS IN THE CHIRAL SU(3) MODEL**

In this section, we derive the dispersion relations for the  $K(\overline{K})$  [\[51\]](#page-7-0) and calculate their optical potentials in asymmetric nuclear matter [\[43\]](#page-7-0). In the present model, the interactions of the kaons and antikaons with the scalar fields (nonstrange,  $\sigma$ , and strange,  $\zeta$ ) and scalar-isovector field  $\delta$ , as well as a vectorial interaction with the nucleons (the so-called Weinberg-Tomozawa interaction), modify the energies for  $K(\overline{K})$  mesons in the medium. It might be noted here that the interaction of the pseudoscalar mesons to the vector mesons, in addition to the pseudoscalar meson-nucleon vectorial interaction, leads

<span id="page-2-0"></span>to a double counting in the linear realization of the chiral effective theory [\[52\]](#page-7-0). Within the nonlinear realization of the chiral effective theories, such an interaction does not arise in the leading or subleading order, but only as a higher order contribution [\[52\]](#page-7-0). Hence the vector meson-pesudoscalar interaction is not considered within the present investigation. In the following, we derive the dispersion relations for the kaons and antikaons and study the dependence of the kaon and antikaon optical potentials on the isospin asymmetric parameter,  $\eta = \frac{1}{2} (\rho_n - \rho_p) / \rho_B$ . For this, we include the effects from isospin asymmetry originating from the scalar-isovector *δ* field as well as vectorial interaction with the nucleons [\[43\]](#page-7-0). In addition, in the present investigation, we consider an isospin-dependent range term arising from the interaction with the nucleons, which was not taken into account in Ref. [\[43\]](#page-7-0).

The interaction Lagrangian modifying the energies of the  $K(\overline{K})$  mesons is given as

$$
\mathcal{L}_{KN} = -\frac{i}{8f_K^2} [3(\bar{N}\gamma^{\mu}N)(\bar{K}(\partial_{\mu}K) - (\partial_{\mu}\bar{K})K) \n+ (\bar{N}\gamma^{\mu}\tau^a N)(\bar{K}\tau^a(\partial_{\mu}K) - (\partial_{\mu}\bar{K})\tau^a K)] \n+ \frac{m_K^2}{2f_K} [(\sigma + \sqrt{2}\zeta)(\bar{K}K) + \delta^a(\bar{K}\tau^a K)] \n- \frac{1}{f_K} [(\sigma + \sqrt{2}\zeta)(\partial_{\mu}\bar{K})(\partial^{\mu}K) + (\partial_{\mu}\bar{K})\tau^a(\partial^{\mu}K)\delta^a] \n+ \frac{d_1}{2f_K^2} (\bar{N}N)(\partial_{\mu}\bar{K})(\partial^{\mu}K) \n+ \frac{d_2}{2f_K^2} [(\bar{p}p)(\partial_{\mu}K^+)(\partial^{\mu}K^-) + (\bar{n}n)(\partial_{\mu}K^0)(\partial^{\mu}K^-)].]
$$
\n(2)

In the above, *K* and  $\bar{K}$  are the kaon  $(K^+, K^0)$  and antikaon  $(K^-, \bar{K^0})$  doublets. In Eq. (2) the first line is the vectorial interaction term (Weinberg-Tomozawa term) obtained from the kinetic term of the Lagrangian [\[43\]](#page-7-0). The second term, which gives an attractive interaction for the *K* mesons, is obtained from the explicit symmetry breaking term [\[42,43\]](#page-7-0). The third term arises within the present chiral model from the kinetic term of the pseudoscalar mesons [\[43\]](#page-7-0). The fourth and fifth terms in Eq. (2) for the *KN* interactions arise from the terms

$$
\mathcal{L}_{(d_1)}^{\text{BM}} = (d_1)/2 \, Tr(u_\mu u^\mu) \, Tr(\bar{B}B) \tag{3}
$$

and

$$
\mathcal{L}_{(d_2)}^{\text{BM}} = d_2 \operatorname{Tr}(\bar{B}u_\mu u^\mu B) \tag{4}
$$

in the  $SU(3)$  chiral model  $[42,43]$ . The last three terms in Eq. (2) represent the range term in the chiral model, with the last term being an isospin asymmetric interaction. The Fourier transformation of the equation-of-motion for kaons (antikaons) leads to the dispersion relations

$$
-\omega^2 + \vec{k}^2 + m_K^2 - \Pi(\omega, |\vec{k}|, \rho) = 0,
$$

where  $\Pi$  denotes the kaon (antikaon) self-energy in the medium.

