PHYSICAL REVIEW C **78**, 024301 (2008)

Analog $E1$ transitions and isospin mixing

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(Received 21 December 2007; revised manuscript received 23 June 2008; published 6 August 2008)

We investigate whether isospin mixing can be determined in a model-independent way from the relative strength of $E1$ transitions in mirror nuclei. The specific examples considered are the $A = 31$ and $A = 35$ mirror pairs, where a serious discrepancy between the strengths of $7/2^- \rightarrow 5/2^+$ transitions in the respective mirror nuclei has been observed. A theoretical analysis of the problem suggests that it ought to be possible to disentangle the isospin mixing in the initial and final states given sufficient information on experimental matrix elements. With this in mind, we obtain a lifetime for the relevant $7/2^-$ state in ^{31}S using the Doppler-shift attenuation method. We then collate the available information on matrix elements to examine the level of isospin mixing for both $A = 31$ and $A = 35$ mirror pairs.

DOI: 10.1103/PhysRevC.78.024301 PACS number(s): 21.10.Tg, 23.20.Js, 27.30.+t

I. INTRODUCTION

The concept of isobaric spin (isospin) was introduced into nuclear physics to represent the fact that the nuclear force is to first order charge independent [1]. Were charge independence obeyed in the strictest sense, then isobaric multiplets would be degenerate in energy, and all bound nuclear states would have a definite and pure isospin. Moreover, there would be an exact correspondence between the wave function of states in an isobaric multiplet. In fact, these degeneracies are lifted by the action of isospin nonconserving interactions, the most important of which is the Coulomb force. The resulting separation of the members of an isobaric multiplet is termed the Coulomb displacement energy (CDE). Taking into account the substantial mass differences arising from the CDE, there are also discrepancies between the excitation energies of states in mirror nuclei as a function of angular momentum at the level of 100 keV [2]; such deviations reflect detailed nuclear structure effects such as the difference in the alignment of proton-proton and neutron-neutron pairs [2] and the electromagnetic spin-orbit interaction [3,4]. A more general question is the extent to which the Coulomb interaction can induce the breakdown of isospin purity, an issue reviewed by Soper [5]. These impurities can be manifested in a range of experimental scenarios including isospin-forbidden particle decays from highly excited, particle-unbound states; perturbation of electromagnetic matrix elements; and nuclear β decay.

Isospin mixing and its effect on β-decay matrix elements is of considerable interest in the context of tests of the Conserved Vector Current (CVC) hypothesis [6]. Such tests concern the β decay of Fermi superallowed emitters for which the log ft value should have a fixed value if the CVC hypothesis is correct. Small isospin breaking effects lead to weak Gamow-Teller decay branches in competition with the dominant superallowed branch and the influence of such branches must be accounted for. Conventionally, the relevant isospin mixing of the ground states is evaluated using shell-model calculations; typical values being $\sim$0.5% for mass 50 (fp-shell nuclei) rising to 1% or higher for $A \approx 70$ (fpg shell) [7]. Approaches that could extract isospin mixing from experimental data in a model-independent manner are, therefore, of considerable interest.

An open question is to what extent isospin mixing may be inferred from the impact it may have on electromagnetic transition rates. Warburton and Weneser [8] reviewed this issue nearly 40 years ago and discussed isospin mixing in the context of a number of selection rules expected for both conjugate and self-conjugate nuclei. Their first rule concerned the fact that, between states with $T_z \neq 0$, electromagnetic transitions must obey the selection rule $\Delta T = 0, \pm 1$. A γ decay from a $T = 2$ to $T = 0$ state, therefore, could only arise due to isospin mixing of $T = 1$ components in initial or final states, or the isotensor component of the Coulomb interaction. In practice, the relevant $T = 2$ states lie at very high excitation energy, and the relevant γ-ray transitions are high in energy. This poses an experimental challenge. Moreover, the fact that such states are particle unbound further complicates the analysis. Warburton and Weneser [8] also pointed out a number of selection rules applying to self-conjugate ($N = Z$) nuclei. For example, because $E1$ transitions are purely isovector in nature, they are strictly forbidden between states of the same isopin in $T_z = 0$ nuclei [9]. This behavior has been examined by Farnea *et al.* [10] in the case of the $5^- \rightarrow 4^+$ transition in ^{64}Ge, where this transition is found to be dominated by its $M2$ component. The $E1$ component of this transition has the very weak strength of $\sim 2.5 \times 10^{-7}$ W.u. Calculations suggest that the level of isospin mixing needed to account for the observed $E1$ transition strength is $\alpha^2 = 2.50\%^{+1.0\%}_{-0.7\%}$ [10]. A second rule relating to self-conjugate nuclei, advanced by Warburton and Weneser [8], is the weakness of $\Delta T = 0$, $M1$ transitions in such nuclei. A good system for searching for the role of isospin mixing, therefore, is an odd-odd $N = Z$ nucleus, because $T = 0$ and $T = 1$ states lie close together near the ground state and accidental degeneracies are likely. Lisetskiy *et al.* [11] made a detailed analysis of γ decays in the odd-odd $N = Z$ nucleus ^{54}Co. In particular, they considered the decays of a doublet of 4^+ states with $T = 0$ and $T = 1$, respectively, to a $T = 0, 3^+$ state. Analysis of the $E2/M1$ multipole mixing ratios for these decays allowed the isospin mixing to be quantified at $\sim 0.2\%$ [11].

