PHYSICAL REVIEW C **77**, 065504 (2008)

Neutron–mirror-neutron oscillations in a trap

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(Received 13 April 2008; published 27 June 2008)

We calculate the rate of neutron–mirror-neutron oscillations for ultracold neutrons trapped in a storage vessel. Recent experimental bounds on the oscillation time are discussed.

DOI: 10.1103/PhysRevC.77.065504 PACS number(s): 12.90.+b, 28.20.−v, 14.20.Dh

I. INTRODUCTION

During the last couple of years we have been witnessing a revival of interest in "mirror particles," "mirror matter," and the "mirror world." The idea of the existence of a hypothetical hidden sector to compensate for mirror asymmetry was first explicitly formulated in Ref. [1]. This subject has a rich history—for a review, see Ref. [2]. The present wave of interest in mirror particles has been to a great extent initiated by the quest for neutron–mirror-neutron oscillations (n–n'). It has been conjectured that n–n' oscillations may play an important role in the propagation of ultra-high-energy cosmic rays and that the oscillation time τ_{osc} may be as small as $\tau_{osc} \sim 1$ s [3]. Implications of mirror particles for cosmology and astrophysics are discussed in a number of papers, e.g., Ref. [4]. Last year the first experimental data on n–n' transitions were published with the results $\tau_{osc} \geqslant 103$ s [5] and $\tau_{osc} \geqslant 414$ s [6]. Possible laboratory experiments to search for n–n' oscillations are discussed in Ref. [7].

Experimental results [5,6] were obtained using the ultracold neutrons (UCN), i.e., neutrons with energy $E < 10^{-7}$ eV stored in a trap. Previously a similar experimental setup was used in the search for neutron–antineutron oscillations (see Ref. [8] and references therein). The crucial difference between n–n' and n–$\bar{n}$ oscillations is that n' freely escapes from the trap while $\bar{n}$ either annihilates on the trap walls or gets reflected. Therefore the formalism developed for n–$\bar{n}$ oscillations cannot be adjusted to treat n–n' transitions. Still the two processes have a common point. This is the problem of a correct quantum mechanical description of the UCN wave function. Most often it is assumed that the wave function of the bottled UCN corresponds to a stationary state of a particle inside a potential well [9,10]. Alternatively, other authors [11] describe oscillations of the trapped neutrons on the basis of the free plane waves. Both pictures do not correspond to the physics of real experiments. The process proceeds in time in three stages.

At the first stage the filling of the trap takes place, then the neutrons are kept inside the trap during the storage time (hundreds of seconds), and finally the neutrons leave the trap to the detectors. Therefore the wave function undergoes a complicated evolution that can hardly be described without resorting to approximations. We first evaluate the neutron–mirror-neutron oscillations using a stationary wave function as the initial state wave function. Then we do the same using a wave packet instead of a stationary wave function.

The article is organized as follows. We start in Sec. II with an analysis of the oscillations in the stationary wave function approach. Transitions take place from one of the trap eigenstates. In Sec. III transitions are considered in the presence of a superimposed magnetic field. A general equation for the transition rate is derived and the limits of weak and strong fields are considered. Sec. IV is devoted to the wave packet formalism. The evolution of the UCN wave packet is encoded using the trap Green's function. The neutron–mirror-neutron transition rate is calculated. In Sec. V the main conclusions are presented and open problems are formulated. The Appendix contains a comparison between the infinite and the finite well models.

