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Effects of ground state correlations on the structure of odd-mass spherical nuclei

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It is well known that the Pauli principle plays a substantial role at low energies because the quasiparticle and phonon operators, used to describe them, are built of fermions and as a consequence they are not ideal bosons. The correct treatment of this problem requires calculation of the exact commutators between the quasiparticle and phonon operators and in this way to take into account the Pauli principle corrections. In addition to the correlations due to the quasiparticle interaction in the ground-state influence the single-particle fragmentation as well. In this article, we generalize the basic equations of the quasiparticle-phonon nuclear model to account for both effects mentioned above. As an illustration of our approach, calculations of the structure of the low-lying states in the odd-mass nuclei $^{131-137}$Ba have been performed.

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I. INTRODUCTION

The generation of exotic nuclei, made possible due to the emergence of the radioactive ion beam accelerators, poses a challenge in front of theoretical and experimental physicists, because new methods need to be developed to understand the behavior of these nuclei far away from the valley of stability. These studies are motivated by the significant changes that take place in the structure of these nuclei. Along with the changes in the shell structure within the mean-field approximation, the many-body effects increase their role as we move away from the magic numbers. One common approach for describing odd-mass nuclei based on the mean-field approximation is to consider the coupling between even-even core excitations and a nucleon outside of the core [1–5].

A fairly good theoretical description of the ground-state correlations (GSC) can be achieved within an extended version of the quasiparticle-phonon model (QPM) [4,6]. Here and further by GSC we imply correlations due to the quasiparticle-phonon interaction in the ground state.

The QPM is widely used for the description of the energies and fragmentation of nuclear excitations. The different versions of the QPM equations for odd-mass spherical nuclei are given in Refs. [7–9]. It has been shown in Refs. [7,10] that corrections due to the action of the Pauli principle are very important for the determination of the energies of some states. In addition to a good agreement with the experimental energies in both the low- and higher-lying region for several odd-mass lead isotopes near ^{208}Pb has been reached in Ref. [11] using the quasiparticle multistep shell-model method. Effects of coupling of different nuclear excitations in even-even nuclei have been considered in Ref. [12]. An interesting recent development in this direction is presented in Ref. [13] where the influence of the relativistic effects of the Dirac sea on the particle-vibration coupling is studied.

However, in none of the above investigations the effects of the ground-state correlations have been considered. As it was proved in Ref. [14], the GSC influence the single-particle fragmentation shifting the strength to higher excitation energies. In their study the operators of the quasiparticles and the phonons are taken as commuting ones, thus neglecting the Pauli principle. Additionally to treat the interaction between the single-particle states near the Fermi level with the vibrating core for nuclei remote from the valley of stability, an extended configurational space that takes into account the correlations in the ground state has to be used.

In this article, we generalize the basic QPM equations for odd-mass spherical nuclei to take account of the effects due to the GSC and the Pauli principle. We treat long-range ground-state correlations by including backward-going quasiparticle-phonon vertices using the equation-of-motion method [15] with explicitly taking into account the Pauli principle. Numerical calculations of the structure of the low-lying states in the odd-mass nuclei $^{131-137}$Ba within the developed approach have been performed.

This article is organized as follows: in Sec. II we give the basic ingredients of the QPM for odd-mass nuclei as well as the new developments that allowed us to treat the GSC. An approximation, presented in Sec. IIIA, makes possible the derivation of an equation for the energies of the states in odd-mass nuclei. In Sec IIIB, a reduction to the "classic" equation is made, where the GSC are not taken into account. Numerical calculations of different quantities of our model as well as the energies and the spectroscopic factors are presented in Sec. IV. Conclusions are drawn in Sec. V.

II. FORMULATION OF THE MODEL

We employ the QPM-Hamiltonian, including an average nuclear field, described by the Woods-Saxon potential, pairing interactions, isoscalar particle-hole residual forces in separable form with the Bohr-Mottelson radial dependence [1]:

$$H = \sum_{\tau}^{(n,p)} \left[\sum_{jm} (E_j - \lambda_\tau) a^{\dagger}_{jm} a_{jm} - \frac{1}{4} G^{(0)}_{\tau} : (P^{\dagger}_0 P_0)^{\tau} : - \frac{1}{2} \sum_{\lambda\mu} \kappa^{(\lambda)} : (M^{\dagger}_{\lambda\mu} M_{\lambda\mu}) : \right]. \tag{1}$$

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The single-particle states are specified by the quantum numbers (jm), E_j are the single-particle energies, λ_τ is the chemical potential, and $G_\tau^{(0)}$ and $\kappa^{(\lambda)}$ are the strengths in the particle-particle and in the particle-hole channels, respectively. The sum goes over protons (p) and neutrons (n) independently and the notation $\tau = \{n, p\}$ is used. The pair creation and the multipole operators entering the normal products in Eq. (1) are defined as follows:

$$P_0^\dagger = \sum_{jm} (-1)^{j-m} a_{jm}^\dagger a_{j-m}^\dagger, \tag{2}$$

$$M_{\lambda\mu}^\dagger = \frac{1}{\sqrt{2\lambda+1}} \sum_{jj'mm'} f_{jj'}^{(\lambda)} \langle jmj'm' \mid \lambda\mu\rangle a_{jm}^\dagger a_{j'm'}, \tag{3}$$

where $f_{jj'}^{(\lambda)}$ are the single-particle radial matrix elements of the residual forces.

