Two-quasineutron states in $^{248}_{98}$Cf and $^{250}_{98}$Cf and the neutron-neutron residual interactions

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Two-quasineutron states in ^{248}Cf and ^{250}Cf were investigated by single-neutron transfer reactions, ^{249}Cf$(d,t)^{248}$Cf and ^{249}Cf$(d,p)^{250}$Cf. The majority of levels observed were assigned to 12 bands in ^{248}Cf and six bands in ^{250}Cf, constructed from the single-particle states in neighboring Cf nuclei. The effective two-body interactions between two odd neutrons coupled outside a deformed core were deduced from the differences in the energies of the bandheads formed by the parallel and antiparallel coupling of the intrinsic spins of the two single-particle states.

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I. INTRODUCTION

Great progress has been made in the synthesis of superheavy elements [1]. At the same time, γ-ray experiments are being carried out to understand the structure of nuclei approaching the superheavy region. Since these nuclei are produced with very low cross sections, these investigations are currently limited to nuclei just above Fm ($Z = 100$). One of the goals of these experiments is to identify one-quasiparticle states in odd-mass nuclei [2–4] and two-quasiparticle states in even-even nuclei [5–7]. Although these experiments provide some spectroscopic information, the activities are too low to yield the detailed spectroscopic data needed to make single-particle assignments. For detailed spectroscopic studies, radioactive samples in kBq amounts and target materials in μg quantities are needed. The heaviest nuclide that is available in such quantities and has a long half-life is $^{249}_{98}$Cf ($t_{1/2} = 351$ yr). In the 1970s, we performed (d,t) and (d,p) reaction experiments on this target to investigate the level structures of ^{248}Cf and ^{250}Cf. The data were not published because of the principal author's untimely death. The recent interest in the single-particle structure of heavy nuclei and the difficulty in repeating such a measurement have led the remaining authors to publish these data some 30 yr after the measurement.

The level structure of ^{250}Cf was previously studied in detail by measuring the radiation emitted during the decay of $^{250}_{99}$Es isomers [8–10] and $^{250}_{97}$Bk [11], and by studying the ^{250}Cf(d,d') reaction [12]. These studies provide the spins and parities of ^{250}Cf levels which allow the interpretation of the ^{249}Cf$(d,p)^{250}$Cf reaction data. Only the ground-state band and a level at 592.2 keV are known in ^{248}Cf from the β^- decay studies of ^{248}Bk [13]. In this work, we report on the structures of ^{248}Cf and ^{250}Cf nuclei and on the neutron-neutron residual interactions obtained from the splitting energies of the bands built by the parallel and antiparallel coupling ($\Sigma = 1, 0$) of the intrinsic spins of the two one-quasiparticle states. The experimental level energies in these nuclei are found to be in good agreement with theoretical [14,15] two-quasiparticle energies. The properties of single-particle states obtained in these studies can be used to characterize the observed levels in transfermium nuclei.

The analysis of data and the theoretical calculations are carried out in exactly the same way as was done for the data on two-quasiparticle states in ^{236}U [16].

II. EXPERIMENTAL METHODS AND RESULTS

The experiments reported in this article were performed in the 1970s with the Argonne FN tandem Van de Graaff accelerator. A 50 μg/cm^2 ^{249}Cf target was prepared by the Argonne electromagnetic isotope separator [17] by the deposition of decelerated ^{249}Cf ions on a 40 μg/cm^2 carbon foil in an area of 1×3 mm. The target was bombarded with a deuteron beam from the accelerator, and the emerging protons and tritons were momentum-analyzed with an Enge split-pole magnetic spectrograph [18]. Protons and tritons were detected with emulsion plates placed at the focal plane of the spectrograph. The plates were developed, and the tracks in the emulsion plates were scanned with an automated machine [19]. A few plates were also scanned manually to reduce the background. Spectra were measured at spectrograph angles of 90°, 120°, and 135°, and at bombarding deuteron energies of 11.0, 12.0, 13.0, and 14.0 MeV. For these measurements, the spectrograph solid angle was 2.0 msr and the spectra resolutions (full width at half maximum) were 12 and 15 keV for the (d,p) and (d,t) reactions, respectively. As mentioned above, the method was essentially that of Ref. [16]. Examples of (d,t) and (d,p) spectra are shown in Figs. 1 and 2. A total of three proton and six triton spectra were analyzed with the peak-fitting program AUTOFIT [20]. Excitation energies, cross sections, and the assignments of the levels are given in Tables I and II. The errors in the relative energies of strongly populated levels are ±1 keV; the errors in the table represent the absolute error. Uncertainties in the absolute cross sections are estimated to be ∼20%. In the tables, only statistical errors are given, which for strong peaks are ∼5%. In ^{249}Cf(d,p) spectra, peaks below

*Deceased.

TABLE I. Excitation energies and differential cross sections measured with the ^{249}Cf(d, t)^{248}Cf reaction at 12.0-MeV deuteron energy. Assignments in parentheses are tentative.

