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Spin dependence of the modified Kramers width of nuclear fission

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A statistical model calculation for the decay of a compound nucleus is presented where the compound nuclear spin dependence of the Kramers modified fission width is included. Specifically, the spin dependences of the frequencies of the harmonic oscillator potentials osculating the rotating liquid-drop model potential at equilibrium and saddle regions are considered. Results for the $^{16}O+^{208}Pb$ system show that the energy dependence of the dissipation strength extracted from fitting experimental data is substantially reduced when the spin dependence of the frequencies is properly taken into account.

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During the last two decades, experimental and theoretical investigations of heavy ion induced fusion-fission reactions at beam energies above Coulomb barriers have made significant contributions to the understanding of the nuclear bulk dynamics at high excitation energies. Specifically, careful analyses of experimental values of multiplicities of pre-scission light particles (mainly neutrons and γ's) [1–8], evaporation residue cross sections [9–11] and mass and kinetic energy distributions of fission fragments [1–3] have established that the fission dynamics of a hot compound nucleus is dissipative in nature. The theoretical analyses are usually performed either by employing the Langevin equation in a dynamical model of nuclear fission [12–14] or by using the statistical model where the fission width includes the effects of dissipation [15]. The later approach is used more frequently [6–9,11] since it is rather straightforward to implement it in a standard statistical model code for the decay of a compound nucleus.

Considering fission as a diffusive process of a Brownian particle across the fission barrier in a viscous medium, Kramers solved the corresponding Fokker-Planck equation with a few simplifying approximations which finally yielded the so-called Kramers modified Bohr-Wheeler expression for fission width as [15,16]

$$\Gamma_K = \frac{\hbar\omega_{\rm gs}}{T}\sqrt{\frac{m_{\rm gs}}{m_{\rm sad}}}\, f_\beta \Gamma_{\rm BW}, \tag{1}$$

where

$$f_\beta = \sqrt{1+\left(\frac{\beta}{2\omega_{\rm sad}}\right)^2} - \frac{\beta}{2\omega_{\rm sad}}.$$

In the above, $\Gamma_{\rm BW}$ is the fission width due to Bohr and Wheeler [17] and β is the strength of the reduced dissipation coefficient. $\omega_{\rm gs}$ and $\omega_{\rm sad}$ are the local frequencies of the harmonic oscillator potentials which osculate the liquid drop model nuclear potential at the ground state and the saddle configurations, respectively, while $m_{\rm gs}$ and $m_{\rm sad}$ are the corresponding inertia parameters. T is the nuclear temperature. The dimensionless quantity $\eta = \beta/2\omega_{\rm sad}$ is often used as a free parameter in order to fit experimental data.

A number of assumptions are usually made while applying Eq. (1) in statistical model calculations. A constant value for the parameter η is usually assumed for all spin values of the compound nucleus (CN). The centrifugal barrier however changes the potential profile at higher values of spin of a CN, which consequently results in a spin dependence of the frequencies $\omega_{\rm gs}$ and $\omega_{\rm sad}$ of the osculating harmonic oscillator potentials [18]. Figure 1 shows the frequencies as a function of spin for the compound nucleus ^{224}Th. The spin dependence of the parameter η is also shown in this figure. Since higher values of angular momentum states are populated at higher excitation energies of a CN formed in a heavy ion induced fusion reaction, the above observation indicates that larger values of η would be required at higher excitation energies. In fact, a strong energy dependence of η had been observed earlier [6,7,11] in a number of statistical model analyses of experimental data. This immediately suggests that the observed energy dependence of η, or at least a part of it, can be accounted for by the above spin dependence of $\omega_{\rm gs}$ and $\omega_{\rm sad}$. We would address this issue in this communication. To this end, we would perform statistical model calculations for pre-scission neutron multiplicities $n_{\rm pre}$ and evaporation residue (ER) cross sections using the fission width as given by Eq. (1) along with the spin dependent values of $\omega_{\rm gs}$ and $\omega_{\rm sad}$. The factor $\hbar\omega_{\rm gs}/T$ in Eq. (1) accounts for the collective vibration of the nucleus in the potential pocket and gives the correct Kramers limit for very small values of dissipation [19]. Though it is usually omitted by many authors we shall keep it in our calculation.