Explicitly, the self-energy  $\Pi(\omega, |\vec{k}|)$  for the kaon doublet arising from the interaction (2) is given as

$$
\Pi(\omega, |\vec{k}|) = -\frac{1}{4f_K^2} [3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)]\omega \n+ \frac{m_K^2}{2f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') \n+ \left[ -\frac{1}{f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2} (\rho_s^p + \rho_s^n) \right. \n+ \frac{d_2}{4f_K^2} ((\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n)) \Big] (\omega^2 - \vec{k}^2),
$$
\n(5)

where the  $\pm$  signs refer to the  $K^+$  and  $K^0$ , respectively. In the above,  $\sigma' = \sigma - \sigma_0$ ,  $\zeta' = \zeta - \zeta_0$ , and  $\delta' = \delta - \delta_0$  are the fluctuations of the scalar-isoscalar fields *σ* and *ζ* and the third component of the scalar-isovector field, *δ*, from their vacuum expectation values. The vacuum expectation value of  $\delta$  is zero ( $\delta$ <sub>0</sub> = 0), because a nonzero value for it will break the isospin symmetry of the vacuum (the small isospin breaking effect coming from the mass and charge difference of the up and down quarks has been neglected here).  $\rho_p$  and  $\rho_n$  are the number densities for the proton and the neutron, and  $\rho_s^p$  and  $\rho_s^n$  are their scalar densities.

Similarly, for the antikaon doublet, the self-energy is calculated as

$$
\Pi(\omega, |\vec{k}|) = \frac{1}{4f_K^2} [3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)]\omega \n+ \frac{m_K^2}{2f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') \n+ \left[ -\frac{1}{f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2} (\rho_s^p + \rho_s^n) \right. \n+ \frac{d_2}{4f_K^2} ((\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n)) \Big] (\omega^2 - \vec{k}^2),
$$
\n(6)

where the  $\pm$  signs refer to the  $K^-$  and  $\bar{K}^0$ , respectively.

The optical potentials are calculated from the energies of the kaons and antikaons using

$$
U(\omega, k) = \omega(k) - \sqrt{k^2 + m_K^2},\tag{7}
$$

where  $m_K$  is the vacuum mass for the kaon (antikaon).

The parameters  $d_1$  and  $d_2$  are calculated from the empirical values of the *KN* scattering lengths for  $I = 0$  and  $I = 1$ channels, given by

$$
a_{KN}(I=0) = \frac{m_K}{4\pi f_K^2 (1 + m_K/m_N)}\times \left[ -\frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_{\sigma}^2} + \sqrt{2} \frac{g_{\zeta N}}{m_{\zeta}^2} - 3 \frac{g_{\delta N}}{m_{\delta}^2} \right) + \frac{(d_1 - d_2)m_K}{2} \right]
$$
(8)

and

$$
a_{KN}(I = 1) = \frac{m_K}{4\pi f_K^2 (1 + m_K/m_N)}\times \left[ -1 - \frac{m_K f_K}{2} \left( \frac{g_{\sigma N}}{m_{\sigma}^2} + \sqrt{2} \frac{g_{\zeta N}}{m_{\zeta}^2} + \frac{g_{\delta N}}{m_{\delta}^2} \right) + \frac{(d_1 + d_2)m_K}{2} \right].
$$
 (9)

These are taken to be [\[40,53,54\]](#page-7-0)

 $a_{KN}(I=0) \approx -0.09$  fm,  $a_{KN}(I=1) \approx -0.31$  fm, (10)

leading to the isospin averaged *KN* scattering length as

$$
\bar{a}_{KN} = \frac{1}{4} a_{KN}(I=0) + \frac{3}{4} a_{KN}(I=1) \approx -0.255 \text{ fm.}
$$
 (11)

The pion-nucleon scattering lengths given by

$$
a_{\pi N}\left(I = \frac{3}{2}\right) = \frac{m_{\pi}}{4\pi f_{\pi}^{2}(1 + (m_{\pi}/m_{N}))} \times \left[-\frac{1}{2} - \frac{g_{\sigma N}}{m_{\sigma}^{2}}m_{\pi}f_{\pi} + \frac{(d_{1} + d_{2})m_{\pi}}{2}\right] (12)
$$

and

$$
a_{\pi N}\left(I = \frac{1}{2}\right) = \frac{m_{\pi}}{4\pi f_{\pi}^{2}(1 + (m_{\pi}/m_{N}))} \times \left[1 - \frac{g_{\sigma N}}{m_{\sigma}^{2}} m_{\pi} f_{\pi} + \frac{(d_{1} + d_{2})m_{\pi}}{2}\right] \tag{13}
$$

are also calculated in the present work.