A further selection rule advanced by Warburton and Weneser [8] is that corresponding $E1$ transitions in conjugate nuclei (i.e., the mirror pair of nuclei, one with A protons and B neutrons, the other with B protons and A neutrons) should have equal strength. They showed that this rule was essentially satisfied for $E1$ transitions in the conjugate nuclei ^{11}C and ^{11}B, while there was a factor of two difference in the $E1$ transition strengths for the decay of the $1/2^+$ excited states in ^{13}C and ^{13}N. There is a very large shift in the energy of the $1/2^+$ state in ^{13}N attributable to the Thomas-Ehrman effect, which relates to the greater radial extent of the proton wave function [12]. Such an effect is especially pronounced for the $s_{1/2}$ orbital [12]. The nonequivalence of the wave functions for the $1/2^+$ states could therefore explain the difference in $E1$ transition strengths. A still more dramatic example was the case of the decay of the $1/2^+$ levels at 8.312 MeV in ^{15}N and 7.550 MeV in ^{15}O. In this case, the $E1$ transition strengths differed by a factor of 400, but here the situation is complex as the state in ^{15}O is unbound and so mixing effects may be very large [8]. Although data on $E1$ transitions was limited at the time Warburton and Weneser [8] wrote their review, the data appeared to show that, for cases where both the decaying states under consideration were bound, the respective $E1$ transitions had nearly equal strength, even where important nuclear structure phenomena like the Thomas-Ehrman shift were at work. Warburton and Weneser [8] commented that it would be very interesting to test this rule for heavier nuclei, where isospin mixing would be expected to be larger; such data did not exist at that time, and little deviation from this rule has shown up since. It was, therefore, of great interest when Ekman *et al.* [3] in their study of the $T = 1/2$ mirror nuclei ^{35}Ar and ^{35}Cl highlighted isospin mixing as the possible origin of the marked difference in the decay branching of the first $7/2^-$ state in the respective nuclei. In ^{35}Ar, an $E1$ transition formed a strong decay branch from the $7/2^-$ state to the $5/2^+$ state, while in the well-studied stable nucleus ^{35}Cl, the analogous transition was almost completely absent. In both nuclei, the respective $7/2^-$ states were well bound. Ekman *et al.* [3] suggested that the cancellation of the $E1$ matrix element in ^{35}Cl arose from isospin mixing, in this case, between the dominant $T = 1/2$ and a weak $T = 3/2$ component. They were unable, however, to quantify this suggestion because absolute transition strengths were not available for the relevant transitions in ^{35}Ar. Behavior similar to that reported for $A = 35$ was also found in the $T = 1/2$ mirror nuclei ^{31}S and ^{31}P [4,13]. Again, the decay pattern of the first $7/2^-$ state was found to change dramatically between the mirror nuclei. In this case, a 2195-keV $E1$ transition clearly present in ^{31}P was found to have no counterpart in the decay scheme of ^{31}S. This is the reverse of the situation in the $A = 35$ mirror pair where the $E1$ was found to disappear in the $T_z = 1/2$ member of the pair. Again, the levels concerned were all particle-bound and so the effects could not be related to the effects of the loosely bound protons. In the $A = 31$ example, as for the $A = 35$ mirror nuclei, the relevant transition strengths were unavailable prior to the present work. The motivation of the present work, therefore, was to obtain transition strengths to quantify this phenomenon as well as to examine how such information could be used to extract the isospin mixing, if present.

II. ISOSPIN MIXING AND ELECTRIC DIPOLE TRANSITIONS IN MIRROR NUCLEI

Let us consider, from a theoretical perspective, whether $E1$ transitions between analog states of mirror nuclei can be used to extract information on isospin mixing. This discussion is formulated in general terms without explicit reference to specific isobaric systems. As will be shown, the main result of this analysis is that, provided sufficient $B(E1)$ values are known experimentally, a model-independent estimate of isospin mixing can be obtained that does not rely on a calculation of $E1$ matrix elements. Having set this challenge to experiment, we review whether sufficient information is available in either the $A = 31$ or $A = 35$ systems to obtain a model-independent determination of isospin mixing.

We begin the discussion with reference to Fig. 1, which shows a generic ensemble of isobaric analog states in $T_z =$

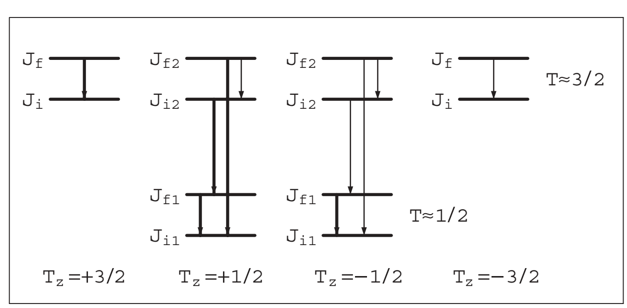

FIG. 1. Schematic figure of isobaric analog states in $T_z = \pm 3/2$ and $\pm 1/2$ nuclei near the $N = Z$ line. The arrows indicate the ten possible $E1$ transitions from where information on the isospin mixing can be extracted. The thick arrows correspond to the $E1$ transitions known in the $A = 31$ and $A = 35$ nuclei (see text).

$\pm 3/2$ and $\pm 1/2$ nuclei near the $N = Z$ line. We assume that the $B(E1)$ values between initial and final low-lying states J_i and J_f in the nuclei with $T_z = +1/2$ and $T_z = -1/2$ are known experimentally. These states have a dominant $T = 1/2$ component, but contain small admixtures of higher-lying states with $T = 3/2$. The problem at hand is the following: What additional experimental information is required to determine the isospin mixing, which, for the present purposes, may be different for the two states involved? In Fig. 1, one such set of higher-lying states is shown; they necessarily have the same angular momenta, J_i and J_f, as their low-lying siblings, but carry predominantly isospin $T = 3/2$. Of course, these higher-lying states may in turn contain $T = 5/2$ admixtures, but as long as these are small they do not affect the subsequent argument and they can be neglected. The figure shows the simplest situation when only one higher-lying state for each angular momentum J_i and J_f mixes with the $T = 1/2$ states. If there are several such higher-lying states, each contribution must be considered separately and gives rise to additional isospin admixtures.

