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ω meson production in pp collisions with a polarized beam

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Model independent formulas are derived for the beam analyzing power A_y and beam to meson spin transfers in $pp \to pp\omega$, taking into consideration all six threshold partial wave amplitudes $f_1, \ldots, f_6$ covering the Ss, Sp, and Ps channels. It is shown that the lowest three partial wave amplitudes f_1, f_2, f_3 can be determined empirically without any discrete ambiguities. Partial information with regard to the amplitudes f_4, f_5, f_6 covering the Ps channel may be extracted, if the measurements are carried through at the double differential level.

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Meson production in N-N collisions has excited considerable interest [1], ever since the measurements [2] in the early 1990s revealed that the total cross section for $pp \to pp\pi^0$ exceeded the then available theoretical estimates [3] by more than a factor of 5. Moreover, a large momentum transfer is involved when an additional particle is produced in the final state, which implies that the features of the N-N interaction is probed at very short distances. These have been estimated [4] to be of the order of 0.53 fm, 0.21 fm, and 0.18 fm for the production of π, ω, and φ, respectively. The experimental studies in the case of pion production have reached a high degree of sophistication [5,6], where the three-body final state is completely identified kinematically and spin observables are measured employing a polarized beam on a polarized target. The Jülich meson exchange model [7], which yielded theoretical predictions closer to data than most other models, has been more successful in the case of charged pion production [6] than with neutral pions [5]. A recent analysis [8] of $\vec{p}\vec{p} \to pp\pi^0$ measurements [5], following a model-independent irreducible tensor approach [9], showed that the Jülich model deviates from the empirically extracted estimates quite significantly for the ${}^3P_1 \to {}^3P_0 p$ and to a lesser extent for the ${}^3F_3 \to {}^3P_2 p$ transitions; this analysis has also emphasized the role of $\triangle$ in the model calculation as the calculations have also been carried out with and without taking into consideration the $\triangle$ contribution. In contrast, the production of isoscalar mesons ω and φ involves only the excited nucleon states [10]. Moreover, the Okubo-Zweig-Iizuka (OZI) rule [11] suppresses φ production relative to ω production. In view of the dramatic violation [12] of this rule observed in $\bar{p}p$ collisions, the ratio $R_{\phi/\omega}$ was measured [13] and it was found to be an order of magnitude larger, after correction for the available phase space, than the theoretical estimate $R_{OZI} = 4.2 \times 10^{-3}$ [14]. The latest experimental estimate [15] is $R_{\phi/\omega} \approx 8 \times R_{OZI}$. The total cross section for $pp \to pp\omega$ was measured [16] at five excess energies ϵ in the range 3.8 MeV to 30 MeV in c.m. The threshold energy dependence up to $\epsilon = 320$ MeV has been studied using several models [17] theoretically. A strong anisotropic angular distribution was reported [18] at $\epsilon = 173$ MeV from an experimental study at the time-of-flight spectrometer TOF of COoler SYnchrotron COSY [19] at Julich. The onset of higher partial waves was seen at a much lower energy in the more recent measurements [20] at the COSY-ANKE facility [21] and also in unpublished work [22] at two values of ϵ higher than [20]. Quark model calculations [23] have also predicted anisotropy in the angular distribution. A set of six partial wave amplitudes have been identified [24] to study the reaction at threshold and near threshold energies covering Ss, Sp, and Ps channels. Taking into consideration only the Ss and Sp amplitudes, the then existing data [13,18,22] on the differential cross section was analyzed [25], where it was also shown that the empirical estimates of the three amplitudes could be obtained from experimental measurements of the differential cross section, ω meson polarization and the analyzing power in a polarized beam and polarized target experiment, for which a proposal had already been made [26]. Very recently the beam analyzing power A_y was measured [27] for the first time. A program to measure the beam to meson spin transfer is underway [28], using the 3π decay mode of ω. In this context, it has recently been shown [29] that the 3π decay mode of ω can be utilized to determine the tensor polarization of ω, but not its vector polarization.

