Neutrino emission due to Cooper-pair recombination in neutron stars reexamined

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Neutrino emission in processes of breaking and formation of neutron and proton Cooper pairs is calculated within the Larkin-Migdal-Leggett approach for a superfluid Fermi liquid. We demonstrate explicitly that the Fermi-liquid renormalization respects the Ward identity and ensures the weak vector-current conservation. The systematic expansion of the emissivities for small temperatures and nucleon Fermi velocity $v_{\mathrm{F},i}$, $i = n, p$, is performed. Both neutron and proton processes are mainly controlled by the axial-vector current contributions, which are not strongly changed in the superfluid matter. Thus, compared to earlier calculations, the total emissivity of processes on neutrons paired in the $1S_0$ state is suppressed by a factor of $\simeq (0.9\text{–}1.2)v_{\mathrm{F},n}^2$. A similar suppression factor ($\sim v_{\mathrm{F},p}^2$) arises for processes on protons.

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I. INTRODUCTION

In minutes or hours after its birth, a neutron star cools down to a temperature $T \sim$ MeV via neutrino transport to the surface and then becomes transparent for neutrinos. Thereafter, for $\sim 10^5$ yr, the cooling is determined by the emissivity of neutrinos produced in direct reactions [1–6]. For such temperatures, neutrons and protons in the neutron star interior are highly degenerate. Therefore, the rate of neutrino production is suppressed by the reaction phase space; the greater the number of nucleons involved, the smaller the phase space.

The most efficient are one-nucleon processes, e.g., $n \to pe\bar{\nu}$, called direct URCA (DU) reactions. Their emissivity is $\varepsilon^{\mathrm{DU}} \sim 10^{27} \times T_9^6\,(n/n_0)^{2/3}\theta(n - n_c^{\mathrm{DU}})\frac{\mathrm{erg}}{\mathrm{cm^3 s}}$ (see Ref. [7]), where $T_9 = T/(10^9\mathrm{K})$, and n is the nucleon density measured in units of the nuclear matter saturation density n_0. The DU processes are operative only when the proton fraction exceeds a critical value of 11–14%. Equations of state constructed from realistic nucleon-nucleon interactions, like Urbana-Argonne one [8], show that this condition is fulfilled only at very high densities. This implies that the DU processes may occur only in the most heavy neutron stars, e.g., with masses $\sim 2\,M_\odot$ for the equation of state [8], where $M_\odot = 2 \times 10^{33}$ g is the solar mass. At $n \sim n_0$ the proton fraction is typically about 3–5%, cf. Fig. 2 in Ref. [9].

In the absence of DU processes, the most efficient ones become the two-nucleon reactions, e.g., $nn \to npe\bar{\nu}$, called modified URCA (MU) processes with the emissivity $\varepsilon^{\mathrm{MU}} \sim 10^{21} \times T_9^8\,(n/n_0)^{2/3}\frac{\mathrm{erg}}{\mathrm{cm^3 s}}$, cf. Ref. [10]. Note the smaller numerical prefactor and the higher power of the temperature for the emissivity of the two-nucleon processes compared with the DU emissivity. In-medium change of the nucleon-nucleon interaction in the spin-isospin particle-hole channel due to pion softening may strongly increase the two-nucleon reaction rates, which, nevertheless, in all relevant cases, remain significantly smaller than that for the DU processes [2,11]. The nucleon bremsstrahlung reactions, such as $nn \to nn\nu\bar{\nu}$ (nB), $np \to np\nu\bar{\nu}$ (npB) and $pp \to pp\nu\bar{\nu}$ (pB), have an order of magnitude smaller emissivity than MU processes.

At low temperatures, the nucleon matter is expected to undergo a phase transition into a state with paired nucleons [12]. The neutron superfluidity and/or proton superconductivity take place below some critical temperatures $T_{\mathrm{c},n}$ and $T_{\mathrm{c},p}$, respectively, which depend on the density. At densities $n < (1\text{–}2)\,n_0$ neutrons are paired in the $1S_0$ state and in the $3P_2$ state at higher densities. Protons are paired in the $1S_0$ state for densities $n \lesssim (2\text{–}4)n_0$. Pairing gaps, Δ_i, are typically $\sim$0.1–1 MeV and depend crucially on details of the interaction in the particle-particle channel, see Fig. 5 in Ref. [13].

The gap in the energy spectrum significantly reduces the phase space of the nucleon processes roughly by the factor $\exp(-\Delta/T)$ for the one-nucleon DU process and $\exp(-2\Delta/T)$ for two-nucleon processes. However, even with inclusion of the nucleon pairing effects, the DU rate is large enough that the occurrence of these processes would lead to an unacceptably fast cooling of a neutron star in disagreement with modern observational soft x-ray data [9,13,14]. This statement has been tested with gaps varying in a broad band allowed by different microscopic calculations. Certainly, DU processes could be less efficient if one kept gaps finite also at high densities. Microscopic calculations do not support this possibility. Thus, according to recent analysis, the DU processes most probably will not occur in typical neutron stars with masses in the range of 1.0–1.5$M_\odot$, based on the cooling and population syntheses scenarios [13–15].

Superfluidity allows for a new mechanism of neutrino production associated with Cooper pair breaking or formation (PBF), e.g., the reaction $n \to n\nu\bar{\nu}$ and $p \to p\nu\,\bar{\nu}$, where one of the nucleons is paired. For $1S_0$ neutron pairing, the PBF emissivity was evaluated first in Ref. [16] in the Bogoliubov ψ-operator technique and then in Refs. [17,18] within the Fermi-liquid approach.

The proton PBF emissivity was estimated in Ref. [17] which accounted for in-medium renormalization of the nucleon weak-interaction vertex due to strong interactions. Mixing of electromagnetic and weak interactions through the electron–electron-hole loop can additionally change the proton vertex [19,20]. There are also relativistic corrections to the axial-vector coupling vertices of the order $v_{\mathrm{F},i}^2$; $v_{\mathrm{F},i}$ are neutron and proton Fermi velocities, with $i = n, p$, [21]. These three effects together resulted in a one or two orders of magnitude enhancement of the proton PBF emissivity over that evaluated with the free vector-current vertex. Thus, one concludes that neutron PBF and proton PBF emissivities can be equally important for neutron star cooling, depending on the parameter choice and, especially, on the relation between gaps Δ_p and Δ_n. The PBF emissivity for the $3P_2$ neutron pairing has been analyzed in Ref. [22].

Following Refs. [2,17,18], the emissivities of the neutron and proton PBF processes were estimated as $\varepsilon^{i\mathrm{PFB}} \sim 10^{28} \times (\Delta_i/\mathrm{MeV})^7 (T/\Delta_i)^{1/2} (n_i/n_0)^{1/3} e^{-2\Delta_i/T} \frac{\mathrm{erg}}{\mathrm{cm}^3\mathrm{s}}$ for $T \ll \Delta_i$ and $i = n, p$. Having a large numerical prefactor and very moderate temperature dependence of the preexponent, these reactions significantly contribute to the neutron star cooling provided gaps are not too small. These processes have been included in the cooling code rather recently [23]. Since then, the PBF reactions have been the main part of any cooling scenario together with the MU processes [13,14,24,25]. Uncertainties in the pairing gaps are large. Therefore, surface temperatures of neutron stars computed in different approaches vary significantly.

Kundu and Reddy [26] and Leinson and Perez [27] made an important observation: all previous calculations of the neutrino reactions in superfluid matter disrespect the Ward identity and, as a consequence, the conservation of the electroweak vector current.

The Ward or in the general case Ward-Takahashi identities impose nontrivial relations between vertex functions and Green's functions, which synchronize any modification of Green's function with a corresponding change in the vertex function. Satisfying these relations ensures that the symmetry properties of the initial theory are preserved in actual calculations. For instance, we start with the theory of weak interactions with a conserved vector current. The current would remain trivially conserved in calculations with only bare vertices and bare Green's functions. In strongly interacting systems, Green's functions change necessarily; but for quasiparticle Green's functions, the current conservation is easily restored by a proper inclusion of short-range correlations in the vertices, cf. development of the Fermi-liquid theory by Migdal [15,28,29]. Following these two simplest cases, Refs. [16,21,22] did not incorporate any medium effects, whereas Ref. [17] used dressed quasiparticle Green's functions together with dressed normal vertices. For the superfluid system, the situation is more peculiar. Since in the superfluid system the nucleon Green's function notoriously differs from the free one, the vector-current vertex must get corrections even if no other interaction between quasiparticles is included. Additional anomalous vector-current vertices disregarded in previous calculations must be properly accounted for. These corrections *cancel exactly the vector-current contributions* to the neutrino emissivity for zero neutrino momenta, cf. Ref. [27].

Assuming that the axial-vector current contributes only little to the PBF emissivity, Ref. [27] claimed that the PBF emissivity calculated in Refs. [16,17,22] is to be suppressed by a factor of $\sim v_{\mathrm{F},n}^4/20 \sim 10^{-3}$ for $n \sim n_0$ for neutrons and by $\sim 10^{-7}$ for protons in the case of $1S_0$ pairing. Such a severe reduction of the neutron and proton PBF emissivities could significantly affect previous results on the neutron star cooling dynamics. Reference [30] revises the results of Ref. [27] by applying expansion in the $\vec{q}^{\,2}$ parameter and putting $v_{\mathrm{F},n} = 0$. Ref. [30] claims that the suppression factor for the neutron PBF emissivity is $\sim T/m^*$, where m^* is the nucleon effective mass. This would reduce the neutron PBF emissivity by a factor of $\sim 5 \times 10^{-3}$ for temperatures $T \sim 0.5 T_{c,n}$, cf. Fig. 5 in Ref. [30].

References [27,30] used the convenient Nambu-Gorkov matrix formalism developed to describe metallic superconductors [31,32]. The price paid for that convenience is that the formalism does not distinguish interactions in the particle-particle and particle-hole channels. Such an approach is, generally speaking, not applicable to the strongly interacting matter present in neutron stars. In nucleon matter at low temperatures, the nn and pp nucleon-nucleon interactions in the particle-particle channel are attractive, whereas in relevant particle-hole channels they are repulsive [17,29,33]. The adequate formalism was developed by Larkin and Migdal for Fermi liquids with pairing at $T = 0$ in Ref. [34] and generalized then by Leggett for $T \neq 0$ in Ref. [35].

In the present paper, using the Larkin-Migdal-Leggett formalism, we analytically calculate neutrino emissivity from the superfluid neutron star matter with the $1S_0$ neutron-neutron and proton-proton pairing. Both normal and anomalous vertex corrections are included. We explicitly demonstrate that the Fermi-liquid renormalization [34] respects the Ward identity and vector-current conservation. Our final estimations of neutron and proton PBF emissivities differ from those in Refs. [27,30]. We find that the main term in the emissivity $\sim v_{\mathrm{F},i}^2$ follows from the axial-vector current, whereas the leading term in the emissivity from the vector current appears only at the $v_{\mathrm{F},i}^4$ order, as in Ref. [27].