In what follows we work in quasiparticle representation determined by the canonical Bogoliubov transformation:

$$a_{jm}^\dagger = u_j \alpha_{jm}^\dagger + (-1)^{j-m} v_j \alpha_{j-m}. \tag{4}$$

The Hamiltonian can be represented in terms of bifermion quasiparticle operators (and their conjugate ones):

$$B(jj';\lambda\mu) = \sum_{mm'} (-1)^{j'+m'} \langle jmj'm' \mid \lambda\mu\rangle \alpha_{jm}^\dagger \alpha_{j'-m'}, \tag{5}$$

$$A^\dagger(jj';\lambda\mu) = \sum_{mm'} \langle jmj'm' \mid \lambda\mu\rangle \alpha_{jm}^\dagger \alpha_{j'm'}^\dagger. \tag{6}$$

The phonon creation operators are defined in the two-quasiparticle space in a standard fashion:

$$Q_{\lambda\mu i}^\dagger = \frac{1}{2}\sum_{jj'} \big[\psi_{jj'}^{\lambda i} A^\dagger(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi_{jj'}^{\lambda i} A(jj';\lambda-\mu\big], \tag{7}$$

where the index λ denotes the multipolarity and μ is its z projection in the laboratory system. The normalization of the one-phonon states reads:

$$\langle|[Q_{\lambda\mu i}, Q_{\lambda'\mu'i'}^\dagger]|\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}. \tag{8}$$

In terms of quasiparticles and phonons the Hamiltonian is rewritten in the following way:

$$H = h_0 + h_{pp} + h_{QQ} + h_{QB}, \tag{9}$$

where its first two single particle terms are:

$$h_0 + h_{pp} = \sum_{jm} \varepsilon_j \alpha_{jm}^\dagger \alpha_{jm}, \tag{10}$$

The last two terms are expressed as:

$$h_{QQ} = -\frac{1}{8}\sum_{\lambda\mu ii'} \mathscr{A}(\lambda ii')[Q_{\lambda\mu i}^\dagger + (-)^{\lambda-\mu} Q_{\lambda-\mu i}] \times [Q_{\lambda-\mu i'}^\dagger + (-)^{\lambda+\mu} Q_{\lambda\mu i'}], \tag{11}$$

$$h_{QB} = -\frac{1}{2\sqrt{2}}\sum_{\lambda\mu ijj'} \frac{\pi_j}{\pi_\lambda}\Gamma(jj'\lambda i)[(-)^{\lambda-\mu} Q_{\lambda\mu i}^\dagger + Q_{\lambda-\mu i}]\, B(jj';\lambda-\mu) + \text{h.c.}, \tag{12}$$

where the following notations are used:

$$\mathscr{A}(\lambda ii') = \frac{X^{\lambda i} + X^{\lambda i'}}{\sqrt{Y^{\lambda i} Y^{\lambda i'}}}, \tag{13}$$

$$\Gamma(jj'\lambda i) = \frac{\pi_\lambda}{\pi_j}\frac{v_{jj'}^{(-)} f_{jj'}^{(\lambda)}}{\sqrt{Y^{\lambda i}}}, \tag{14}$$

$$X^{\lambda i} = \sum_{jj'} \frac{\big[f_{jj'}^{(\lambda)} u_{jj'}^{(+)}\big]^2 \varepsilon_{jj'}}{\varepsilon_{jj'}^2 - \omega_{\lambda i}^2}, \tag{15}$$

$$Y^{\lambda i} = \sum_{jj'} \frac{\big[f_{jj'}^{(\lambda)} u_{jj'}^{(+)}\big]^2 \varepsilon_{jj'}\omega_{\lambda i}}{\big(\varepsilon_{jj'^2} - \omega_{\lambda i}^2\big)^2}, \tag{16}$$

where:

$$v_{jj'}^{(-)} = u_j u_{j'} - v_j v_{j'}, \quad u_{jj'}^{(+)} = u_{j'} v_j + u_j v_{j'}$$

and $\pi_j = \sqrt{(2j+1)}$.

The model wave function of an odd-mass spherical nucleus is taken in the form [14]:

$$\Psi_\nu(JM) = O_{JM\nu}^\dagger|\rangle, \tag{17}$$

where:

$$O_{JM\nu}^\dagger = C_{J\nu}\alpha_{JM}^\dagger + \sum_{j\lambda i} D_j^{\lambda i}(J\nu) P_{j\lambda i}^\dagger(JM) - E_{J\nu}\tilde{\alpha}_{JM} - \sum_{j\lambda i} F_j^{\lambda i}(J\nu)\tilde{P}_{j\lambda i}(JM), \tag{18}$$

with

$$P_{j\lambda i}^\dagger(JM) = [\alpha_{jm}^\dagger Q_{\lambda\mu i}^\dagger]_{JM} \tag{19}$$

and ˜ stands for time conjugate according to the convention: $\tilde{P}_{j\lambda i}(JM) = (-1)^{J-M} P_{j\lambda i}(J-M)$.

We apply the equation-of-motion method to the excitation operator (18):

$$\langle|\{\delta O_{JM\nu}, H, O_{JM\nu}^\dagger\}|\rangle = \eta_{J\nu}\langle|\{\delta O_{JM}, O_{JM}^\dagger\}|\rangle. \tag{20}$$

Following the linearization procedure [15], at the final state of calculation of the matrix elements, we consider the ground state of the even-even nucleus to be a vacuum state for both operators α_{JM} and $Q_{\lambda\mu i}$.