The purpose of this communication is to extend the model independent theoretical approach [24,25] to examine (i) the analyzing power [27] with a polarized beam and (ii) the beam to meson spin transfer [28], taking into consideration all the six Ss, Sp, Ps threshold partial wave amplitudes. We also focus attention on the empirical determination of these amplitudes from such measurements [27,28], when they are taken together with the measurements of the unpolarized differential cross section and ω polarization.

The notation $({}^{2s+1}L_j)_i \to ({}^{2s+1}L_j)_f l$ used in [5], to designate the partial wave amplitudes in the context of $pp \to pp\pi^o$, is by itself inadequate to describe completely the partial wave amplitudes for $pp \to pp\omega$, since ω has spin 1 in contrast to the spin zero of the pion. Therefore, one has to either employ the notations introduced earlier in [24] or generalize the notations used in [5] to $({}^{2s+1}L_j)_i \to ({}^{2s+1}L_j)_f {}^3 l_{j_\omega}$, together with the understanding that the vector addition of j_ω and j_f yields $j_i = j$ in order to conserve the total angular momentum j in the reaction. To facilitate comparison of the two different notations, we may now change the symbols l_i, l_f, L, S of [24] to $L_i, L_f, \mathcal{L}, \mathcal{S}$, respectively, and note that the orbital angular momenta and spins have been added in [24] in a L-S coupling scheme, in contrast to the generalization to [5] suggested above which corresponds to j-j coupling. We may, therefore, express the matrix elements $R_{\mathcal{LS}} = \langle((lL_f)\mathcal{L}(1s_f)\mathcal{S})j\|\boldsymbol{T}\|(L_i s_i)j\rangle$ of the on-energy-shell transition matrix $\boldsymbol{T}$ given by Eq. (3) of [24] in terms of $M_{j_\omega j_f} = \langle((l1)j_\omega(L_f s_f)j_f)j\|\boldsymbol{T}\|(L_i s_i)j\rangle$ through

$$R_{\mathcal{LS}} = [\mathcal{L}][\mathcal{S}]\sum_{j_\omega j_f}[j_\omega][j_f]\begin{Bmatrix} l & 1 & j_\omega \\ L_f & s_f & j_f \\ \mathcal{L} & \mathcal{S} & j \end{Bmatrix} M_{j_\omega j_f}, \qquad (1)$$

and enumerate the lowest six threshold partial wave amplitudes covering the Ss, Sp, Ps channels in the two schemes as $R_1, \ldots, R_6$ and $M_1, \ldots, M_6$ respectively. Using Eq. (1), we have

$$\begin{aligned} &R_k = M_k; \quad k = 1, 2, 3; \quad R_4 = -M_4, \\ &R_5 = \tfrac{1}{2}(M_5 + \sqrt{3}M_6); \quad R_6 = \tfrac{1}{2}(\sqrt{3}M_5 - M_6). \end{aligned} \qquad (2)$$

The R_k as well as the M_k are functions of c.m. energy E at which the reaction takes place and the invariant mass W of the two protons system in the final state. Let E_ω and $\boldsymbol{q}$ denote the energy and momentum of the ω meson in the c.m. frame, while $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ denote, respectively, the initial and final relative momenta between the two protons in their respective c.m. frames such that (q, θ, φ), $(p_i, \theta_i, \varphi_i)$, $(p_f, \theta_f, \varphi_f)$ denote the polar co-ordinates of $\boldsymbol{q}$, $\boldsymbol{p}_i$, $\boldsymbol{p}_f$, respectively. The E_ω, q, p_i, and p_f are known, if E and W are given. Introducing the factor $F = [\frac{W E_\omega (E-E_\omega) q p_f}{4(2\pi)^5 p_i}]^{1/2}$, depending purely on the kinematical variables and $g_k = (-1)^{\mathcal{L}+L_i+s_i-j}[j]^2[\mathcal{S}][s_f]^{-1}$, we may express $f_k = F\, T_k$ of [24,25], as

$$f_k = K\, g_k\, R_k; \quad k = 1, \ldots, 6, \qquad (3)$$

where $K = ((4\pi)^3/\sqrt{3})\, F$ is common for all k. They are listed in Table I for ready reference.