An observation of central importance to what follows is that the electric dipole operator, to a very good approximation, has isovector character; that is, it transforms as a pure vector under rotations in isospin space. Its isoscalar part is totally absent from an $E1$ transition internal to the nucleus. This implies that, under the assumption of isospin symmetry, there exist definite relations between transitions between isobaric analog states. In particular, the B(E1) values of transitions between corresponding states in mirror nuclei are identical and, more generally, those between isobaric analog states are proportional with proportionality factors that are related to isospin Clebsch-Gordan coefficients. If isospin were an exact symmetry, all $E1$ transition strengths could be expressed in terms of only four matrix elements reduced in angular momentum and in isospin that we denote as

$$M_{2T_f,2T_i} \equiv \langle J_f; T_f ||| T^{(1)}(E1) ||| J_i; T_i \rangle. \tag{1}$$

The triple bars indicate that the dependence on the initial and final angular momentum projections M_i and M_f and on the isospin projection T_z has been factored out. This dependence is contained in the Clebsch-Gordan coefficients $\langle J_f M_f \, 1\mu | J_i M_i \rangle$ and $\langle T_f T_z \, 10 | T_i T_z \rangle$ (or, depending on the convention, corresponding $3j$ symbols), where we use the fact that the $E1$ transition operator has isovector character as indicated with its superscript (1). The initial and final angular momenta J_i and J_f are the same for all four reduced matrix elements, but the isospins T_i and T_f can be 1/2 or 3/2. The explicit expression for an arbitrary $B(E1)$ value in terms of the triply barred reduced matrix elements is

$$B(E1; J_i T_i T_z \to J_f T_f T_z) = \frac{1}{2J_i + 1} \langle J_f; T_f T_z \| T_0^{(1)}(E1) \| J_i; T_i T_z \rangle^2, \tag{2}$$

with

$$\langle J_f; T_f T_z \| T_0^{(1)}(E1) \| J_i; T_i T_z \rangle = (-)^{T_f - T_z} \begin{pmatrix} T_f & 1 & T_i \\ -T_z & 0 & T_z \end{pmatrix} M_{2T_f,2T_i}, \tag{3}$$

where the symbol between brackets is a $3j$ symbol. The conclusion is that, if isospin is an exact symmetry, the $B(E1)$ values of the ten transitions indicated in Fig. 1 can be expressed in terms of four quantities: M_{11}, M_{13}, M_{31}, and M_{33}. Note, in particular, that M_{31} is different from M_{13}.

If one allows for isospin mixing between the low- and high-lying J_i and J_f states, a further two unknowns are introduced that can be denoted as mixing angles ϕ_i and ϕ_f. The true low- and high-lying initial states $|J_{1i}\rangle$ and $|J_{2i}\rangle$ can be expressed as follows in terms of the isospin eigenstates:

$$\begin{aligned} |J_{1i}\rangle &= \cos\phi_i |J_i; T = 1/2\rangle + \sin\phi_i |J_i; T = 3/2\rangle, \\ |J_{2i}\rangle &= -\sin\phi_i |J_i; T = 1/2\rangle + \cos\phi_i |J_i; T = 3/2\rangle, \end{aligned} \tag{4}$$

and similarly for the final states $|J_{1f}\rangle$ and $|J_{2f}\rangle$ in terms of the mixing angle ϕ_f. As a consequence, the $B(E1)$ values of the ten transitions are modified and now depend on the four matrix elements M_{kl} as well as on the two mixing angles ϕ_i and ϕ_f. We find that the $B(E1)$ values in the $T_z = +1/2$ nucleus are given by

$$\begin{aligned} &B(E1; J_{1i} \to J_{1f}) \\ &\quad = \frac{1}{6(2J_i+1)} \bigg(M_{11} \cos\phi_f \cos\phi_i - \frac{M_{33}}{\sqrt{10}} \sin\phi_f \sin\phi_i \\ &\qquad - M_{13} \cos\phi_f \sin\phi_i + M_{31} \sin\phi_f \cos\phi_i \bigg)^2, \\ &B(E1; J_{1i} \to J_{2f}) \\ &\quad = \frac{1}{6(2J_i+1)} \bigg(-M_{11} \sin\phi_f \cos\phi_i - \frac{M_{33}}{\sqrt{10}} \cos\phi_f \sin\phi_i \\ &\qquad + M_{13} \sin\phi_f \sin\phi_i + M_{31} \cos\phi_f \cos\phi_i \bigg)^2, \\ &B(E1; J_{2i} \to J_{1f}) \\ &\quad = \frac{1}{6(2J_i+1)} \bigg(M_{11} \cos\phi_f \sin\phi_i + \frac{M_{33}}{\sqrt{10}} \sin\phi_f \cos\phi_i \\ &\qquad + M_{13} \cos\phi_f \cos\phi_i + M_{31} \sin\phi_f \sin\phi_i \bigg)^2, \\ &B(E1; J_{2i} \to J_{2f}) \\ &\quad = \frac{1}{6(2J_i+1)} \bigg(M_{11} \sin\phi_f \sin\phi_i - \frac{M_{33}}{\sqrt{10}} \cos\phi_f \cos\phi_i \\ &\qquad + M_{13} \sin\phi_f \cos\phi_i - M_{31} \cos\phi_f \sin\phi_i \bigg)^2. \end{aligned} \tag{5}$$

In the $T_z = -1/2$ nucleus, the same expressions apply but with the replacements $M_{kl} \mapsto -M_{kl}$, if $k = l$, and $M_{kl} \mapsto M_{kl}$, if $k \neq l$. Finally, in the $T_z = \pm 3/2$ nuclei the $B(E1)$ values are given by

$$B(E1; J_i \to J_f) \frac{3}{20\,(2J_i+1)} (M_{33})^2. \tag{6}$$

If at least six (appropriate) $B(E1)$ values are known, these equations determine the matrix elements M_{kl} and the mixing angles ϕ_i and ϕ_f. If five $B(E1)$ values are known, they determine a relation between ϕ_i and ϕ_f.

To illustrate our method, we assume that the five transitions that are indicated with thick arrows in Fig. 1 have known $B(E1)$ values. We can then eliminate the four unknown matrix elements M_{kl} and we obtain the following equation in ϕ_i and ϕ_f:

$$\begin{aligned}&3[M(-1/2,1,1)-M(+1/2,1,1)\\&\quad+M(+1/2,1,1)\cos 2\phi_f+M(+1/2,1,1)\cos 2\phi_i\\&\quad-M(+1/2,1,2)\sin 2\phi_f-M(+1/2,2,1)\sin 2\phi_i]\\&=4M(+3/2,1,1)\sin\phi_f\sin\phi_i,\end{aligned} \quad (7)$$

where $M(T_z,k,l)$ is a shorthand notation for the square root of a measured $B(E1)$ value in a T_z nucleus according to

$$M(T_z,k,l)\equiv\pm\sqrt{B(\mathrm{E1};J_{ik}\rightarrow J_{fl})}. \quad (8)$$

Equation (7) defines a relation between the mixing angles ϕ_i and ϕ_f that is *independent of all theoretical matrix elements*M_{kl}.

