Prompt proton decay and deformed bands in ⁵⁶Ni

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High-spin states in the doubly magic N = Z nucleus ⁵⁶Ni have been investigated with three fusion-evaporation reaction experiments. New γ -ray transitions are added, and a confirmation of a previously suggested prompt proton decay from a rotational band in ⁵⁶Ni into the ground state of ⁵⁵Co is presented. The rotational bands in ⁵⁶Ni are discussed within the framework of cranked Nilsson-Strutinsky calculations.

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I. INTRODUCTION

The experimentally observed shell gaps associated with the magic numbers are important building blocks within nuclear structure. These are well described by the nuclear shell model. In this model, the N = Z = 28 nucleus ⁵⁶Ni is doubly magic. To describe the shell gap at N = Z = 28, it is necessary to include the spin-orbit force in the nuclear mean field potential. It causes a splitting within the fp shell: the energetically favored $j = \ell + s$ orbit $1f_{7/2}$ separates from the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbitals, also called the upper fp shell, thus creating the shell gap at N = Z = 28.

The experimentally observed spherical energy levels in ⁵⁶Ni are very well described by shell-model calculations [1,2] using modern-day interactions. Furthermore, two rotational bands in ⁵⁶Ni are known [3]. The first band is readily explained as being built by four-particle–four-hole excitations (4p-4h) from the $1f_{7/2}$ orbit into the upper fp orbitals. It is accurately predicted by shell-model calculations [2,3] and by Monte Carlo shell-model calculations [4]. States at even higher excitation energies, however, are formed by exciting nucleons into the next orbital, the $1g_{9/2}$ subshell. It is not straight forward to include the $1g_{9/2}$ orbital in the spherical shell-model calculations [5], leading to an inability of conventional shell-model calculations to describe certain classes of excited states and the evolution of collectivity at high spins. Indeed, the second rotational band observed

in ⁵⁶Ni is interpreted to involve excitations into the $1g_{9/2}$ orbital. Previously, cranked Hartree-Fock and Hartree-Fock-Bogoliubov methods with Skyrme interaction [3] as well as cranked Nilsson-Strutinsky calculations [6] have successfully described the second experimental rotational band, along with Monte Carlo shell-model calculations using the full fp shell including the $1g_{9/2}$ orbital [7].

For neutron-deficient nuclei in the mass $A \sim 60$ region, proton emission can compete with ordinary γ decay because of the low Coulomb barrier. Proton emission has been observed from well-deformed excited rotational bands, for instance, in ⁵⁸Cu [8]. In this decay, often referred to as the prompt proton decay, the initial state is well deformed, and through the emission of a monoenergetic proton, spherical states in the daughter nucleus are populated. In Ref. [3], weak evidence of a prompt proton decay from the second rotational band in ⁵⁶Ni was reported. This article presents firm confirmation of the proton emission.

Fusion evaporation reactions have previously been utilized to study ⁵⁶Ni. Spherical yrast and yrare states with energies up to $E_x \sim 15$ MeV and spin and parity of $I^{\pi} = 14^+$ were observed [1,3]. The present experimental data comprise the statistics from several similar experiments (Sec. II). The focus of the analysis is to investigate the high-spin rotational bands (Sec. III A) and the suggested prompt proton decay (Sec. III B) in ⁵⁶Ni. The experimentally observed states are discussed in the framework of the cranked Nilsson-Strutinsky approach in Sec. IV, adding some new information to the results previously published [6].

II. EXPERIMENTS AND DATA ANALYSIS

Excited states in ⁵⁶Ni were produced in three different experiments, which are summarized in Table I. Experiment 1 was performed at Lawrence Berkeley National Laboratory and formed the basis for the results on ⁵⁶Ni in Ref. [3].

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TABLE I. Details of the fusion-evaporation reaction experiments that are the basis of the present work. The lower part indicates the number of detectors.