II. STATIONARY WAVE FUNCTION APPROACH

The problem of neutron–mirror-neutron oscillations in free space can be solved by diagonalization of the time-dependent two-channel Schrodinger equation with the result [12]

$$|\psi_{n'}(t)|^2 = \frac{4\varepsilon^2}{\omega^2 + 4\varepsilon^2} \exp(-\Gamma_\beta t) \sin^2\left(\frac{1}{2}\sqrt{\omega^2 + 4\varepsilon^2} t\right), \quad (1)$$

where $\omega = E_n - E_{n'} = |\mu_n| B$ is the energy difference between the neutron and the mirror neutron due to the superimposed magnetic field (the mirror neutron does not interact with "our" magnetic field), $\varepsilon = \tau_{osc}^{-1}$ is the mixing parameter, and Γ_β is the neutron β-decay width. In arriving at Eq. (1) the spatial part of the wave function was factored out making use of the fact that in free space the wave functions of n and n' are of the same form. In the trap, however, the situation is different: the neutron is confined while for the mirror neutron the trap walls do not exist. As already mentioned in the Introduction, the description of the trapped UCN is a nontrivial problem. The naive guess would be that inside the trap the neutron wave function corresponds to a discrete eigenstate. Here we assume that the neutron wave function is that of a particle in a potential well with the boundary conditions corresponding (in the first approximation) to a complete reflection.

To make calculations tractable and transparent we consider the following simple model of a trap. Let it be a one-dimensional square well of width $L = 1$ m with walls at $x = 0$

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and $x = L$, i.e., the potential of the form

$$U(x) = \begin{cases} V, \; x < 0 \\ 0, \; 0 < x < L \; . \\ V, \; x > L \end{cases} \quad (2)$$

The height of the potential well depends on the material it is made of with the typical value $V = 2 \cdot 10^{-7}$ eV, which is used in the following calculations. For such a well the limit for stored UCN velocity is 6.2 m/s. The number of discrete levels in such a trap can be estimated as

$$M \simeq \frac{L\sqrt{2mV}}{\pi} \simeq \frac{10^8}{\pi}. \quad (3)$$

We choose the UCN energy to be $E = 0.8 \cdot 10^{-7}$ eV. This energy corresponds to a level with quantum number $j \simeq 2 \cdot 10^7$. Positions and eigenfunctions of such highly excited states in a finite-depth potential are very close to the same quantities in the infinite well (except for the levels close to the upper edge of the well; we do not consider such levels). The finite-depth corrections are considered in the Appendix. The eigenvalues and eigenfunctions for the infinite well are

$$E_j = \frac{\pi^2 j^2}{2mL^2}, \quad k_j = \frac{\pi j}{L}, \quad j = 1, 2, 3 \cdots \quad (4)$$

$$\varphi_j(x) = \sqrt{\frac{2}{L}} \sin k_j x. \quad (5)$$

Another important quantity characterizing highly excited states is the classical frequency $\omega_{\rm cl}$ [11]

$$\omega_{\rm cl} = \frac{\pi^2}{mL^2} j = \frac{2\pi}{\tau_{\rm cl}} = \delta E_j, \quad (6)$$

where $\tau_{\rm cl}$ is the time of the classical period and

$$\delta E_j = E_{j+1} - E_j \simeq 0.8 \cdot 10^{-14}\,{\rm eV}$$

is the level spacing. Levels with $j \gg 1$ are almost equidistant. In the semiclassical limit we may also define the trap crossing time τ,

$$\tau = \frac{\tau_{\rm cl}}{2} = \frac{mL}{k_j} \simeq 0.26 \text{ s}, \quad (7)$$

for $j = 2 \cdot 10^7$. Next we calculate the rate of $(n\text{–}n')$ oscillations for the neutron at the jth discrete level. The neutron and mirror neutron wave functions in a two-component basis are

$$\tilde{\varphi}_j(x) = \sqrt{\frac{2}{L}} \sin k_j x \begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv \varphi_j(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (8)$$

$$\tilde{f}_p(x) = \frac{1}{\sqrt{2\pi}} e^{ipx} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv f_p(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (9)$$

where $-\infty < p < +\infty$. The $(n\text{–}n')$ system is described by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{W} = \begin{pmatrix} \frac{k^2}{2m} + U & 0 \\ 0 & \frac{p^2}{2m} \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon \\ \varepsilon & 0 \end{pmatrix}. \quad (10)$$