In the present statistical model calculation, we shall consider evaporation of neutrons, protons, α particles and statistical giant dipole γ rays as the decay channels of an excited CN in addition to fission. The particle and GDR γ partial decay widths are obtained from the standard Weisskopf formula as given in [13]. A time-dependent fission width will be used in order to account for the build-up time or the transient time period that elapses before the stationary value of the Kramers modified width is reached [20]. A parameterized form of the dynamical fission width is given as [21]

$$\Gamma_f(t) = \Gamma_K[1-\exp(-2.3t/\tau_f)], \tag{2}$$

where

$$\tau_f = \frac{\beta}{2\omega_{\rm gs}^2}\ln\left(\frac{10B_f}{T}\right)$$

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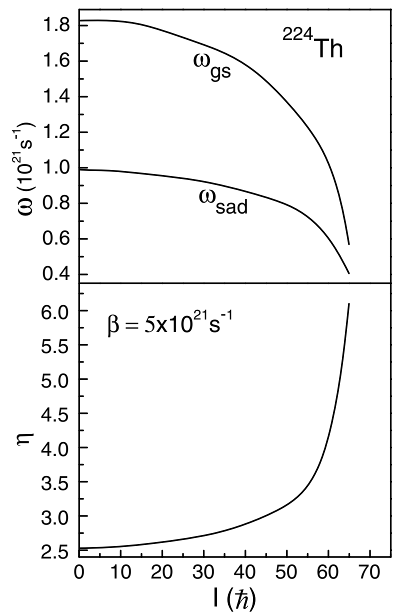

FIG. 1. Compound nuclear spin (l)-dependence of osculating frequencies at ground state ($\omega_{\rm gs}$) and saddle point ($\omega_{\rm sad}$), and dissipation parameter $\eta = \beta/2\omega_{\rm sad}$.

is the transient time period and B_f is the spin-dependent fission barrier. Though a recent work [22] provides a more accurate description of time-dependent fission widths, we have used Eq. (2) in the present work in order to compare our results with the earlier works. In the above definition of the fission width, fission is considered to have taken place when the CN crosses the saddle point deformation. During transition from saddle to scission, the CN can emit further neutrons, which would contribute to the pre-scission multiplicity. The saddle-to-scission time period is given as [23]

$$\tau_{\rm ss} = \tau_{\rm ss}^0 \left(\sqrt{1 + \left(\frac{\beta}{2\omega_{\rm sad}}\right)^2} + \frac{\beta}{2\omega_{\rm sad}} \right),$$

where $\tau_{\rm ss}^0$ is the nondissipative saddle-to-scission time interval and its value is taken from [24]. We have also calculated the multiplicity of neutrons emitted from the fission fragments ($n_{\rm post}$) assuming symmetric fission.

The Bohr-Wheeler fission width $\Gamma_{\rm BW}$ is obtained as a phase space integral over all the available states at the saddle point [25],

$$\Gamma_{\rm BW} = \frac{1}{2\pi\rho_1(E_i, J_i)} \int_0^{E_i - B_f} \rho_2(E_i - B_f - E, J_i)\,dE, \quad (3)$$

where ρ_1 is the level density at the initial state (E_i, J_i) and ρ_2 is the level density at the saddle point. The angular momentum dependent fission barrier B_f is calculated from the finite-range liquid drop model for the nuclear potential [26] and the rigid rotator values for moment of inertia. The spin distribution of the CN is assumed to follow the usual Fermi distribution, the parameters of which are obtained by fitting the experimental fusion cross sections. The level density parameter is taken from the work of Ignatyuk *et al.* [27], who proposed a form that reflects the nuclear shell structure effects at low excitation energies and is given as follows:

$$a(E_{\rm int}) = \tilde{a}\left(1 + \frac{f(E_{\rm int})}{E_{\rm int}}\delta W\right) \quad (4)$$

with

$$f(E_{\rm int}) = 1 - \exp\left(-\frac{E_{\rm int}}{E_D}\right),$$

where $E_{\rm int}$ is the thermal energy of the CN, δW is the shell correction taken as the difference between the experimental and liquid drop model masses, E_D accounts for the rate at which the shell effects melt away with increase of excitation energy, and $\tilde{a}$ is the asymptotic value to which the level density parameter approaches with increasing excitation energy of the CN. $\tilde{a}$ depends on the nuclear mass and shape in a fashion similar to that of the liquid drop model [28]. The particle and γ partial decay widths for a given spin of the CN are calculated using the value of the shape-dependent level density parameter at the corresponding ground state deformation of the compound nucleus. Similarly, the fission width for a given CN spin is calculated using the level density parameter value at the corresponding saddle point.