Note that the measured transition strengths do not provide information on the sign of the quantities $M(T_z,k,l)$. If five measured $B(E1)$ values are known, this leads in principle to $2^5=32$ different choices. The problem can be simplified as follows. First, we have that $M(-1/2,1,1)\approx -M(+1/2,1,1)$, so we may assume that these matrix elements have the opposite sign. This also follows from Eq. (7) for $\phi_i=\phi_f=0$ (no isospin mixing). Furthermore, without loss of generality, we may choose the sign of one matrix element, and we take $M(+1/2,1,1)$ and $-M(-1/2,1,1)$ as positive. We still are left with $2^3=8$ possible choices of the signs of $M(1/2,1,2)$, $M(+1/2,2,1)$, and $M(+3/2,1,1)$. A convenient way to run through all eight possibilities is the following. We can always adopt a convention in Eq. (4) such that $\cos\phi_i$, $\sin\phi_i$, $\cos\phi_f$, and $\sin\phi_f$ are positive, which corresponds to the domains $0\leqslant\phi_i\leqslant\pi/2$ and $0\leqslant\phi_f\leqslant\pi/2$. A change of sign of $M(+1/2,1,2)$ in Eq. (7) is equivalent to the substitution $\phi_f\mapsto\pi-\phi_f$. Similarly, changing the sign of $M(+1/2,2,1)$ is equivalent to $\phi_i\mapsto\pi-\phi_i$. Finally, $M(+3/2,1,1)\mapsto -M(+3/2,1,1)$ corresponds to either $\phi_i\mapsto\phi_i+\pi$ or $\phi_f\mapsto\phi_f+\pi$. We, therefore, conclude that the entire set of solutions is scanned if $0\leqslant\phi_i\leqslant\pi$ and $0\leqslant\phi_f\leqslant 2\pi$ or if $0\leqslant\phi_i\leqslant 2\pi$ and $0\leqslant\phi_f\leqslant\pi$. Beyond these boundaries, solutions will repeat themselves. If, as is expected, $|\sin\phi_i|$ and $|\sin\phi_f|$ are small, it is more convenient to scan the domains $-\pi/2\leqslant\phi_i\leqslant\pi/2$ and $-\pi/2\leqslant\phi_f\leqslant 3\pi/2$. In this convention there are two physical regions, either $(\phi_i,\phi_f)\approx(0,0)$ or $(\phi_i,\phi_f)\approx(0,\pi)$.

From an examination of the theoretical background to this problem, it is clear that at least five $B(E1)$ values are required to assess the magnitude of the isospin mixing and six $B(E1)$ values to fix the mixing angles without ambiguity. The present work focuses on the $A=31$ and $A=35$ cases. In the former example, a $B(E1)$ value is known for the $7/2^-\rightarrow 5/2^+$ transition in ^{31}P from a previous lifetime measurement [14], but lifetimes were only known for a few low-lying states in ^{31}S. The motivation of the present experimental work, therefore, was to obtain lifetimes for levels in ^{31}S to extract $B(E1)$ values for the transitions of interest.

III. LIFETIME MEASUREMENTS FOR THE $A=31$ MIRROR NUCLEI

Lifetimes of excited states in ^{31}S have been obtained in the present work using the Doppler Shift Attenuation Method (DSAM). An earlier study of the mirror nuclei ^{31}S and ^{31}P used the ^{20}Ne + ^{12}C reaction [4]. While it would have been desirable to repeat this reaction, which was known to give a good population of the $7/2^-$ state of interest, it was found that it was difficult to get a reliable adhesion between a carbon foil and a thick target backing. It was, therefore, decided to change to the ^{16}O + ^{16}O reaction and make use of metal oxide on metal targets. The Tandem accelerator from the ATLAS facility at Argonne National Laboratory accelerated an ^{16}O beam to an energy of 29 MeV. The beam bombarded a 530 μg/cm^2 thick target of nickel monoxide on a backing of 3.5 mg/cm^2 of nickel. The resulting γ radiation was detected using the Gammasphere array [15] consisting of 100 high-purity germanium detectors. In this array, there were 17 different angular ring positions that could be used to obtain DSAM lineshapes. The γ-ray coincidence data were sorted into a series of matrices with γ rays detected in all detectors on one axis and γ rays detected in a specific ring on the other.

The Monte Carlo DSAM code "lineshape" was employed to fit the observed lineshapes and determine lifetimes [16]. The slowing-down process in the target and backing was modeled using the SRIM2008 stopping powers [17]. In a DSAM analysis, the general procedure is to fit the lineshape in angle-sorted spectra gated by transitions lying above the transition of interest to remove the effect of side feeding. The low population of the $T_z=1/2$ nucleus ^{31}S, however, meant that it was not possible to gate on transitions above the level of interest when obtaining lineshape spectra for the decay of the $7/2^-$ level. It was, therefore, necessary to gate on transitions below and include the effects of side feeding. The feeding of the $7/2^-$ level in ^{31}S comes principally from a 1926-keV γ ray deexciting a $9/2^-$ state and a 2383-keV γ ray de-exciting an $11/2^-$ level. In the case of ^{31}P, the nucleus is populated to much higher spin and the feeding pattern is complex. Ionescu-Bujor *et al.* [18] have recently reported lifetimes for some of these high-lying high spin states that feed the $7/2^-$ level. In some cases, these exceed 1 ps. Fortunately, these states do not appear to be populated in ^{31}S in our study and the 1926- and 2383-keV transitions decay from states with relatively short lifetimes. Because the population of levels above appeared negligible, we fit the lineshapes corresponding to the 1926- and 2383-keV transitions using lineshape spectra produced by gating below and obtained effective lifetimes of 245(45) and 180(35) fs for the $9/2^-$ and $11/2^-$ levels, respectively. These are in reasonable conformity with the lifetimes of the mirror states in ^{31}P, which have lifetimes of 55(17) and 120(50) fs, respectively. Lineshape spectra for the 1166-keV $7/2-\rightarrow 5/2^+$ transition were obtained by summing spectra gated by the 1249- and 2036-keV transitions. Figure 2 shows the lineshape of the 1166-keV transition at 70°, 90°, and