In all calculations the exact commutation relations between the quasiparticle and phonon operators are considered:

$$[\alpha_{jm}, Q_{\lambda\mu i}^\dagger] = \sum_{j'm'} \langle jmj'm'|\lambda\mu\rangle \psi_{jj'}^{\lambda i}\alpha_{j'm'}^\dagger. \tag{21}$$

The normalization condition of the wave function reads

$$\langle|\{O_{JM\nu}, O_{JM\nu}^+\}|\rangle = C_{J\nu}^2 + E_{J\nu}^2 + \sum_{j\lambda i}\big[D_j^{\lambda i}(J\nu)\big]^2 + \sum_{j\lambda i}\big[F_j^{\lambda i}(J\nu)\big]^2 + \sum_{j\lambda ij'\lambda'i'}\big[D_j^{\lambda i}(J\nu)D_{j'}^{\lambda' i'}(J\nu) + F_j^{\lambda i}(J\nu)F_{j'}^{\lambda' i'}(J\nu)\big]\mathscr{L}_J(j\lambda i|j'\lambda' i') = 1. \tag{22}$$

The equation-of-motion leads to the following system of linear equations for each state with quantum numbers JM:

$$\begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \times \begin{pmatrix} C_{J\nu} \\ D_{j'}^{\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'}^{\lambda'i'}(J\nu) \end{pmatrix} = \eta_{J\nu} \begin{pmatrix} C_{J\nu} \\ D_j^{\lambda i}(J\nu) + \sum_{j'\lambda'i'} D_{j'}^{\lambda'i'}(J\nu)\mathcal{L}_J(j\lambda i|j'\lambda'i') \\ -E_{J\nu} \\ -F_j^{\lambda i}(J\nu) - \sum_{j'\lambda'i'} F_{j'}^{\lambda'i'}(J\nu)\mathcal{L}_J(j\lambda i|j'\lambda'i') \end{pmatrix}. \quad (23)$$

The explicit expressions for the quantities entering the formulas above will be considered one by one.

$$\mathcal{L}_J(j\lambda i|j'\lambda'i') = \pi_\lambda \pi_{\lambda'} \sum_{j_1} \psi_{j_1 j}^{\lambda'i'} \psi_{j_1 j'}^{\lambda i} \begin{Bmatrix} j' & j_1 & \lambda \\ j & J & \lambda' \end{Bmatrix}, \quad (24)$$

$$\begin{aligned} V(Jj\lambda i) &= \langle | \{[\alpha_{JM}, H], P_{j\lambda i}^{+}(JM)\} | \rangle \\ &= -\frac{1}{\sqrt{2}}\Gamma(Jj\lambda i) - \frac{1}{\sqrt{2}} \sum_{j'\lambda'i'} (\mathcal{T}_J(j\lambda i; j'\lambda'i') \\ &\quad + \mathcal{L}_J(j\lambda i|j'\lambda'i'))\Gamma(Jj'\lambda'i'). \end{aligned} \quad (25)$$

As a result of the application of the equation-of-motion method, the matrix elements $V(Jj\lambda i)$ between quasiparticle and quasiparticle⊗phonon states differ from the ones obtained earlier [7] by an additive term containing $\mathcal{T}_J(j\lambda i|j'\lambda'i')$:

$$\mathcal{T}_J(j\lambda i|j'\lambda'i') = \pi_\lambda \pi_{\lambda'} \sum_{j_1} \psi_{j_1 j'}^{\lambda i} \varphi_{j_1 j}^{\lambda'i'} \begin{Bmatrix} j' & j_1 & \lambda \\ j & J & \lambda' \end{Bmatrix}. \quad (26)$$

The matrix elements $W(Jj\lambda i)$ appear after the introduction of the backward-going terms in the operator (18) and they present a central issue in this work.

$$\begin{aligned} W(Jj\lambda i) &= \langle | \{[\alpha_{JM}^{+}, H], \tilde{P}_{j\lambda i}^{+}(JM)\} | \rangle \\ &= -\frac{1}{4}\frac{\pi_\lambda}{\pi_J} \sum_{i'\tau_0} \mathcal{A}_{\tau_0}(\lambda i i')\varphi_{Jj}^{\lambda i'} \\ &\quad - \frac{1}{4} \sum_{\lambda' j' i' i'' \tau_0} \mathcal{A}_{\tau_0}(\lambda' i' i'') \frac{\pi_{\lambda'}}{\pi_J} [\varphi_{Jj'}^{\lambda' i'} \mathcal{L}_J(j\lambda i|j'\lambda' i'') \\ &\quad - \psi_{Jj'}^{\lambda' i''} \mathcal{T}_J(j\lambda i|j'\lambda' i')]. \end{aligned} \quad (27)$$

The calculation of the diagonal matrix elements yields:

$$K_J(j\lambda i|j'\lambda i') = \tfrac{1}{2}[I_J(j\lambda i|j'\lambda'i') + I_J(j'\lambda'i'|j\lambda i)], \quad (28)$$

where

$$I_J(j\lambda i|j'\lambda'i') = \langle | \{P_{j\lambda i}(JM), [H, P_{j'\lambda'i'}^{+}(JM)]\} | \rangle \quad (29)$$

and

$$\begin{aligned} I_J(j\lambda i|j'\lambda'i') + I_J(j'\lambda'i'|j\lambda i) &= 2\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}(\omega_{\lambda i} + \varepsilon_j) + \mathcal{L}_J(j'\lambda'i'|j\lambda i)(\varepsilon_{j'j} \\ &\quad + \omega_{\lambda'i'} + \omega_{\lambda i}) - \mathcal{R}_J(j\lambda i|j'\lambda'i'). \end{aligned} \quad (30)$$

