Dependence of two-nucleon momentum densities on total pair momentum

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Two-nucleon momentum distributions are calculated for the ground states of ^{3}He and ^{4}He as a function of the nucleons' relative and total momenta. We use variational Monte Carlo wave functions derived from a realistic Hamiltonian with two- and three-nucleon potentials. The momentum distribution of pp pairs is found to be much smaller than that of pn pairs for values of the relative momentum in the range 300–500 MeV/c and vanishing total momentum. However, as the total momentum increases to 400 MeV/c, the ratio of pp to pn pairs in this relative momentum range grows and approaches the limit 1/2 for ^{3}He and 1/4 for ^{4}He, corresponding to the ratio of pp to pn pairs in these nuclei. This behavior should be easily observable in two-nucleon knock-out processes, such as $A(e, e'pN)$.

In a recent letter, we studied the role of tensor forces on the correlations between pairs of nucleons in light nuclei [1]. In that work we reported calculations of the relative momentum distribution of pp and pn pairs with vanishing total momentum. We found that the strong spatial-spin-isospin correlations induced by the tensor force lead to large differences in the pp and pn distributions at moderate values of the relative momentum in the pair. These differences have been observed in a two-nucleon knockout experiment on ^{12}C at Jefferson Laboratory (JLab) [2].

In this rapid communication, we report an extension of our calculations to finite total momentum of the correlated pair for ^{3}He and ^{4}He nuclei. This is motivated by a preliminary analysis of data on ^{3}He from the CEBAF large acceptance spectrometer (CLAS) collaboration at JLab [3]. We find that the large differences in pp and pn distributions gradually diminish as the center-of-mass momentum increases, until it approaches the ratio of pp to pn pairs for a given whole nucleus.

The probability of finding two nucleons with relative momentum $\mathbf{q}$ and total momentum $\mathbf{Q}$ in isospin state $T M_T$ in the ground state of a nucleus is proportional to the density

$$
\begin{aligned}
\rho_{T M_T}(\mathbf{q}, \mathbf{Q}) = {} & \frac{A(A-1)}{2(2J+1)} \sum_{M_J} \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \cdots d\mathbf{r}_A \, d\mathbf{r}'_1 \, d\mathbf{r}'_2 \\
& \times \psi^\dagger_{J M_J}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{r}_3, \ldots, \mathbf{r}_A) \, e^{-i\mathbf{q}\cdot(\mathbf{r}_{12}-\mathbf{r}'_{12})} \\
& \times e^{-i\mathbf{Q}\cdot(\mathbf{R}_{12}-\mathbf{R}'_{12})} P_{T M_T}(12) \\
& \times \psi_{J M_J}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_A),
\end{aligned}
\tag{1}
$$

where $\mathbf{r}_{12} \equiv \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{R}_{12} \equiv (\mathbf{r}_1 + \mathbf{r}_2)/2$, and similarly for $\mathbf{r}'_{12}$ and $\mathbf{R}'_{12}$. Here $P_{T M_T}(12)$ is the isospin projection operator, and $\psi_{J M_J}$ denotes the nuclear wave function in spin and spin-projection state $J M_J$. The normalization is

$$
\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{Q}}{(2\pi)^3} \rho_{T M_T}(\mathbf{q}, \mathbf{Q}) = N_{T M_T},
\tag{2}
$$

where $N_{T M_T}$ is the number of NN pairs in state $T M_T$. Obviously, integrating $\rho_{T M_T}(\mathbf{q}, \mathbf{Q})$ over only $\mathbf{Q}$ gives the probability of finding two nucleons with relative momentum $\mathbf{q}$, regardless of their pair momentum $\mathbf{Q}$ (and vice-versa).

For this study we use variational Monte Carlo (VMC) wave functions, derived from a realistic Hamiltonian consisting of the Argonne v_{18} two-nucleon [4] and Urbana-IX three-nucleon [5] interactions (AV18/UIX). The double Fourier transform in Eq. (1) is computed by Monte Carlo (MC) integration. A standard Metropolis walk, guided by $|\psi_{J M_J}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_A)|^2$, is used to sample configurations [6]. For each configuration a two-dimensional grid of Gauss-Legendre points, x_i and X_j, is used to compute the Fourier transform. Instead of just moving the ψ' position ($\mathbf{r}'_{12}$ and $\mathbf{R}'_{12}$) away from a fixed ψ position ($\mathbf{r}_{12}$ and $\mathbf{R}_{12}$), both positions are moved symmetrically away from $\mathbf{r}_{12}$ and $\mathbf{R}_{12}$, so Eq. (1) becomes

$$
\begin{aligned}
\rho_{T M_T}(\mathbf{q}, \mathbf{Q}) = {} & \frac{A(A-1)}{2(2J+1)} \sum_{M_J} \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \cdots d\mathbf{r}_A \, d\mathbf{x} \, d\mathbf{X} \\
& \times \psi^\dagger_{J M_J}(\mathbf{r}_{12} + \mathbf{x}/2, \mathbf{R}_{12} + \mathbf{X}/2, \mathbf{r}_3, \ldots, \mathbf{r}_A) \\
& \times e^{-i\mathbf{q}\cdot\mathbf{x}} \, e^{-i\mathbf{Q}\cdot\mathbf{X}} P_{T M_T}(12) \, \psi_{J M_J}(\mathbf{r}_{12} - \mathbf{x}/2, \mathbf{R}_{12} \\
& - \mathbf{X}/2, \mathbf{r}_3, \ldots, \mathbf{r}_A).
\end{aligned}
\tag{3}
$$