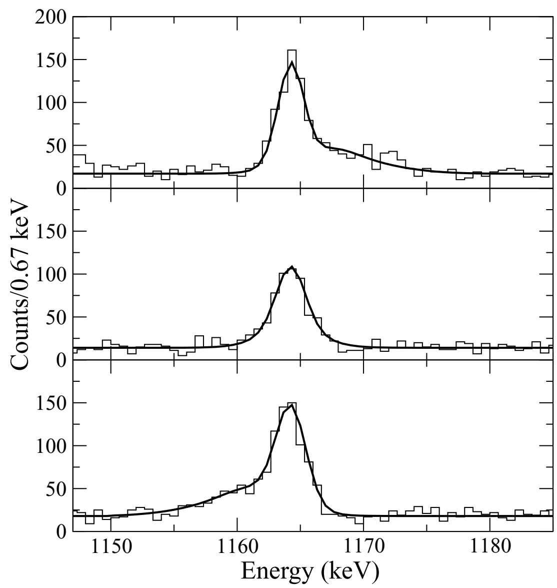

FIG. 2. Lineshapes for the 1166-keV γ ray in ^{31}S shown for 70° (top), 90° (middle), and 110° (bottom). The fit obtained by the lineshape program is the thick line. The spectra were obtained by gating on the 1249- and 2036-keV transitions below the γ ray of interest.

110°. Fitting these lineshapes for the 1166-keV transition, incorporating side feeding from the two discrete transitions, led to a lifetime for the $7/2^-$ state in ^{31}S of 1.03(21) ps.

To examine the reliability of the methodology of gating below the transition of interest, a lifetime was obtained for the corresponding $7/2^-$ state in ^{31}P which decays by a 1136-keV γ ray for which the lifetime had been previously measured to be 0.59(3) ps [14]. First, a gate was set on the 2394-keV transition above the level of interest and lineshapes corresponding to the 1136-keV transition were fit. A lifetime of 0.66(13) ps was obtained. Then, by using the sum of gated spectra of the 2029- and 1266-keV transitions below the level of interest, the lineshape of 1136-keV transition was fitted with a consideration of the estimated effective lifetimes contributed by the 2071-, 2365-, and 2394-keV transitions. In this case, the lifetime extracted was 0.72(8) ps, consistent with that obtained through gating above.

IV. $B(E1)$ TRANSITION STRENGTHS

Having obtained a lifetime for the $7/2^-$ level in ^{31}S, we are now able to calculate $B(E1)$ transition rates for the transitions deexciting both $7/2^-$ states in ^{31}P *and* ^{31}S (see Table I). We are only able to set an upper limit on the $B(E1)$ strength for the unobserved 2215-keV γ ray. It should be noted that this ignores possible $M2$ admixtures, which become more likely as the $E1$ strength is attenuated. In any case, the $B(E1)$ strength for the $7/2^- \rightarrow 5/2^+$ transition in ^{31}P exceeds that of the analogous transition in ^{31}S by at least a factor of 40.

To put the present observations in context, let us return to the case of the $A = 35$ mirror nuclei, where seemingly behavior opposite that of the $A = 31$ case is observed: a prominent $7/2^- \rightarrow 5/2^+ E1$ transition is observed in the $T_z = -1/2$ nucleus, ^{35}Ar, whereas this branch is seemingly very weak in the mirror nucleus, ^{35}Cl. In fact, the measured lifetime for the $7/2^-$ state in ^{35}Cl indicates that the latter transition has the extremely weak transition strength of $1.4(3) \times 10^{-8}$ W.u. This transition also exhibits a significant $M2$ component ($B(M2) = 6.3(37) \times 10^{-3}$ W.u.) [19]), which is perhaps not surprising given the extremely small $E1$ matrix element. The lifetime of the $7/2^-$ level in ^{35}Ar is presently unknown. The approach taken by Ekman *et al.* [3] was to assume that the $M2$ transition from the $7/2^-$ state has the same transition strength in each case ($B(M2) = 0.28$ W.u.). Following this approach, $B(E1)$ for the 1446-keV transition in ^{35}Ar is 3×10^{-5}, which is 2000 times larger than the analogous transition in ^{35}Cl. Ekman *et al.* [3] assume that isospin mixing is taking place and that there are contributions to the matrix elements diagonal and nondiagonal in T that have similar magnitude (1.5×10^{-5} W.u.) and that cancel in the case of ^{35}Cl and sum in the case of ^{35}Ar. We note that this analysis is not as detailed as that presented in the present work. Moreover, while the analysis by Ekman *et al.* [3] appears reasonable qualitatively, it does lead to the conclusion that the two components diagonal and nondiagonal in T would have to differ by less than 0.1 %, which would be an astonishing coincidence. Clearly, a possible weakness in this analysis may be the assumption that the $M2$ transitions in the mirror nuclei have the same strength. Unlike $E1$ transitions, $M2$ transitions have both an isoscalar and an isovector part. Warburton and Weneser [8] suggest a "quasi-rule" for such transitions that, so long as they are relatively strong, they should be of near-equal transition strength in conjugate nuclei. However, the $M2$ transitions in this case are not strong. Prosser and Harris [19] calculated $M2$ transition rates for the $A = 35$ mirror nuclei and predicted that the $7/2^- \rightarrow 3/2^+$ transition in ^{35}Ar should have $B(M2) = 0.0032$ W.u., compared to $B(M2) = 0.185$ W.u. in ^{35}Cl. If we use this predicted $B(M2)$ strength for ^{35}Ar, in conjunction with the $M2$ branching ratios as measured by Ekman *et al.* [3], then we conclude that the 1446-keV transition would have $B(E1) = 6.7(20) \times 10^{-7}$ W.u. This reanalysis suggests that the two $E1$ transitions differ in transition strength by a factor of 50 rather than by a factor of 2000 as suggested by Ekman *et al.* [3]. Both $E1$ transitions are seen to be extremely weak, cf. $B(E1) \approx 2.5 \times 10^{-7}$ W.u for the $5^- \rightarrow 4^+$ transition

TABLE I. Table of $B(E1)$ transition rates obtained for ^{31}S from lifetimes measured in the present work and ^{31}P from the literature. The 2215-keV transition is presently unobserved; an upper limit on its branching ratio of 2% is obtained from previous work [4]. In all cases, the transitions are assumed to have a negligble $M2$ component.