Formulae for all the observables are derived here in terms of the f_k. One may use Eqs. (2) to express the R_k in Eq. (3) in terms of the M_k, so that the pp system in the final state is conveniently expressed in the form used in elastic NN scattering and pion production in NN collisions. To facilitate comparison with the information already known on the NN interaction. Defining

$$f_{ij} = f_i + \frac{1}{\sqrt{10}} f_j; \quad f'_{ij} = f_i - \frac{2}{\sqrt{10}} f_j, \qquad (4)$$

TABLE I. The threshold partial wave amplitudes in terms of the reduced matrix elements of the on-energy-shell $\boldsymbol{T}$ matrix.

Initial pp state	Final $pp\omega$ state	Partial wave amplitudes
${}^3P_1$	$({}^1Ss){}^3\mathcal{S}_1$	$f_1 = -3\sqrt{3}KR_1 = -3\sqrt{3}KM_1$
${}^1S_0$	$({}^1Sp){}^3\mathcal{P}_0$	$f_2 = -\sqrt{3}KR_2 = -\sqrt{3}KM_2$
${}^1D_2$	$({}^1Sp){}^3\mathcal{P}_2$	$f_3 = -5\sqrt{3}KR_3 = -5\sqrt{3}KM_3$
${}^1S_0$	$({}^3Ps){}^3\mathcal{P}_0$	$f_4 = -KR_4 = KM_4$
${}^1D_2$	$({}^3Ps){}^3\mathcal{P}_2$	$f_5 = -5KR_5 = -\frac{5K}{2}(M_5 + \sqrt{3}M_6)$
${}^1D_2$	$({}^3Ps){}^5\mathcal{P}_2$	$f_6 = -\frac{5\sqrt{5}}{\sqrt{3}}KR_6 = -\frac{5\sqrt{5}K}{2}(\sqrt{3}M_5 - M_6)$

with $i, j = 2, 3$ or $4, 5$, the unpolarized double differential cross section for $pp \to pp\omega$ may be expressed, following [25], as

$$\frac{d^2\sigma_o}{dW d\Omega_f d\Omega} = \frac{1}{4} Tr(\mathcal{M}\mathcal{M}^\dagger) \equiv d^2\sigma_0, \qquad (5)$$

where $\mathcal{M} = F\,\boldsymbol{T}$ denotes the reaction matrix and $\mathcal{M}^\dagger$ denotes the hermitian conjugate of $\mathcal{M}$. We have

$$d^2\sigma_0 = \frac{1}{768\pi^3}[(\alpha_0 + 0.9\alpha_2\cos^2\theta) + 9(\zeta_0 + \zeta_2\cos^2\theta_f)], \qquad (6)$$

where α_0, α_2, ζ_0 and ζ_2 are given by

$$\alpha_0 = |f_1|^2 + 3|f_{23}|^2; \quad \alpha_2 = |f_3|^2 - 2\sqrt{10}\Re(f_2 f_3^*), \qquad (7)$$

$$\zeta_0 = |f_{45}|^2 + \tfrac{9}{50}|f_6|^2; \quad \zeta_2 = |f'_{45}|^2 - \zeta_0. \qquad (8)$$