$\mathcal{R}_J(j\lambda i|j'\lambda'i')$ stands for the quantity:

$$\begin{aligned} \mathcal{R}_J(j\lambda i|j'\lambda'i') &= \frac{1}{4}\sum_{i_1\tau_0} [\mathcal{A}_{\tau_0}(\lambda i_1 i)\mathcal{L}_J(j'\lambda'i'|j\lambda i_1) \\ &\quad + \mathcal{A}_{\tau_0}(\lambda' i_1 i')\mathcal{L}_J(j\lambda i j'\lambda' i_1)] \\ &\quad + \frac{1}{4}\sum_{\lambda_1 i_1 i_2 j_1 \tau_0} \mathcal{A}_{\tau_0}(\lambda_1 i_1 i_2) \\ &\quad \times [\mathcal{L}_J(j\lambda i|j_1\lambda_1 i_1)\mathcal{L}_J(j'\lambda'i'; j_1\lambda_1 i_2) \\ &\quad + \mathcal{L}_J(j'\lambda'i'|j_1\lambda_1 i_1)\mathcal{L}_J(j\lambda i|j_1\lambda_1 i_2)]. \end{aligned} \quad (31)$$

The quantities $\mathcal{L}_J(j\lambda i|j'\lambda'i')$ (24), $\mathcal{T}_J(j\lambda i|j'\lambda'i')$ (26) and $\mathcal{R}_J(j\lambda i|j'\lambda'i')$ (31) vanish if the Pauli principle is not respected.

III. APPROXIMATIONS

A. General

As has been shown in Ref. [7], $\mathcal{L}_J$ (24) are alternating quantities and their diagonal values are much greater than the nondiagonal ones. This is natural from the physical point of view as the Pauli principle is violated most probably in the configurations, formed by identical quasiparticles. The same is valid for the new quantities $\mathcal{T}_J$ (26).

$$\mathcal{L}_J(j\lambda i|j'\lambda'i') = \mathcal{L}(Jj\lambda i)\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}, \quad (32)$$

$$\mathcal{T}_J(j\lambda i|j'\lambda'i') = \mathcal{T}(Jj\lambda i)\delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}, \quad (33)$$

where

$$\mathcal{L}(Jj\lambda i) = \pi_\lambda^2 \sum_{j'} \left(\psi_{j'j}^{\lambda i}\right)^2 \begin{Bmatrix} j & j' & \lambda \\ j & J & \lambda \end{Bmatrix}, \quad (34)$$

$$\mathcal{T}(Jj\lambda i) = \pi_\lambda^2 \sum_{j'} \psi_{j'j}^{\lambda i}\varphi_{j'j}^{\lambda i} \begin{Bmatrix} j & j' & \lambda \\ j & J & \lambda \end{Bmatrix}. \quad (35)$$

It can be seen, that in this approximation the vertices:

$$V(Jj\lambda i) = -\frac{1}{\sqrt{2}}[1 + \mathcal{L}(Jj\lambda i) + \mathcal{T}(Jj\lambda i)]\Gamma(Jj\lambda i). \quad (36)$$

are renormalized by the factor $[1 + \mathcal{L}(Jj\lambda i) + \mathcal{T}(Jj\lambda i)]$. In configurations with a strong Pauli principle violation the quantities $\mathcal{L}(Jj\lambda i)$ go to -1 and $\mathcal{T}(Jj\lambda i)$ go to 0. The role of the term $\mathcal{T}(Jj\lambda i)$ in the renormalization becomes more important as the phonon collectivity increases.

As with the vertices V, the vertices W are renormalized now by the factor $[1 + \mathcal{L}(Jj\lambda i) - \mathcal{L}(jJ\lambda i)]$:

$$W(Jj\lambda i) = -\frac{1}{4}\frac{\pi_\lambda}{\pi_J}[1 + \mathcal{L}(Jj\lambda i) - \mathcal{L}(jJ\lambda i)] \times \sum_{i'\tau_0} \mathcal{A}_{\tau_0}(\lambda i i')\varphi_{Jj}^{\lambda i'} \quad (37)$$

and again in configurations with a strong Pauli principle violation the quantities $\mathcal{L}(Jj\lambda i)$ go to -1 and $\mathcal{L}(jJ\lambda i)$

go to 0. The results for V and W show that configurations with $\mathscr{L}(Jj\lambda i)$ close to -1 must be excluded from the configurational space.

The diagonal matrix elements (28) become:

$$K_J(j\lambda i j'\lambda' i') = \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}[1+\mathscr{L}(Jj\lambda i)][\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)], \quad (38)$$

where the quantities

$$\mathscr{R}(Jj\lambda i) = \frac{\mathscr{R}_J(j\lambda i|j\lambda i)}{1+\mathscr{L}(Jj\lambda i)}. \quad (39)$$

play a very important role as they shift the values of the poles and this shift depends on the extent of the Pauli principle violation [7].