In the next section, we present the general expression for the emissivity of the neutron PBF processes formulated in terms of the imaginary part of the current-current correlator for weak processes on neutral currents. Then within the Fermi-liquid approach to a superfluid, we introduce Green's functions and the gap equation. In Sec. III we formulate and solve Larkin-Migdal equations for vertices and apply them to calculating the imaginary part of the current-current correlator. The neutrino emissivities of the neutron and proton PBF processes are calculated in Secs. IV and V, respectively. Conclusions are formulated in Sec. VI. The values of Landau-Migdal parameters are shortly reviewed in Appendix A. In Appendix B, we compare our results with those of previous works.

II. GENERAL EXPRESSIONS: EMISSIVITY, GREEN'S FUNCTIONS, AND PAIRING GAPS

A. Neutrino emissivity

The weak neutrino-neutron and neutrino-proton interactions on neutral currents are described by the effective low-energy Lagrangian

$$\mathcal{L} = \frac{G}{2\sqrt{2}} \sum_{i=n,p} \left(V_i^\mu - A_i^\mu\right) l_\mu, \tag{1}$$

where $l_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$ is the lepton current, and $V_i^\mu = g_V^{(i)}\bar{\Psi}_i\,\gamma^\mu\Psi_i$ and $A_i^\mu = g_A^{(i)}\bar{\Psi}_i\,\gamma^\mu\gamma_5\Psi_i$ stand for nucleon (neutron or proton) vector and axial-vector currents with nucleon bispinors Ψ_i. The coupling constants are $g_V^{(n)} = g_V = -1$, $g_V^{(p)} = c_V = 1 - 4\sin^2\theta_{\rm W} \simeq 0.04$ and $g_A^{(p)} = -g_A^{(n)} = g_A = 1.26$. The Fermi constant is $G \approx 1.2\times 10^{-5}$ GeV^{-2}. For the nonrelativistic nucleons, $V_i^\mu \approx \psi_i^\dagger(p')(1, (\vec{p}\,' + \vec{p}\,)/2m)\psi_i(p)$ and $A^\mu \approx \psi_i^\dagger(p')(\vec{\sigma}(\vec{p}\,'+\vec{p}\,)/2m, \vec{\sigma})\psi_i(p)$, where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, are the Pauli matrices acting on nucleon spinors ψ_i, and $\vec{p}\,'$ and $\vec{p}$ are outgoing and incoming momenta, and m is the mass of the free nucleon, cf. Ref. [36].

Neutrino emissivity for one neutrino species can be calculated as

$$\varepsilon_{\nu\bar{\nu}} = \frac{G^2}{8}\int \frac{d^3q_1}{(2\pi)^3\,2\omega_1}\frac{d^3q_2}{(2\pi)^3\,2\omega_2}\,\omega\, f_{\rm B}(\omega)\,2\Im\sum\chi(q), \tag{2}$$

where $q = (\omega, \vec{q}\,) = q_1 + q_2$, $q_{1,2} = (\omega_{1,2}, \vec{q}_{1,2})$ are four-momenta of the outgoing neutrino and antineutrino, $f_{\rm B}(\omega) = 1/(\exp(\omega/T) - 1)$ are Bose occupations, and $\Im\chi$ is the imaginary part of the susceptibility of the nucleon matter to weak interactions, i.e., the Fourier transform of the current-current correlator $\langle(V_\mu(x)l^\mu(x) - A_\mu(x)l^\mu(x))(V^\nu(y)l_\nu^\dagger(y) - A^\nu(y)l_\nu^\dagger(y))\rangle$, for weak processes. The sum in Eq. (2) is taken over the lepton spins.

According to the optical theorem, $\Im\chi$ can be expressed as a sum of squared matrix elements of all available reactions with all possible intermediate states, $\sum|M|^2$. A particular contribution to $\sum|M|^2$ can be also calculated within the Bogoliubov ψ-operator approach for a given form of the nucleon-nucleon interaction, as it has been done in Refs. [16,22]. In this approach, however, an account of further in-medium modifications of nucleon propagators and interaction vertices is obscured by the danger of double counting. The Green's function technique for Fermi liquid [29,33] is more suitable for such extensions, as demonstrated in Refs. [11,17]. We will follow the Green's function approach for superfluid Fermi liquids [29,34]. As a simplification, we will focus on the low-temperature limit $T \ll \Delta$. The temperature dependence enters through nucleon occupation factors $\propto e^{-\Delta/T}(1 + O(T^2/\Delta^2))$ and also as $1 + O(T^2/\epsilon_{\rm F}^2)$ corrections in the low-temperature expansion of standard Fermi integrals, when the high-energy region $\epsilon \gg \Delta$ is dominating in the integrals. Since the boson occupation factor f_B in Eq. (2) generates already the leading exponent $e^{-2\Delta/T}$, we can evaluate $\Im\chi$ for $T = 0$; see also discussion below.

B. Nucleon Green's functions and pairing gaps

The nucleon Green's function for the interacting system in a normal state (n.s.), i.e. without pairing, is given by the Schwinger-Dyson equation, which in the momentum representation reads

$$\widehat{G}_{\rm n.s.}(p) = \widehat{G}_0(p) + \widehat{G}_0(p)\widehat{\Sigma}_{\rm n.s.}(p)\,\widehat{G}_{\rm n.s.}(p),$$

with $\widehat{G}_0(p) = G_0(\epsilon, \vec{p})\hat{\mathbf{1}} = \hat{\mathbf{1}}/(\epsilon - \epsilon_p + i0\,{\rm sgn}\epsilon)$, where $\hat{\mathbf{1}}$ is the unity matrix in the spin space. All information on the interaction is incorporated in the nucleon self-energy $\widehat{\Sigma}_{\rm n.s.}$, being a functional of the Green's function $\widehat{G}_{\rm n.s.}$. In absence of the spin-orbit interaction, the full Green's function is also diagonal in the spin space, i.e., $\widehat{G}_{\rm n.s.} = G_{\rm n.s.}\hat{\mathbf{1}}$. For strongly interacting systems, such as dense nucleon matter, the exact calculation of $G_{\rm n.s.}$ is an extremely difficult task. However, for strongly degenerate nucleon systems at temperatures T much less than the neutron and proton Fermi energies $\epsilon_{{\rm F},i}$, $i = n, p$, fermions are only slightly excited above the Fermi sea. So, the full Green's function of the normal state is given by the sum of the pole term and a regular part:

$$G_{\rm n.s.}(p) = \frac{a}{\epsilon - \epsilon_p + i0\,{\rm sgn}\epsilon} + G_{\rm reg}(p), \tag{3}$$

where the excitation energy is counted from the nucleon chemical potential μ, $\epsilon_p = p^2/(2m^*) - \mu$, $\mu \simeq \epsilon_{\rm F} = p_{\rm F}^2/(2m^*)$ for the low temperatures under consideration, and $p_{\rm F}$ is the Fermi momentum. The effective mass and the nontrivial pole residue are determined by the real part of the self-energy, as $a^{-1} = 1 - (\partial\Re\Sigma_{\rm n.s.}/\partial\epsilon)_{\rm F}$ and $1/m^* = a(1/m + 2\partial\Re\Sigma_{\rm n.s.}/\partial p^2)_{\rm F}$. The subscript F indicates that the corresponding quantities are evaluated at the Fermi surface ($\epsilon, \epsilon_p \to 0$). According to Ref. [29], only the pole part of $G_{\rm n.s.}$ is relevant to the description of processes happening in a weakly excited Fermi system. The regular part can be absorbed by the renormalization of the particle-particle and particle-hole interactions at the Fermi surface. The quantities m^* and a can be expressed through the Landau-Migdal parameters characterizing the fermion interaction at the Fermi surface at zero energy-momentum transfer. The imaginary part of the self-energy, $\Im\Sigma_{\rm n.s.}$, can be omitted in the pole term of Green's function (3) in the low-temperature limit (quasiparticle approximation).

In a system with pairing, a new kind of process such as transition of a particle into a hole and a condensate pair and vice versa becomes possible. The one-particle–one-hole irreducible amplitudes of such processes can be depicted [34] as in Fig. 1. Besides the normal Green's functions for quasi-particles $i\hat{G}$

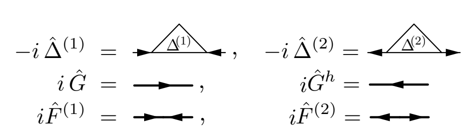

FIG. 1. Diagrams depicting the amplitudes of transition of a particle into a hole and a condensate pair and vice versa and normal and anomalous Green's functions.

and holes $i\hat{G}^h$, one introduces anomalous Green's functions $i\hat{F}^{(1)}$ and $i\hat{F}^{(2)}$; their diagrammatic notations are given in Fig. 1. The full normal and anomalous Green's functions are related by the Gor'kov equations

$$\begin{aligned}\hat{G}(p) &= \hat{G}_{\rm n.s.}(p) + \hat{G}_{\rm n.s.}(p)\hat{\Delta}^{(1)}(p)\hat{F}^{(2)}(p),\\ \hat{F}^{(2)}(p) &= \hat{G}^h_{\rm n.s.}(p)\hat{\Delta}^{(2)}(p)\hat{G}(p).\end{aligned} \quad (4)$$

The second equation involves the normal-state Green's function of the hole (superscript h), which in the absence of a spin-orbit interaction is simply $i\hat{G}^h_{\rm n.s.}(p) = i\hat{G}_{\rm n.s.}(-p)$. In the case of the $1S_0$ pairing, the spin structures of the anomalous Green's functions and the transition amplitudes are simple: $\hat{\Delta}^{(1)} = \hat{\Delta}^{(2)} = \Delta i\,\sigma_2$ and $\hat{F}^{(1)} = \hat{F}^{(2)} = Fi\sigma_2$. Equations (4) are to be completed by the equation for the amplitude $\hat{\Delta}^{(1)}(p)$,

$$[\hat{\Delta}^{(1)}]^a_b = \int \frac{d^4p'}{(2\pi)^4 i}[\widehat{V}(p,p')]^{ac}_{bd} \times \left[\hat{G}(p')\hat{\Delta}^{(1)}(p')\hat{G}^h_{\rm n.s.}(p')\right]^d_c, \quad (5)$$

where $\widehat{V}$ stands for a two-particle irreducible potential, which determines the full in-medium particle-particle scattering amplitude. The potential $\widehat{V}$ can be separated in the scalar and spin-spin interactions defined as

$$[\widehat{V}]^{ac}_{bd} = V_0\,(i\sigma_2)^a_b(i\sigma_2)^c_d + V_1(i\sigma_2\vec{\sigma})^a_b(\vec{\sigma}i\sigma_2)^c_d.$$