Experiment	GS54 1	GSFMA42 2	GSFMA138 3	
Beam	³⁶ Ar	³⁶ Ar	³⁶ Ar	
Target	²⁸ Si	²⁸ Si	²⁸ Si	
E_{beam} (MeV)	143	148	142	
Gammasphere	82	86	77	
Microball	complete	65 elements	16 elements	
Neutron shell	15	20	30	
LuWuSiA	_	wall	box + wall	
No. of pixels	_	1024	2048	
FMA	_	_	yes	
Ion chamber	_	_	yes	

Experiments 2 and 3 were performed at Argonne National Laboratory. The experiments utilized fusion-evaporation reactions in slightly inverse kinematics, all producing the compound nucleus ⁶⁴Ge. The residual nucleus to be studied here, ⁵⁶Ni, is obtained through evaporation of only two α particles. This implies (i) a low production cross section but (ii) ensures entry states at high excitation energies E_x and angular momentum I^{π} . The relative reaction cross section for production of ⁵⁶Ni is as low as some 0.02%.

The experimental setup employed the Microball detector [9] to detect and identify charged particles through pulse-shape discrimination techniques. In conjunction with Microball, the Lund Washington University Silicon Array (LuWuSiA) was used [8,10,11].

It consists of up to eight silicon strip $\Delta E \cdot E$ telescope detectors for detection of charged particles. Figure 1 displays the charged particle detectors for experiment 3. Four of the telescopes, forming the so-called box, are situated around the target position and cover angles between 40° and 120°. The four remaining telescopes covering the forward angles

 $(5^{\circ} < \Theta < 40^{\circ})$ form the wall. The angle Θ is given relative to the beam axis. The *E* detectors have active areas of 61×61 mm and are about 1 mm thick. Each *E* detector has 32 strips which are mutually combined to yield 16 electronic channels. The ΔE detectors are 50×50 mm in size and are about 65 μ m thin with 16 strips each. The signals from the ΔE and *E* detectors can be combined, thus creating pixels which are approximately 3×3 mm² in size. Experiment 2 utilized a wall similar to the one in Fig. 1, which had 1024 pixels. Further details can be found in Ref. [12]. In experiment 3, the maximum number of 2048 pixels for LuWuSiA was used. More details can be found in Refs. [10,11].

In experiments 1, 2, and 3, γ rays were detected by the Gammasphere array [13], comprising 86, 82, and 77 detectors, respectively. In experiments 2 and 3, the Heavimet collimators were removed from the Gammasphere detectors to provide γ -ray multiplicity and sum-energy measurements [14]. In experiment 3, the evaporated neutrons were detected by the neutron shell [15], and the fragment mass analyzer (FMA) [16] and ion chamber [17] were utilized to separate and identify different reaction products from each other.

In the analysis, discrimination between protons and α particles is vital in creating clean particle gated γ -ray spectra. Hence, each Mircoball event was associated with time, energy, and charge-ratio signals obtained through pulse-shape techniques. These signals were plotted in three two-dimensional spectra, and protons and α particles were identified only after fulfilling gate conditions in all three maps. For the LuWuSiA, unequivocal particle identification was obtained in three two-dimensional matrices over (i) the energy loss in the ΔE detectors versus the energy in the *E* detectors and (ii) the time in the ΔE (*E*) detector versus energy in the ΔE (*E*) detector.

Particles were identified if they fulfilled the gate conditions in all three maps. To maximize the number of used events in the data for experiments 2 and 3, different combinations of hits in the same LuWuSiA telescope were analyzed. The types of events considered are shown in Table II.



FIG. 1. (Color online) Charged particle detectors for experiment 3. The two remaining rings of the Microball detector [9] covering the backward angles are shown in yellow. The silicon $\Delta E(E)$ detectors of LuWuSiA are shown in purple (green). The beam enters from the left and hits the target, and the reaction products exit through the exit collimator to the right, into the fragment mass analyzer (FMA). The units are in mm.

For instance, the type of event called E in Table II, can occur if

- (i) one particle has sufficiently low energy to be fully stopped in the ΔE detector and another particle penetrates the ΔE detector and hits the *E* detector as well, or
- (ii) one of the particles has an energy smaller than the threshold energy for detection of the E detector, or
- (iii) one of the particles is scattered off the E detector, or
- (iv) by chance, the particles both penetrate the ΔE detector but hit the same *E* strip.