The pion-nucleon and kaon-nucleon  $\Sigma$  coefficients,  $\Sigma_{\pi N}$ and  $\Sigma_{KN}$ , are also calculated within the present chiral effective model using the coupling constants for the interactions of nucleons and pseudoscalar mesons with the scalar mesons.

#### **IV. RESULTS AND DISCUSSIONS**

The present calculations use the following model parameters. The values  $g_{\sigma N} = 10.6$  and  $g_{\zeta N} = -0.47$  are determined by fitting vacuum baryon masses. The other parameters as fitted to the asymmetric nuclear matter saturation properties in the mean-field approximation are  $g_{\omega N} = 13.3$ ,  $g_{\rho N} = 5.5$ ,  $g_4 = 79.7$ ,  $g_{N\delta} = 2.5$ ,  $m_\zeta = 1024.5$  MeV,  $m_\sigma = 466.5$  MeV, and  $m_\delta = 899.5$  MeV. The value of  $g_{\rho N}^2/4\pi \approx 2.4$  of the present work, may be compared with the value of 2.6 [\[55\]](#page-7-0) and a range of values of 2.1 to 3.4 [\[56\]](#page-7-0) in the literature. The value of the  $\omega$  meson-nucleon coupling,  $g_{\omega N}^2/4\pi \approx 14$  in the present investigation, is the same as that in Ref. [\[48\]](#page-7-0), whereas this coupling was taken to be around 24 in Ref. [\[50\]](#page-7-0). The coefficients  $d_1$  and  $d_2$ , calculated from the empirical values of the *KN* scattering lengths for  $I = 0$  and  $I = 1$  channels (10), are  $5.5/m_K$  and  $0.66/m_K$ , respectively. Using these parameters, the symmetry energy defined as

$$
a_4 = 2\frac{d^2 E}{d\eta^2}|_{\eta=0} \tag{14}
$$

has a value of  $a_4 = 31.7$  MeV at a saturation nuclear matter density of  $\rho_0 = 0.15$  fm<sup>-3</sup>. Figure 1 shows the density



FIG. 1. The symmetry energy,  $a_4$ , plotted as a function of the baryon density,  $\rho_B/\rho_0$ .

dependence of the symmetry energy, which increases with density similar to previous calculations [\[57\]](#page-7-0).

The values of the pion-nucleon scattering lengths [\[58–63\]](#page-7-0) are calculated in the present model, with the values of  $d_1$ and  $d_2$  as obtained by fitting the kaon-nucleon scattering lengths. Their values for  $I = 3/2$  and  $I = 1/2$ , given by Eqs. (12) and (13) are obtained as  $a_{\pi N}(I = \frac{3}{2}) = -0.1474$  fm



FIG. 2. (Color online) The kaon energies [for  $K^+$  in (a) and for  $K^0$  in (b)] in MeV plotted as functions of the baryon density,  $\rho_B/\rho_0$ , for different values of the isospin asymmetry parameter,  $\eta = \frac{1}{2}(\rho_n - \rho_n)$  $\rho_p$ *)*/ $\rho_B$ .



FIG. 3. (Color online) The energies of the antikaons [for *K*<sup>−</sup> in (a) and for  $\bar{K}^0$  in (b)], at zero momentum as functions of the baryon density  $(\rho_B/\rho_0)$ , are plotted for different values of the isospin asymmetry parameter, *η*.