Solution of the Gor'kov equations (4) is straightforward. The relevant pole parts of Green's functions are

$$G(p) = \frac{a\,(\epsilon+\epsilon_p)}{\epsilon^2 - E_p^2 + i0\,{\rm sgn}\epsilon},\quad F(p) = \frac{-a\Delta}{\epsilon^2 - E_p^2 + i0\,{\rm sgn}\epsilon}, \quad (6)$$

where $E_p^2 = \epsilon_p^2 + \Delta^2$. Integrations over the internal momenta in fermion loops, e.g., over p' in Eq. (5), involve energies far off the Fermi surface. One may renormalize [29,33] the interaction ($\widehat{V} \to \widehat{\Gamma}^\xi$) in such a manner that integrations go over the region near the Fermi surface and only the quasiparticle (pole) term in Green's function (6) is operative. The advantage of the Fermi-liquid approach is that all expressions enter renormalized amplitudes rather than the bare potentials. For $|\vec{p}\,| \simeq p_{\rm F} \simeq |\vec{p}\,'|$, the effective interaction amplitude is a function of only the angle between $\vec{p}$ and $\vec{p}\,'$. The amplitude in the particle-particle channel is parametrized as

$$[\widehat{\Gamma}^\xi]^{ac}_{bd} = \Gamma^\xi_0(\vec{n},\vec{n}')(i\sigma_2)^a_b(i\sigma_2)^c_d + \Gamma^\xi_1(\vec{n},\vec{n}')(i\sigma_2\vec{\sigma})^a_b(\vec{\sigma}\,i\sigma_2)^c_d,$$

and the interaction in the particle-hole channel is

$$[\widehat{\Gamma}^\omega]^{ac}_{bd} = \Gamma^\omega_0(\vec{n},\vec{n}')\delta^a_b\,\delta^c_d + \Gamma^\omega_1(\vec{n},\vec{n}')(\vec{\sigma})^a_b(\vec{\sigma})^c_d.$$

Here and below, $\vec{n} = \vec{p}/|\vec{p}\,|$ and $\vec{n'} = \vec{p}'/|\vec{p'}\,|$. Superscript ω indicates that the amplitude is taken for $|\vec{q}\,\vec{v}_{\rm F}| \ll \omega$ and $\omega \ll \epsilon_{\rm F}$, where ω and $\vec{q}$ are transferred energy and momentum. Amplitudes $\Gamma^{\xi,\omega}_0$, $\Gamma^{\xi,\omega}_1$ are expanded in the Legendre polynomials.

Integrating over the internal momenta in loops, we can separate the part accumulated in the vicinity of the Fermi surface $\int \frac{2\,d^4p}{(2\pi)^4 i} \simeq \int \frac{d\Omega_{\vec{p}}}{4\pi}\int d\Phi_p$ with $\int d\Phi_p = \rho\int_{-\infty}^{+\infty}\frac{d\epsilon}{2\pi\,i}\int_{-\infty}^{+\infty} d\epsilon_p$, where $\rho = \frac{m^* p_{\rm F}}{\pi^2}$ is the density of states at the Fermi surface.

After the Fermi-liquid renormalization, Eq. (5) reduces to

$$\begin{aligned}\Delta(\vec{n}) &= -A_0\left\langle\Gamma^\xi_0(\vec{n},\vec{n}')\Delta(\vec{n}')\right\rangle_{\vec{n}'},\\ A_0 &= \int d\Phi_p G_{\rm n.s.}(p)G^h_{\rm n.s.}(p)\theta(\xi-\epsilon_p) \approx a^2\rho\ln(2\xi/\Delta),\end{aligned} \quad (7)$$

where we denoted $\langle\ldots\rangle_{\vec{n}} = \int\frac{d\Omega_{\vec{n}}}{4\pi}(\ldots)$ and $\xi \sim \epsilon_{\rm F}$. One usually determines the gap supposing $\xi = \epsilon_{\rm F}$.

III. CURRENT-CURRENT CORRELATOR, EQUATIONS FOR VERTICES, AND VECTOR-CURRENT CONSERVATION

A. Current-current correlator

Applying the theory of Fermi liquids with pairing [29,34], we can present contributions to the susceptibility χ in terms of the diagram shown in Fig. 2. Here the dashed line relates to the Z boson coupled to the neutral lepton currents; vertices on the left are the bare vertices following from the Lagrangian (1). The right-hand-side vertices $\hat{\tau}$, $\hat{\tau}^h$, $\hat{\tau}^{(1)}$, and $\hat{\tau}^{(2)}$ are the full vertices determined by the diagrams shown in Fig. 3. The blocks in Fig. 3 correspond to the two-particle irreducible interaction in the particle-particle channel, Γ^ξ, and the particle-hole irreducible interaction in the particle-hole channel, Γ^ω. We emphasize that only chains of bubble diagrams are summed up in this particular formulation. Thus, the imaginary part of χ accounts only for one-nucleon processes. To include two-nucleon processes within a quasiparticle approximation, one should add diagrams with self-energy insertions to Green's functions and iterate the Landau-Migdal amplitudes $\Gamma^{\omega,\xi}$ in Fig. 3 in the horizontal channel [17,37]. In the general case for particles with widths, the interpretation of different processes contributing to $\Im\chi$ is more peculiar and needs another resummation scheme [37].

Taking the imaginary part of χ, we cut a diagram through two fermion lines. Cuts of neutron lines correspond to neutron PBF processes, and those of proton lines correspond to proton PBF processes. Since the neutron density in a neutron star is much higher than the proton density, we can drop all diagrams where proton lines are uncut from the set of the bubble chains included in Fig. 2.

References [16,21,22] considered only the first two diagrams in Fig. 2 with bare vertices. References [2,17] treated those two diagrams with full vertices on the right, whereas one must consider all four diagrams with the full vertices.

Vector and axial-vector currents contribute to χ separately, i.e., $\chi = \chi_V + \chi_A$, where

$$\chi_a = {\rm Tr}\int\frac{d^4p}{(2\pi)^4 i}\hat{\tau}^\omega_a\big\{\hat{G}_+\hat{\tau}^\dagger_a\hat{G}_- + \hat{F}^{(1)}_+\hat{\tau}^{h\dagger}_a\hat{F}^{(2)}_- + \hat{G}_+\hat{\tau}^{(1)\dagger}_a\hat{F}^{(2)}_- + \hat{F}^{(1)}_+\hat{\tau}^{(2)\dagger}_a\hat{G}_-\big\},\quad a = V, A. \quad (8)$$

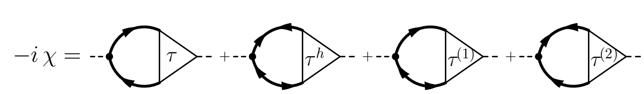

FIG. 2. Diagrams of contributions to the susceptibility χ in Eq. (2).

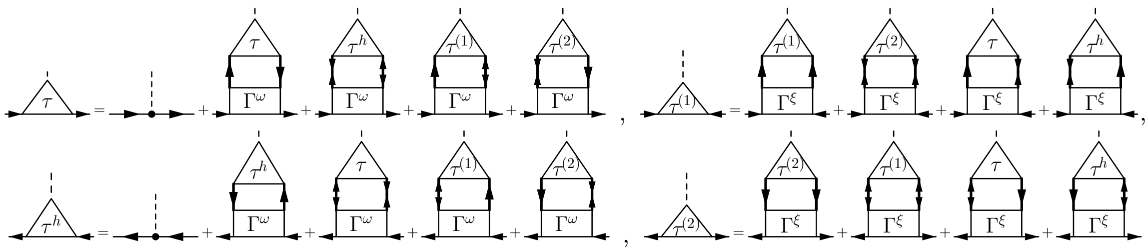

FIG. 3. Graphical representation of dressed vertices in Fig. 2.

Here and below, we use the short-hand notations $G_\pm = G(p \pm q/2)$ and the analogous one for the $F_\pm$ Green's functions. All left vertices in Fig. 2 are the "bare" vertices τ_a^ω; after the Fermi-liquid renormalization [29,33], $\tau_a^\omega = [1 + \Gamma_0^\omega (G_+G_-)^\omega]\tau_a^0$, which involves the particle-hole effective interaction Γ_0^ω, integrated with off-pole parts of Green's functions $(G_+G_-)^\omega = \lim_{\vec{q}\to 0} \int \frac{2d^4p}{(2\pi)^4 i} G_+G_-$, and τ_a^0 follows from Eq. (1). The difference between τ_a^0 and τ_a^ω can be cast [29] in terms of a local charge of the quasiparticle $e_a = a\tau_a^\omega/\tau_a^0$. Then

$$\hat{\tau}_V^\omega = g_V\big(\tau_{V,0}^\omega l_0 - \vec{\tau}_{V,1}^\omega \vec{l}\big), \quad \tau_{V,0}^\omega = \frac{e_V}{a}, \quad \vec{\tau}_{V,1}^\omega = \frac{e_V}{a}\vec{v},$$
$$\hat{\tau}_A^\omega = -g_A\big(\vec{\tau}_{A,1}^\omega \vec{\sigma} l_0 - \tau_{A,0}^\omega \vec{\sigma}\vec{l}\big), \quad \tau_{A,0}^\omega = \frac{e_A}{a}, \quad \vec{\tau}_{A,1}^\omega = \frac{e_A}{a}\vec{v}. \tag{9}$$

For the vector current, $e_V = 1$ and the vertices τ_V^ω and $\vec{\tau}_V^\omega$ satisfy the Ward identity $\omega\tau_{V,0}^\omega - \vec{q}\vec{\tau}_{V,1}^\omega = G_{\rm n.s.}^{({\rm pole}),-1}(p + q/2) - G_{\rm n.s.}^{({\rm pole}),-1}(p - q/2)$, with the pole part of the normal-state Green's function $G_{\rm n.s.}^{({\rm pole})} = G_{\rm n.s.} - G_{\rm reg}$. The local charge for the axial-vector current differs from the unity varying in different parametrizations as $e_A \simeq 0.8$–0.95, as it follows from studies of the Gamow-Teller transitions in nuclei; see Refs. [29,38,39] and references therein.