The challenge is to achieve an accurate particle identification for events other than type A. Also for types B-F, particle identification is based on the appropriate time and energy relations for a given particle type (i.e., proton or α). The conditions were determined by the signals for an A event, i.e., the same time and energy conditions were utilized on the other types of events. B and C events are easily treated as signals from protons, and α particles are rather well separated for the wall. For the box, the signals are not equally well separated. Events of type D were given special attention, as there are several ways of combining the two ΔE signals with the two E detector signals. For instance, one of the ΔE signals was combined with one of the E detector signals. If they matched the particle identification conditions in both the $\Delta E \cdot E$ energy matrix and the ΔE and E energy-time matrices, these signals were associated with each other, and the particle type was clearly identified. Events of type E (F) were first checked to see if the strips with hits in the $\Delta E(E)$ were neighboring strips. If so, the energy deposited in them was summed, and it was assumed that these events corresponded to the particle either hitting the middle of the strips or scattering from one to the other strip. Then these types of events could be treated as being of type A. If the strips were not neighboring, they were treated as a combination of a types A and B in the case of E, and for type F as a sum of types A and C. The different combinations were investigated, and if they fulfilled the appropriate conditions, the particle was identified. Otherwise, the event was disregarded.

The detection efficiency for protons in experiment 3 is 65% and for α particles 40%, including the various types of hits in Table II. Experiment 2 had slightly lower efficiencies, while experiment 1, utilizing the full Microball detector, had detection efficiencies of 80% for protons and 65% for α particles.

TABLE II. Different types of events considered for LuWuSiA in experiments 2 and 3. These events must, of course, occur within the same ΔE -E telescope, and the numbers indicate the number of strips with signals in them.

	А	В	С	D	Е	F
ΔE	1	1	_	2	2	1
Ε	1	_	1	2	1	2

The γ -ray energy resolution was optimized using an eventby-event kinematic reconstruction method to reduce the effect of the Doppler broadening due to the evaporated particles. The energy and direction of the charged particles was registered in either the Microball or LuWuSiA. It allows reconstruction of the momenta and direction of the recoiling residual nuclei, and hence it is possible to obtain the angle between the direction of the recoil and the germanium detectors of Gammapshere on an event-by-event basis. In turn, this permits a more precise Doppler correction of the γ -ray energies. This led to a significantly improved energy resolution of 13 keV for a γ -ray energy of 3 MeV in the $2\alpha 1p$ evaporation channel [18] in experiment 3. Performing a standard Doppler correction will result in a full width at half maximum (FWHM) of \sim 34 keV for the 3 MeV γ ray in the 2 α 1p channel, which can be substantially lowered by defining the kinematics with Microball. Interestingly, for experiment 3, the limiting factor for further improving the FWHM of the γ rays is the finite opening angle of the germanium detectors [18].

The events were sorted offline into various E_{ν} projections, E_{γ} - E_{γ} matrices, and E_{γ} - E_{γ} - E_{γ} cubes subject to the appropriate evaporated particle conditions. The cube was analyzed using the Radware software package [19]. In parallel, E_{γ} - E_{γ} matrices gated on various ⁵⁶Ni γ rays were utilized. The main contaminating reaction channel in the γ -ray spectra are events for which one proton escaped detection. In the present case, this is the $2\alpha 1p$ channel leading to ⁵⁵Co. For every ⁵⁶Ni nucleus, about 30 ⁵⁵Co nuclei are produced. Another class of contaminants arose from genuine 2α events, but from undesired fusion evaporation reactions between the beam and oxygen contaminants in the target ${}^{36}\text{Ar} + {}^{16}\text{O} \rightarrow {}^{44}\text{Ti} + 2\alpha$, or carbon buildup on the target in the course of the experiments ${}^{36}\text{Ar} + {}^{12}\text{C} \rightarrow {}^{40}\text{Ca} + 2\alpha$. To decrease the influence of the most significant contaminant, ⁵⁵Co, appropriate E_{γ} - E_{γ} matrices as well as E_{γ} spectra were subtracted from the ⁵⁶Ni analysis objects. The spectrum analysis also employed the spectrum analysis code TV [20].