The system of equations (23) can be transformed into the following one [14]:

$$\left[\begin{pmatrix}\varepsilon_J & 0\\ 0 & -\varepsilon_J\end{pmatrix}+\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}\right]\begin{pmatrix}C_{J\nu}\\ -E_{J\nu}\end{pmatrix} = \eta_{J\nu}\begin{pmatrix}C_{J\nu}\\ -E_{J\nu}\end{pmatrix}, \quad (40)$$

where

$$M_{11} = \sum_{j\lambda i}\frac{1}{[1+\mathscr{L}(Jj\lambda i)]}\left\{\frac{V^2(Jj\lambda i)}{\eta_{J\nu}-[\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)]} + \frac{W^2(Jj\lambda i)}{\eta_{J\nu}+\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)}\right\}, \quad (41)$$

$$M_{22} = \sum_{j\lambda i}\frac{1}{[1+\mathscr{L}(Jj\lambda i)]}\left\{\frac{W^2(Jj\lambda i)}{\eta_{J\nu}-[\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)]} + \frac{V^2(Jj\lambda i)}{\eta_{J\nu}+\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)}\right\}, \quad (42)$$

$$M_{12} = \sum_{j\lambda i}\frac{V(Jj\lambda i)W(Jj\lambda i)}{[1+\mathscr{L}(Jj\lambda i)]}\left\{\frac{1}{\eta_{J\nu}+\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)} - \frac{1}{\eta_{J\nu}-[\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)]}\right\} = M_{21}, \quad (43)$$

leading to the equation

$$M_{12}M_{21} = (\varepsilon_J+M_{11}-\eta_{J\nu})(M_{22}-\varepsilon_J-\eta_{J\nu}). \quad (44)$$

B. Limit cases and analysis

The equation (44) can be approximated by the following one

$$\varepsilon_J-\eta_{J\nu} \approx -M_{11} - \frac{M_{12}^2}{|2\varepsilon_J-(M_{22}-M_{11})|}. \quad (45)$$

Therefore, neglecting the backward amplitudes, i.e., setting $W(Jj\lambda i)=0$, (45), immediately reduces to the secular equation obtained earlier [7]:

$$\varepsilon_J-\eta_{J\nu} = \sum_{j\lambda i}\frac{V^2(Jj\lambda i)}{\{[\omega_{\lambda i}+\varepsilon_j-\mathscr{R}(Jj\lambda i)]-\eta_{J\nu}\}[1+\mathscr{L}(Jj\lambda i)]}. \quad (46)$$

One significant difference for the solutions $\eta_{J\nu}$ of the equation (45) in comparison to equation (46), comes from the second term in the right-hand side of the expression (41) that contributes to a shift of the first solution of equation (45) to higher energies. The second term of the right-hand side of equation (45) also contributes in the same direction but to much smaller extent. The shift in energy becomes larger as the interaction between the quasiparticles and phonons increases. A critical value for this interaction exists in both Eqs. (45) and (46). In the case of Eq. (45) the increase in the interaction strength due to the additive terms in M_{11} containing $W(Jj\lambda i)$ would shift the first solution toward the first pole, whereas in the case of Eq. (45) the solution moves in the opposite direction.

IV. NUMERICAL RESULTS

To give a qualitative picture of the effect on the structure of the low-lying states, imposed by both the backward-going amplitudes and the Pauli principle, numerical calculations for the isotopes ^{131}Ba, ^{133}Ba, ^{135}Ba, and ^{137}Ba have been performed. This chain enters the transitional region where the anharmonic effects play a gradually increasing role at low and mainly at intermediate energies. Thus the results presented in this section may lack some accuracy because the wave function (17) does not contain configurations that account for these effects. The pairing constants G_τ are fitted in the standard way to reproduce the odd-even mass differences. This procedure is consistent with the study in Ref. [16] where it has been pointed out that the quadrupole correlation energy varies slightly with the mass number and therefore the odd-even mass differences would remain relatively unaffected by the quadrupole-quadrupole correlations. The obtained pairing gaps are in a good agreement (within 10%) with the values predicted by the empirical formula $\Delta \approx \frac{12}{\sqrt{A}}$ MeV [1].

Our calculations include quadrupole and octupole phonons. The strength parameter $\kappa^{(2)}$ in the Hamiltonian is adjusted so that the odd energy spectrum of the low-lying states is reasonably close to the experimental values, whereas $\kappa^{(3)}$ is fixed by the experimental energy of the first octupole state of the neighboring even-even nucleus. As a result, the calculated energies of the first quadrupole states in ^{130}Ba, ^{132}Ba, ^{134}Ba, and ^{136}Ba within the studied models have values that are much higher than the experimental ones. Moreover, if $\kappa^{(2)}$ is fixed within the model that takes account of the GSC, the values of ω_{2_1} are systematically greater about 10% as compared to the model where the backward amplitudes are not considered. It is worth mentioning that after the introduction of the anharmonic effects the energy of these states would decrease. It is well known that the BCS theory violates particle number conservation (PNC). However, for spherical nuclei with developed pairing the effects due to PNC are weak [4], which has also been confirmed by our calculations. For the low-lying states in the nuclei under consideration the deviation of the number of particles is less than 1–2%.

As we move away from the magic number 82 for the neutron subsystem, the correlations in the ground state tend to increase along with the quantities $W(Jj\lambda i)$. This trend is

TABLE I. Values of the matrix elements $V^2(Jj\lambda i)$ and $W^2(Jj\lambda i)$ calculated for $J^\pi = 1/2^+, 3/2^+, 11/2^-, 5/2^+, 7/2^+$ at the lowest poles.