Here the polar angles of the x and X grids are also sampled by MC integration, with one sample per pair. This procedure is similar to that adopted most recently in studies of the ^{3}He$(e, e'p)d$ and ^{4}He$(\vec{e}, e'\vec{p})^3$H reactions [7] and has the advantage of very substantially reducing the statistical errors originating from the rapidly oscillating nature of the integrand for large values of q and Q.

The present method is computationally intensive, because complete Gaussian integrations must be performed for each of the configurations sampled in the random walk. The large range of values of x and X required to obtain converged results, especially for ^{3}He, require fairly large numbers of points; we used grids of up to 96 and 80 points for x and X, respectively.

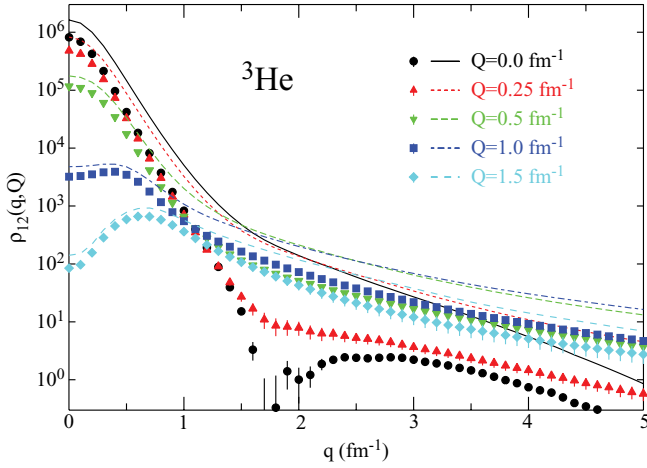FIG. 1. (Color online) The pn (lines) and pp (symbols) momentum distributions in ^{3}He as functions of the relative momentum q at total pair momentum Q from 0 to 2 fm^{-1}.

We also sum over all pairs in a given MC sample instead of just a single pair.

The pn and pp distributions at five values of the total momentum Q, with $\mathbf{Q}\|\mathbf{q}$, are shown as functions of the relative momentum q for ^{3}He in Fig. 1 and for ^{4}He in Fig. 2. The statistical errors due to the MC integration are displayed only for the pp pairs; they are comparable for the pn pairs. When the total momentum vanishes, there is a node in the pp relative momentum distribution just below 2 fm^{-1}, while the pn distribution has a broad shoulder in this region. Integration over the relative momenta in the range 1.5–2.5 fm^{-1} gives a ratio of pp to pn pairs $R_{pp/pn} = 0.014 \pm 0.004$ for ^{3}He, compared to 1/2 for the whole nucleus integrated over all $\mathbf{q}$ and $\mathbf{Q}$. For ^{4}He and $Q = 0$, the value is $R_{pp/pn} = 0.023 \pm 0.006$, compared to 1/4 for the whole nucleus.

The much greater magnitude of the pn momentum distribution is due to the strong correlations induced by tensor components in the underlying NN potential. When $Q = 0$, the pair and residual $(A - 2)$ system are in a relative S wave.
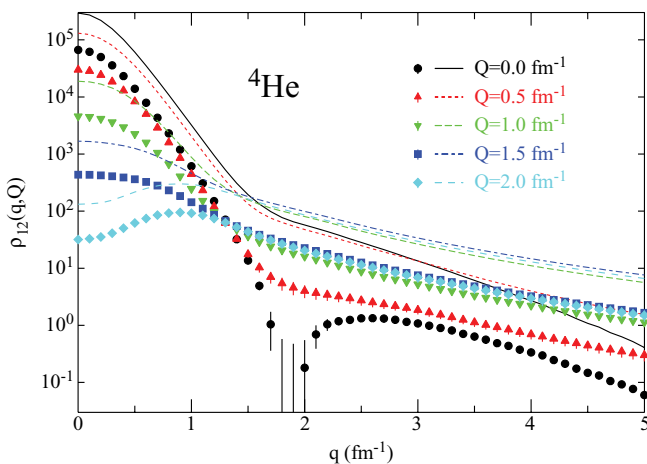

FIG. 2. (Color online) The pn (lines) and pp (symbols) momentum distributions in ^{4}He as functions of the relative momentum q at total pair momentum Q from 0 to 2 fm^{-1}.