We have chosen the system ^{16}O+^{208}Pb for our calculation mainly because of two reasons. Firstly, experimental data on $n_{\rm pre}$ and ER cross section over a wide range of beam energy are available for this system [9,10] and, secondly, it has been theoretically investigated extensively in the past [7,29]. We first show the calculated values of neutron multiplicities along with the experimental data in Fig. 2 for different values of the reduced dissipation coefficient β. We have performed two sets of calculations. In one set, the fission widths are

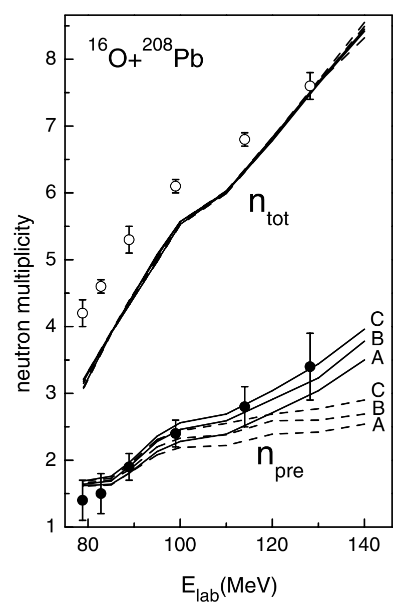

FIG. 2. Pre-scission ($n_{\rm pre}$) and total ($n_{\rm tot}$) neutron multiplicities. The experimental values (filled circles) are from [4]. The solid and dashed lines are statistical model calculations with and without spin dependence of frequencies, respectively. A, B, and C denote results with $\beta = 4, 5,$ and 6 (in 10^{21} s^{-1}), respectively. The $n_{\rm tot}$ values obtained with different β's from the two sets of calculations are almost indistinguishable.

calculated using the spin-dependent frequency values while they are obtained with spin-independent frequency values (set equal to the $l = 0$ values) in the other set of calculation. It is observed that n_{pre} calculated with spin-dependent frequencies for a given β has larger dependence on the initial beam energy than those calculated with constant ωs. In fact, a reasonable agreement with the experimental values can be obtained with $\beta = 6$ (in 10^{21} s^{-1}) in the former calculation. In what follows, we shall study the dependence of β on the initial excitation energy of the CN and not on its instantaneous values which decreases with time due to successive particle and γ emissions. Though the later would have been more desirable, the former can still provide us the gross features of energy dependence which would be adequate for our present purpose. We have subsequently extracted the β values by fitting the experimental multiplicity separately at each value of incident energy in order to compare the initial excitation energy dependence of β from the two sets of calculations. Figure 3 shows the results. The initial excitation energy dependence of β obtained with spin-dependent frequencies is much weaker compared to that obtained with constant values of the frequencies. The total neutron multiplicity ($n_{\text{tot}} = n_{\text{pre}} + n_{\text{post}}$) is also plotted in Fig. 2. Since the initial excitation energy of the nuclear system (CN plus fission fragments) is essentially carried away by the pre-scission and fission-fragment neutrons, n_{tot} values are not sensitive to β as can be seen from this figure.

We shall next show in Fig. 4 the ER excitation functions calculated with different values of β along with the experimental cross sections. In addition to the total ER cross sections, the ER cross sections with ($\sigma(\alpha, xn, yp)$) and without ($\sigma(xn, yp)$) α emission are also plotted in this figure. It is observed that unlike the results for n_{pre}, the difference between the ER cross sections from the two sets of calculations, with and without the spin dependence of the frequencies, is small. This can be explained as follows. Since evaporation residues are

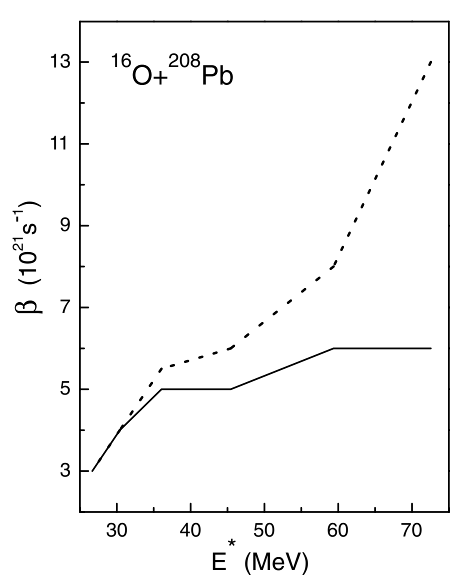

FIG. 3. Initial excitation energy (E^*) dependence of β. The solid and dashed lines correspond to fitted values obtained with and without spin dependence of frequencies, respectively.