The states (8) and (9) are the eigenstates of $\hat{H}_0$; therefore it is convenient to use the interaction representation. The probability of finding at time t a mirror neutron instead of a neutron reads

$$P_{nn'} = \int dp |\langle \tilde{f}_p| \exp\left\{-i \int_0^t dt' \hat{W}_{\rm int}(t')\right\} |\tilde{\varphi}_j\rangle|^2, \quad (11)$$

where $\hat{W}_{\rm int}(t) = e^{i\hat{H}_0 t} \hat{W} e^{-i\hat{H}_0 t}$. In the first order of perturbation theory we get

$$P_{nn'} = \int dp |\langle \tilde{f}_p| \int_0^t dt' \hat{W}_{\rm int}(t') |\tilde{\varphi}_j\rangle|^2$$
$$= \varepsilon^2 \int dp \left| \int_0^t dt' e^{-i(E_j - E_p)t'} \right|^2 |\langle f_p|\varphi_j\rangle|^2, \quad (12)$$

where $E_j = \frac{k_j^2}{2m}$, $\quad E_p = \frac{p^2}{2m}$. The time-dependent integral is a standard one,

$$w(E_p) = \left| \int_0^t dt' e^{-i(E_j - E_p)t'} \right|^2 = \frac{4 \sin^2\left[\frac{(E_p - E_j)t}{2}\right]}{(E_p - E_j)^2}. \quad (13)$$

The overlap of the wave functions reads

$$g_j(p) = |\langle f_p|\varphi_j\rangle|^2 = \frac{4k_j^2}{\pi L \left(p^2 - k_j^2\right)^2} \sin^2\left(\frac{pL + \pi j}{2}\right),$$
$$j = 1, 2, \ldots. \quad (14)$$

From Eqs. (12), (13), and (14) we obtain

$$P_{nn'} = \varepsilon^2 \int_{-\infty}^{+\infty} dp g_j(p) w(E_p). \quad (15)$$

It is convenient to change the integration from dp to dE_p taking into account that $g(p) = g(-p)$.

Then

$$P_{nn'} = 2m\varepsilon^2 \int dE_p \frac{g(E_p) w(E_p)}{p}, \quad (16)$$

where the factor 2 comes from the fact that two plane waves $e^{\pm ipx}$ correspond to the same energy E_p. Both functions $g(E_p)$ and $w(E_p)$ peak at $E_p = E_j$. According to Eqs. (14) and (13) the widths ΔE_p^g and ΔE_p^w of the corresponding maxima are

$$\Delta E_p^g \simeq \pi/\tau, \quad \Delta E_p^w \simeq 4\pi/t, \quad (17)$$

with τ being the trap crossing time. At times $t \gg \tau$ we may substitute by $g(E_p)/p$ its value at $p = k_j$ and take it out of the integral (16). From Eq. (14) one gets $g(E_j) = L/4\pi$. The remaining integration in Eq. (16) can be extended to $(-\infty < E_p < +\infty)$, yielding $2\pi t$. Collecting all pieces together we obtain

$$P_{nn'} = \varepsilon^2 \tau t. \quad (18)$$

At very short times $t \ll \tau$ the function $w(E_p)$ becomes smoother than $g(E_p)$. Hence $w(E_p)$ can be taken out of the integral (16). The remaining integral is time independent while $w(E_p) \sim t^2$. As a result $P_{nn'} \sim \varepsilon^2 t^2$ and we cannot define the transition probability per unit time [12]. On the other hand, Eq. (18) is valid only for times shorter than the neutron β-decay time t_β because we defined the eigenstate (8) neglecting the β decay. The condition $\tau \ll t \ll t_\beta$ has been satisfied with fair accuracy in experiments [5,6].

III. STATIONARY APPROACH WITH MAGNETIC FIELD INCLUDED

The search for n–n' oscillations in experiments with bottled UCN is based on the comparison of UCN storage with and without a superimposed magnetic field [5,6]. It is assumed that there is no mirror magnetic field in the laboratory and therefore the interaction of the neutron with the magnetic field lifts the degeneracy and thus suppresses the oscillations.