Excitation energy (keV)	$d\sigma/d\Omega(\mu b/sr)$[a]			Level assignment K, I^π
	90°	120°	135°	
0			<1	$0, 0^+$
44 ± 2	2.5 ± 0.9	0.6 ± 0.1	2.6 ± 0.5	$0, 2^+$
138 ± 2	3.5 ± 0.7	1.5 ± 0.2	3.3 ± 0.8	$0, 4^+$
290 ± 2	4.1 ± 0.8	3.9 ± 0.3	6.3 ± 1.1	$0, 6^+$
490 ± 2	3.3 ± 0.9	2.0 ± 0.3	8.5 ± 0.9	$0, 8^+$
593 ± 1	13.2 ± 1.7	11.3 ± 0.8	10.7 ± 1.0	$2, 2^-$
630 ± 1	13.3 ± 1.5	15.2 ± 0.9	16.5 ± 1.7	$2, 3^-$
677 ± 1	17.8 ± 2.2	16.0 ± 0.9	19.4 ± 1.6	$2, 4^-$
735 ± 1	11.6 ± 1.6	16.5 ± 0.8	18.1 ± 1.6	$2, 5^-$
779 ± 2		0.8 ± 0.2		$(0, 10^+)$
806 ± 1	6.7 ± 1.0	10.2 ± 0.5	11.0 ± 1.7	$2, 6^-$
885 ± 1	2.0 ± 0.7	3.7 ± 0.4		$2, 7^-$
979 ± 2		0.9 ± 0.2		$2, 8^-$
1021 ± 2		0.6 ± 0.2		
1048 ± 2		1.1 ± 0.2		
1079 ± 2		1.1 ± 0.2		
1112 ± 2		1.9 ± 0.3		
1179 ± 2		1.5 ± 0.2		
1261 ± 2		7.6 ± 0.8	9.0 ± 2.6	$8, 8^-$
1293 ± 2		3.4 ± 0.6		
1319 ± 2		1.8 ± 0.5		
1351 ± 2	5.5 ± 1.5	9.4 ± 0.9	12.3 ± 2.3	$8, 9^-$
1391 ± 2		2.6 ± 0.6	4.1 ± 1.2	
1432 ± 2	1.9 ± 0.5			
1463 ± 1	65 ± 3	81 ± 3	78 ± 6	$5, 5^-$
1477 ± 2	7.8 ± 1.7		5.2 ± 2.1	$2, 2^-$
1509 ± 1	7.8 ± 1.2	11.8 ± 1.0	12.5 ± 2.5	$2, 3^-$
1530 ± 1	23.0 ± 1.7	29.7 ± 1.5	29.2 ± 3.5	$5, 6^-$
1557 ± 1	7.7 ± 1.2	12.9 ± 1.0	12.7 ± 2.5	$2, 4^-$
1577 ± 1	34.3 ± 2.1	48.0 ± 1.8	55 ± 5	$7, 7^-$
1605 ± 1	7.5 ± 1.2	16.2 ± 1.2	18.0 ± 1.5	$5, 7^-$
1621 ± 1	8.2 ± 1.3	13.7 ± 1.2	20.3 ± 1.7	$2, 5^-$
1640 ± 1	49.0 ± 2.4	58.5 ± 2.1	57.3 ± 2.5	$4, 4^-$
1663 ± 1	13.6 ± 1.4	18.8 ± 2.5	28.8 ± 1.8	$7, 8^-$
1686 ± 3	6.1 ± 1.2	11.2 ± 1.3	6.3 ± 1.2	$2, 6^-$
1698 ± 2	21.1 ± 1.7	23.5 ± 1.6	29.9 ± 2.1	$4, 5^-$
1731 ± 2	2.2 ± 0.9	6.3 ± 0.7	7.6 ± 1.1	$5, 8^-$
1766 ± 2	7.9 ± 0.5	15.3 ± 1.1	16.4 ± 1.6	$4, 6^-$
1781 ± 3	4.4 ± 0.8	10.1 ± 1.0	8.7 ± 1.2	$7, 9^-$
1839 ± 3	2.2 ± 0.8		4.7 ± 1.2	$(5, 9^-)$
1852 ± 1	4.0 ± 0.9	7.7 ± 1.0	6.8 ± 1.4	$4, 7^-$
1927 ± 1	63 ± 3	90 ± 3	106 ± 5	$5, 5^+$
1946 ± 3	4.5 ± 0.9	2.6 ± 0.6	6.3 ± 1.8	$4, 8^-$
1968 ± 1	8.0 ± 1.0	13.5 ± 1.1	21.5 ± 2.4	
1992 ± 1	5.4 ± 0.9	8.9 ± 0.9	17.1 ± 2.2	$5, 6^+$
2018 ± 3	4.4 ± 0.9	4.4 ± 0.7	9.8 ± 1.8	
2072 ± 1	48 ± 3	74 ± 4	85 ± 9	$4, 4^+$
2105 ± 1		4.2 ± 1.2	5.7 ± 1.2	$(3, 4^-)$
2131 ± 1	6.0 ± 1.2	16.0 ± 2.0	15.6 ± 1.7	$4, 5^+$
2161 ± 2			5.9 ± 1.2	$(3, 5^-)$
2184 ± 2	8.7 ± 1.8	15.2 ± 1.2	15.8 ± 1.7	$6, 6^-$
2207 ± 1	1.7 ± 0.8	5.6 ± 0.8	7.0 ± 1.3	$4, 6^+$
2241 ± 1	10.3 ± 1.2	24.6 ± 1.5	29.9 ± 2.7	$7, 7^+$
2262 ± 1	5.5 ± 1.0	9.6 ± 1.1	14.2 ± 2.0	$(6, 7^-)$
2281 ± 2	4.8 ± 1.0	10.5 ± 1.1	15.1 ± 2.1	$2, 2^+$
2314 ± 2	5.3 ± 1.1	7.7 ± 0.9	9.8 ± 1.8	$2, 3^+$
2368 ± 2	2.8 ± 1.1	4.3 ± 1.2	5.7 ± 1.5	$(2, 4^+)$
2463 ± 2			5.9 ± 1.0	
2492 ± 2	3.3 ± 0.8	5.0 ± 0.8	3.9 ± 1.0	
2512 ± 1	8.0 ± 1.0	20.6 ± 1.5	19.9 ± 1.7	$3, 3^+$
2533 ± 1	4.7 ± 0.9	14.1 ± 1.2	11.6 ± 1.4	
2557 ± 1		8.0 ± 1.0	7.8 ± 1.2	$3, 4^+$
2580 ± 1	0.6 ± 0.8	3.3 ± 0.7	2.5 ± 0.9	
2602 ± 1	9.0 ± 1.2	30.6 ± 1.7	23.9 ± 1.8	$6, 6^+$
2634 ± 2		2.1 ± 0.6		$(3, 5^+)$
2682 ± 2		1.8 ± 0.6		$(6, 7^+)$

[a]The uncertainties contain statistical errors only.

the 1210-keV peak had very few counts, which corresponds to an upper limit of 5 μb/sr cross section for each level.

The goal of the experiment was to use angular distributions of cross sections for level spin determination. However, because the cross section for each level contains more than one partial wave and the absolute cross sections do not have high precision, angular distributions were not used. The presence of the same peak in all the spectra measured at different angles provides confidence that the peaks observed belong to the nucleus of interest.

III. CALCULATION OF DIFFERENTIAL CROSS SECTIONS

In a one-neutron transfer reaction on an even-even target, the differential cross section to a level with spin j depends on c_{jK}^2, where c_{jK} is the expansion coefficient of the Nilsson [21] wave function. Each member of a rotational band in an odd-neutron nucleus is populated by one partial wave l. The differential cross section is given by [22]

$$d\sigma/d\Omega = 2P_K c_{jK}^2 \sigma_{\rm DW},$$

where j is the angular momentum of the transferred neutron, $\sigma_{\rm DW}$ is the single-particle cross section calculated with distorted-wave Born approximation (DWBA) theory, and $P_K = U_K^2$ for the (d, p) reaction and $P_K = V_K^2$ for the (d, t) reaction.