As the angular distribution of γ rays is symmetric with respect to the reaction plane, the spins and parities of the states were determined by utilizing yields measured by Ge detectors placed at different angles with respect to the beam axis. Some of the detectors make up a "pseudo"-ring placed at an average angle of 83° and other detectors can form a pseudo-ring at an effective angle of 30° [21]. Particle gated $\gamma\gamma$ matrices with γ rays detected at 30° (alternatively, 83°) versus γ rays detected anywhere in the array were generated for all the experiments. From these matrices, the intensity ratios

$$R_{30-83} = \frac{I_{\gamma}(30^{\circ})}{I_{\gamma}(83^{\circ})}$$

can be obtained [21]. These allow spins and parities for excited states to be determined because γ rays of different multipolarities have different R_{30-83} values. Stretched quadrupole transitions are predicted to have $R_{30-83} \sim 1.3$, whereas $R_{30-83} \sim 0.8$ indicates a stretched dipole transition. Significant deviations from the latter values indicate E2/M1 mixing. The statistics, though combined from three experiments, were too low to allow an analysis of directional correlations of oriented states (DCO ratios).



FIG. 2. Relevant high-spin part of the experimental level scheme of ⁵⁶Ni from the present work. The focus lies on the new γ -ray transitions at high excitation energy. The level scheme is obtained from the combined data of the $2\alpha 1 p$ and the 2α evaporation channels. The widths of the arrows are proportional to the relative intensities of the γ rays. The ground state and the first three excited states of ⁵⁵Co are also included, relative to the ground state of ⁵⁶Ni [25].

III. RESULTS

A. $\gamma\gamma$ coincidence analysis

The level scheme resulting from the present analysis is displayed in Fig. 2, including γ rays from the 2α and sometimes the $2\alpha 1 p$ reaction channel. It is necessary to include $2\alpha 1 p$ data to obtain the correct yield for one of the structures (see discussion below and Sec. III B). In Fig. 2, three structures can be seen; the yrast structure (marked "yrast"), the first rotational band (marked "SD1"), and the second rotational band (marked "SD2"). The number of observed γ rays and excited states has been extended with respect to Ref. [1]. The transitions and their placement in the level scheme have been determined by $\gamma \gamma$ coincidences, sum-energy relations of transitions, and their relative intensities. The relative intensities of the γ rays were adjusted and determined through the relation between the 1224 and 3626 keV transitions, which both feed the 2700 keV level. The latter state is set to only decay with one γ ray of 100% relative intensity. The

intensities of the remaining γ rays were determined with the help of the Radware program Xmlev [19], as it makes it possible to exclude contaminants through simultaneously applying several γ gates. Spins and parities of the states have been determined from the R_{30-83} ratios and yrast state considerations. The results are summarized in Table III.

The yrast band has been extended with a 3897 keV γ ray placed on top of the 14735 keV level. The γ ray can be seen along with some other previously known γ rays in ⁵⁶Ni in Fig. 3. Experimentally, the level at $E_x = 18632$ keV decaying with the 3897 keV γ ray cannot be given a definite spin-parity assignment, as the yield of the γ ray is too low. As this γ ray is clearly seen in Fig. 3 while no other new transitions could be found, it is likely that the level it proceeds from is the yrast 16⁺ level. Hence, a tentative assignment of spin and parity (16⁺) is suggested. This is concurrent with results from a state-of-the-art shell-model calculation using the code ANTOINE [22,23], with the GXPF1 interaction [24] allowing six active particles to be excited into the so-called upper fp

TABLE III. Energies of the excited states in ⁵⁶Ni, as well as the energies and relative intensities of the γ rays placed in the level scheme, their angular distribution ratios and multipole assignments, and to the very right, the spins and parities of initial and final states. The values are determined from the combined statistics in the 2α and $2\alpha 1 p$ channels.