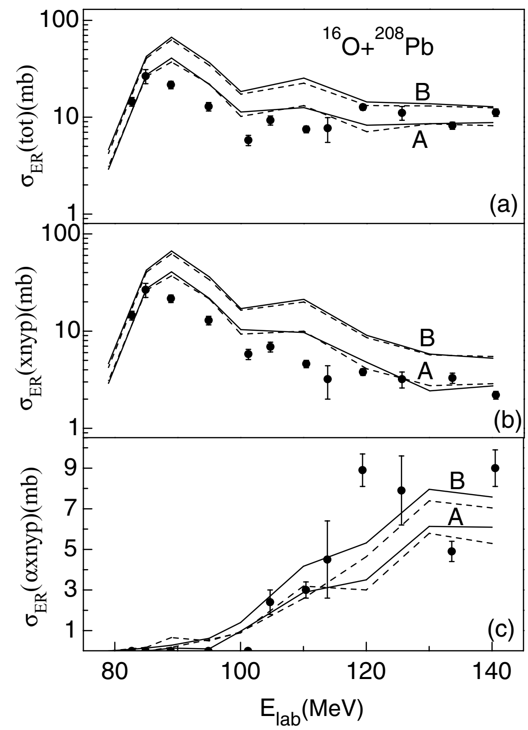

FIG. 4. Evaporation residue cross sections. The total ER cross sections are plotted in the top (a) panel. The middle (b) and the lower (c) panels show the cross sections of evaporation residues formed in ($xnyp$) and ($\alpha xnyp$) channels, respectively. The experimental values (filled circles) are from [9]. The solid and dashed lines are statistical model calculations with and without spin dependence of frequencies, respectively. A and B denote results with $\beta = 1$ and 2 (both in 10^{21} s^{-1}), respectively.

preferably formed from compound nuclei with lower spin values while a CN with a higher spin is more likely to undergo fission, the spin dependence of frequencies (see Fig. 1) will affect the fission probability more strongly than the ER cross section. In particular, this feature is expected to be more prominent for highly fissile systems like ^{224}Th where residues are mostly formed from CN with very small values of angular momentum, which results in a marginal spin dependence of residue formation as shown in Fig. 4. On the other hand, fission probabilities and particularly those at higher excitation energies where high spin states are populated are expected to be more sensitive to the spin dependence of frequencies as we find in the calculated values of pre-scission neutrons in Fig. 3 in the above.

We have further calculated the average number of α particles emitted by the evaporation residues. The experimental and the statistical model predictions are given in Fig. 5. From Figs. 4 and 5, we find that a value of 1 (in 10^{21} s^{-1}) for β can account for all the ER related processes in a satisfactory manner.

We thus arrive at two values for β, both energy independent, in order to separately fit the neutron multiplicities and ER cross sections. Similar observations have been made earlier in both dynamical [30] and statistical [7] model calculations. In order to reproduce both n_{pre} multiplicities and ER cross sections,

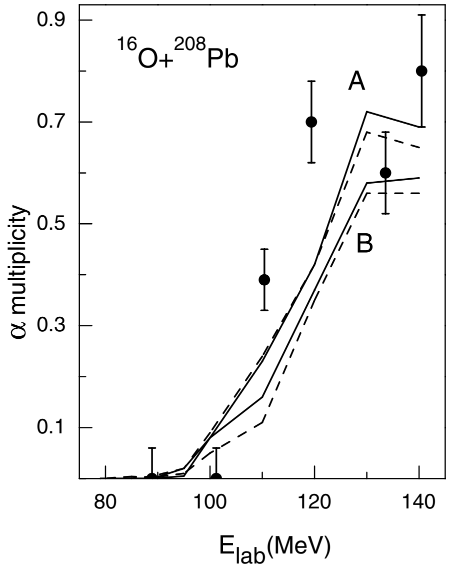

FIG. 5. α multiplicities from evaporation residues. Experimental values are from [9] and the solid and dashed lines are statistical model calculations with and without spin dependence of frequencies, respectively. A and B denote results with $\beta = 1$ and 2 (both in 10^{21} s^{-1}), respectively.