In a one-neutron transfer reaction on an odd-neutron target, the cross section for the population of a member of the two-quasineutron band contains contributions from more than one partial wave l. The differential cross section is given by [16,23]

$$d\sigma/d\Omega = \sum_j P_K |\langle I_i j K_i (K_f - K_i) | I_f K_f \rangle|^2 c_{jK}^2 \sigma_{\rm DW},$$

where I_i, I_f are the target and product nuclear spins, K_i, K_f are their projections on the nuclear symmetry axis, and the term in angular brackets is the Clebsch-Gordan coefficient. The DWBA cross section $\sigma_{\rm DW}$ was calculated with the computer

TABLE II. Excitation energies and differential cross sections measured with the ^{249}Cf$(d,p)^{250}$Cf reaction at 12.0-MeV deuteron energy. Assignments in parenthesis are tentative. Assignments $(5,5^-)_{pp}$, $(5,6^-)_{pp}$, and $(5,7^-)_{pp}$ represent two-quasiproton states; these were identified in the ^{249}Bk$(\alpha,t)^{250}$Cf reaction [30].

Excitation energy (keV)	$d\sigma/d\Omega(\mu b/sr)$[a] 90°	120°	135°	Level assignment K, I^π
1210 ± 3	13.8 ± 1.8	33 ± 4	22.3 ± 2.2	2, 2⁻ + 1, 3⁻
1227 ± 3		∼6		
1245 ± 3		∼7		2, 2⁺ + 2, 3⁻
1256 ± 2	141 ± 7	143 ± 9	108 ± 7	4, 4⁻
1268 ± 3		∼10		0, 0⁺
1295 ± 3	11.4 ± 1.6	25.5 ± 2.8	22.0 ± 2.3	0, 2⁺
1310 ± 3	60 ± 3	58 ± 4	58 ± 4	4, 5⁻
1335 ± 4	9.4 ± 1.4	14.5 ± 2.2	11.6 ± 1.8	0, 3⁻
1379 ± 3	34.4 ± 3.2	53 ± 5	32.6 ± 2.1	4, 6⁻
1396 ± 3	38 ± 4	45 ± 5	30 ± 2	$(5,5^-)_{pp}$
1428 ± 3	11.7 ± 3.2	8 ± 3	14.2 ± 2.7	3, 3⁻
1456 ± 3	27 ± 4	30 ± 4	32 ± 4	$(5,6^-)_{pp}$ + 4, 7⁻
1478 ± 2	180 ± 10	160 ± 9	121 ± 7	5, 5⁻
1498 ± 2	118 ± 8	140 ± 9	114 ± 7	6, 6⁻
1530 ± 3	5.6 ± 2.8	8 ± 2	10.6 ± 2.7	$(5,7^-)_{pp}$
1547 ± 2	62 ± 5	72 ± 5	69 ± 5	5, 6⁻
1573 ± 3	26 ± 3	36 ± 5	44 ± 4	6, 7⁻
1596 ± 2	34 ± 4	59 ± 6	51 ± 5	8, 9⁻
1626 ± 2	93 ± 6	121 ± 6	91 ± 6	5, 7⁻ + 3, 3⁻
1662 ± 3	32 ± 5	35 ± 5	33 ± 4	6, 8⁻
1678 ± 3	26 ± 5	36 ± 6	28 ± 4	3, 4⁻
1719 ± 4		29 ± 5	23.6 ± 4.3	5, 8⁻
1736 ± 3	23 ± 6	37 ± 5	23.4 ± 4.4	3, 5⁻
1775 ± 6	16 ± 5	17 ± 4	13.3 ± 2.4	
1802 ± 4	14 ± 5	21.4 ± 4.3	9.1 ± 2.2	3, 6⁻
1843 ± 4	15.8 ± 2.8	11.5 ± 3.5	4.0 ± 1.9	
1878 ± 3	39 ± 4	46 ± 9	27 ± 3	
1901 ± 4	17 ± 3	30 ± 8	17.8 ± 2.5	
1944 ± 3	34 ± 4	57 ± 11	27 ± 4	
1955 ± 3	57 ± 5	44 ± 10	40 ± 4	
2005 ± 5	69 ± 6	77 ± 14	60 ± 25	
2017 ± 3	246 ± 10	242 ± 22	262 ± 69	(5, 5⁺)
2045 ± 5		44 ± 10	56 ± 29	
2074 ± 3	135 ± 7	159 ± 18	136 ± 46	(4, 4⁺)
2091 ± 4	24 ± 4	43 ± 11	53 ± 30	
2131 ± 4	27 ± 5	35 ± 7	41 ± 6	
2150 ± 4	31 ± 5	43 ± 7	44 ± 6	
2519 ± 5	62 ± 7	100 ± 10	94 ± 10	(9, 9⁻)
2596 ± 5	61 ± 7	80 ± 10	74 ± 10	
2616 ± 5	56 ± 7	98 ± 10	94 ± 10	
2656 ± 5	55 ± 9	58 ± 7	45 ± 6	
2720 ± 5	∼50	60 ± 8	55 ± 8	

[a]The uncertainties contain statistical errors only.

code DWUCK4 [24] using the parameters of the optical-model potential listed in Table III. The values of c^2_{jK} were calculated [25] by the Nilsson model with parameters $\mu = 0.45$, $\kappa = 0.05$, and $\beta = 0.25$. The contributions of Coriolis mixing to cross sections and excitation energies were taken into account when important.

TABLE III. Parameters of the optical-model potential $V(r) = -V(1+\exp X)^{-1} + iW'(1+\exp X')^{-1} - iW''$, where $X = (r - r_0A^{1/3})/a$ and $X' = (r - r'_0A^{1/3})/a'$.