In magnetic field B the energy of the trapped neutron becomes equal to

$$E_j = \frac{k_j^2}{2m} + \mu B, \tag{19}$$

where $\mu = -\mu_n = 1.91\mu_N(\mu_N = e/2m_p)$.

Inclusion of the magnetic field does not alter the functions $w(E_p)$ and $g(p)$ given by Eqs. (13) and (14). There is, however, an important difference between our present considerations and those of the previous section. As we see from Eq. (19), $w(E_p)$ now peaks at $p = \pm\sqrt{k_j^2 + 2m\mu B}$ while the maximum of $g(p)$ is as before at $p = \pm k_j$. As a result instead of Eq. (16) we obtain

$$P_{nn'} = \frac{4\varepsilon^2 t}{(\mu B)^2 \tau \sqrt{1 + \frac{2m\mu B}{k_j^2}}} \times \begin{cases} \cos^2 \frac{k_j L}{2}\sqrt{1 + \frac{2m\mu B}{k_j^2}}, & j = 1, 3, \ldots \\ \sin^2 \frac{k_j L}{2}\sqrt{1 + \frac{2m\mu B}{k_j^2}}, & j = 2, 4 \cdots \end{cases} \tag{20}$$

This equation can be simplified by taking into account that the quantities (μB) and $k_j^2/2m$ differ by many orders of magnitude. In Refs. [5] and [6], the value of the magnetic field varied in the interval $(1-2)nT \leqslant B \leqslant (\text{few})\,\mu T$, which corresponds to 10^{-16} eV $\lesssim \mu B \lesssim 10^{-13}$ eV, while $k_j^2/2m \simeq 10^{-7}$ eV (note that the unshielded Earth magnetic field corresponds to $\mu B \simeq 3 \cdot 10^{-12}$ eV $\ll \frac{k_j^2}{2m}$).

Therefore Eq. (20) easily reduces to

$$P_{nn'} \simeq 4\varepsilon^2 \frac{t}{\tau} \frac{\sin^2\left(\frac{1}{2}\mu B \tau\right)}{(\mu B)^2}, \tag{21}$$

where τ is the trap crossing time. For our model of the trap described in Sec. II we have $\tau/2 \simeq 2 \cdot 10^{14}$ eV^{-1}. Therefore in the limit of the weak magnetic field $B \simeq nT$, Eq. (21) yields

$$P_{nn'} \simeq \varepsilon^2 \tau t, \tag{22}$$

as expected [see Eq. (18)]. In the opposite limit of strong magnetic field $B \simeq$ (few) μT we have to take into account that the quantities τ and B in Eq. (21) experience fluctuations leading to rapid oscillations of the function $\sin^2(\frac{1}{2}\mu B\tau)$. In particular, the crossing time τ may vary because of either changes in L at each collision or variations in the neutron velocity. Substituting the rapidly oscillating quantity in Eq. (21) by its mean value equal to 1/2 we obtain the equation describing the neutron–mirror-neutron transitions in the strong magnetic field,

$$P_{nn'} = \varepsilon^2 \frac{2t}{(\mu B)^2 \tau}. \tag{23}$$

IV. THE WAVE PACKET APPROACH

We now turn to the question formulated in the Introduction, namely, to the problem of the UCN wave function evolution and to the calculation of the oscillations in the wave packet approach. To get physically transparent results and to avoid numerical calculations suited to a concrete experiment we assume that the UCN coming to the trap from the source are described by the Gaussian wave packets [13].

The wave packet moving from the left and for $t = 0$ centered at $x = 0$ is given by the expression

$$\Psi_k(x, t=0) = (\pi a^2)^{-1/4} \exp\left\{-\frac{(x-x_0)^2}{2a^2} + ikx\right\}, \tag{24}$$

where a is the width of the wave packet and k is its central momentum. The normalization of the wave packet (23) corresponds to one particle in the entire one-dimensional space,

$$\int_{-\infty}^{+\infty} dx\, |\Psi_k(x, t=0)|^2 = 1. \tag{25}$$

Let the UCN energy be equal to the value chosen in Sec. II, $E = 0.8 \cdot 10^{-7}$ eV, and let the beam resolution be equal to $\Delta E/E = 10^{-3}$. Thus the set of parameters to be used is[1]

$$\begin{aligned} E &= 0.8 \cdot 10^{-7}\ \text{eV}, \\ \lambda &= \frac{2\pi}{k} \simeq 10^{-5}\ \text{cm}, \\ a &\simeq 3.2 \cdot 10^{-3}\ \text{cm}. \end{aligned} \tag{26}$$