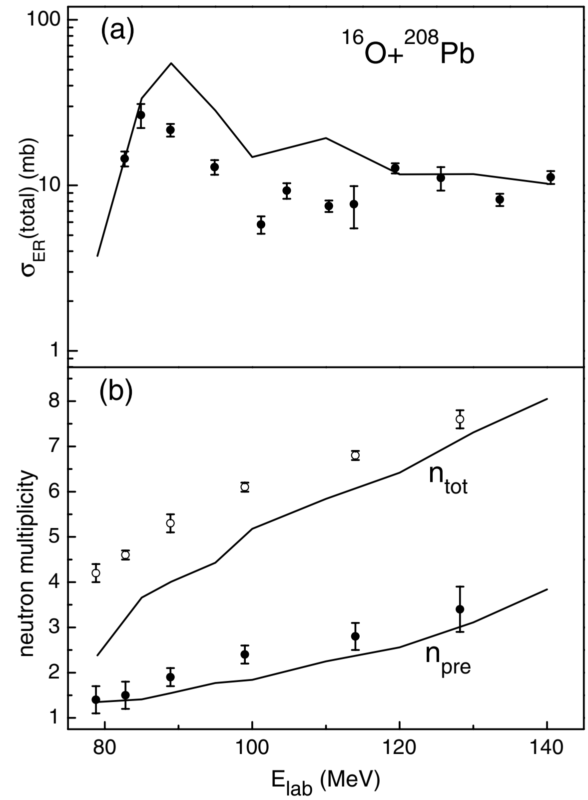

FIG. 6. Simultaneous fit to experimental pre-scission neutron multiplicities and ER cross sections by statistical model calculation using $\beta_{in} = 1.5$ and $\beta_{out} = 15$ (both in 10^{21} s^{-1}) using spin-dependent frequencies.

phenomenological form factors for the dissipation strength have been suggested where dissipation is weak at small deformations of the CN and becomes many times larger at large deformations [30]. Such choices are motivated by the facts that the ER cross sections essentially portray the pre-saddle fission dynamics whereas additional neutrons can be emitted during transition of the CN from the saddle configuration to the scission. It is however also possible that one of the reasons for a strong dissipation at large deformations is to account for the enhanced neutron emission from the neutron-rich neck region. This aspect however requires further investigations [31]. A shape-dependent dissipation has also been obtained in a microscopic derivation of one-body dissipation where the chaotic nature of the single particle motion was considered [32] giving rise to a suppression of dissipation strength for small CN deformations. In the present work, we shall consider the following shape-dependent β for our calculation [7]. A small value for $\beta(\beta_{in})$ will be used within the saddle point in order to calculate the fission width whereas a larger value (β_{out}) will be used beyond the saddle point to give the saddle-to-scission transition time. Figure 6 shows a simultaneous fit to both n_{pre} and ER cross sections with $\beta_{in} = 1.5$ and $\beta_{out} = 15$ (both in 10^{21} s^{-1}). It should be noted here that similar conclusions regarding the dissipation strengths were made in [7] where spin-independent values of the frequencies were used and consequently, an additional temperature dependence was found necessary in order to reproduce experimental data.

In summary, we have investigated a specific aspect of the fission width due to Kramers, namely its spin dependence arising out of the change in the shape of the liquid drop model potential with angular momentum. The present work shows that the energy dependence of the dissipation strength extracted from fitting experimental data is substantially reduced when the change in shape of the fission barrier with increasing spin of a compound nucleus is properly taken into account. We thus conclude that this spin-dependent effect should be included in a statistical model analysis employing Kramers modified fission width in order to deduce the correct strength and energy dependence of the phenomenological dissipation.

[1] H. Rossner, D. Hilscher, D. J. Hinde, B. Gebauer, M. Lehmann, M. Wilpert, and E. Mordhorst, Phys. Rev. C **40**, 2629 (1989).

[2] D. J. Hinde, D. Hilscher, H. Rossner, B. Gebauer, M. Lehmann, and M. Wilpert, Phys. Rev. C **45**, 1229 (1992).