State	Nuclide	Pole's structure	V^2	W^2	$\mathscr{R}$	$1+\mathscr{L}$
$1/2^+$	^{131}Ba	$2d_{3/2}\otimes 2_1$	0.0361	1.302	−0.36	0.93
	^{133}Ba	$2d_{3/2}\otimes 2_1$	0.1225	2.1	−0.43	0.932
	^{135}Ba	$2d_{3/2}\otimes 2_1$	0.4	0.837	−0.11	0.99
	^{137}Ba	$2d_{3/2}\otimes 2_1$	0.56	0.49	−0.053	0.9948
$3/2^+$	^{131}Ba	$2d_{3/2}\otimes 2_1$	0.006	0.941	−0.005	0.9855
	^{133}Ba	$2d_{3/2}\otimes 2_1$	0.0445	1.72	0.072	1.031
	^{135}Ba	$2d_{3/2}\otimes 2_1$	0.218	0.556	0.0092	1
	^{137}Ba	$2d_{3/2}\otimes 2_1$	0.31	0.327	0.0048	1.0016
$11/2^-$	^{131}Ba	$1h_{11/2}\otimes 2_1$	0.00686	2.132	−0.90	0.801
	^{133}Ba	$1h_{11/2}\otimes 2_1$	0.0493	3.92	−1.25	0.753
	^{135}Ba	$1h_{11/2}\otimes 2_1$	0.086	3.55	−1.19	0.76
	^{137}Ba	$1h_{11/2}\otimes 2_1$	0.3	2.91	−0.84	0.8451
$5/2^+$	^{131}Ba	$2d_{3/2}\otimes 2_1$	0.0266	0.101	−0.04	1
	^{133}Ba	$2d_{3/2}\otimes 2_1$	0.047	0.14	−0.068	0.986
	^{135}Ba	$2d_{3/2}\otimes 2_1$	0.0784	0.052	−0.010	1
	^{137}Ba	$2d_{3/2}\otimes 2_1$	0.1	0.029	−0.0056	1
$7/2^+$	^{131}Ba	$1h_{11/2}\otimes 3_1$	0.037	0.22	0.21	1.015
	^{133}Ba	$2d_{3/2}\otimes 2_1$	0.118	0.439	−0.68	0.874
	^{135}Ba	$2d_{3/2}\otimes 2_1$	0.262	0.16	−0.19	0.9743
	^{137}Ba	$2d_{3/2}\otimes 2_1$	0.325	0.09	−0.093	0.989

presented in Table I, where $W(Jj\lambda i)$ are evaluated only at the lowest poles. It turns out that the contribution from the terms in Eq. (41), corresponding to configurations lying at higher energies, diminishes because of the increased values of the poles and the weakened interaction between the quasiparticles in the ground state. Characteristic feature of the QPM is that quantities related to both the pairing and the multipole-multipole interactions enter the expressions for the interaction vertices producing some competitive effects between them. These effects are essential for the understanding of the behavior of $V(Jj\lambda i)$ and $W(Jj\lambda i)$ along the isotopic chain. Having the lowest quasiparticle energies, the states $1h_{11/2}$, $3s_{1/2}$, and $2d_{3/2}$ experience the greatest part of the interaction with the remaining quasiparticles in the ground state.

The quantities $\mathscr{R}(Jj\lambda i)$ and $\mathscr{L}(Jj\lambda i)$ show a strong dependence on the degree of collectivity of the vibrational states in the neighboring even-even nuclei. As seen in Table I, their values increase as we move away from the magic number of the neutron subsystem. As far as we study the low-lying states only, it is mainly the first quadrupole state that influences them.

For reasons of conciseness, we introduce the following notations, indicating the different variants of the model:

(i) QPM—standard model as given in Ref. [4]
(ii) QPM_P—model, including only the Pauli principle [see Eq. (46)]
(iii) QPM_BCK—model, including backward amplitudes but not the Pauli principle, i.e., the $\mathscr{L}$, $\mathscr{T}$, and $\mathscr{R}$ are set to zero
(iv) QPM_BCK_P—model, including backward amplitudes and the Pauli principle [see Eq. (23)].

Solving the systems of Eqs. (23), one can find the structure of the wave functions (17) and the energies of the excited states. Working in a diagonal approximation for $\mathscr{L}_J$ and $\mathscr{T}_J$, this system reduces to a generalized eigenvalue problem. In Figs. 1 and 2 a comparison between the experimental values of the energies and the theoretical calculations within different versions of the model is presented. In Fig. 3 we show the migration of the lowest energies of the quasiparticle states and of the states as calculated within the QPM_BCK_P version of the model for the isotopes under study.

We restrict the calculation to the six neutron levels $1/2^+, 3/2^+, 9/2^-, 11/2^-, 5/2^+, 7/2^+$. As mentioned above, the values of $\kappa^{(2)}$ are determined by the spectrum of the odd-mass nuclei. To perform a comparative study of the levels' positions, we fix the values of $\kappa^{(2)}$ in QPM_BCK_P and keep them constant in the calculations within the other versions of the model.

The level ordering, presented in the third column in Fig. 1, generally agrees with the one obtained in Ref. [10]. The results, presented in these figures, support the conclusion following from Eq. (45) for states near the Fermi level. As the first solutions, obtained after the introduction of the backward-going terms become closer to the first poles and consequently closer to the second solutions the gaps between the first and the second states with signatures $J^\pi = 1/2^+, 3/2^+, 11/2^-$ are significantly reduced. For the states $5/2^+$ and $7/2^+$ the effect of the GSC is less important because their energies are well above the Fermi level and the values of $W(Jj\lambda i)$ are therefore small (see Table I). The intruder state $9/2^-$ in ^{131}Ba deserves special attention. This state is practically a pure quasiparticle⊗phonon one with structure $[1h_{11/2}\otimes 2_1^+]_{9/2^-}$. The significant reduction of the energy of this state is due