The differential cross section given by Eq. (5) or (6) may be multiplied by W/E to yield $(d^2\sigma_o/dE_\omega d\Omega_f d\Omega)$. If $\boldsymbol{P}$ denotes the polarization of the proton beam the spin density matrix ρ^i characterizing the initial state may be written as

$$\rho^i = \tfrac{1}{4}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{P}), \qquad (9)$$

while the density matrix ρ^f for the final state is defined in terms of its elements

$$\rho^f_{\chi_f, \chi'_f} = \langle s_f m_f; m|\mathcal{M}\rho^i\mathcal{M}^\dagger|s'_f m'_f; m'\rangle, \qquad (10)$$

where $\chi_f \equiv (s_f, m_f, m)$. The differential cross section for $p(\vec{p}, \omega)pp$ is given by

$$d^2\sigma = Tr\rho^f = d^2\sigma_o[1 + \boldsymbol{P}\cdot\boldsymbol{A}], \qquad (11)$$

where the analyzing power

$$\boldsymbol{A} = \frac{1}{128\sqrt{6}\,\pi^3}\beta_1(\hat{\boldsymbol{q}} \times \hat{\boldsymbol{p}}_i); \quad \beta_1 = \Im(f_1^* f_{23}), \qquad (12)$$

is transverse to the reaction plane and hence has a single component $A_y = -\frac{1}{128\sqrt{6}\,\pi^2}\Im(f_1^* f_{23})\sin\theta$, with respect to a right handed frame whose z-axis is along the beam, $\boldsymbol{p}_i$ while $\boldsymbol{q}$ lies in the reaction plane, i.e., z-x plane so that the azimuthal angle φ of $\boldsymbol{q}$ is zero. Throughout this paper we use this frame, which may perhaps be referred to as the Madison Frame [30]. It is to be noted that the analyzing power is independent of (θ_f, φ_f) and as such gets multiplied by the factor 4π, if the measurements are made at the single differential level.

Moreover, and importantly, $\boldsymbol{A}$ does not involve any of the Ps partial wave amplitudes f_4, f_5, or f_6

The density matrix ρ^ω characterizing the spin state of the ω produced, when the beam is polarized, is defined in terms of its elements

$$\rho^\omega_{m,m'} = \sum_{s_f m_f} \rho^f_{s_f m_f m; s_f m_f m'} \tag{13}$$

$$= \frac{Tr\rho^\omega}{3} \sum_{k=0}^{2} (-1)^q C(1k1; m' - qm)[k] t^k_q, \tag{14}$$

where the Fano statistical tensors t^k_q with $k = 1, 2$ define, respectively, the vector and tensor polarizations of the ω meson. If the beam is unpolarized, we may set $\boldsymbol{P} = 0$ in Eq. (9) and denote the resulting Fano statistical tensors by $(t^k_q)_0$. We have $(t^1_0)_0 = 0$ and

$$C_0\,(t^1_{\pm1})_0 = \frac{9\,i\,\alpha_3\ \sin 2\theta}{2\sqrt{10}} + \frac{9\,i\,\zeta_3\ \sin 2\theta_f e^{\pm i\varphi_f}}{4}, \tag{15}$$

$$C_0\,(t^2_0)_0 = \frac{\alpha_4 - 9\alpha_5 \cos^2\theta}{\sqrt{6}} - \frac{9(\zeta_4 - \zeta_5 \cos^2\theta_f)}{\sqrt{6}}, \tag{16}$$

$$C_0\,(t^2_{\pm1})_0 = \pm\frac{3\alpha_6\ \sin 2\theta}{2} \mp \frac{9\zeta_6\ \sin 2\theta_f e^{\pm i\varphi_f}}{4}, \tag{17}$$

$$C_0\,(t^2_{\pm2})_0 = -\frac{3\alpha_7\ \sin^2\theta}{2} + \frac{9\zeta_7\ \sin^2\theta_f e^{\pm 2i\varphi_f}}{4}, \tag{18}$$

where $C_0 = 256\sqrt{3}\pi^3\, d^2\sigma_0$, and

$$\alpha_3 = \Im(f_2 f_3^*), ; \quad \alpha_4 = |f_1|^2 + 3|f_{23}|^2, \tag{19}$$