The condition $a \gg \lambda$ ensures the localization of the wave packet. Note that the above value of E corresponds to the level E_j with a very high quantum number $j \simeq 2 \cdot 10^7$. Next we estimate the number of levels within ΔE. One has

$$\Delta j = \frac{\Delta E}{\omega_{\text{cl}}} = \frac{v(\Delta k)}{\omega_{\text{cl}}} = \frac{L}{\pi a} \simeq 10^4. \tag{27}$$

The large number of levels forming the wave packet is a necessary condition for the trapped wave packet to be localized (in free space this condition reads $a \gg \lambda$, see above). The time evolution of the initial wave packet (24) proceeds according to the following law,

$$\Psi_k(x, t) = \int dx' G(x, t; x', 0)\Psi_k(x', 0), \tag{28}$$

where $G(x, t; x', t')$ is the trap Green's function. In the infinite well approximation we may use the spectral decomposition of the Green's function over the set of eigenfunctions (5) and write

$$\Psi_k(x, t) = \sum_{j=1}^{\infty} e^{-iE_j t} \varphi_j(x) \int_0^L dx' \varphi_j^*(x')\Psi_k(x', 0). \tag{29}$$

[1]The problem of the choice of the wave packet parameters is addressed in the next section.

The width of the wave packet (29) increases with time according to

$$a' = a\left[1+\left(\frac{t}{ma^2}\right)^2\right]^{1/2} \simeq a\left(\frac{t}{ma^2}\right),$$

where for our model the spreading time is $ma^2 \simeq 1.7 \cdot 10^{-2}$ s and $t/ma^2 \simeq 60t/s$. A so-called collapse time t_c [14] corresponds to $a' = L$ and is equal to $t_c \simeq 500$ s. At $t = t_c$ the wave packet spreads uniformly over the entire well and the stationary regime considered in Sec. II sets in —see [15]. We note in passing that there is another time scale in the problem, the so-called revival time $t_{\rm rev} = 4mL^2/\pi \simeq 2 \cdot 10^7$ s when the wave packet regains its initial shape (see Ref. [14] and references therein).

The initial wave packet $\Psi_k(x,0)$ contains only a right running wave [see Eq. (24)]. The trapped wave packet (29) contains both right and left running waves; i.e., it correctly describes reflections from the trap walls. We assume that the point x_0 [see Eq. (24)] is not in the immediate vicinity of the trap walls; i.e., x_0 is at least a few times of a away from the walls. Then the integration in Eq. (29) can be extended to the entire one-dimensional space. This yields

$$F(k,k_j;L,a,x_0) \equiv \int_{-\infty}^{+\infty} dx' \varphi_j^*(x')\Psi_k(x',0) = i\left(\frac{a\sqrt{\pi}}{L}\right)^{1/2} \times \left\{\exp\left[-\frac{a^2(k-k_j)^2}{2} + i(k-k_j)x_0\right] - \exp[\cdots k_j \to -k_j \cdots]\right\}. \tag{30}$$