$E_{\rm exc}$	E_{γ}	$I_{\rm rel}$	R_{30-83}	Mult.	I_i^{π}	I_f^{π}
(keV)	(keV)	(%)		ass.	(ħ)	$(\dot{\hbar})$
2700(1)	2700(1)	100(4)	1.3(1)	<i>E</i> 2	2^{+}	0^{+}
3924(1)	1224(1)	89(5)	1.2(1)	E2	4+	2^{+}
5316(1)	1392(1)	78(5)	1.2(1)	E2	6^{+}	4^{+}
5350(1)	2650(1)	3(1)	1.6(3)	$\Delta I = 0$	2^{+}	2^{+}
	5351(2)	5(1)	1.1(2)	E2	2^{+}	0^+
6326(1)	976(1)	8(2)	1.1(1)	E2	4+	2^{+}
	2402(1)	3(1)	_	$\Delta I = 0$	4+	4^{+}
	3626(1)	12(1)	1.4(3)	E2	4^{+}	2^{+}
7652(1)	1326(1)	23(4)	1.2(1)	E2	6^{+}	4^{+}
	3729(2)	1(1)	_	E2	6^{+}	4^{+}
7955(1)	2638(1)	36(4)	1.3(2)	E2	8^{+}	6^{+}
9309(2)	1657(1)	25(3)	1.1(1)	E2	8^+	6^{+}
9735(2)	845(2)	1(1)	_	(E2)	7	(5)
	2083(2)	1(1)	-	$(\Delta I = 1)$	7	6^+
9418(2)	1463(1)	28(3)	1.4(1)	E2	10^{+}	8^+
10935(2)	1626(1)	13(2)	0.8(1)	$\Delta I = 1$	9	8^+
	1200(1)	25(3)	1.2(2)	E2	9	7
11296(2)	1987(1)	13(2)	1.3(1)	E2	10^{+}	8^+
12358(2)	2940(1)	12(1)	1.2(3)	E2	12^{+}	10^{+}
12508(2)	1212(1)	3(1)	0.7(4)	-	11	10^{+}
	1573(1)	38(3)	1.5(1)	E2	11	9
13578(3)	2282(2)	11(2)	1.2(2)	E2	12^{+}	10^{+}
13644(2)	2349(2)	3(1)	_	-	(12 ⁺) ^a	10^{+}
	4226(2)	2(1)	-	-	(12 ⁺) ^a	10^{+}
14454(2)	1946(1)	35(5)	1.2(2)	E2	13	11
14735(2)	2377(2)	9(3)	1.2(3)	E2	14^{+}	12^{+}
16357(3)	2779(3)	3(1)	0.7(2)	$\Delta I = 1$	13	12^{+}
16772(3)	2318(2)	28(2)	1.2(1)	E2	15	13
18632(3)	3897(4)	<1	-	_	(16^{+})	14^{+}
19520(3)	2748(4)	14(2)	1.5(1)	E2	17	15
22458(3)	2938(4)	2(1)	-	-	_	(17)

^aData taken from Ref. [1].

shell. The theoretically predicted 16^+ level has an energy of 18839 keV. Hence the difference between the suggested experimental (16^+) level and the theoretical level is ~ 200 keV, which is smaller than the mean level deviation [1], thus supporting the tentative experimental assignment.

The 13644 keV level decays with two γ rays feeding the spherical yrast structure with a known 4226 keV γ ray and the SD1 band with a new γ ray of 2349 keV. This level has a tentative spin and parity of (12⁺), which is only 66 keV apart from the 12⁺, 13578 keV level in SD1. It is interesting to note that levels of the same spin and parity with similar energies should interact and displace each other by typically 200–300 keV.

The 2779 keV γ ray on top of SD1 has a R_{30-83} value of 0.7(2), which corresponds to a $\Delta I = 1$ transition. Hence, the 16357 keV level has a spin of I = 13, and the 2779 keV γ ray

is not a member of the rotational band SD1. In Ref. [3] this level was given a tentative spin-parity of (14^+) , but the present R_{30-83} value rules it out.

A new connecting transition between SD1 and SD2 has been found. It proceeds from the 12508 keV level in SD2 to the 11296 keV level in SD1. This transition has an intensity of 3(1) and an R_{30-83} value of 0.7(4). This value is consistent with an 11 \rightarrow 10⁺, $\Delta I = 1$ transition. Note that $\Delta I = 1$ between SD1 and SD2 is fixed via the R_{30-83} value of the 1626 keV γ ray.