$$\alpha_5 = |f_2|^2 + \frac{3}{10}|f_3|^2 - \frac{2}{\sqrt{10}}\Re(f_2 f_3^*), \tag{20}$$

$$\alpha_6 = |f_2|^2 - \frac{1}{5}|f_3|^2 - \frac{1}{\sqrt{10}}\Re(f_2 f_3^*); \quad \alpha_7 = |f_{23}|^2, \tag{21}$$

$$\zeta_3 = \Im\left[f'_{45}\left(f_{45} + \frac{3}{5\sqrt{2}} f_6\right)^*\right], \tag{22}$$

$$\zeta_4 = \frac{1}{2}\zeta_0 + \frac{9}{5\sqrt{2}}\Re(f_{45} f_6^*), \tag{23}$$

$$\zeta_5 = |f'_{45}|^2 + \zeta_4, \tag{24}$$

$$\zeta_6 = \Re\left[f'_{45}\left(f_{45} + \frac{3}{5\sqrt{2}} f_6\right)^*\right], \tag{25}$$

$$\zeta_7 = \zeta_0 - \frac{6}{5\sqrt{2}}\Re(f_{45} f_6^*), \tag{26}$$

which are bilinears in the partial wave amplitudes $f_1, \ldots, f_6$.

When the beam is polarized, i.e., $\boldsymbol{P} \neq 0$ in Eq. (9), we may likewise derive formulas for $C\,t^k_q$, where $C = 256\sqrt{3}\pi^3\, d^2\sigma$. Correspondingly, the Cartesian components $P^\omega_i, i = x, y, z$ of the vector polarization and $P^\omega_{ij}, i, j = x, y, z$ of the tensor polarization of the ω meson with spin-1 are given, following [30], in terms of the Fano statistical tensors t^1_q and t^2_q respectively, while $(P^\omega_i)_0, (P^\omega_{ij})_0$ are given likewise by $(t^1_q)_0$ and $(t^2_q)_0$, respectively. The Cartesian components of the beam to meson spin transfers may then be defined following [31] through

$$C\,P^\omega_i = C_0 \sum_{j=x,y,z} \left((P^\omega_i)_0 + K^i_j P_j\right), \tag{27}$$

$$C\,P^\omega_{ij} = C_0 \sum_{k=x,y,z} \left((P^\omega_{ij})_0 + K^{ij}_k P_k\right). \tag{28}$$

The nonzero spin transfers K^i_j are given by

$$C_0 K^x_x = C_0\,K^y_y = -\beta_4 \cos\theta, \tag{29}$$

$$C_0 K^z_x = \sqrt{2}\,\beta_2\ \sin\theta, \tag{30}$$

$$C_0 K^z_z = \frac{1}{\sqrt{3}}\beta_3. \tag{31}$$

The nonzero spin transfers K^{ij}_k are given by

$$C_0\,K^{xx}_y = -2\sqrt{2}\,\beta_1\ \sin\theta, \tag{32}$$

$$C_0\,K^{yy}_y = \sqrt{2}\,\beta_1\ \sin\theta, \tag{33}$$

$$C_0 K^{zz}_y = \sqrt{2}\,\beta_1\ \sin\theta, \tag{34}$$

which add up to zero and

$$C_0\,K^{xz}_y = -C_0 K^{yz}_x = -\frac{3}{\sqrt{2}}\beta_5\ \cos\theta, \tag{35}$$

$$C_0\,K^{xy}_x = \frac{3}{\sqrt{2}}\beta_1\ \sin\theta. \tag{36}$$

It may be noted that the spin transfers are also independent of (θ_f, φ_f). Apart from β_1 given by Eq. (12), we have