[3] D. J. Hinde, H. Ogata, M. Tanaka, T. Shimoda, N. Takahashi, A. Shinohara, S. Wakamatsu, K. Katori, and H. Okamura, Phys. Rev. C **39**, 2268 (1989).

[4] H. Rossner, D. J. Hinde, J. R. Leigh, J. P. Lestone, J. O. Newton, J. X. Wei, and S. Elfström, Phys. Rev. C **45**, 719 (1992).

[5] R. Butsch, M. Thoennessen, D. R. Chakrabarty, M. G. Herman, and P. Paul, Phys. Rev. C **41**, 1530 (1990).

[6] D. J. Hofman, B. B. Back, and P. Paul, Phys. Rev. C **51**, 2597 (1995).

[7] I. Dioszegi, N. P. Shaw, I. Mazumdar, A. Hatzikoutelis, and P. Paul, Phys. Rev. C **61**, 024613 (2000).

[8] I. Dioszegi, N. P. Shaw, A. Bracco, F. Camera, S. Tettoni, M. Mattiuzzi, and P. Paul, Phys. Rev. C **63**, 014611 (2000).
[9] K.-T. Brinkmann, A. L. Caraley, B. J. Fineman, N. Gan, J. Velkovska, and R. L. McGrath, Phys. Rev. C **50**, 309 (1994).
[10] C. R. Morton, D. J. Hinde, J. R. Leigh, J. P. Lestone, M. Dasgupta, J. C. Mein, J. O. Newton, and H. Timmers, Phys. Rev. C **52**, 243 (1995).
[11] B. B. Back, D. J. Blumenthal, C. N. Davids, D. J. Henderson, R. Hermann, D. J. Hofman, C. L. Jiang, H. T. Penttilä, and A. H. Wuosmaa, Phys. Rev. C **60**, 044602 (1999).
[12] Y. Abe, S. Ayik, P.-G. Reinhard, and E. Suraud, Phys. Rep. **275**, 49 (1996).
[13] P. Fröbrich and I. I. Gontchar, Phys. Rep. **292**, 131 (1998).
[14] A. V. Karpov, P. N. Nadtochy, D. V. Vanin, and G. D. Adeev, Phys. Rev. C **63**, 054610 (2001).
[15] H. A. Kramers, Physica (Amsterdam) **4**, 284 (1940).
[16] Pierre Grange, Nucl. Phys. **A428**, 37c (1984).
[17] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[18] J. P. Lestone, Phys. Rev. C **51**, 580 (1995).
[19] V. M. Strutinsky, Phys. Lett. **B47**, 121 (1973)
[20] P. Grange, Li Jun-Qing, and H. A. Weidenmüller, Phys. Rev. C **27**, 2063 (1983).
[21] K. H. Bhatt, P. Grange, and B. Hiller, Phys. Rev. C **33**, 954 (1986).
[22] B. Jurado, C. Schmitt, K.-H. Schmidt, J. Benlliure, and A. R. Junghans, Nucl. Phys. **A747**, 14 (2005).
[23] H. Hofmann and J. R. Nix, Phys. Lett. **B122**, 117 (1983).
[24] P. Grange, S. Hassani, H. A. Weidenmüller, A. Gavron, J. R. Nix, and A. J. Sierk, Phys. Rev. C **34**, 209 (1986).
[25] I. I. Gontchar and P. Fröbrich, Nucl. Phys. **A551**, 495 (1993).
[26] A. J. Sierk, Phys. Rev. C **33**, 2039 (1986).
[27] A. V. Ignatyuk, M. G. Itkis, V. N. Okolovich, G. N. Smirenkin, and A. Tishin, Sov. J. Nucl. Phys. **21**, 612 (1975).
[28] W. Reisdorf, Z. Phys. A **300**, 227 (1981).
[29] G. Chaudhuri and S. Pal, Eur. Phys. J. A **18**, 9 (2003).
[30] P. Fröbrich, I. I. Gontchar, and N. D. Mavlitov, Nucl. Phys. **A556**, 281 (1993).
[31] N. Carjan, P. Talou, and O. Serot, Nucl. Phys. **A792**, 102 (2007).
[32] S. Pal and T. Mukhopadhyay, Phys. Rev. C **54**, 1333 (1996).