and  $a_{\pi N}(I = \frac{1}{2}) = 0.1823$  fm, respectively. This determines the isoscalar and isovector scattering lengths for  $\pi N$ scattering  $(a_{+} = (a_{\pi N}(I = \frac{1}{2}) + 2a_{\pi N}(I = \frac{3}{2}))/3$  and  $a_{-} =$  $(a_{\pi N}(I = \frac{1}{2}) - a_{\pi N}(I = \frac{3}{2}))/3$  to be  $a_{+} = -0.0266/m_{\pi}$  and  $a_0 = 0.078/m_\pi$ . These may be compared with the results of  $a_{+} = -0.0029/m_{\pi}$  and  $a_{-} = 0.0936/m_{\pi}$  derived from pionic atoms [\[61\]](#page-7-0), the values  $a_{+} = -0.0012/m_{\pi}$  and  $a_{-} =$ 0.0895/ $m_{\pi}$  using the empirical values of the  $\pi$ <sup>-</sup>*p* and  $\pi$ <sup>-</sup>*d* scattering lengths [\[62\]](#page-7-0), and the values  $a_{+} = -0.0001/m_{\pi}$  and  $a_0 = 0.0885/m_\pi$  from pionic deuterium shift as quoted in Ref. [\[63\]](#page-7-0).

The  $\pi N$  and *KN* sigma coefficients,  $\Sigma_{\pi N}$  and  $\Sigma_{KN}$ , are also calculated within the model. They turn out to be 44 and 725 MeV for the set of parameters used in the present calculations.

The kaon and antikaon properties were studied in the isospin symmetric hadronic matter within the chiral SU(3) model in Ref. [\[42\]](#page-7-0). The contribution from the vector interaction (Weinberg-Tomozawa term) leads to a drop in the antikaon energy, whereas they are repulsive for the kaons. The scalar meson exchange term arising from the scalar-isoscalar fields (*σ* and *ζ*) is attractive for both *K* and  $\bar{K}$ . The first term of the range term of Eq. [\(2\)](#page-2-0) is repulsive whereas the second term has an attractive contribution for the isospin symmetric matter [\[42\]](#page-7-0) for both kaons and antikaons. The third term of the range term has an isospin asymmetric contribution.

The contributions from (i) the last term of the Weinberg-Tomozawa term, (ii) the scalar-isovector  $\delta$  field, and (iii) the  $d_2$ term in the interaction Lagrangian given by Eq. [\(2\)](#page-2-0) introduce an



FIG. 4. (Color online) The kaon energies [for  $K^+$  in (a), (c), and (e) and for  $K^0$  in (b), (d), and (f)], as compared to the isospin symmetric case, plotted as functions of the momentum for various values of the baryon density,  $\rho_B$ , and for different values of the isospin asymmetry parameter, *η*.

isotopic asymmetry in the *K* and  $\bar{K}$  energies. For  $\rho_n > \rho_n$ , in the kaon sector,  $K^+(K^0)$  has negative (positive) contributions from *δ*. The *δ* contribution from the scalar exchange term is positive (negative) for  $K^+$  ( $K^0$ ), whereas that arising from the range term has the opposite sign and dominates over the former contribution.

In Fig. 2, the energies of the  $K^+$  and  $K^0$  at zero momentum are plotted for different values of the isospin asymmetry parameter, *η*, at various densities. For  $\rho_B = \rho_0$  the energy of  $K^+$  is seen to drop by about 7 MeV at zero momentum when  $\eta$  changes from 0 to 0.5. On the other hand, the  $K^0$ energy is seen to increase by about 27 MeV for  $\eta = 0.5$ from the isospin symmetric case of  $\eta = 0$ . The reason for this opposite behavior for the  $K^+$  and  $K^0$  on the isospin asymmetry originates from the vectorial (Weinberg-Tomozawa) *δ* meson contribution as well as from the isospin-dependent range term ( $d_2$  term) contributions. For  $K^+$ , the  $\eta$  dependence of the energy is seen to be less sensitive at higher densities, whereas the energy of  $K^0$  is seen to have a larger drop from the  $\eta = 0$ case as we increase the density.

For the antikaons, the  $K^{-}(\bar{K^{0}})$  energy at zero momentum is seen to increase (drop) with  $\eta$  as we increase the density as seen in Fig. 3. The sensitivity of the isospin asymmetry dependence of the energies is seen to be larger for *K*<sup>−</sup> with density, whereas it becomes smaller for  $K^0$  at high densities.



FIG. 5. (Color online) The antikaon energies [for *K*<sup>−</sup> in (a), (c), and (e) and for  $\bar{K}^0$  in (b), (d), and (f)], as compared to the isospin symmetric case, plotted as functions of the momentum for different values of the isospin asymmetry parameter,  $\eta$ , and for various values of the baryon density,  $\rho_B$ .

The mass drop as modified in the isospin asymmetry in neutron star matter will have relevance for the onset of antikaon (*K*<sup>−</sup> and  $\bar{K}^0$ ) condensation.