Particle	V MeV	r_0 fm	a fm	r_C fm	W' MeV	r'_0 fm	a' fm	W'' MeV
d	60	1.5	0.6	1.5	15	1.5	0.6	
p	57	1.3	0.5	1.3	8	1.3	0.5	
t	155	1.2	0.7	1.2	0	1.3	0.65	160
n		1.25	0.65					

IV. DISCUSSION

A. Level assignment in ^{248}Cf

Single-particle states in odd-mass nuclei adjacent to ^{248}Cf and in their isotones had been studied previously by one-neutron transfer reactions [26,27]. The observed signatures of cross sections are found to be in agreement with the theoretical signatures. These single-particle states can couple to produce two-quasiparticle states in ^{248}Cf, and their excitation energies, and in particular their ordering, can be estimated. Since one component of the two-quasiparticle states populated in the ^{249}Cf$(d,t)^{248}$Cf reaction must be the ^{249}Cf ground state 9/2⁻[734] configuration, only states formed by the coupling of this state with other neighboring states are considered. Also, since the (d,t) reaction populates predominantly hole states that have large values of V_K^2, only states below the Fermi surface are considered. The orbitals originating from the $j_{15/2}$ state have very low cross section because of the high l value, and hence the two-quasiparticle states involving the $j_{15/2}$ state are not expected to be populated with sufficient intensity to be observed. In the ^{249}Cf$(d,t)^{248}$Cf reaction, the single-particle orbitals from which the neutron is picked up are the same as the orbitals excited in the ^{248}Cm$(d,t)^{247}$Cm, ^{246}Cm$(d,t)^{245}$Cm [27], and ^{244}Pu$(d,t)^{243}$Pu [28] reactions. Thus the following configurations are expected in ^{248}Cf in order of increasing excitation energy: $\{9/2^-[734]; 5/2^+[622]\}_{2^-,7^-}$, $\{9/2^-[734]; 7/2^+[624]\}_{1^-,8^-}$, $\{9/2^-[734]; 1/2^+[631]\}_{4^-,5^-}$, $\{9/2^-[734]; 1/2^-[501]\}_{4^+,5^+}$, $\{9/2^-[734]; 3/2^+[631]\}_{3^-,6^-}$, $\{9/2^-[734]; 5/2^-[503]\}_{2^+,7^+}$, and $\{9/2^-[734]; 3/2^-[501]\}_{3^+,6^+}$. The level structure of ^{248}Cf and the two-quasineutron assignments deduced in the present study are shown in Fig. 3. Signatures, which are relative cross sections of the members of rotational bands, calculated with the Nilsson wave functions, are compared with the experimental signatures in Figs. 4 and 5. The agreement between experiment and theory is excellent.

Only the $K, I^\pi = 2, 2^-$ state in ^{248}Cf was known previously from the decay of ^{248}Bk [13]. We have identified several of the members of the $\{9/2^-[734]; 5/2^+[622]\}_{2^-,7^-}$, $\{9/2^-[734]; 1/2^+[631]\}_{4^-,5^-}$, and $\{9/2^-[734]; 1/2^-[501]\}_{4^+,5^+}$ rotational bands, and the cross sections of these bands are comparable to the cross sections of

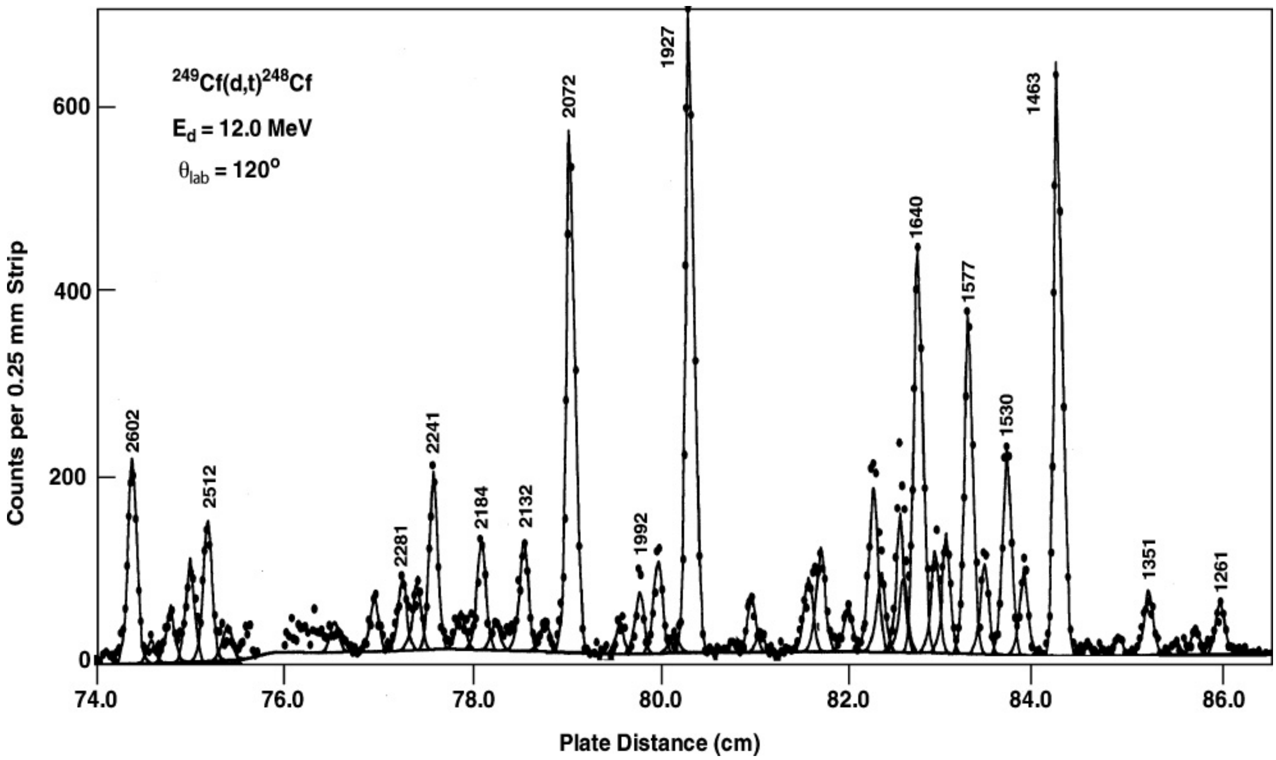

FIG. 1. Triton spectrum from the ^{249}Cf$(d,t)^{248}$Cf reaction measured with an Enge split-pole magnetic spectrograph at 120°. The incident deuteron energy was 12.0 MeV. The tracks on the emulsion plates were counted by an automatic machine [19]. Energies of strong peaks and band heads are given in keV. Energies of unlabeled peaks are given in Table I.

corresponding bands in the odd-neutron nuclei ^{245}Cm and ^{243}Pu. For these reasons, the above assignments are quite firm. The assignments $\{9/2^-[734]; 5/2^-[503]\}_{2^+,7^+}$, $\{9/2^-[734]; 3/2^-[501]\}_{3^+,6^+}$, and $\{9/2^-[734]; 3/2^+[631]\}_{3^-,6^-}$ are also quite probable because these single-particle states were identified in ^{243}Pu [28] and their relative energies and relative cross sections in the two nuclei are quite similar.