B. Larkin-Migdal equations for full vertices

Consider first one sort of nucleons, e.g., neutron. At the Fermi surface, the full vertices $\hat{\tau}$, $\hat{\tau}^h$, $\hat{\tau}^{(1)}$, and $\hat{\tau}^{(2)}$ can be treated as functions of out-going momentum $\vec{q}$ and the nucleon Fermi velocity $\vec{v} = v_{\rm F}\vec{n}$, $\vec{n} = \vec{p}/p$. Their general structures are

$$\begin{aligned}
\tau_V &= g_V\big(\tau_{V,0}l_0 - \vec{\tau}_{V,1}\vec{l}\big), \quad \tau_V^h = g_V\big(\tau_{V,0}l_0 + \vec{\tau}_{V,1}\vec{l}\big),\\
\tau_V^{(1)} &= -\tau_V^{(2)} = -g_V\big(\widetilde{\tau}_{V,0}l_0 - \vec{\widetilde{\tau}}_{V,1}\vec{l}\big)i\sigma_2,\\
\tau_A &= -g_A\big(\vec{\tau}_{A,1}\vec{\sigma}l_0 - \tau_{A,0}\vec{\sigma}\vec{l}\big),\\
\tau_A^h &= -g_A\big(-\vec{\tau}_{A,1}\vec{\sigma}^{\rm T}l_0 - \tau_{A,0}\vec{\sigma}^{\rm T}\vec{l}\big),\\
\tau_A^{(1)} &= +g_A\big(\vec{\widetilde{\tau}}_{A,1}\,\vec{\sigma}l_0 - \widetilde{\tau}_{A,0}\vec{\sigma}\,\vec{l}\big)i\sigma_2,\\
\tau_A^{(2)} &= -g_A i\sigma_2\big(\vec{\widetilde{\tau}}_{A,1}\,\vec{\sigma}l_0 - \widetilde{\tau}_{A,0}\vec{\sigma}\,\vec{l}\big).
\end{aligned} \tag{10}$$

Superscript T denotes matrix transposition.

As follows from the diagrammatic representation of Fig. 3, the full vertices obey the Larkin-Migdal equations [34]:

$$\begin{aligned}
&\tau_{a,0}(\vec{n},q)\\
&\quad = \tau_{a,0}^\omega(\vec{n},q) + \big\langle \Gamma_a^\omega(\vec{n},\vec{n}')\,[L(\vec{n}',q;P_{a,0})\tau_{a,0}(\vec{n}',q)\\
&\qquad + M(\vec{n}',q)\widetilde{\tau}_{a,0}(\vec{n}',q)]\big\rangle_{\vec{n}'},\\
&\widetilde{\tau}_{a,0}(\vec{n},q)\\
&\quad = -\big\langle \Gamma_a^\xi(\vec{n},\vec{n}')\,[(N(\vec{n}',q)+A_0)\widetilde{\tau}_{a,0}(\vec{n}',q)\\
&\qquad + O(\vec{n}',q;P_{a,0})\tau_{a,0}(\vec{n}',q)]\big\rangle_{\vec{n}'},\\
&\vec{\tau}_{a,1}(\vec{n},q)\\
&\quad = \vec{\tau}_{a,1}^\omega(\vec{n},q) + \big\langle \Gamma_a^\omega(\vec{n},\vec{n}')\,[L(\vec{n}',q;P_{a,1})\vec{\tau}_{a,1}(\vec{n}',q)\\
&\qquad + M(\vec{n}',q)\vec{\widetilde{\tau}}_{a,1}(\vec{n}',q)]\big\rangle_{\vec{n}'},\\
&\vec{\widetilde{\tau}}_{a,1}(\vec{n},q)\\
&\quad = -\big\langle \Gamma_a^\xi(\vec{n},\vec{n}')\,[(N(\vec{n}',q)+A_0)\vec{\widetilde{\tau}}_{a,1}(\vec{n}',q)\\
&\qquad + O(\vec{n}',q;P_{a,1})\,\vec{\tau}_{a,1}(\vec{n}',q)]\big\rangle_{\vec{n}'},
\end{aligned} \tag{11}$$

where $a = V, A$ and $P_{V,0} = -P_{V,1} = -P_{A,0} = P_{A,1} = 1$. To write the one set of equations for both vector and axial-vector weak currents, we introduced new notation for the effective interaction $\Gamma_a^{\omega,\xi} = \Gamma_0^{\omega,\xi}$, if $a = V$, and $\Gamma_a^{\omega,\xi} = \Gamma_1^{\omega,\xi}$, if $a = A$. Functions L, M, N, and O are defined as

$$\begin{aligned}
L(\vec{n},q;P) &= \int d\Phi_p[G_+G_- - (G_+G_-)^\omega - F_+F_-P]\\
&= a^2\rho\left[\frac{\vec{q}\vec{v}}{\omega - \vec{q}\vec{v}}(1-g(z)) - g(z)(1+P)\Big/2\right],\\
M(\vec{n},q) &= \int d\Phi_p[G_+F_- - F_+G_-]\\
&= -a^2\rho\frac{\omega+\vec{q}\vec{v}}{2\Delta}g(z),\\
N(\vec{n},q) &= \int d\Phi_p\big[G_+G_-^h - \big(G_pG_p^h\big)\theta(\xi-\epsilon_p) + F_+F_-\big]\\
&= a^2\rho\frac{\omega^2 - (\vec{q}\vec{v})^2}{4\Delta^2}g(z),\\
O(\vec{n},q;P) &= -\int d\Phi_p[G_+F_- + F_+G_-^h P]\\
&= a^2\rho\left[\frac{\omega+\vec{q}\vec{v}}{4\Delta} + \frac{\omega-\vec{q}\vec{v}}{4\Delta}P\right]g(z),
\end{aligned} \tag{12}$$

$$g(z^2) = \int_{-1/2}^{+1/2} \frac{dx}{4z^2x^2 - z^2 + 1 + i\,0}$$
$$= -\frac{\operatorname{arcsinh}\sqrt{z^2-1}}{z\sqrt{z^2-1}} - \frac{i\pi\theta(z^2-1)}{2z\sqrt{z^2-1}}, \qquad (13)$$
$$z^2 = \frac{\omega^2 - (\vec{q}\vec{v})^2}{4\Delta^2} > 1, \quad \vec{v} = v_{\rm F}\vec{n}.$$

Expressions (12) and (13) are derived for $T = 0$. For finite temperatures, $T < T_c$, all expressions in Eq. (12), except for L, hold as well, but with $g(z^2) \to g(z^2, T)$ and $\Delta \to \Delta(T)$. Generalization of the expression for L requires the introduction of one more temperature-dependent integral besides g. Such expressions were derived by Leggett in Ref. [35]. As follows from these expressions, there arises an essential simplification in the limit of low temperatures, $T \ll \Delta$. We exploit the fact that to calculate the PBF emissivity, we need only the imaginary part of the current-current correlator $\Im\chi$. Since $\omega > 2\Delta$ for the PBF kinematics, the emissivity is exponentially suppressed by $e^{-2\Delta/T}$ stemming from the Bose occupation factor $f_{\rm B}(\omega)$ in Eq. (2). Therefore, we may take $\Im\chi \propto \Im g(T = 0)$, since it is already multiplied by the term vanishing for $T \to 0$. The not accounted for temperature corrections in $\Im\chi$ prove to be $\sim 1 + O(e^{-\Delta/T}(1 + T^2/\Delta^2)) + O(T^2/\epsilon_{\rm F}^2)$. The latter term follows from the expansion of Fermi integrals when the integration goes over energy regions far from the Fermi surface. Such corrections are small in the limit $T \ll \Delta$ and we omit them.

Using vertices (9) and (10) in Eq. (8), the correlators χ_V and χ_A can be expressed as

$$\chi_V(q) = g_V^2\langle(l_0 - \vec{v}\vec{l})\,(l_0^\dagger\chi_{V,0}(\vec{n},q) - \vec{\chi}_{V,1}(\vec{n},q)\vec{l}^{\,\dagger})\rangle_{\vec{n}},$$
$$\chi_A(q) = g_A^2\langle(l_0\vec{v} - \vec{l})(l_0^\dagger\,\vec{\chi}_{A,1}(\vec{n},q) - \chi_{A,0}(\vec{n},q)\vec{l}^{\,\dagger})\rangle_{\vec{n}},$$
$$\chi_{a,0}(\vec{n},q) = L(\vec{n},q;P_{a,0})\,\tau_{a,0}(\vec{n},q) + M(\vec{n},q)\,\widetilde{\tau}_{a,0}(\vec{n},q),$$
$$\vec{\chi}_{a,1}(\vec{n},q) = L(\vec{n},q;P_{a,1})\,\vec{\tau}_{a,1}(\vec{n},q) + M(\vec{n},q)\,\vec{\widetilde{\tau}}_{a,1}(\vec{n},q).$$

C. Solution for vector and axial-vector parts of the current-current correlator

It is natural to expect that first and higher Legendre harmonics of $\Gamma^{\omega,\xi}_{0,1}(\vec{n},\vec{n}')$ are smaller than the zero-th ones because of the centrifugal factor [29]. This allows us to retain only zero harmonics $\Gamma^{\omega,\xi}_{0,1}(\vec{n},\vec{n}') = \Gamma^{\omega,\xi}_{0,1} = \text{const.}$, expressed through dimensionless Landau-Migdal parameters as [29]

$$\Gamma_0^{\omega,\xi} = \frac{f^{\omega,\xi}}{a^2\rho(n_0)}, \quad \Gamma_1^{\omega,\xi} = \frac{g^{\omega,\xi}}{a^2\rho(n_0)}. \qquad (14)$$

In isospin asymmetric matter, f^ω and g^ω are different for interactions between two neutrons ($f^\omega_{nn}, g^\omega_{nn}$), two protons ($f^\omega_{pp}, g^\omega_{pp}$), and a neutron and proton ($f^\omega_{np}, g^\omega_{np}$). Note that values f^ω_{nn}, f^ω_{pp} are necessarily positive, the requirement of the stability of the nucleon matter, whereas corresponding values in the particle-particle channel f^ξ_{nn}, f^ξ_{pp} are negative, otherwise there would be no $1S_0$ pairing. In this respect, our derivations differ from those that do not distinguish interactions in particle-hole and particle-particle channels and use Nambu-Gorkov formulations with one bare potential ($V < 0$ in our case). Empirical constraints on the values of the Landau-Migdal parameters are given in Appendix A.