Then we can calculate the transition probability $P_{nn'}$ following the procedure described in Sec. II. Instead of the wave function (8) we now have

$$\Psi_k(x,t) = \sum_{j=1}^{\infty} e^{-iE_jt}\varphi_j(x)F_j(k), \tag{31}$$

with $F_j(k)$ being the shorthand notation for the function $F_j(k,k_j;L,a,x_0)$ defined by Eq. (30). The normalization condition for $F_j(k)$ reads

$$\sum_j |F_j(k)|^2 = 1. \tag{32}$$

In line with Eq. (15) and following the arguments presented after Eq. (16), we write

$$P_{nn'} = \varepsilon^2 \sum_{j,l} F_j(k)F_l^*(k)e^{\frac{i}{2}(E_l-E_j)t} \times \int_{-\infty}^{+\infty} dp \left[\frac{2\sin\frac{(E_p-E_j)t}{2}}{(E_p-E_j)}\right] \times \left[\frac{2\sin\frac{(E_p-E_l)t}{2}}{(E_p-E_l)}\right]\langle f_p|\varphi_j\rangle\langle\varphi_l|f_p\rangle. \tag{33}$$

Consider first the contribution $P_{nn'}^{(1)}$ of the diagonal terms with $j = l$. We have

$$P_{nn'}^{(1)} = \varepsilon\sum_j |F_j(k)|^2 \int_{-\infty}^{+\infty} dp \frac{4\sin\frac{(E_p-E_j)t}{2}}{(E_p-E_j)^2}\langle f_p|\varphi_j\rangle\langle\varphi_j|f_p\rangle = \varepsilon^2 \sum_j |F_j(k)|^2 \frac{2m}{k_j}2\pi t\frac{L}{4\pi} = \varepsilon^2\langle\tau\rangle t, \tag{34}$$

with $\langle\tau\rangle$ being the weighted crossing time

$$\langle\tau\rangle = \sum_j |F_j(k)|^2\tau(k_j), \tag{35}$$

and $\tau(k_j) = mL/k_j$. Next we turn to the contribution $P_{nn''}^{(2)}$ of the nondiagonal terms in Eq. (32). In this case we are dealing with a two-hump function with maxima at $E_p = E_j$ and $E_p = E_l$. Therefore we may write

$$P_{nn'}^{(2)} = 4\pi m\varepsilon^2 \left\{\sum_j \frac{F_j(k)}{k_j}\sum_l F_l^*(k)e^{\frac{1}{2}(E_l-E_j)t} \times \left[\frac{2\sin\frac{(E_j-E_l)t}{2}}{(E_j-E_l)}\right]\langle f_j|\varphi_j\rangle\langle\varphi_l|f_j\rangle + (j \leftrightarrow l)\right\}. \tag{36}$$

Replacing summation over l by integration we obtain

$$P_{nn'}^{(2)} \simeq 8\varepsilon^2 \sum_j |F_j|^2\frac{m^2L^2}{k_j^2} = 8\varepsilon^2\langle\tau^2\rangle, \tag{37}$$

where

$$\langle\tau^2\rangle = \sum_j |F_j|^2\tau_j^2. \tag{38}$$

Collecting the two contributions (34) and (37) together we get the final result

$$P_{nn'} = \varepsilon^2\langle\tau\rangle t\left(1 + 8\frac{\langle\tau^2\rangle}{\langle\tau\rangle t}\right). \tag{39}$$

V. CONCLUSIONS

We have calculated the rate of neutron–mirror-neutron oscillations for trapped UCN. Two types of UCN wave functions were used: the stationary solution for a particle inside a potential well and the Gaussian wave packet. Calculations were performed in the first-order perturbation theory. This approximation is legitimate provided $P_{nn'} \ll 1$. From Eqs. (22) and (23) it follows that this condition holds for $\tau_{nn'} \gg 7$ s and $\tau_{nn'} \gg 0.1$ s in the weak and strong magnetic fields, respectively. Obviously the first-order perturbation theory describes the transition of the neutron into the mirror neutron. The inverse process appears only in the second order in ε. For the analysis of the experiments in Refs. [5] and [6], the first-order perturbation theory is a fair approximation.

The above analysis has been performed for a simple one-dimensional trap. We think that such a model correctly describes the principal features of the process. Generalization to the three-dimensional rectangular trap is trivial. The simplest way to generalize our result to the trap with arbitrary geometry is to substitute the crossing time τ with the effective crossing time corresponding to a given trap geometry.

Experimental data [5,6] were analyzed using the free space equation (1) with the time t being limited by the crossing time τ.