Nucleus	E_γ (keV)	$I_i \rightarrow I_f$	$B(E1)$ (W.u.)
^{31}S	1166	$7/2^- \rightarrow 5/2^+_2$	$6.1(1.2) \times 10^{-4}$
	2215	$7/2^- \rightarrow 5/2^+_1$	$<1.8 \times 10^{-6}$
^{31}P	1136	$7/2^- \rightarrow 5/2^+_2$	$4.5(8) \times 10^{-4}$
	2195	$7/2^- \rightarrow 5/2^+_1$	$8.2(6) \times 10^{-5}$

in ^{64}Ge [10]. Moreover, the variation in transition strength between the mirror nuclei is now of an order similar to that observed in the $A = 31$ mirror pair, albeit it is the $T_z = 1/2$ member that has the weaker $B(E1)$ strength in $A = 35$. Clearly, it would be very worthwhile to measure the lifetime of the $7/2^-$ state in ^{35}Ar to verify the predicted $B(M2)$ value. On the basis of the predicted $B(M2)$ value, this lifetime should be around 350 ps.

As discussed above, to begin to determine the level of isospin mixing, we need additional matrix elements to determine the isospin mixing, we now require at least three more matrix elements corresponding to $T = 3/2 \rightarrow T = 1/2$, $T = 1/2 \rightarrow T = 3/2$, and $T = 3/2 \rightarrow T = 3/2$ transitions. For completeness, we review the available data for both the $A = 31$ and $A = 35$ cases.

For the $T = 3/2 \rightarrow T = 1/2$ component, we can make use of a recent series of detailed (p,γ) studies on ^{30}Si and ^{34}S [20]. These measurements have identified the $1f_{7/2}$ isobaric analog states in both ^{31}P and ^{35}Cl [20]. Their γ widths are 1.63(25) and 1.37(20) eV, respectively. These values may be combined with accurate branching ratios for the $1f_{7/2}$ resonance in ^{31}P [21] and ^{35}Cl [19], leading to $B(E1)$ values for the $7/2^-, T = 3/2 \rightarrow 5/2^+_1, T = 1/2$ transitions of $2.0(3) \times 10^{-4}$ W.u. in ^{31}P and $3.3(7) \times 10^{-5}$ W.u. in ^{35}Cl.

While the $1f_{7/2}$ analog states appear unique, which leads to a relatively simple extraction of the relevant matrix elements, the situation appears significantly more complex when we attempt to obtain the $T = 1/2 \rightarrow T = 3/2$ component. In this case, we need to examine $5/2^+, T = 3/2 \rightarrow 7/2^-, T = 1/2$ transitions, but this is not straightforward because in both ^{31}P and ^{35}Cl there are a number of known (p,γ) resonances with $J^\pi = 5/2^+$. The question naturally arises, then, as to which resonances to consider. To first order, we would expect the most significant isospin mixing to take place between analog states. Analog states should be connected by strong $M1$ transitions. In ^{31}P, the first $5/2^+, T = 3/2$ resonance is split into two components at 8.032 and 8.105 MeV. These components have a negligible γ branch to the $T = 1/2, 7/2^-$ state [22]. There is a second $5/2^+$ resonance split into a further five fine structure components between 9.009 and 9.131 MeV [23]. These levels have a more significant branch to the $T = 1/2, 7/2^-$ state. If we sum all of these component γ branches, we obtain $B(E1) \approx 3.4 \times 10^{-4}$ W.u. [23]. While the lower pair of resonances has a very weak $M1$ decay to the $5/2^+$ state at 2234 keV, the two $5/2^+$ resonances at 9.067 and 9.116 MeV have strong $M1$ branches to the $5/2^+$ state. The upper set of resonances, therefore, appears the most relevant for determining the required $E1$ transition strength. To summarize, then, we have considered only the upper set of resonances around 9 MeV as the relevant analog to the $T = 1/2, 5/2^+$ state and used the sum of the $E1$ branches from this set of states in our analysis. The $T = 1/2 \rightarrow T = 3/2$ and $T = 3/2 \rightarrow T = 1/2$ matrix elements in ^{31}P, therefore, appear to be of similar order, i.e., $B(E1) \sim 10^{-4}$ W.u. Indeed, these are typical values for isovector $E1$ transitions.

Following a similar procedure, if we sum the measured $B(E1)$ strengths for $5/2^+$ resonances in the ^{34}S(p,γ) reaction that correspond to states at 8.216, 8.893, and 9.081 MeV in ^{35}Cl, we obtain $\approx 4.3 \times 10^{-3}$ W.u. [24]. Each of these resonances gives a similar strength to the total $B(E1)$ probability. The 8.893-MeV resonance has a strong $M1$ branch to the $5/2^+, T = 1/2$ state. In this case, therefore, the $T = 1/2 \rightarrow T = 3/2$ matrix element appears to considerably exceed the $T = 3/2 \rightarrow T = 1/2$ matrix element.

To obtain matrix elements for the $T = 3/2 \rightarrow T = 3/2$ component, we need to turn to the $T_z = 3/2$ nuclei, ^{35}S and ^{31}Si, because in the $T_z = 1/2$ nuclei it would be difficult to observe such isoscalar transitions in competition with transitions to the $T = 1/2$ states. In the case of ^{35}S, we have an upper limit only for the strength of the $5/2^+, T = 3/2 \rightarrow 7/2^-, T = 3/2$ transition, with $B(E1) < 1.5 \times 10^{-3}$ W.u. The $5/2^+$ state involved is the analog of the upper of the set of $5/2^+$ resonances in ^{35}Cl, which we considered when obtaining the $T = 1/2 \rightarrow T = 3/2$ matrix element.

For ^{31}Si, the situation is more complicated because there is a measured $B(E1)$ value for the $5/2^+, T = 3/2 \rightarrow 7/2^-, T = 3/2$ transition of $6.0(13) \times 10^{-4}$ W.u. The $5/2^+$ state involved here, though, corresponds to the lower set of $T = 3/2, 5/2^+$ resonances in ^{31}P, which had negligible $E1$ branches to the $T = 1/2, 7/2^-$ state, as well as weak $M1$ transitions to the lowest $5/2^+, T = 1/2$ state. This inconsistency cautions against the use of this set of values in any calculation. The extent of our knowledge of relevant $B(E1)$ strengths in the $A = 31$ and 35 systems is summarized in Fig. 3.