For the angular-independent amplitudes (only zero-th harmonics are included) the Larkin-Migdal equations (11) get simple solutions:

$$\tau_{a,0}(q) = \gamma_a(q;P_{a,0})\tau^\omega_{a,0},$$
$$\gamma_a^{-1}(q;P) = 1 - \Gamma^\omega_a\langle\mathcal{L}(\vec{n},q;P)\rangle_{\vec{n}},$$
$$\mathcal{L}(\vec{n},q;P) = L(\vec{n},q;P) - \frac{\langle O(\vec{n},q;P)\rangle_{\vec{n}}}{\langle N(\vec{n},q)\rangle_{\vec{n}}}M(\vec{n},q), \qquad (15)$$
$$\widetilde{\tau}_{a,0}(q) = -\frac{\langle O(\vec{n},q;P_{a,0})\rangle_{\vec{n}}}{\langle N(\vec{n},q)\rangle_{\vec{n}}}\tau_{a,0}(q),$$

where $\Gamma^{\omega,\xi}_a$ are given in Eq. (14). We have exploited here the relation $1 = -\Gamma^\xi_0\langle A_0\rangle$ following from the gap equation (7). Although integrals in Eq. (11) do not produce terms $\propto\vec{v}$ for constant $\Gamma^{\omega,\xi}_a$, the vector vertices $\vec{\tau}_{a,1}$ and $\vec{\widetilde{\tau}}_{a,1}$ gain new terms proportional to $\vec{q}$; thus, $\vec{\tau}_{a,1}(\vec{n},q) = \vec{\tau}^{\,\omega}_{a,1}(\vec{n},q) + \vec{n}_q\tau^{(q)}_{a,1}(q)$ and $\vec{\widetilde{\tau}}_{a,1}(\vec{n},q) = \vec{n}_q\,\widetilde{\tau}^{(q)}_{a,1}(q)$, where $\vec{n}_q = \vec{q}/|\vec{q}\,|$ and

$$\tau^{(q)}_{a,1}(q) = \gamma_a(q;P_{a,1})\Gamma^\omega_a\langle\widetilde{\mathcal{L}}(\vec{n},q;P_{a,1})(\vec{n}\vec{n}_q)\rangle_{\vec{n}},$$
$$\widetilde{\tau}^{(q)}_{a,1}(q) = -\frac{\langle O(\vec{n},q;P_{a,1})\rangle}{\langle N(\vec{n},q)\rangle_{\vec{n}}}\tau^{(q)}_{a,1} - \frac{\langle O(\vec{n},q;P_{a,1})(\vec{n}\vec{n}_q)\rangle_{\vec{n}}}{\langle N(\vec{n},q)\rangle_{\vec{n}}}, \qquad (16)$$
$$\widetilde{\mathcal{L}}(\vec{n},q;P) = L(\vec{n},q;P) - \frac{\langle M(\vec{n},q)\rangle_{\vec{n}}}{\langle N(\vec{n},q)\rangle_{\vec{n}}}O(\vec{n},q;P).$$

Note that $g(z) \propto z^{-2} \to 0$ for $|z^2| \to \infty$, as follows from Eq. (13) and $g(z)$ being zero in case of a nonsuperfluid medium ($\Delta = 0$). Then anomalous vertices vanish, and the factors γ_V and γ_A acquire the standard form for the normal Fermi liquid [29]$\gamma_V^{-1} = 1 + \frac{\rho}{\rho(n_0)}f^\omega\Phi(\omega,\vec{q}\,)$ and $\gamma_A^{-1} = 1 + \frac{\rho}{\rho(n_0)}g^\omega\Phi(\omega,\vec{q}\,)$ with the Lindhard function $\Phi(\omega,\vec{q}\,) = \langle\vec{q}\vec{v}/(\vec{q}\vec{v} - \omega)\rangle_{\vec{n}}$ having customary asymptotics $\Phi(\omega \gg |\vec{q}\,\vec{v}\,|) \simeq -\vec{q}^{\,2}\vec{v}^{\,2}/(3\omega^2)$, $\Phi(\omega \ll |\vec{q}\,\vec{v}\,|) \simeq 1$.

With Eqs. (9), (15), and (16), we cast $\chi^\mu_a = (\chi_{a,0}, \vec{\chi}_{a,1})$ as

$$\chi_{a,0}(\vec{n},q) = \gamma_a(q;P_{a,0})\,\mathcal{L}(\vec{n},q;P_{a,0}),$$
$$\vec{\chi}_{a,1}(\vec{n},q) = \vec{v}\gamma_a(q;P_{a,1})\,\mathcal{L}(\vec{n},q;P_{a,1}) + \delta\vec{\chi}_{a,1}(\vec{n},q),$$
$$\delta\vec{\chi}_{a,1}(\vec{n},q) = \frac{M(\vec{n},q)}{\langle N(\vec{n}',q)\rangle_{\vec{n}'}}\langle O(\vec{n}',q;P_{a,1})(\vec{v} - \vec{v}\,')\rangle_{\vec{n}'} \qquad (17)$$
$$+ \mathcal{L}(\vec{n},q;P_{a,1})\gamma_a(q;P_{a,1})$$
$$\times\,\Gamma^\omega_a\langle\widetilde{\mathcal{L}}(\vec{n}',q;P_{a,1})(\vec{v}\,' - \vec{v}\,)\rangle_{\vec{n}'}.$$

Here γ_a are precisely those nucleon-nucleon correlation factors that were introduced in Ref. [17]. They depend on Landau-Migdal parameters in the particle-hole reaction channels.

D. Vector-current conservation

Now we are in the position to verify that the correlator of the vector current χ^μ_V supports the current conservation. First we note that there are convenient relations

$$
\begin{aligned}
&\langle \mathcal{L}(\vec n, q; P) - \widetilde{\mathcal{L}}(\vec n, q; P)\rangle_{\vec n} \\
&\quad = 0, \\
&\langle \omega \mathcal{L}(\vec n, q; \pm 1) - \widetilde{\mathcal{L}}(\vec n, q; \mp 1)\,(\vec q \vec v\,)\rangle_{\vec n} \\
&\quad = 0, \\
&\langle \vec q \vec \chi_{a,1}(\vec n, q)\rangle_{\vec n} \\
&\quad = \gamma_a(P_{a,1})\langle(\vec q \vec v\,)\widetilde{\mathcal{L}}(\vec n, q; P_{a,1})\rangle_{\vec n}\,, \\
&\langle(\vec q \vec v\,)\,[\omega \mathcal{L}(\vec n, q; +1) - (\vec q \vec v\,)\mathcal{L}(\vec n, q; -1) \\
&\qquad -\vec q \delta \vec \chi_{V,1}(\vec n, q, \Gamma^\omega = 0)]\rangle_{\vec n} \\
&\quad = \vec q^{\,2} a^2 n/m^*.
\end{aligned} \tag{18}
$$

These relations help us establish important properties of the vector-current correlators (17):

$$
\begin{aligned}
&\langle \omega\, \chi_{V,0} - \vec q \vec \chi_{V,1}\rangle_{\vec n} \\
&\quad = \gamma_V(q;+1)\gamma_V(q;-1)\omega\Gamma_V^\omega\,\langle \mathcal{L}(\vec n, q; +1)\rangle(\langle \mathcal{L}(\vec n, q; -1)\rangle \\
&\qquad - \langle \mathcal{L}(\vec n, q; +1)\rangle) = O\big(f^\omega\, g \vec q^{\,6} v_{\rm F}^6/\omega^6\big), \\
&\Im\langle(\vec q \vec v\,)(\omega\, \chi_{V,0} - \vec q \vec \chi_{V,1})\rangle_{\vec n} \\
&\quad = \omega\Gamma_V^\omega \langle \mathcal{L}(\vec n, q; +1)\rangle\langle(\vec q \vec v\,)[\gamma_V(q;+1)\mathcal{L}(\vec n, q; +1) \\
&\qquad - \gamma_V(q;-1)\mathcal{L}(\vec n, q; -1)]\rangle \\
&\quad = O\big(f^\omega\, g \vec q^{\,6} v_{\rm F}^6/\omega^6\big).
\end{aligned} \tag{19}
$$

Here we use the expansion of $\mathcal{L}$ and $\widetilde{\mathcal{L}}$ in the series for $|\vec q| v_{\rm F}/\omega \ll 1$:

$$
\begin{aligned}
\mathcal{L}(\vec n, q; +1) &= \frac{yx}{1-yx} + g y^2\left(\tfrac{1}{3} - x^2\right) \\
&\quad \times \left[1 + yx + y^2\left(\tfrac{1}{3} + x^2\right)\right] + O(gy^5), \\
\mathcal{L}(\vec n, q; -1) &= \frac{yx}{1-yx} - gyx(1 + yx + y^2x^2 + y^3x^3) \\
&\quad + O(gy^5), \\
\widetilde{\mathcal{L}}(\vec n, q; +1) &= \frac{yx}{1-yx} - gyx(1 + y^2x^2) \\
&\quad - gy^2\big(\tfrac{1}{3} - x^2\big)\big[1 - y^2\big(\tfrac{1}{3} + x^2\big)\big] + O(gy^5), \\
\widetilde{\mathcal{L}}(\vec n, q; -1) &= \frac{yx}{1-yx} - gy^2x^2(1 + y^2x^2) \\
&\quad - gxy^3\big(\tfrac{1}{3} - x^2\big) + O(gy^6), \\
y &= q v_{\rm F}/\omega, \quad x = (\vec q \vec v\,)/|\vec q| v_{\rm F}, \\
g &= g\left[\frac{\omega^2}{\Delta^2}(1 - y^2x^2)\right].
\end{aligned} \tag{20}
$$

Relations (19) demonstrate that the imaginary part of the vector-current correlator calculated with full vertices (15) and (16) is transverse,

$$
\Im\langle \tau_\mu^\omega \chi_V^\nu\rangle_{\vec n} q_\nu = O\big(f^\omega g \vec q^{6} v_{\rm F}^6/\omega^6\big), \tag{21}
$$

at least up to terms of the higher order than $f^\omega(|\vec q|\, v_{\rm F}/\omega)^5 g$, which are beyond the Fermi-liquid approximation for Green's functions (3) and (6). This ensures conservation of the vector current. Note that for $\Gamma^\omega = 0$ or $g = 0$, we have $\Im\langle \tau_\mu^\omega \chi_V^\nu\rangle_{\vec n} q_\nu \equiv 0$, and then the vector current is conserved exactly. Since $g \propto \Delta(T)$ and $\Delta = 0$ for $T > T_c$, we have proven in passing that the vector current is conserved exactly above T_c.

To prove the transversality of the real part of the vector-current correlator, it would be necessary to include the tadpole diagram contribution where the coupling originates from the "gauging" of the kinetic term, $\psi \vec\nabla^2 \psi/2m^*$, of an effective nonrelativistic nucleon Hamiltonian.

Some comments on the approximations done in previous works would be here appropriate. In all previous works, the vector-current contribution was considered as the dominant term for the case of $1S_0$ pairing. Expressions for the PBF emissivity in Refs. [16,21,22] can be recovered if we put $\Im\chi_{V,0}(\vec n, q) = \Im L(\vec n, q; +1)$ and $\Im\chi_{\vec V,1} = 0$. The result of Refs. [2,17] is obtained by taking $\Im\chi_{V,0}(\vec n, q) = \Im L(\vec n, q; +1)/[1 + \Gamma_0^\omega \Re L(\vec n, q; +1)]^2$ and also $\Im\chi_{\vec V,1} = 0$. Setting $\Gamma^\omega = 0$ in the limit $\omega \ll \Delta$ and $\vec q = 0$, we reproduce the expressions of Ref. [27].

Note that relations (18) do not hold with $\mathcal{L}$ replaced by L and, hence, the transversality relation (21) is spoiled if one ignores the anomalous vertex terms.