$$\beta_2 = \Re(f_1 f_{23}^*), ; \quad \beta_3 = |f_1|^2, \tag{37}$$

$$\beta_4 = \Re(f_1 f_{23}^*), ; \quad \beta_5 = \Im(f_1 f'^*_{23}). \tag{38}$$

It is to be noted that all the β are independent of f_4, f_5, f_6. As such neither the beam analyzing power nor the spin transfers provide any information on the Ps amplitudes. Moreover, since f_4, f_5 and f_6 lead to the production of s-wave meson, their presence contributes only to the isotropic terms with respect to θ. Measurements of $d^2\sigma_0$ and $(t^k_q)_0$, using an unpolarized beam but at the double differential level yield ζ_0 and ζ_2 given by Eq. (8) and $\zeta_3, \ldots, \zeta_7$ given by Eqs. (22)–(26). It may be noted that ζ's are independent of f_1, f_2, f_3 and are bilinears involving only the Ps amplitudes f_4, f_5, f_6. It may also be noted that f_4, f_5, f_6 lead to a triplet state of the two nucleons in the final state, whereas, f_1, f_2, f_3 lead to a singlet spin state and as such the two sets do not mix, when no observations are made with regard to the spins of the two nucleons in the final state. Clearly,

$$|f'_{45}|^2 = \zeta_0 + \zeta_2 = \zeta_5 - \zeta_4, \tag{39}$$

$$|f_{45}|^2 + \frac{9}{50}|f_6|^2 = \frac{1}{4}(2\zeta_4 + 3\zeta_7) = \zeta_0, \tag{40}$$

$$\Re(f_{45} f_6^*) = \frac{5\sqrt{2}}{24}(2\zeta_4 - \zeta_7), \tag{41}$$

$$\Re\left[f'_{45}\left(f_5 + \frac{1}{\sqrt{5}} f_6\right)^*\right] = \frac{\sqrt{10}}{3}(\zeta_6 - \zeta_5 + \zeta_4), \tag{42}$$

$$\Im\left[f'_{45}\left(f_5 + \frac{1}{\sqrt{5}} f_6\right)^*\right] = \frac{\sqrt{10}}{3}\zeta_3. \tag{43}$$

Since an overall phase is arbitrary one may assume f'_{45} to be real and positive and determine $|f_5 + \frac{1}{\sqrt{5}} f_6|^2$ empirically. If $|f_{45}|^2$ or $|f_6|^2$ is known the above information is sufficient to determine f_4, f_5, f_6 empirically except for an overall phase. Note that f_6 is the only amplitude with $\mathcal{S} = 2$ and if one sets $f_6 = 0$, f_4 and f_5 can be determined empirically except for an overall phase.

In order to determine $|f_1|^2, |f_2|^2, |f_3|^2$ empirically, one need not have to carry out measurements at the double differential level. On integration with respect to $d\Omega_f$, one may drop all the terms involving ζ provided α_0 and α_4 in Eqs. (6) and (16) are replaced by $\alpha'_0 = \alpha_0 + 3(3\zeta_0 + \zeta_2)$ and $\alpha'_4 = \alpha_4 + 1.5(3\zeta_7 - \zeta_0)$, respectively, in the expressions for the differential cross section and $(t^2_0)_0$ at the single differential level and C and C_0 gets divided by 4π. The β's remain unchanged.Thus one can determine $\alpha_2, \ldots, \alpha_7$ from the measurements of $d\sigma_0$ and $(t^k_q)_0$ with respect to θ alone employing an unpolarized beam.

It is readily seen that β_1 can be determined not only from the analyzing power given by Eq. (12) but also from any of the spin transfers given by Eqs. (32), (33), (34), and (36). One can determine β_5 from Eq. (35). The empirical estimates of the bilinears $\beta_2, \beta_3, \beta_4$ are obtainable from Eqs. (30), (31), (29), respectively. One may thus readily determine empirically

$$|f_1|^2 = \beta_3, \tag{44}$$

$$|f_2|^2 = \tfrac{1}{90}[50\,\alpha_5 + 40\alpha_6 - 7\alpha_2], \tag{45}$$

$$|f_3|^2 = \tfrac{1}{9}[20\,(\alpha_5 - \alpha_6) - \alpha_2]. \tag{46}$$

Without any loss of generality one may assume f_1 to be real and express $f_2 = |f_2|\,\exp(i\varphi_2)$, $f_3 = |f_3|\,\exp(i\varphi_3)$ so that one can determine