Using the matrix elements for the $A = 31$ and $A = 35$ cases shown in Fig. 3, we can solve Eq. (7) graphically. It is important to remember that, in the $A = 31$ isobars, the results of the analysis will be rendered uncertain because we do not have information on the relevant B(E1) value in ^{31}Si. In the $A = 35$ isobars, on the other hand, we do not have an experimental matrix element for the $T = 1/2 \rightarrow T = 1/2$ component in ^{35}Ar, for which we have been forced to rely on scaling from a predicted $B(M2)$ value [19].

Figure 4 illustrates the allowed values of the isospin mixing angles ϕ_i and ϕ_f in the $A = 31$ nuclei. Because five $B(E1)$ values are known experimentally, only a relation between the mixing angles can be established that corresponds to a curve in the domains $-\pi/2 \leqslant \phi_i \leqslant \pi/2$ and $-\pi/2 \leqslant \phi_f \leqslant 3\pi/2$. In addition, only an upper limit is known for the $B(E1; 7/2^- \rightarrow 5/2^+)$ value in ^{31}S, and hence consistency with Eq. (7) is imposed for the range $0 \leqslant B(E1; 7/2^- \rightarrow 5/2^+) \leqslant 1.9 \times 10^{-6}$ W.u. This leads to the two closely spaced curves in Fig. 4 that meander through the entire allowed domain; the small region between one curve and its immediately adjacent one defines the mixing angles consistent with Eq. (7). We see from Fig. 4 that none of the allowed solutions goes through the regions with $|\sin\phi_{i,f}| < 0.1$, which corresponds with isospin mixing smaller than 1% in both states. We conclude that no coherent picture is obtained from the $A = 31$ data as regards $E1$ transitions and isospin mixing, which is probably due to our inconsistent use of analog transitions, as pointed out above.

The situation is more encouraging in the $A = 35$ nuclei. Again, many different values of (ϕ_i, ϕ_f) are consistent with Eq. (7), but we may focus our attention on the regions $(\phi_i, \phi_f) \approx (0, 0)$ or $(\phi_i, \phi_f) \approx (0, \pi)$. A band of allowed

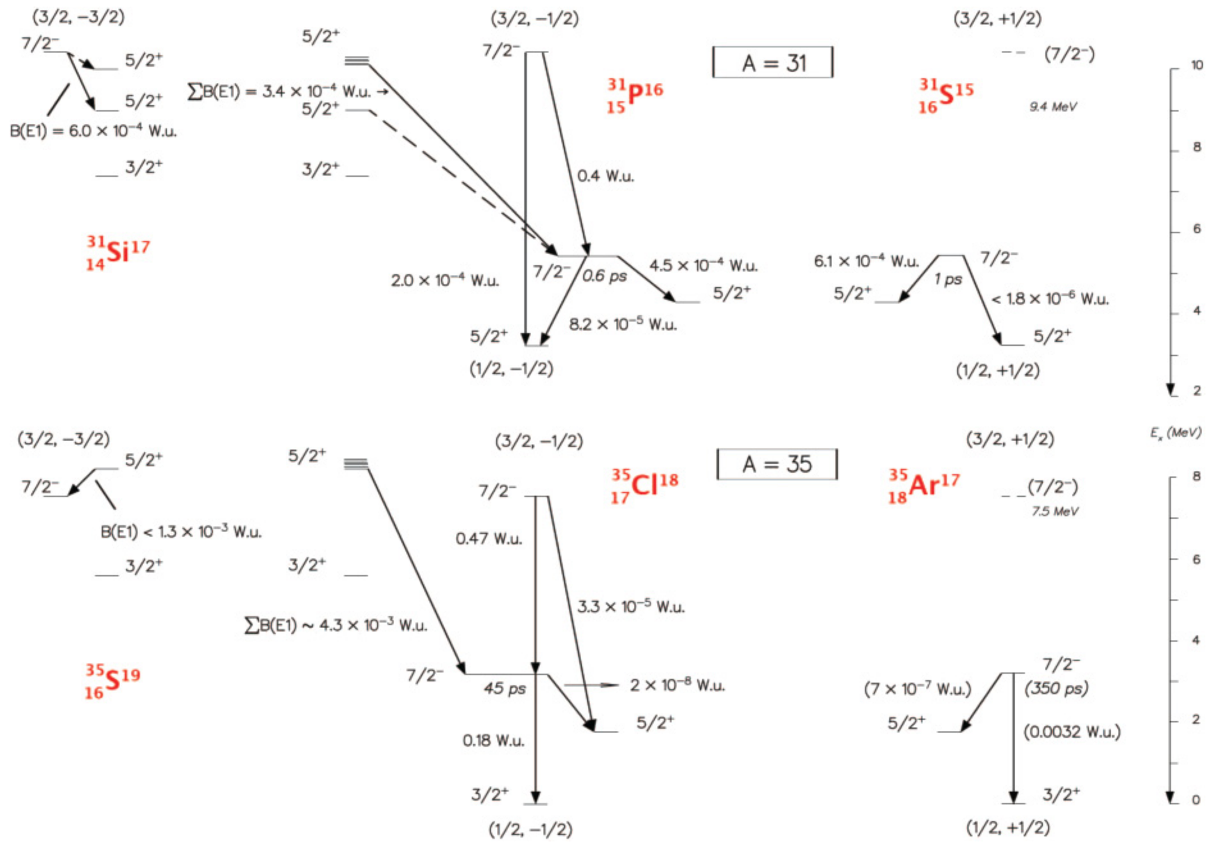

FIG. 3. (Color online) Simplified level schemes showing the states of interest in the $A = 31$ and $A = 35$ systems. The states are labeled with their isospin using the convention (T, T_z). Transition strengths are shown where known. The manner in which these strengths were arrived at is described in the text, in particular, those for the decay of the $T = 1/2, 7/2^-$ state in ^{35}Ar, which were calculated not measured.