IV. NEUTRINO EMISSION VIA NEUTRON PBF

After correlators (17) are established, it remains to take the sum over the lepton spins and integrate over the leptonic phase space in Eq. (2). The latter can be easily done with the help of the Lenard integral [40]

$$
\begin{aligned}
&\int \frac{d^3q_1}{(2\pi)^3\, 2\omega_1}\frac{d^3q_2}{(2\pi)^3\, 2\omega_2}\sum\{l^\mu l^{\dagger\nu}\}\delta^{(4)}(q_1 + q_2 - q) \\
&\quad = \frac{1}{48\pi^5}(q^\mu q^\nu - g^{\mu\nu}q^2)\theta(\omega)\theta(\omega^2 - \vec q^{\,2}).
\end{aligned} \tag{22}
$$

Now the neutrino emissivity (2) can be cast as

$$
\begin{aligned}
\varepsilon_{\nu\nu} &= \varepsilon_{\nu\nu,V} + \varepsilon_{\nu\nu,A}, \\
\varepsilon_{\nu\nu,a} &= \frac{G^2}{8} g_a^{*2}\int_0^\infty d\omega\omega f_{\rm B}(\omega)\int_0^\omega \frac{d|\vec q|\vec q^{\,2}}{6\pi^4}\frac{\kappa_a}{a^2} \\
&= \frac{G^2 g_a^{*2}}{240\pi^4}\int_0^\infty d\omega\, \omega^6 f_{\rm B}(\omega)\, Q_a(\omega),
\end{aligned} \tag{23}
$$

$$
Q_a(\omega) = \frac{5}{\omega^5}\int_0^\omega d|\vec q|\vec q^{\,2}\frac{\kappa_a}{a^2}, \tag{24}
$$

$$
\begin{aligned}
\kappa_a &= \int \frac{d^3q_1}{2\omega_1}\frac{d^3q_2}{2\omega_2}\delta^{(4)}(q_1 + q_2 - q) \\
&\quad \times \frac{3}{4\pi}\Im\sum \chi_a(q).
\end{aligned} \tag{25}
$$

In κ_a the sum is taken over the lepton spins. Shortening notations we introduced in Eq. (23), effective couplings $g_a^* = e_a\, g_a$. The particular normalization of the quantity Q_a is chosen so that for $Q_a(\omega) = Q^0(\omega)$ with

$$
Q^{(0)}(\omega) = -\rho\Im g(\omega^2/4\Delta^2), \tag{26}
$$

we obtain the expression for the neutron PBF emissivity

$$\epsilon_{\nu\nu}^{(0n)} = \frac{4\rho_n\, G^2\Delta_n^7}{15\,\pi^3} I\left(\frac{\Delta_n}{T}\right), \quad I(z) = \int_1^\infty \frac{dy\, y^5}{\sqrt{y^2-1}} e^{-2zy}, \tag{27}$$

which coincides with the old result [16,17] after the replacement $e^{-2zy} \to \frac{1}{(e^{zy}+1)^2}$. From now on we restore, when necessary, subscripts n or p to distinguish neutron and proton PBF processes, respectively.

A. Emissivity on vector current

For the vector current, we have

$$\begin{aligned}\kappa_V &= \Im[\vec q^{\,2}\langle\chi_{V,0}(\vec n,q)\rangle_{\vec n} + \langle(\vec q\vec v)\vec q\,\vec\chi_{V,1}(\vec n,q)\rangle_{\vec n} \\ &\quad + (\omega^2-\vec q^{\,2})\langle\vec v\,\vec\chi_{V,1}(\vec n,q)\rangle_{\vec n} - \omega\,\langle(\vec q\vec v\,)\,\chi_{V,0}(\vec n,q)\rangle_{\vec n} \\ &\quad - \omega\langle\vec q\,\vec\chi_{V,1}(\vec n,q)\rangle_{\vec n}].\end{aligned} \tag{28}$$

Using relations (17) and (19), we can simplify Eq. (28) as

$$\kappa_V = (\vec q^{\,2}-\omega^2)\,\Im\langle\chi_{V,0}(\vec n,q) - \vec v\vec\chi_{V,1}(\vec n,q)\rangle_{\vec n}. \tag{29}$$

Both scalar and vector components in Eq. (29) are of the order $v_{\rm F}^4$,

$$\Im\langle\chi_{V,0}(\vec n,q)\rangle \approx -\frac{4\,\vec q^{\,4}v_{\rm F}^4}{45\omega^4}\,a^2\rho\Im g\left(\frac{\omega^2}{4\Delta^2}\right) > 0,$$

$$\Im\langle\vec v\vec\chi_{V,1}(\vec n,q)\rangle \approx -\frac{2\vec q^{\,2}v_{\rm F}^4}{9\omega^2}\,a^2\rho\Im g\left(\frac{\omega^2}{4\Delta^2}\right) > 0.$$

We have put $\gamma_V \to 1$ since $\gamma_V \simeq 1 + O(f_{nn}^\omega v_{\rm F}^2)$.

Note that the first term in Eq. (29), $\propto \Im\chi_{V,0}$, would give a negative contribution to Q_V. The full expression for the reaction probability becomes positive only because of the presence of *the vector component of the vertex* [second term in Eq. (29)]. This is because we used Ward identities, which impose relations between zero-th and vector components. However, if one keeps in Eq. (28) only the first term related to the zero-th component of the vertex and drops other terms, as was done in early works, the expression for the reaction probability would be also positive.

Then in terms of Q_V we get

$$Q_V^n \simeq \tfrac{4}{81} v_{{\rm F},n}^4 Q^{(0n)}(\omega). \tag{30}$$

Finally for the neutron PBF emissivity on the vector current, we obtain (for one neutrino flavor)

$$\epsilon_{\nu\nu,V}^{n{\rm PFB}} \simeq \epsilon_{\nu\nu}^{(0n)} g_V^2\, \tfrac{4}{81} v_{{\rm F},n}^4. \tag{31}$$

Note that even though Ref. [27] used the approximation $\omega \ll \Delta_n$, which is not fulfilled in the PBF case, our expression (31) deviates only slightly from the corresponding result obtained in Ref. [27].

The authors of Ref. [30] calculated the susceptibility χ_V including only the zero-th component, $\chi_{V,0}$, for $v_{\rm F}=0$, performing an expansion for small $\vec q$. They found the leading term $\propto \vec q^{\,2}/2m$. However, it has the opposite sign [see Eq. (48) in Ref. [30]] to the second term, $\propto I_B$ [in Eq. (40) of Ref. [30]], which would yield the reaction probability if bare vertices were used. Note also that the key equations (35) and (38)–(45) in Ref. [30] differ from the Larkin-Migdal equations (12) (for $T \ll \Delta$ as supposed in Ref. [34], and for $v_{\rm F}=0$ as assumed in Ref. [30]). As follows from Eqs. (12) and (15), our expression for $\langle\mathcal{L}(\vec n,q,+1)\rangle_{\vec n} \propto q^4 v_{{\rm F},n}^2$ vanishes if $v_{{\rm F},n}\to 0$, although $\vec q^{\,2}/m^*$ terms were present in the original loop integrals.

Further comparison of our results with the results of other works is relegated to Appendix B.

B. Emissivity on axial-vector current

Now we focus on the process on the axial-vector current. Then

$$\begin{aligned}\kappa_A &= \Im[\vec q^{\,2}\langle\vec v\,\vec\chi_{A,1}(\vec n,q)\rangle_{\vec n} + (3\omega^2-2\,\vec q^{\,2})\langle\chi_{A,0}(\vec n,q)\rangle_{\vec n} \\ &\quad - \omega\langle\vec q\,\vec\chi_{A,1}(\vec n,q)\rangle_{\vec n} - \omega\,\langle(\vec q\vec v\,)\,\chi_{A,0}(\vec n,q)\rangle_{\vec n}].\end{aligned} \tag{32}$$

The last two crossing terms in the squared brackets cannot be eliminated. Keeping only terms $\propto v_{\rm F}^2$, we cast Eq. (32) as

$$\begin{aligned}\kappa_A &= \Im\big[\vec q^{\,2}v_{\rm F}^2\langle L(\vec n,q;+1)\rangle_{\vec n} + (3\omega^2-2\vec q^{\,2})\langle L(\vec n,q;-1)\rangle_{\vec n} \\ &\quad - \omega^2\langle L(\vec n,q;-1)\rangle_{\vec n} - \omega\langle(\vec q\,\vec v\,)L(\vec n,q;-1)\rangle_{\vec n}\big] \\ &\approx -a^2\rho\, v_{\rm F}^2\vec q^{\,2}\left[1+\left(1-\frac{2}{3}\frac{\vec q^{\,2}}{\omega^2}\right)-\frac{2}{3}\right]\Im g\left(\frac{\omega^2}{4\Delta^2}\right).\end{aligned} \tag{33}$$

As in the case with the vector current, by simplifying we could put $\gamma_A = 1$, since $\gamma_A \simeq 1 + O\left(g_{nn}^\omega v_{\rm F}^2\right)$.

The contribution of the axial-vector current to the neutrino emissivity is determined by

$$Q_A^n(\omega) \simeq \left(1+\frac{11}{21}-\frac{2}{3}\right) v_{{\rm F},n}^2\, Q^{(0n)}(\omega). \tag{34}$$

The second term in round brackets of Eq. (34) has been mentioned already in Ref. [16] and then recovered in Ref. [22]. Our coefficient (11/21) is twice as large as that presented in those works. We notice that the integral $I_s/2 = (u'v - v'u)^2$, where u and v are coefficients of the Bogoliubov transformation, is in Ref. [22] twice as large as that in Ref. [16]. In agreement with the former evaluation, we arrive at the coefficient 11/21 rather than at 11/42, as presented in Refs. [16,22]. The first term in Eq. (34) (for $m^*=m$) is the same as in Ref. [22], which calculated this relativistic correction for the first time. The factor $(m^*/m)^2$ does not arise in our calculations, since the mass renormalization is performed everywhere, including the vertices. Otherwise, the Ward identity would not hold for the renormalized "bare" vertex τ_μ^ω. The third term related to the time-space component product was not considered before.

Finally for the neutron PBF emissivity on the axial-vector current, we obtain (for one neutrino flavor)

$$\epsilon_{\nu\nu,A}^{n{\rm PBF}} \simeq \tfrac{6}{7} g_A^{*\,2} v_{{\rm F},n}^2 \epsilon_{\nu\nu}^{(0n)}. \tag{35}$$

The resulting emissivity is the sum of contributions (31) and (35),

$$\epsilon_{\nu\nu}^{n{\rm PBF}} = \epsilon_{\nu\nu,V}^{n{\rm PBF}} + \epsilon_{\nu\nu,A}^{n{\rm PBF}} \simeq \epsilon_{\nu\nu,A}^{n{\rm PBF}}. \tag{36}$$

The axial-vector term, being $\propto v_{\rm F}^2$, is now the dominating contribution. Thus, the ratio of the emissivity of the neutron

PFB obtained here to the emissivity calculated in Refs. [16,22], where the main contribution was due to the vector current, is

$$R(n\text{PFB}) = \frac{\epsilon_{\nu\nu}^{n\text{PBF}}}{\epsilon_{\nu\nu}^{(0n)}} \simeq \frac{6}{7} g_A^{*2} v_{\text{F},n}^2 = F_n\, v_{\text{F},n}^2.$$

For $n = n_0 = 0.17\text{fm}^{-3}, m^* = 0.8m$, we estimate $F_n \simeq 0.9$–1.2, $v_{\text{F},n} \simeq 0.36$, and $R \simeq 0.12$–0.15. For $n = 2n_0, m^* = 0.7m$, R increases up to 0.24–0.32. This is in drastic contrast with estimations $R(n\text{PFB}) \sim 10^{-3}$ in Ref. [27] (being actually valid only for the rate of the partial vector-current contributions, rather than for the full emissivities) and $R(n\text{PFB}) \simeq 5 \times 10^{-3}$ obtained in Ref. [30].