The two-quasiparticle state $\{9/2^-[734]; 7/2^+[624]\}_{8^-}$ observed in ^{246}Cm at 1180 keV [29], and the state corresponding to this configuration should have similar energy in ^{248}Cf. Since the major components of the $7/2^+[624]$ wave function are $j = 9/2$ and $j = 11/2$ states, only 8^-, 9^-, and 10^- members of the 8^- band should be populated in the (d,t) reaction. We calculate cross sections of 13, 11, and 3 μb/sr at 120°

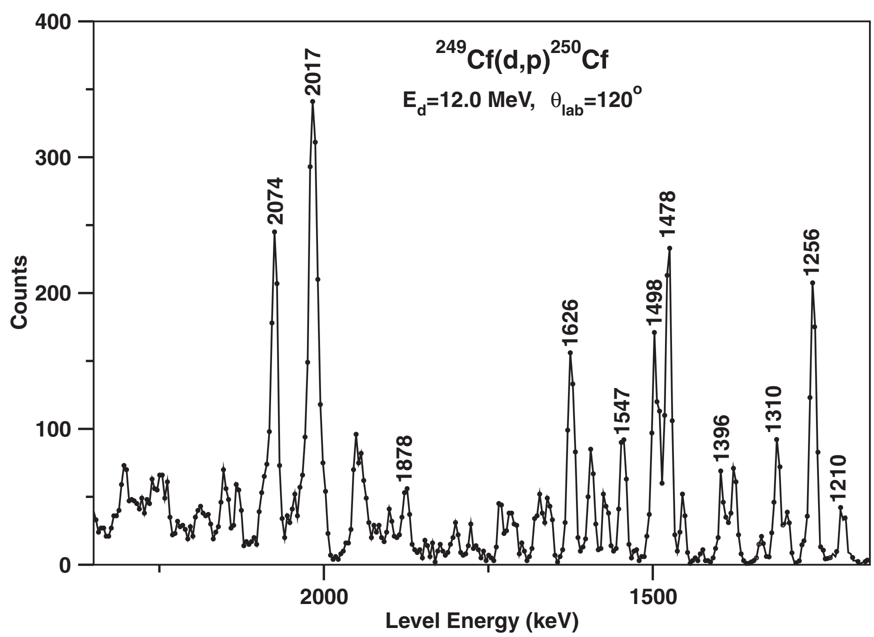

FIG. 2. Proton spectrum from the ^{249}Cf$(d,p)^{250}$Cf reaction measured with an Enge split-pole magnetic spectrograph at 120°. The incident deuteron energy was 12.0 MeV. The tracks on the emulsion plates were counted manually. Energies of strong peaks and band heads are given in keV. The energies of unlabeled peaks are included in Table II.

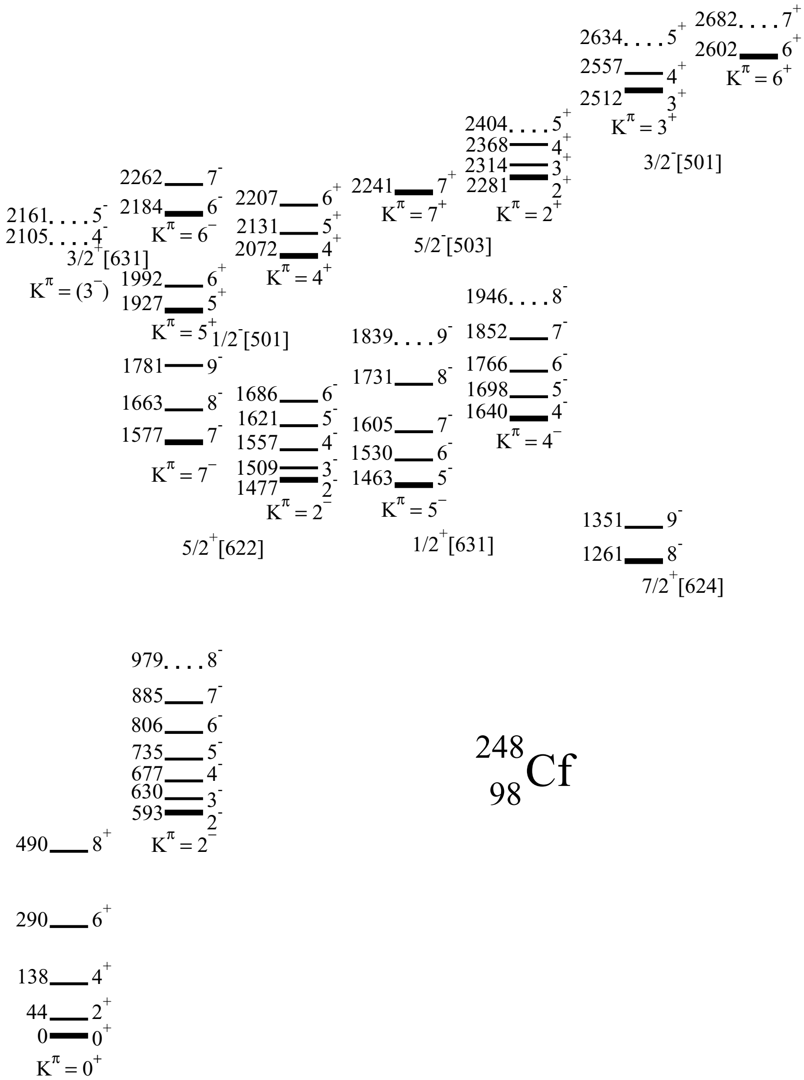

FIG. 3. Level scheme of ^{248}Cf obtained in the present work. Level energies in keV are given on the left side and I^π values on the right. K^π values were obtained by coupling the $9/2^-[734]$ Nilsson state (the ground state of ^{249}Cf) and the state shown next to the band. Assignments to levels represented by dashed lines are tentative.

for the 8^-, 9^-, and 10^- members of the band, respectively. The only states around this energy that have the expected cross sections are the 1261- and 1351-keV levels, and thus we assign these two levels to the 8^- and 9^- members of the $\{9/2^-[734]; 7/2^+[624]\}_{8^-}$ band.