(ϕ_i, ϕ_f) values going through the former region is indicated in Fig. 5. The middle line is the solution of Eq. (7) with the (largely) experimental matrix elements $M(T_z, k, l)$ in the $A = 35$ isobars. The outer lines are consistent with this solution to within 1σ deviation where the errors on all $B(E1)$ values have been taken into account. The sensitivity to the errors on the different $B(E1)$ values varies strongly. For example, the solution is largely insensitive to the $B(E1; 5/2^+, 1/2 \to 7/2^-, 1/2)$ value in ^{35}S, and the currently known upper limit suffices for the present purpose; that is, the error on this $B(E1)$ value does not contribute significantly to the error in the (ϕ_i, ϕ_f) plot. For reducing the latter error, a better precision is required for the $7/2^-, 1/2 \to 5/2^+, 1/2$ transition in ^{35}Ar and the $7/2^-, 3/2 \to 5/2^+, 1/2$ and $5/2^+, 3/2 \to 7/2^-, 1/2$

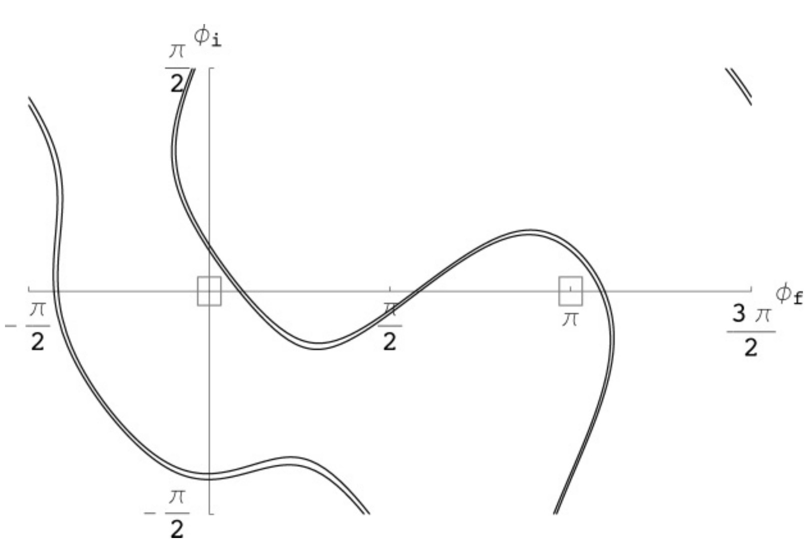

FIG. 4. Correlation plot between the isospin mixing angles ϕ_i and ϕ_f consistent with Eq. (7) with experimental information as available in the $A = 31$ nuclei. The solution of Eq. (7) meanders through the entire region $-\pi/2 \leqslant \phi_i \leqslant \pi/2$ and $-\pi/2 \leqslant \phi_f \leqslant 3\pi/2$. The two closely spaced lines correspond to the solution for the upper limit of the $B(E1; 7/2^- \to 5/2^+)$ value in ^{31}S and for a zero value for this quantity, respectively. The region of mixing ($|\sin \phi_{i,f}| < 0.1$) is indicated with the gray rectangles.

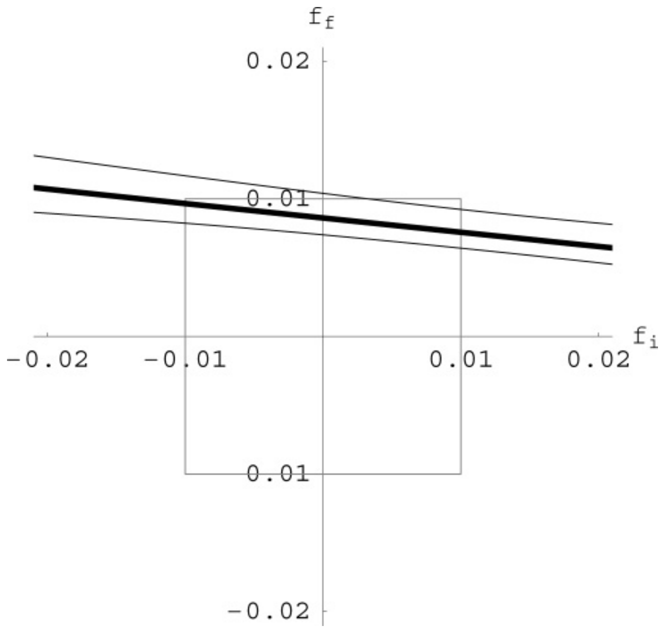

FIG. 5. Correlation plot between the isospin mixing angles ϕ_i and ϕ_f consistent with Eq. (7) and the experimental information available for the $A = 35$ nuclei. The thick line is the solution of Eq. (7) and the outer lines are consistent with it to within 1σ deviation. The region of small mixing ($|\sin\phi_{i,f}| < 0.01$) is indicated with the gray rectangle.

transitions in ^{35}Cl. Moreover, an additional $B(E1)$ value is needed for pinning down the mixing angles unambiguously.

V. CONCLUSIONS

In conclusion, we examined theoretically whether isospin mixing in bound nuclear levels can be obtained from consideration of $E1$ transition strengths in analog systems. Five $B(E1)$ values are required to determine a relation between the mixing angles of the initial and final states, while an additional $B(E1)$ value would be required to determine the mixing of each state individually. We obtained a $B(E1)$ value for the $7/2^-, 1/2 \rightarrow 5/2^+, 1/2$ transition in ^{31}S and collated known $B(E1)$ values for the $A = 31$ and $A = 35$ mirror pairs. A solution was obtained in the $A = 35$ case, which was consistent with less than 1% isospin mixing for both levels, using a $B(E1)$ value for the $7/2^-, 1/2 \rightarrow 5/2^+, 1/2$ transition in ^{35}Ar obtained from scaling to a calculated $B(M2)$ value for a transition from the same $7/2^-$ level. Further work to determine this matrix element experimentally would be very valuable in confirming these initial conclusions. Moreover, measurements of additional matrix elements in both systems, which would likely involve challenging measurements perhaps involving radioactive beams, are clearly desirable to extract the isospin mixing of the individual states.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy, Office of Nuclear Physics, under Contract DE-AC02-06CH11357, and by the UK EPSRC.

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