V. NEUTRINO EMISSION VIA PROTON PBF

Now we turn to the proton PBF processes. If protons were the only particles in the neutron star medium, we could use the results obtained above for the neutron PBF and just replace $g_V^{(n)} \to g_V^{(p)} = c_V$, $v_{\text{F},n} \to v_{\text{F},p}$, $f_{nn} \to f_{pp}$, $g_{nn} \to g_{pp}$, and $e_a^n \to e_a^p$.

A. Emissivity on vector current

Since the bare vertex now yields $(g_V^{(p)})^2 = c_V^2 \simeq 0.002$ rather than $(g_V^{(n)})^2 = 1$ as in the neutron case, one could naively think that the emissivity of the proton PBF process on the vector current is suppressed by a factor of $\sim 10^{-3}(\Delta_p/\Delta_n)^{13/2}\, e^{2(\Delta_p-\Delta_n)/T}$ compared to the emissivity of the neutron PBF process. For imaginary purely proton matter, we would find in the vector channel,

$$\epsilon_{\nu\nu,V} \simeq \epsilon_{\nu\nu}^{(0p)} c_V^2 \frac{4}{8} v_{\text{F},p}^4, \tag{37}$$

$$\epsilon_{\nu\nu}^{(0p)} = \frac{4\rho_p\, G^2 \Delta_p^7}{15\pi^3} I\left(\frac{\Delta_p}{T}\right). \tag{38}$$

In addition to the fact that the emissivity is already suppressed in the vector channel by the factor $\frac{1}{20} v_{\text{F},p}^4$, Leinson and Perez [27] found extra suppression. They included the interaction of protons via photons. This produces new contributions to the susceptibility χ of the type shown in Fig. 4(a).

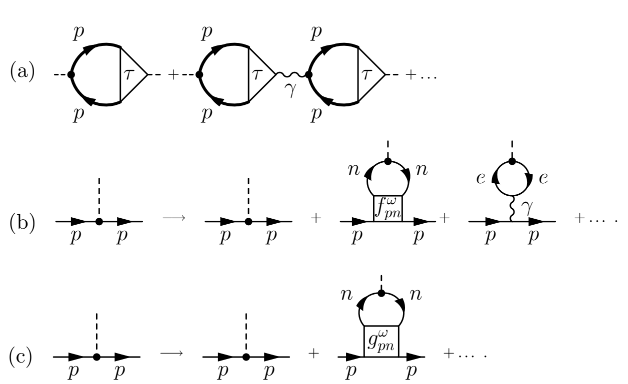

FIG. 4. (a) Contributions to proton PBF due to a photon exchange; (b) in-medium modification of the proton vector current; (c) in-medium modification of the proton axial current.

Dots assume infinite summation of the bubble chains with all four types of the vertices shown in Fig. 2. The wavy line is the dressed photon Green's function. Simplifying, Ref. [27] used the static Coulomb potential instead. To elucidate the origin of differences in our estimations from those in Ref. [27], we will use the same approximations. Effectively, the summation leads to the replacement [27]

$$\tau_{V,0} \to \frac{\tau_{V,0}}{\epsilon_{\text{C}}(q)}, \tag{39}$$

where $\epsilon_{\text{C}}(q) \simeq 1 + \omega_{\text{pl}}^2/\omega^2$ is the dielectric constant, $\omega_{\text{pl}}^2 = 4\pi\, e^2 n_p/m^*$ is the proton plasma frequency with $e^2 = 1/137$. Setting $m^* = m, \omega \simeq 2\Delta_p$, [$\omega \simeq 2\Delta + O(T)$ for the PFB processes], $\Delta \simeq 1.76T_c$ and $T_c \sim 1$ MeV for $x_p = n_p/n \sim 0.03$ at $n = n_0$, cf. Fig. 2 of Ref. [9], we obtain $\epsilon_{\text{C}}(q) \simeq 1 + 0.3 \sim 1$. This result is disagrees with the estimation of Ref. [27], where applying their result to the neutron star matter, the authors put $n_p = n_0$ and $\omega \simeq T_c$, which resulted in the estimation $\epsilon_{\text{C}}(q) \sim 10^2$. Thus, the suppression factor of $< 10^{-6}$ of the emissivity of the proton PBF process quoted in Ref. [27] is misleading. Note that correction of the vertex in Eq. (40) affects only the process on the vector current, since the photon is the vector particle.

For neutron star matter, the replacement (39) is not sufficient, since protons are embedded into an electron liquid of the same concentration and into a much more dense neutron liquid. Renormalization of the weak vector interaction of protons in this case can be taken into account, if we replace the bare coupling τ_V^ω as shown in Fig. 4(b). Dots stand for other graphs not shown explicitly, such as the $\Delta(1232)n$-loop term. Simplifying, we ignore these rather small correction terms. The second graph has been incorporated in Refs. [2,17], which results in the shift

$$c_V \to c_V - f_{np}^\omega \rho^{-1}(n_0) \Re L_{nn} \gamma\left(f_{nn}^\omega\right),$$

where $\gamma^{-1}(f_{nn}^\omega) = 1 - f_{nn}^\omega \rho^{-1}(n_0)\Re L_{nn}$ and $L_{nn} = \langle L(\vec{n}, q; g = 0)\rangle_{\vec{n}}/a^2 = \rho\langle \vec{q}\,\vec{v}/(\omega - \vec{q}\,\vec{v})\rangle_{\vec{n}}$ is the Lindhard function. This correction (although $\propto v_{\text{F}}^2$) leads to the strong enhancement of the tiny bare vertex. A numerically larger correction comes from the electron–electron-hole polarization term (the third graph). Such a possibility has been discussed in Ref. [19] for the process of a possible massive photon decay, and then it was taken into account in the proton PBF emissivity in Ref. [20]. Altogether these corrections can be incorporated into the resulting expression for the emissivity of the proton PBF process with the help of the replacement, cf. Ref. [4],

$$c_V^2 \to \mathcal{F}_p \simeq \epsilon_{\text{C}}^{-2}(q)\left[\frac{f_{np}^\omega}{\rho(n_0)} \Re L_{nn} \gamma(f_{nn}^\omega) + 0.8 C_{ve}\right]^2. \tag{40}$$

Here $C_{ve} = 1$ is the electron weak vector coupling. Thus we find

$$\epsilon_{\nu\nu,V}^{p\text{PBF}} \simeq \epsilon_{\nu\nu}^{(0p)}\, \mathcal{F}_p \tfrac{4}{81} v_{\text{F},p}^4, \tag{41}$$

where the prefactor $\mathcal{F}_p \sim 1$. Finally we obtain an estimate

$$R_V[p/n] = \frac{\epsilon_{\nu\nu,V}^{p\text{PBF}}}{\epsilon_{\nu\nu,V}^{n\text{PBF}}} \sim x_p^{4/3}\left(\frac{\Delta_p}{\Delta_n}\right)^{13/2} e^{2(\Delta_n-\Delta_p)/T}. \tag{42}$$

The ratio $R_V[p/n]$ is sensitive to the values of the proton and neutron gaps as functions of density $\Delta_{n,p}(n)$, the proton fraction x_p, and the temperature T.

B. Emissivity on axial-vector current

Now we consider the axial-vector channel. Photon exchange does not contribute in this channel. The main correction to the vertex comes from the iteration of the nn loops, see Fig. 4(c). Simplifying, we will suppress correlation factors such as $\gamma^2(g^\omega_{nn}) \simeq 1 + O(g^\omega_{nn} v^2_{\mathrm{F},n})$. Thus we obtain

$$\epsilon^{p\mathrm{PBF}}_{\nu\nu,A} \simeq \epsilon^{(0p)}_{\nu\nu}\,\tfrac{6}{7} g^{*2}_A v^2_{\mathrm{F},p}. \tag{43}$$

Comparison with Ref. [22] can be done quite similar to that performed above for neutrons.

We conclude that

$$\epsilon^{p\mathrm{PBF}}_{\nu\nu} = \epsilon^{p\mathrm{PBF}}_{\nu\nu,V} + \epsilon^{p\mathrm{PBF}}_{\nu\nu,A} \simeq \epsilon^{p\mathrm{PBF}}_{\nu\nu,A}. \tag{44}$$

For the ratio $R[p/n]$ we find

$$R[p/n] = \frac{\epsilon^{p\mathrm{PBF}}_{\nu\nu}}{\epsilon^{n\mathrm{PBF}}_{\nu\nu}} \simeq \frac{\epsilon^{p\mathrm{PBF}}_{\nu\nu,A}}{\epsilon^{n\mathrm{PBF}}_{\nu\nu,A}} \sim x_p^{4/3}\left(\frac{\Delta_p}{\Delta_n}\right)^{13/2} e^{(\Delta_n-\Delta_p)/T}. \tag{45}$$

The ratio $R[p/n]$ is sensitive to the choice of pairing gaps, temperature, and proton fraction x_p and can be both $\lesssim 1$ and $\gtrsim 1$.

VI. CONCLUSIONS

In this paper, we recalculated neutrino emissivity via neutron and proton pair breaking and formation processes. We used the Larkin-Migdal-Leggett Fermi-liquid approach to strongly interacting systems with pairing. Compared to the Nambu-Gorkov formalism, the Larkin-Migdal-Leggett approach allows for different interactions in the particle-particle and the particle-hole channels, as is the case for nuclear matter.

To be specific, we focused our discussion on the $1S_0$ pairing. We support the statement of Ref. [27] that medium effects essentially modify vector-current vertices. Only the careful account of these effects allows one to fulfill the Ward identity and to protect conservation of the vector current. Compared with the emissivity calculated in Ref. [16], the partial contribution to the emissivity on the neutron vector current proved to be dramatically suppressed, roughly by a factor of $\sim 0.1 \times v^4_{\mathrm{F},n}$, where $v_{\mathrm{F},n}$ is the neutron Fermi velocity, cf. Ref. [27]. A similar suppression factor arises for the partial contribution to the emissivity on the proton vector current, provided one replaces the neutron Fermi velocity by the proton Fermi velocity. Electron–electron-hole and neutron–neutron-hole polarization effects play a crucial role in the latter estimation. Proton–proton-hole polarization effects are suppressed (these statements are at variance with the estimations in Ref. [27]).