$$\sin\varphi_2 = \frac{-2\beta_1 + \beta_5}{3|f_1||f_2|}; \quad \cos\varphi_2 = \frac{2\beta_2 + \beta_4}{3|f_1||f_2|}, \tag{47}$$

$$\sin\varphi_3 = \frac{\sqrt{10}(\beta_5 - \beta_1)}{3|f_1||f_3|}; \quad \cos\varphi_3 = \frac{\sqrt{10}(\beta_2 - \beta_4)}{3|f_1||f_3|}. \tag{48}$$

Thus it is possible to empirically determine f_1, f_2, f_3 along with their relative phases purely from measurements with respect to θ and without any discrete ambiguities. It may perhaps be pointed out that $|f_1|^2, |f_2|^2, |f_3|^2, |f_{23}|^2$, and $|f'_{23}|^2$ as well as the relative phase between f_2 and f_3 can also be determined using

$$\Re(f_2 f_3^*) = \frac{5}{9\sqrt{10}}[2\,(\alpha_5 + \alpha_6) - \alpha_2], \tag{49}$$

$$\Im(f_2 f_3^*) = \alpha_3, \tag{50}$$

from the measurements employing an unpolarized beam. However, the determination of α_3 in Eq. (50) involves a measurement of the vector polarization of ω. The measurement of the vector polarization cannot be carried out using the dominant 3π decay mode of ω [29]. It may also be pointed out that the measurement of the beam analyzing power and the tensor polarization of ω employing a polarized beam determine only $\sin\varphi_2$ and $\sin\varphi_3$, whereas, determination of $\cos\varphi_2$ and $\cos\varphi_3$ in Eqs. (47) and (48) necessarily involves measurements of the vector polarization of ω employing a polarized beam.

It was pointed out earlier [24] that the decay mode $\omega \to \pi^0\gamma$ with the branching ratio of 8.9% may be utilized to measure vector polarization of ω. It is encouraging to note that WASA [32] at COSY is expected to facilitate the experimental study of $pp \to pp\omega$ via the detection of $\omega \to \pi^0\gamma$ decay. It is to be noted, however, that the determination of the vector polarization of ω involves measuring the circular polarization asymmetry of the radiation, whereas the angular distribution of the intensity of the radiation provides information on the tensor polarization.

The determination of the relative phase of f_{23} with f_1 without any trigonometric ambiguity involves determination of β_1 and β_2, i.e., the measurement of the analyzing power given by Eq. (12) or any of the the spin transfers given by Eqs. (32), (33), (34) and the spin transfers given by Eq. (36). Likewise the determination of the relative phase between f_1 and f'_{23} involve the determination of spin transfers $K^x_x = K^y_y$ and hence β_4 in Eq. (29) and $K^{xz}_y = -K^{yz}_x$ and hence β_5 in Eq. (35). It is also interesting to note that $|f_1|^2 = \beta_3$ in Eq. (37) is directly determined by the spin transfer K^z_z given by Eq. (31). With $|f_1|^2$ thus known, $|f_{23}|^2$ and $|f'_{23}|^2$ can be determined directly from the measurements of the unpolarized differential cross section at $\theta = \pi/2$ and $\theta = 0$ or π.

Finally, we may note that the measurement [27] of A_y compatible with zero does not necessarily imply $f_2 = f_3 = 0$, but may indicate also that the relative phase between f_1 and f_{23} is zero. Since the already observed anisotropy in the angular distribution of the unpolarized differential cross section invalidates the assumption that $f_2 = f_3 = 0$, it is very likely that the measurement [27] at $\epsilon = 129$ MeV indicates only that the relative phase of f_{23} with respect to f_1 is zero at that energy.

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