Equation (1) contains only time dependence because the spatial parts of n and n' wave functions were factored out using the fact that in free space the coordinate wave functions of n and n' have the same form. For bottled UCN the situation is different. The neutron is confined inside the trap while the mirror neutron freely crosses the trap walls. Therefore the use of Eq. (1) to describe oscillations of trapped UCN seems questionable.[2] However, our accurate approach justifies the analysis of the experimental data based on Eq. (1) [5,6]. This can be explained by the semiclassical character of the UCN motion inside the trap of macroscopic size. In the stationary approach the typical UCN energy corresponds to the states with $j \gg 1$, i.e., to the semiclassical part of the spectrum. In the wave packet formalism the classical limit corresponds to $(\Delta j) \sim (j)^{1/2} \to \infty$ [16]. The wave pacets (24) and (26) are close to this limit. For UCN with extremely low energy, $E \lesssim 10^{-16}$ eV, the oscillation pattern changes. We shall consider this question elsewhere. The fraction of UCN with such energies in experiments is negligible. Another point that deserves a dedicated study is decoherence of the UCN wave function and subsequent randomization of the oscillation process. This might occur because dephasing of the wave function caused by collisions with the trap walls and with the residual gas.

ACKNOWLEDGMENTS

We thank A. P. Serebrov who drew our attention to the problem and with whom B.K. had numerous enlightening discussions. Useful remarks were gained from L. B. Okun, I. B. Khriplovich, A. Gal, O. M. Zherebtsov, A. I. Frank, and V. A. Novikov. B.K. thanks Yuri Kamyshkov for the hospitality and support extended at the International Workshop on B-L Violation at Berkeley in September of 2007. B.K. also thanks V. A. Gordeev and the organizers of the XLII St. Petersburg Winter School where a preliminary presentation of this work was made. Financial support from Grants RFBR 06-02-17012, NS-4961.2008.2, NS-4568.2008.2, RFBR 07-02-00830, and RFBR-08-02-00494 is gratefully acknowledged. O.L. is also grateful to the Dynasty Foundation for financial support.

APPENDIX

The calculations presented above were performed for the infinite well model of a trap. Here we consider the finite potential and show that there is only a minor difference between the two models. Consider the potential well defined by Eq. (2). Matching the logarithmic derivatives of the wave functions at $x = 0$ and $x = L$ we obtain the eigenvalue equation

$$k'_j L = \pi j - 2 \arcsin \frac{k'_j}{\sqrt{2mV}} \tag{A1}$$

(the notation k_j is kept for $k_j = \pi j/L$). The small parameter in the problem is

$$\delta = \left(\frac{2}{mVL^2}\right)^{1/2} \simeq 2 \cdot 10^{-8}. \tag{A2}$$

Expanding Eq. (A1) with respect to δ, we obtain

$$k'_j \simeq \frac{\pi j}{L}(1-\delta), \quad E'_j \simeq \frac{\pi^2 j^2}{2mL^2}(1-2\delta). \tag{A3}$$

Therefore, the levels in the finite well are shifted relative to the infinite well levels by

$$E_j - E'_j \simeq 4 \cdot 10^{-15} \text{ eV}. \tag{A4}$$

From Eq. (A3) it follows that the spectrum in the finite well (2) is the same as in the somewhat wider infinite well

$$L' = L(1+\delta). \tag{A5}$$

In the finite well the wave function penetrates into classically forbidden regions inside the trap walls. However, neutron–mirror-neutron transitions inside the walls may be neglected because both the penetration depth d and the collision time τ_{coll} are small: $d \sim 10^{-6}$ cm, $\tau_{\text{coll}} \sim 10^{-8}$ s.

[2] To describe the free space experiments by Eq. (1) one still has to supplement it by a boundary condition at $x = 0$ where the reactor is placed. Otherwise at $t = \pi\tau_{nn'}/2$ the reactor becomes a source of mirror neutrons. We are grateful to L. B. Okun and M. I. Vysotsky for drawing our attention to this.

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