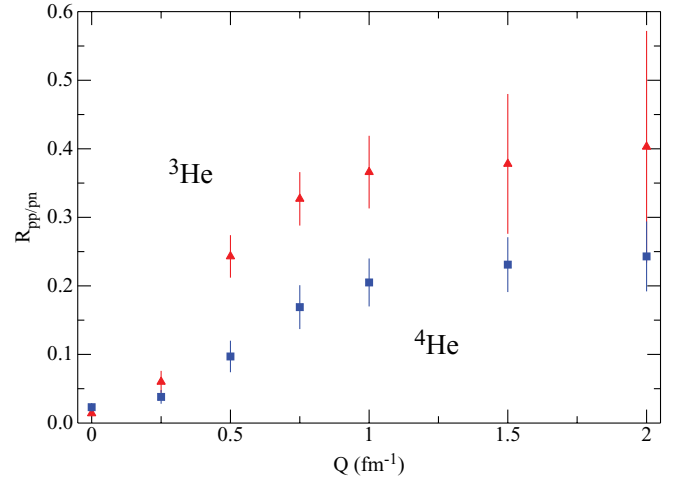
FIG. 3. (Color online) The ratio of pp to pn pairs integrated over relative momentum $\mathbf{q}\|\mathbf{Q}$ of 1.5 to 2.5 fm^{-1} as a function of total momentum Q; red triangles are for ^{3}He and blue squares are for ^{4}He.

Hence, in ^{3}He (^{4}He), whose spin-parity is $\frac{1}{2}^+$ (0^+), pn pairs are predominantly in $T = 0$ and $^3S_1-{}^3D_1$ (deuteron-like) states, while pp pairs are in $T = 1$ and $^1S_0$ (quasi-bound) states [8]. The D-wave component of the deuteron-like pairs fills in the node in the S-wave momentum distribution.

However, for $Q > 0$, the two clusters may have nonzero orbital angular momentum and hence the pp and pn pairs are no longer constrained to be in the quasi-bound or deuteron-like states. Thus the S-wave node in the pp pairs can be filled in by higher angular momentum states. Figures 1 and 2 show that this does happen; the node in the pp relative momentum distribution is rather rapidly filled in as Q increases. Consequently $R_{pp/pn}$ increases as Q increases, as shown in Fig. 3 for q integrated over 1.5–2.5 fm^{-1}.

The most direct evidence for tensor correlations in nuclei comes from measurements of the deuteron structure functions and tensor polarization by elastic electron scattering [9]. In essence, these measurements have mapped out the Fourier transforms of the charge densities of the deuteron in states with spin projections ± 1 and 0, showing that they are very different. In other processes, such as ^{2}H$(d, \gamma)^4$He [10] at very low energy or proton knock-out from a polarized deuteron [11], the effects of tensor correlations are more subtle and their presence is not easily isolated in the experimental data. This is because of corrections from initial or final state interactions and many-body terms in the transition operators.

Some of these corrections will also affect, for instance, the cross sections for $(e, e'pn)$ and $(e, e'pp)$ knock-out processes in back-to-back kinematics. However, one would expect the contributions due to final state interactions in the pn and pp reactions, both between the nucleons in the pair and between these and the nucleons in the residual $(A - 2)$ system, to be of similar magnitude for relative momenta in the range 300–500 MeV/c. In particular, charge-exchange processes have been estimated to give small ($\leqslant 10\%$) corrections to these ratios in ^{12}C at JLab kinematics [2]. Such processes, which are induced by interactions between the knocked-out pair and the residual cluster, could change an initial pn pair on its way

out of the nucleus into the (detected) pp pair, thus increasing the $(e, e'pp)$ to $(e, e'pn)$ cross section ratios. Lastly, leading terms in the electromagnetic two-body current vanish in pp because of their isospin structure [12]. Of course, they will contribute in pn, but are not expected to produce large effects.

The recent experiment at JLab referred to earlier has measured the ratio of $^{12}C(e, e'pn)$ to $^{12}C(e, e'pp)$ cross sections in back-to-back kinematics for relative momenta in the range 300–500 MeV/c [2] to be $\simeq 10$. These measurements have corroborated the results of an earlier analysis of a Brookhaven National Laboratory experiment, which measured cross sections for (p, pp) and (p, ppn) processes on ^{12}C in similar kinematics [13]. The observed enhancement in the pn to pp ratio is in agreement with the predictions of Refs. [1,14] and beautifully demonstrates the crucial role that the tensor force plays in shaping the short-range structure of nuclei.

It would be interesting to extend these measurements to other nuclei. In ^{3}He and ^{4}He, one would expect the node in the pp momentum distribution to be filled in by interaction effects in the final state [7]. However, the ratio of pp to pn cross sections in the range (300–500) MeV/c should still reflect the dominance of the pn momentum distribution at these values of relative momenta. In fact, the analysis of JLab CLAS data on ^{3}He mentioned above suggests that this is indeed the case [3]. These data also seem to confirm the rapid rise of the pp to pn ratio with increasing total pair momentum, predicted in Fig. 3 of the present work.

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