The cross sections, measured at 14.0-MeV deuteron energy, were summed over all the members of each rotational band. The ratios R of the experimental to the calculated summed cross sections were averaged over the three angles of measurement. The ratios of these average cross section ratios of the $K_<$ band, $R(K_<)$, to those of $K_>$ band, $R(K_>)$ are given in Table IV, and these range from 0.88 to 1.23, close to the expected value of 1.0.

B. Level assignment in ^{250}Cf

For the ^{249}Cf$(d, p)^{250}$Cf reaction, the relevant orbitals are the same as those populated in the ^{250}Cf$(d, p)^{251}$Cf reaction [26], and thus the relative energies of two-quasiparticle states in ^{250}Cf can be estimated from the measured energies of single-particle states in ^{251}Cf. Since the ground state of ^{249}Cf is $9/2^-[734]$, only those two-quasiparticle states which contain one neutron in the $9/2^-[734]$ orbital will be populated. The spins and parities of high-spin states in ^{250}Cf were determined in the decay studies of the 8.6-h ^{250}Es [8]. In that work, $K^\pi = 4^-, 5^-, 5^-$, and 6^- were assigned to the 1256-, 1396-, 1478-, and 1498-keV levels, respectively, and their two-quasiparticle configurations were also deduced. The ^{249}Bk(α, t) reaction established [30] the 1396-keV state as corresponding to the $\{7/2^+[633]; 3/2^-[521]\}_{5^-}$ two-quasiproton configuration. The 6^- and 7^- members of the band built on the 1396-keV state were identified at 1456 and 1530 keV in this reaction. Using the previous decay data and the measured reaction cross sections, we made assignments to the ^{250}Cf levels. The spin and parity assignments are summarized in Fig. 6.

The $1/2^-[750]$ band identified in the ^{250}Cf$(d, p)^{251}$Cf reaction [26] has the largest cross section. Thus the $\{9/2^-[734]; 1/2^-[750]\}_{4^+,5^+}$ bands should receive the largest cross sections in the ^{249}Cf$(d, p)^{250}$Cf reaction. We observe the two strongest peaks in Fig. 2 at 2017 and 2074 keV; hence these two states should belong to the $\{9/2^-[734]; 1/2^-[750]\}_{4^+,5^+}$ bands. The calculated cross section of the $K, I^\pi = 5, 5^+$ level is larger than the calculated value for the $4, 4^+$ level thus favoring a $5, 5^+$ assignment to the 2017-keV level. However,

TABLE IV. Splitting energies and cross-section ratios for the bands formed in ^{248}Cf and ^{250}Cf by the coupling of the single-particle states given in column 2. $\Delta E = E(K_<) - E(K_>$, and $R(K_<)/R(K_>)$ is the ratio of R values for the $K_<$ and $K_>$ bands.

Nuclide	$\Omega_1[N_1 n_{z1} \Lambda_1] + \Omega_2[N_2 n_{z2} \Lambda_2]$	Σ	K_f^π	E_x (keV)	ΔE (keV)	$R(K_<)/R(K_>)$ at 14.0 MeV
^{248}Cf	$9/2^-[734] \pm 5/2^+[622]$	1	7^-	1577 ± 1	−100	1.23 ± 0.16
		0	2^-	1477 ± 2		
			2^-	593 ± 1		
	$9/2^-[734] \pm 1/2^+[631]$	1	4^-	1640 ± 1	177	0.88 ± 0.09
		0	5^-	1463 ± 1		
	$9/2^-[734] \pm 1/2^-[501]$	1	4^+	2072 ± 1	145	1.12 ± 0.11
		0	5^+	1927 ± 1		
	$9/2^-[734] \pm 3/2^+[631]$	1	6^-	2184 ± 2		
	$9/2^-[734] \pm 5/2^-[503]$	1	2^+	2281 ± 2	40	1.15 ± 0.15
		0	7^+	2241 ± 1		
	$9/2^-[734] \pm 3/2^-[501]$	1	6^+	2602 ± 1	−90	1.22 ± 0.21
		0	3^+	2512 ± 1		
^{250}Cf	$9/2^-[734] \pm 1/2^+[620]$	1	5^-	1478 ± 2	−222	0.87 ± 0.07[a]
		0	4^-	1256 ± 2		
	$9/2^-[734] \pm 3/2^+[622]$	1	3^-	1626 ± 2	128	0.91 ± 0.10[a]
		0	6^-	1498 ± 2		

[a]Obtained from the summed cross sections averaged over the three angles of measurement at 12.0 MeV.

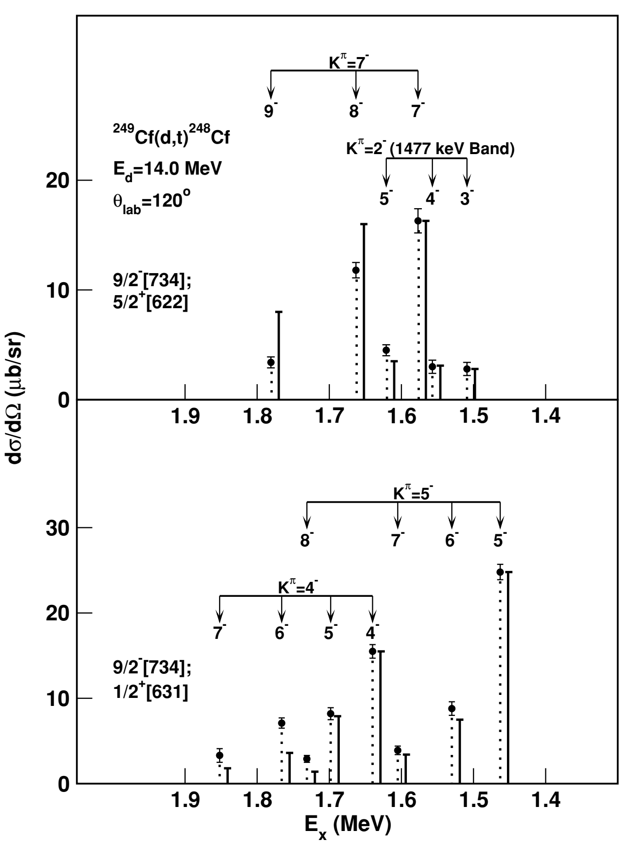

FIG. 4. Comparison of calculated cross sections (solid lines) with the measured cross sections (dashed lines) for the ^{249}Cf$(d,t)^{248}$Cf reaction at $\theta_{\text{lab}} = 120^\circ$ and $E_d = 14.0$ MeV. The calculated values are normalized at the cross sections of $I^\pi = 3^-, 7^-, 5^-$, and 4^- levels for the $K^\pi = 2^-, 7^-, 5^-$, and 4^- bands, respectively.