The energies of the kaons and antikaons, with respect to the isospin symmetric case, for different values of the isospin asymmetric parameter *η* are plotted as functions of the momentum in Figs. 4 and 5. The energies of the kaons and antikaons are plotted for densities  $\rho_B = \rho_0$ ,  $2\rho_0$ , and  $4\rho_0$ in the same figures. These are seen to be more sensitive to momentum as we increase the isospin parameter. The momentum dependence turns out to be stronger for higher densities, and in particular, the effect seems to be more significant for  $K^0$  (as compared to  $K^+$ ) and  $K^-$  (as compared to  $\bar{K}^0$ ).

The qualitative behavior of the isospin asymmetry dependencies of the energies of the kaons and antikaons are also reflected in their optical potentials plotted in Fig. 6 for the kaons and in Fig. 7 for the antikaons at selected densities. The different behavior of the  $K^+$  and  $K^0$ , as well as for the  $K^-$  and  $\bar{K}^0$  optical potentials in the dense asymmetric nuclear matter, should be observed in their production as well as propagation in isospin asymmetric heavy-ion collisions. In particular an experimental study of the  $K^-/\bar{K}^0$  ratio (and its dependence on the kaon momenta) might be a promising tool to investigate the isospin effects discussed here. The effects of the isospin asymmetric optical potentials could thus be observed in



FIG. 6. (Color online) The kaon optical potentials [for  $K^+$  in (a), (c), and (e) and for  $K^0$  in (b), (d), and (f)] in MeV, plotted as functions of the momentum for various baryon densities,  $\rho_B$ , and for different values of the isospin asymmetry parameter, *η*.

nuclear collisions at the Compressed Baryonic Matter (CBM) experiment at the GSI Facility for Antiproton and Ion Research (GSI-FAIR), where experiments with neutron-rich beams are planned to be carried out.

## **V. SUMMARY**

To summarize, within a chiral SU(3) model we have investigated the density dependence of the  $K$ ,  $\bar{K}$ -meson optical potentials in asymmetric nuclear matter, arising from the interactions with nucleons (originating from a vectorial Weinberg-Tomozawa interaction, an isospin symmetric range term, and an isospin asymmetric range term) and scalar mesons (from a scalar exchange as well as a range term). The properties of the light hadrons—as studied in this model—modify the  $K(\overline{K})$ -meson properties in the hadronic medium. The model with parameters fitted to reproduce the properties of hadron masses in vacuum, nuclear matter saturation properties and low-energy *KN* scattering data, takes into account all terms up to the next to leading order arising in chiral perturbative expansion for the interactions of  $K(\bar{K})$  mesons with baryons. The  $\pi N$  scattering lengths are also calculated for the fitted set of model parameters as  $a_{+} = -0.0266/m_{\pi}$  and  $a_{-} =$  $0.078/m<sub>\pi</sub>$  and have been compared with the other results in the literature [\[61–63\]](#page-7-0). The  $\pi N$  and  $KN \Sigma$  coefficients are calculated within the present chiral effective model and have values of  $\Sigma_{\pi N} = 44$  MeV and  $\Sigma_{KN} = 725$  MeV.

<span id="page-6-0"></span>

FIG. 7. (Color online) The antikaon optical potentials [for *K*<sup>−</sup> in (a), (c), and (e) and for  $K^0$  in (b), (d), and (f)] in MeV, plotted as functions of the momentum for various baryon densities,  $\rho_B$ , and for different values of the isospin asymmetry parameter, *η*.

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There is a significant density dependence of the isospin asymmetry on the optical potentials of the kaons and antikaons. The results can be used in heavy-ion simulations that include mean fields for the propagation of mesons [\[51\]](#page-7-0). The different potentials of kaons and antikaons can be particularly relevant for neutron-rich heavy-ion beams at the CBM experiment at GSI-FAIR, Germany, as well as at the experiments at the proposed Rare Isotope Accelerator (RIA) laboratory, USA. The  $K^-/\bar{K}^0$  ratio for different isospin of projectile and target is a promising observable to study these effects. Furthermore, the medium modification of antikaons due to isospin asymmetry in dense matter can have important consequences, for example, on the onset of antikaon condensation in the bulk charge neutral matter in neutron stars. The effects of hyperons as well as finite temperatures on optical potentials of kaons and antikaons and their possible implications on the neutron star phenomenology as well as heavy-ion collision experiments are the intended topics of future investigation.

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