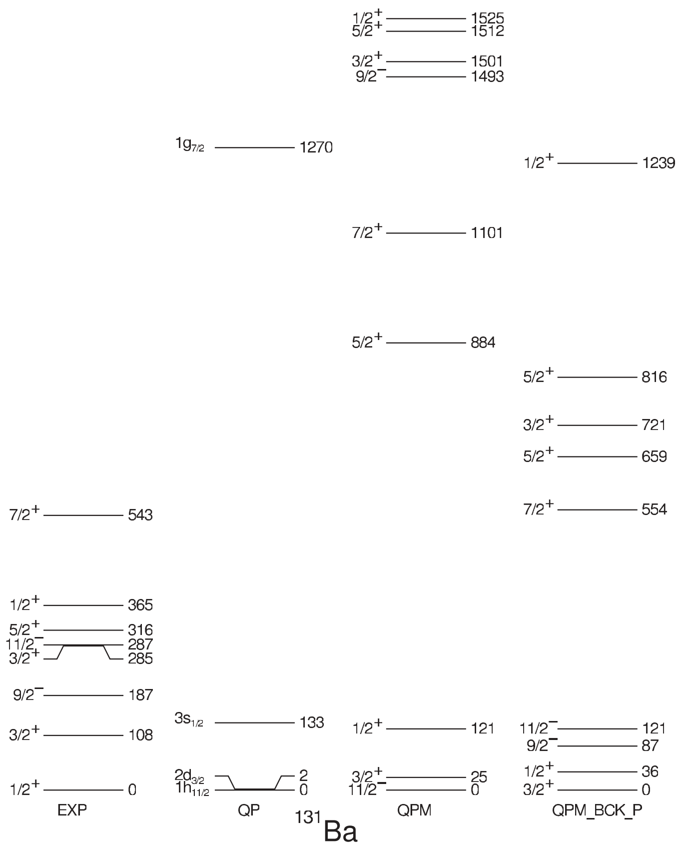

FIG. 1. Low-lying energy spectrum of ^{131}Ba (in keV).

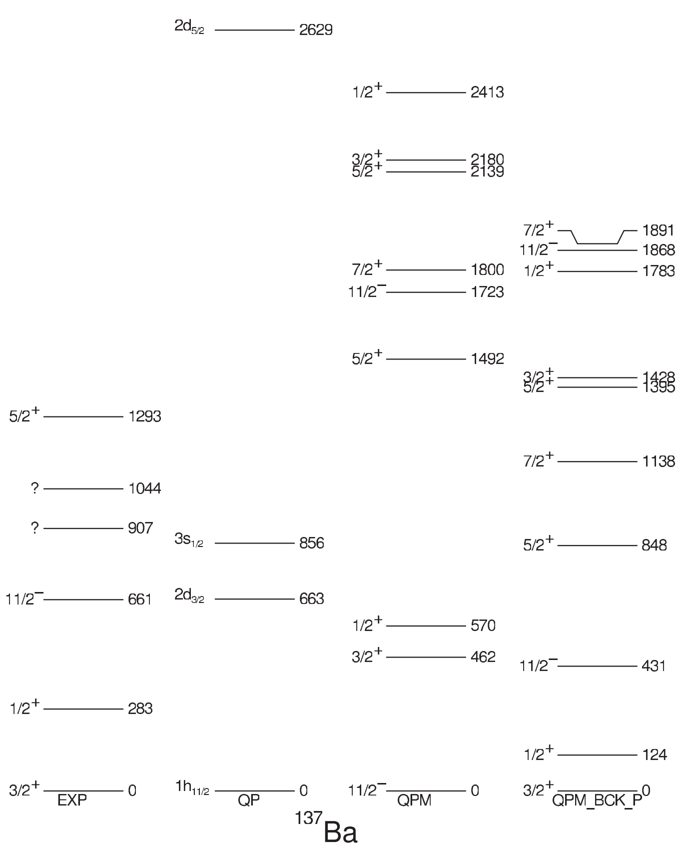

FIG. 2. Low-lying energy spectrum of ^{137}Ba (in keV).