We have to note that the observed cancellation of v^0_{F} terms in the vector-current vertices is required by the gauge invariance, whereas cancellation of v^2_{F} terms in the vector current vertices is rather accidental. Therefore, it is not excluded that in some particular situations, a cancellation of the v^2_{F} terms in the vector-current vertices will be lifted. For example, such a situation could happen in the case of pairing in a complex multicomponent system. Also it could occur when fermions of one species are involved in the pairing with two different gaps, e.g., see the case discussed in Ref. [41].

The dominating contribution to the neutron and proton pair breaking and formation emissivity comes from the weak axial-vector current. Finally, the neutron pair breaking and formation emissivity proves to be suppressed compared to that of Ref. [16] by a factor of ∼0.12–0.15 at nuclear saturation density and ∼0.24–0.32 at twice nuclear saturation density. For the proton pair breaking and formation, the emissivity deviates only by a factor close to unity from the expression used previously in Ref. [22]. Our findings differ from those in Refs. [27,30], in which the authors concluded that the neutron and proton pair breaking and formation emissivities are dramatically suppressed. The modifications of the neutron and proton pair breaking and formation reaction rates that we found are probably not strong enough to essentially influence the previously computed values of surface temperatures of neutron stars.

One may raise the question of how much the emissivity of other relevant neutrino processes might be changed if the medium effects in the presence of nucleon pairing are correctly included. Although we did not perform the corresponding cumbersome calculations, let us formulate our conjectures as follows.

In the reactions with charged currents, as the direct URCA and the modified URCA processes, the transferred neutrino energy is $\omega \simeq \mu_e = p_{\mathrm{F},p} \gg 2\Delta$. Therefore the anomalous Green's functions are taken in the limit $\omega \gg 2\Delta$. In this limit, the g function tends to zero [as follows from the corresponding asymptotic in Eq. (13)]. The effects of normal correlations and pion softening were evaluated in Refs. [2,4,11,17], resulting in significant enhancement of two-nucleon reaction rates. Specifics of the superfluid matter manifest themselves in reactions with charged currents mainly through the phase-space suppression factors.

The two-nucleon bremsstrahlung processes induced by the neutral currents are similar to the pair breaking and formation reactions. In a normal phase, the emissivities are governed by the axial-vector current [10]. For axial-vector vertices, there are no contributions from the anomalous vertices. Therefore, we do not expect a strong suppression of these rates in a superfluid phase (the amplitude remains of the order v^0_{F}) except the natural phase-space suppression estimated in previous works. As in the case of the two-nucleon reactions induced by the charged currents, nucleon short-range correlations and pion softening significantly influence the reaction rates, cf. Refs. [2,11].

Our findings are relevant also for calculations of the quark-pair breaking and formation processes and other quark propagation processes in the color-superconducting medium, which use bare-loop results, e.g., see Refs. [42,43]. Note that since the pairing gaps in the color superconductors can be rather large, $2\Delta > \mu_e$, both reaction rates on neutral and charged currents (URCA) might be affected. Reference [26] considered neutrino scattering off breaking pairs in a color-flavor-locked medium within the Nambu-Gorkov formalism. However, they included only the zero-th component of the vertex, and their expressions for the current-current correlator in the nonrelativistic limit do not coincide with the Larkin-Migdal-Leggett expressions that we have reproduced above.

An interesting observation was made recently in Ref. [44]. A natural explanation of the superburst ignition would require a strong suppression of the neutron pair breaking and formation emissivity for low baryon densities. For $n \sim 10^{12}$ g/cm^3 we estimate a suppression factor of $\sim 0.1(n/n_0)^{2/3} \sim 0.003$.

Another relevant issue is related to absorption and scattering processes of low-energy ($\omega \lesssim$ few MeV) neutrinos and antineutrinos on nuclei. In absence of the electron Fermi sea, the correlation effects may manifest themselves in reactions on both neutral and charged currents. There are different sources for neutrinos of such energies, e.g., reactor neutrinos are good candidates. The Sun and supernova neutrinos also have pronounced low-energy tails. The observation of supernova neutrinos might provide us with unique information on the core collapse and on the compact star formation and cooling [45]. Geoneutrinos and antineutrinos from the progenies of U, Th, and ^{40}K decays inside the Earth reveal information about the whole planet's content of radioactive elements [46]. Finally, verifying the existence of the relic neutrino sea with temperature $T_\nu/T_\gamma = (4/11)^{1/3}$ represents one of the main challenges of the modern cosmology [47].

From a general point of view, our results strongly support the conclusion of Refs. [2,4,11,48] about the essential role played by different medium effects in the neutrino evolution of neutron stars, as was demonstrated in the framework of the "nuclear-medium cooling scenario" [13,14,23]. Without the proper inclusion of medium effects, it is difficult to reach sound conclusions. Further investigations in this direction are required.

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APPENDIX A: VALUES OF LANDAU-MIGDAL PARAMETERS

The isospin symmetry of strong interactions imposes the following relations between different Landau-Migdal parameters: $f_{nn} = f_{pp} = f + f'$, $g_{nn} = g_{pp} = g + g'$, $f_{np} = f_{pn} = f - f'$, and $g_{np} = g_{pn} = g - g'$. By definition, the parameters depend only on the directions of nucleon momenta before and after collision and can be written in terms of the Legendre polynomials, e.g., $f(\vec{n}, \vec{n}') = \sum_l f_l P_l(\cos\theta_{\vec{n}\vec{n}'})$ and analogously for f', g, and g'. Values of f_l, f'_l, g_l, g'_l should be extracted from an analysis of atomic nucleus data or they should be calculated. The situation is simplified by the fact that physical quantities depend only on the values of the first two harmonics.

Unfortunately, the extracted values vary from work to work in rather broad limits because of attempts by authors to obtain the best description of the experiment they study by modifying parametrizations of the NN interaction. Therefore, it is difficult to compare directly the results of different studies and to extract the unique information about all Landau-Migdal parameters. In some approximations, these parameters can be calculated using different representations of a microscopic nucleon-nucleon interaction [49,50], but results depend on the choice of the interaction. Thus, most of the Landau-Migdal parameters are poorly known up to now.

In Ref. [29], the following values of the parameters are quoted: $f_0^\omega \simeq 0.25$, $f_0'^\omega \simeq 1$, $g_0^\omega \simeq 0.5$, $g_0'^\omega \simeq 1$. Calculations in Ref. [51] give the values $f_0^\omega \simeq 0$, $f_0'^\omega \simeq 0.5$–0.6, $g_0^\omega \simeq 0.05 \pm 0.1$, and $g_0'^\omega \simeq 1.1 \pm 0.1$. In Ref. [52], the value g_{pp}^ω was fixed from the data on the two-neutrino double β decays and single β decays, as $g_{0,pp}^\omega \simeq 1$, in favor of the choice of Ref. [51]. Ref. [30] calculated $f_0^\xi \simeq -0.47$ and $g_0^\xi \simeq +0.46$ using Cogny DSI forces, whereas Ref. [39] extracted a different value $f_0^\xi \simeq -(0.25$–$0.33)$ from the analysis of the atomic nucleus data. First Legendre harmonics f_1^ω, $f_1'^\omega$ are related to the value of the effective nucleon mass [29]. Values $g_{1,pp}^\omega = -g_{1,pn}^\omega \simeq -0.11$ are estimated from the analysis of decay energies, and the Gamow-Teller strength distributions in neutron-rich short-lived nuclides [53].

APPENDIX B: CALCULATION WITH BARE VERTICES IN THE LOOP

Let us now comment on results of previous calculations of the neutron PBF emissivity. If, as in previous calculations [16, 22], one neglects contributions from the anomalous vertices $\tilde{\tau}$, i.e., $M \to 0$ and $\mathcal{L} \to L$ in Eq. (17), and puts $\gamma_a = 1$, one may recover the results of calculations with bare vertices (b.v.). Expression (28) for κ_V becomes

$$\kappa_V^{\mathrm{b.v.}} = \Im\big[\vec{q}^{\,2}\langle L(\vec{n}, q; +1)\rangle_{\vec{n}} + v_{\mathrm{F}}^2\,\langle(\omega^2 + (\vec{q}\vec{n})^2 - \vec{q}^{\,2}) \times L(\vec{n}, q; -1)\rangle_{\vec{n}} - \omega\langle(\vec{q}\vec{v}\,) \times (L(\vec{n}, q; +1) + L(\vec{n}, q; -1))\rangle_{\vec{n}}\big]. \tag{B1}$$

In the axial-vector channel, the corresponding quantity is $\kappa_A^{\mathrm{b.v.}} \propto v_{\mathrm{F}}^2$. Therefore we will also keep the v_{F}^2 corrections in Eq. (B1). In the calculations of Refs. [16,22], such terms were dropped. The second term in Eq. (B1) can be indeed neglected, as being $\propto v_{\mathrm{F}}^4$, since $\langle L(\vec{n}, q; -1)\rangle_{\vec{n}} \propto v_{\mathrm{F}}^2$. The third crossing term of the order v_{F}^2 disregarded in previous calculations can be dropped only if conditions (19) hold; that is not the case for the bare vector current-current correlator. We keep this term here. For $|\vec{q}\,|v_{\mathrm{F}} \ll \omega$ (i.e. for $v_{\mathrm{F}} \ll 1$ since $|\vec{q}\,| \lesssim \omega$) we get

$$\frac{\kappa_V^{\mathrm{b.v.}}}{a^2\rho} \approx -\vec{q}^{\,2}\Im g\left(\frac{\omega^2 - (\vec{v}\,\vec{q}\,)^2}{4\Delta^2}\right) - \frac{\vec{q}^{\,4}v_{\mathrm{F}}^2}{3\omega^2}\Im g\left(\frac{\omega^2}{4\Delta^2}\right) + \frac{2}{3}\vec{q}^{\,2}v_{\mathrm{F}}^2\,\Im g\left(\frac{\omega^2}{4\Delta^2}\right). \tag{B2}$$

The first line in Eq. (B2) comes from the expansion of the first term in Eq. (B1). The first term in the second line follows from the crossing term in Eq. (B1). Using (B2) we calculate

$$Q_V^{\rm b.v.} = \left(1 + \frac{5}{21}v_{\rm F}^2 - \frac{2}{3}v_{\rm F}^2\right) Q^{(0)}(\omega).$$

Dropping the $v_{\rm F}^2$ corrections and using expression (26) for $Q_V^{(0)}$ one may reproduce the vector current contribution to the emissivity obtained in Refs. [16,22] (and Refs. [2,17], provided one sets there $\gamma_V = 1$).

The expression for the emissivity of neutron PBF on the axial-vector current calculated with the bare vertices in the loop does not deviate from that in Eq. (34) derived above with the full vertices. Note, however, that second and third terms in Eq. (34) differ from those used in previous calculations.

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