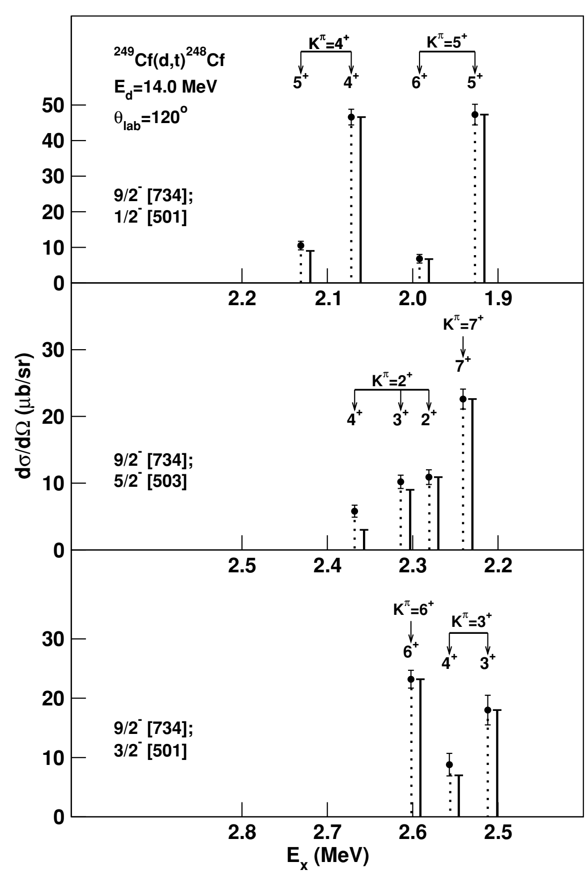

FIG. 5. Same as Fig. 4, but for the positive-parity bands in ^{248}Cf. The calculated cross sections were normalized at the measured values of the $I^\pi = 5^+, 4^+, 7^+, 2^+, 3^+$, and 6^+ levels for the $K^\pi = 5^+, 4^+, 7^+, 2^+, 3^+$, and 6^+ bands, respectively.

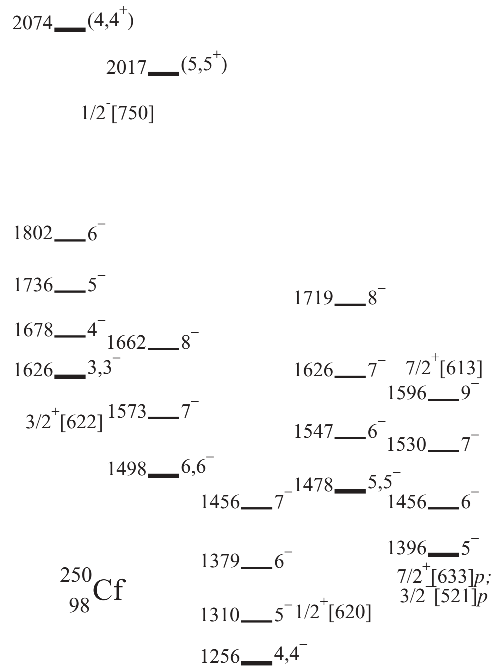

FIG. 6. Same as Fig. 3, but for the level scheme of ^{250}Cf obtained in the present work. The notation $7/2^+[633]p; 3/2^-[521]p$ represents a two-quasiproton configuration.

according to the Gallagher-Moszkowski rule [31,32], the $4, 4^+$ level should lie lower and hence the 2017-keV level should have $4, 4^+$ assignment. Also, the 4^+ and 5^+ bands will have appreciable mixing because of their closeness and moderately large Coriolis matrix element. Thus the assignments of the 2017- and 2074-keV levels are uncertain. In Fig. 6, we have given a tentative assignment of $5, 5^+$ to the 2017-keV state.

The $9/2^+[604]$ state, which has a large cross section in the ^{250}Cf$(d, p)^{251}$Cf reaction, has been identified in ^{251}Cf [33] at 974.0 keV. The state $\{9/2^-[734]; 9/2^+[604]\}_{9^-}$ formed by coupling this state with the ^{249}Cf ground state is expected at ~2500 keV. We observe two states at 2519 and 2616 keV which have the expected energy, cross section, and angular distribution for large l value. Either of the states could be the $9, 9^-$ state. We give the assignment of $K, I^\pi = 9, 9^-$ to the 2519-keV level because its energy is nearer the expected energy.

Another particle state which is expected to be populated strongly in ^{250}Cf is the $7/2^+[613]$ orbital which will couple to the $9/2^-[734]$ band to produce a $K^\pi = 8^-$ band. Calculations give a cross section of 33 μb/sr at 90° for each of the $I = 8$ and $I = 9$ members of the $K^\pi = 8^-$ band. Because of its large cross section (24 μb/sr) and its expected energy, we have assigned the 1596-keV state to the 9^- member of the 8^- band. The 8^- member is not identified. Most likely it is masked by one of the stronger peaks.

The double ratios of cross sections, $R(K_<)/R(K_>)$, for the ^{249}Cf$(d, p)^{250}$Cf reaction measured at a deuteron energy of 12.0 MeV are included in Table IV, and these agree with the expected value of 1.0.

The energies of two-quasiparticle states in ^{248}Cf and ^{250}Cf were calculated by the Dubna group [14,15], and these are compared with the experimental values in Table V. For the $\{9/2^-[734]; 1/2^+[631]\}_{4^-,5^-}$ states in ^{248}Cf, the theoretical [14] energy is calculated for the centroid. Also, the table includes the rotational constants of the bands in ^{248}Cf and ^{250}Cf, which are quite similar to the values measured in other even-even actinide nuclei. More precise values of rotational constants in ^{250}Cf were obtained in the decay scheme studies [8].