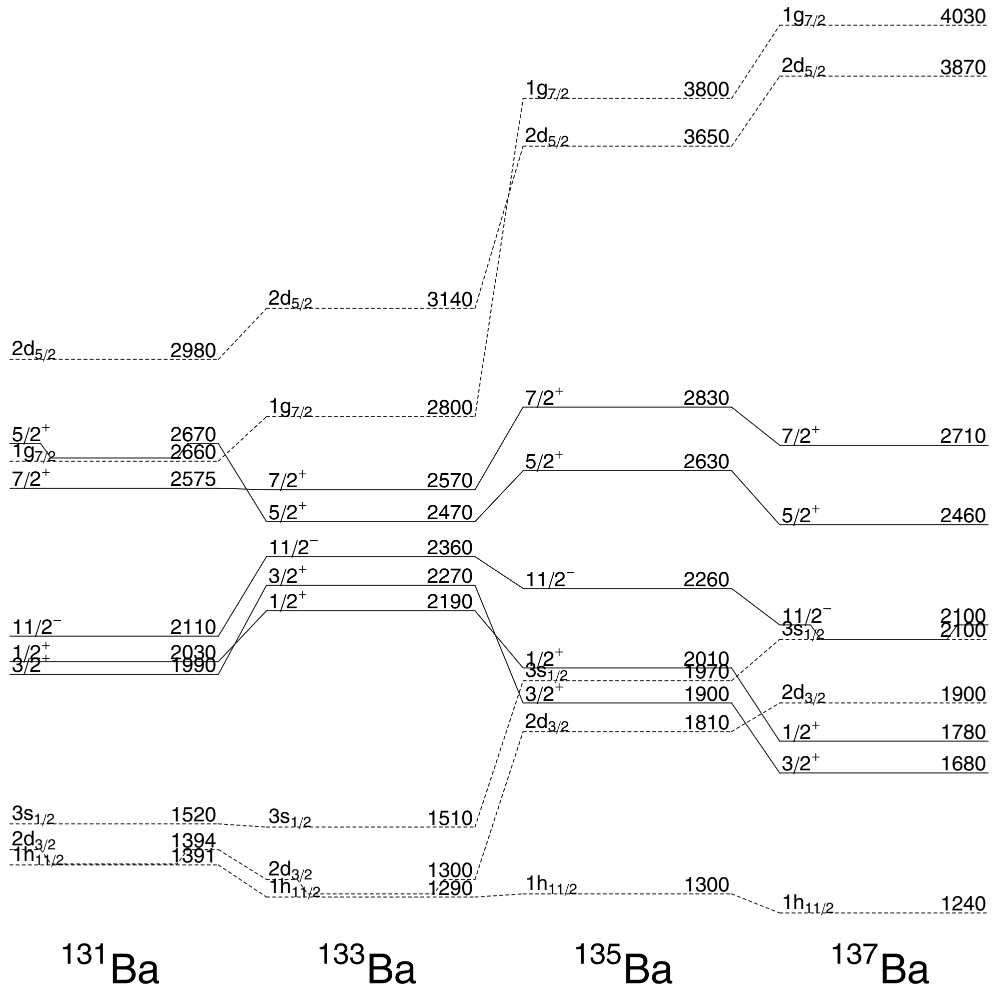

FIG. 3. Comparison of the lowest QPM_BCK_P (solid lines) and one-quasiparticle (dashed lines) eigenvalues referenced from the ground state of the corresponding even-even nucleus (in keV).

to the Pauli principle correction. Hence the inclusion of this correction is essential for the correct ordering of the first several levels. This state is the most important element for fixing the energy of the quadrupole phonon in ^{130}Ba, whereas for the other isotopes in the chain this state is absent in the lower part of the energy spectrum. This peculiarity explains the deviation in ^{131}Ba from the smooth trend in the energies observed in ^{137}Ba, ^{135}Ba, and ^{133}Ba (see Fig. 3) and in the quantities in Table I.

Along with the experimental energies, our calculations provide a reasonable description of the spectroscopic factors for (d, p) reactions (Table II). The observed differences in

TABLE II. Experimental [17] and theoretical spectroscopic factors for the (d, p) reaction of the lowest states with $J^\pi = 1/2^+, 3/2^+$ for ^{131}Ba, ^{133}Ba, ^{135}Ba, and ^{137}Ba.

State	Nuclide	Exp	QPM	QPM_P	QPM_BCK	QPM_BCK_P
$1/2^+$	^{131}Ba	0.265	0.265	0.268	0.271	0.266
	^{133}Ba	0.18	0.176	0.182	0.246	0.231
	^{135}Ba	0.2	0.078	0.079	0.136	0.134
	^{137}Ba	0.09	0.070	0.035	0.087	0.085
$3/2^+$	^{131}Ba	0.257	0.4	0.41	0.39	0.39
	^{133}Ba	0.3	0.29	0.297	0.34	0.34
	^{135}Ba	0.35	0.1	0.1	0.17	0.17
	^{137}Ba	0.17	0.045	0.045	0.11	0.11

the rightmost four columns in this table are due to the quasiparticles in the ground state that additionally modify the single-particle occupation numbers giving us a good idea of the effect due to the backward amplitudes.

Finally, we examine the effect of the GSC and the Pauli principle on the fragmentation of the single-particle states among complex quasiparticle⊗phonon states. In the QPM_BCK versions of the model, the spectroscopic factors for the (d, p) and (d, t) reactions are written as follows:

$$S_{J\nu}^{(d,p)} = (C_{J\nu}u_J - E_{J\nu}v_J)^2, \quad S_{J\nu}^{(d,t)} = (C_{J\nu}v_J + E_{J\nu}u_J)^2. \tag{47}$$

We notice that serious deviations from the expressions for these quantities within the standard QPM $(C_{J\nu}^2 u_J^2, C_{J\nu}^2 v_J^2)$ may occur because of their nonquadratic behavior with respect to u_J and the presence of the backward amplitudes $E_{J\nu}$. Again, in the case when the core has a magic number of nucleons the expressions (47) yield the classical quantities because of the stepwise behavior of u_J and v_J in these nuclei.

We examine levels only in the vicinity of the Fermi level, as for them the interaction in the ground state is stronger than for those lying at higher energies. Furthermore, the values of $\mathscr{R}(Jj\lambda i)$ that effectively result in a shift of the poles thus changing the gap between the pure quasiparticle states and the quasiparticle⊗phonon states, together with the renormalization factors $[1 + \mathscr{L}(Jj\lambda i)]$ exert influence on the single-particle fragmentation as well.

V. CONCLUSION

In this article we derived analytical expressions for the forward and the backward quasiparticle-phonon interaction vertices in odd-mass nuclei. The resulting equation for the energies of the nuclear states is a generalization of the QPM equation in the case where the backward quasiparticle and quasiparticle⊗phonon amplitudes are not taken into account. The comparison between our theoretical calculations and the experimental data for the energies and the spectroscopic factors of the low-lying states in ^{131}Ba, ^{133}Ba, ^{135}Ba, and ^{137}Ba has shown that to describe the structure of such states in odd-mass nuclei far from the magic numbers one needs to take into account the Pauli principle and the ground-state correlations effects simultaneously. To improve this approach a self-consistent description of the mean field with more realistic effective nucleon-nucleon forces is desirable.

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