TABLE V. Excitation energies and rotational constants of bands in ^{248}Cf and ^{250}Cf. Theoretical level energies are from Refs. [14,15]. For the $\{9/2^-[734]; 1/2^+[631]\}$ states, the theoretical energy represents the energy of the centroid of the 5^- and 4^- states.

Isotope	State	K^π	E_{exp} (keV)	E_{theory} (keV)	$\hbar^2/2\Im_0$ (keV)
^{248}Cf	ground state	0^+	0	0	7.02 ± 0.01
		2^-	593	700	5.83 ± 0.06
	$9/2^-[734]; 5/2^+[622]$	2^-	1477	1350	5.89 ± 0.23
		7^-	1577	1500	6.01 ± 0.23
	$9/2^-[734]; 1/2^+[631]$	5^-	1463		5.47 ± 0.17
		4^-	1640	1600	5.87 ± 0.17
	$9/2^-[734]; 1/2^-[501]$	5^+	1927		5.42 ± 0.30
		4^+	2072		6.12 ± 0.13
	$9/2^-[734]; 3/2^+[631]$	6^-	2184		5.61 ± 0.33
	$9/2^-[734]; 5/2^-[503]$	2^+	2281		6.21 ± 0.44
	$9/2^-[734]; 3/2^-[501]$	3^+	2512		5.63 ± 0.30
^{250}Cf	ground state	0^+	0	0	7.05 ± 0.01
		2^-	872	800	5.66 ± 0.01
	$9/2^-[734]; 1/2^+[620]$	4^-	1256	1160	5.58 ± 0.05
		5^-	1478	1500	5.73 ± 0.06
	$7/2^+[633]; 3/2^-[521]$	5^-	1396	1350	5.16 ± 0.04
	$9/2^-[734]; 3/2^+[622]$	6^-	1498	1550	5.76 ± 0.08
		3^-	1626		5.84 ± 0.01

TABLE VI. Splitting energies (in keV) in ^{248}Cf and ^{250}Cf, and their comparisons with theoretical values. Splitting energy = $\Delta E - (\hbar^2/2\Im_0 K_< - \hbar^2/2\Im_0 K_>) = \langle K_<|V_{12}|K_<\rangle - \langle K_>|V_{12}|K_>\rangle$.

Nuclide	State	Exp.	Schiffer	Rosenfeld I $r_0 = 1.2$ fm	Rosenfeld II $r_0 = 1.5$ fm
^{250}Cf	$\{9/2^-[734]; 1/2^+[620]\}_{4^-,5^-}$	−215		−78	−163
	$\{9/2^-[734]; 3/2^+[622]\}_{3^-,6^-}$	143	98	125	236
^{248}Cf	$\{9/2^-[734]; 5/2^+[622]\}_{2^-,7^-}$	−70	−134	−70	−122
	$\{9/2^-[734]; 1/2^+[631]\}_{4^-,5^-}$	181	138	109	196
	$\{9/2^-[734]; 1/2^-[501]\}_{4^+,5^+}$	148	61	63	121
	$\{9/2^-[734]; 5/2^-[503]\}_{2^+,7^+}$	71	56	86	167
	$\{9/2^-[734]; 3/2^-[501]\}_{3^+,6^+}$	−73	−77	−58	−108

C. Energy splitting between states with parallel and antiparallel coupling

When two unpaired neutrons in a deformed even-even nucleus are coupled, the projections of their spins on the nuclear symmetry axis, Ω_1 and Ω_2, can combine to produce two states with $K_> = \Omega_1 + \Omega_2$ and $K_< = |\Omega_1 - \Omega_2|$. These two states split because of the residual neutron-neutron interactions. Thus by measuring the energy difference between the states with parallel and antiparallel coupling, one can obtain information on the two-body effective interaction between two unpaired neutrons in a deformed nucleus. As discussed in Ref. [16], the energy splitting ΔE is given by the expression

$$\Delta E = E(K_<) - E(K_>) = \hbar^2/2\Im_0 K_< - \hbar^2/2\Im_0 K_> + \langle K_<|V_{12}|K_<\rangle - \langle K_>|V_{12}|K_>\rangle,$$

where $\Im_0$ is the moment of inertia and V_{12} is the the effective residual two-body interaction. Using the interaction given in Ref. [16], we calculated the energy splittings between the triplet and singlet states, and these are given in Table VI. The calculations were performed with Schiffer mixture of forces [34,35] and Rosenfeld mixture of forces [36], and both give reasonable agreement with the data.

The order of the two bands built by coupling two single-particle states are in agreement with the Gallagher-Moszkowski rule [31,32] which predicts that when two neutrons couple to form two-quasiparticle states, the state with $\Sigma = 0$ should lie below the state with $\Sigma = 1$.

V. SUMMARY

Two-quasiparticle states in ^{248}Cf and ^{250}Cf were investigated by (d,t) and (d,p) reactions on a ^{249}Cf target. Previous decay scheme studies and the signatures of cross sections measured in the present work were used to characterize the states. Several two-quasineutron states, in which one neutron occupies the $9/2^-[734]$ orbital, have been identified in these two nuclei. Energy splittings between parallel and antiparallel coupling were measured for seven pairs of states in these two nuclei. Measured differences in energy matrix elements $\langle K_<|V_{12}|K_<\rangle - \langle K_>|V_{12}|K_>\rangle$ are found to be in good agreement with theoretical values.

The energies of two-quasiparticle states and singlet-triplet splitting energies measured in ^{248}Cf and ^{250}Cf provide well-established data in the heaviest nuclei, which can be used to determine the parameters of theoretical models to calculate such energies in the heavier transfermium nuclei. Recently, two-quasiparticle $K^\pi = 8^-$ isomers were identified in ^{250}Fm [37] and ^{252}No [38,39] at 1199 and 1254 keV, respectively. These energies are in excellent agreement with the value of 1261 keV in the isotone ^{248}Cf obtained in the present work, where the (d,t) reaction cross sections establish the configuration of this state as $9/2^-[734]; 7/2^+[624]$. Two-quasiparticle states have also been observed in ^{254}No [7,40]. The splitting energies deduced in the isotone ^{250}Cf were used to calculate the energies of these two-quasiparticle states.

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