On top of the rotational structure SD2, a γ ray of 2938 keV is added. Its R_{30-83} value cannot be determined because of low statistics. The γ -ray spectrum displaying the SD2 band can be seen in Fig. 4. It is noteworthy that only a small fraction of the intensity in the 2748 keV transition is seen in the 2938 keV transition. It is therefore suggested that the 19518 keV level is populated through several low-intensity γ rays (cf. Sec. IV).

B. Proton- γ coincidence analysis

An E_{γ} - E_{γ} matrix was created from the data taken in experiments 2 and 3, with the conditions that the γ -ray transitions are coincident with another γ ray of SD2 and belong to either the 2α or the $2\alpha 1p$ channel. In case of the $2\alpha 1 p$ channel, the proton had to have an energy smaller than 3.2 MeV. By selecting and summing the contribution from each γ -ray transition in SD2, the expected γ decay into SD1 via a γ ray of 1626 keV can be seen in the resulting spectra as in Fig. 4. The γ -decay sequence continues with the 1657 keV, 1326 keV γ rays until the ground state of ⁵⁶Ni is reached. However, an additional γ ray of 1200 keV is observed (cf. Fig. 4) that is not in apparent coincidence with any of the known low-lying γ -ray transitions of SD1 or the yrast structure. The summed intensity of the 1626 and 1200 keV γ rays accounts for the yield of the 1573 keV γ ray, see Table III. The R_{30-83} value of the 1200 keV γ -ray transition indicates an E2 character, it hence proceeds to an I = 7 level at 9735 keV. Moreover, two γ rays of 845 and 2083 keV are observed in weak coincidence with the 1200 keV γ ray. These were reported in Ref. [3], but even with the increased statistics in comparison to Ref. [3], these still remain tentatively assigned to ⁵⁶Ni.

Additionally, using only the data from the $2\alpha 1p$ channel, a proton energy E_p versus γ -ray energy E_{γ} matrix can be constructed with the demand that at least one additional γ ray in the event belonged to SD2. By selecting the γ rays of SD2 in this E_{γ} - E_p matrix, their proton spectrum can be created. The summed contribution from each γ ray is seen in the solid black curve in Fig. 5. For comparison, the proton energy spectra corresponding to the known $11/2^- \rightarrow 7/2^$ ground state transition in ⁵⁵Co is also included in the figure. This red dashed curve has a continuous energy distribution, as expected for evaporated protons. The former proton energy curve, on the other hand, has a clear peak at 2.54(3) MeV (cf. Fig. 5).

The masses of both ⁵⁶Ni and ⁵⁵Co are known [25], hence the energy of the proton can be calculated, taking into account



FIG. 3. Sum of spectra in coincidence with transitions in the yrast structure, 2700, 1224, 1392, 2638, 1463, 2940, and 2377 keV, which are marked by their energy in keV. The added transition of 3897 keV on top of the yrast structure is labeled as well. The peaks marked with stars are previously known γ rays in ⁵⁶Ni [1]. The used $\gamma \gamma$ matrix demands an additional coincidence with one of the 2700, 1224, 1392, 2638, and 1463 keV γ -ray transitions. The background was subtracted by a small portion of the contaminating $2\alpha 1 p$ channel. The resolution of the spectrum is 4 keV per channel.

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the energy of the I = 7 level in ⁵⁶Ni, that is,

$$E_p = E_x({}^{56}\text{Ni}) - Q_p \frac{M({}^{55}\text{Co})}{M({}^{56}\text{Ni})}$$

$$\approx E_x({}^{56}\text{Ni}) - (M({}^{55}\text{Co}) + m_H - M({}^{56}\text{Ni})) \frac{M({}^{55}\text{Co})}{M({}^{56}\text{Ni})}$$

$$\approx 2.53 \text{ MeV}.$$

which is consistent with the peak position in Fig. 5.

The FWHM of the proton peak is 250 keV. To optimize the resolution of the proton peak, an event-by-event kinematical correction was performed if the detected proton had an initial energy smaller than 3.2 MeV. The method is described in Ref. [8].

It is also possible to demand a coincidence with the proton peak, thus producing the corresponding γ -ray spectrum in



Fig. 6 from the E_{γ} - E_p matrix. In the figure, the sequence of the 1200, 1573, 1946, 2318, and 2748 keV γ rays is clearly seen. Noteworthily, virtually no other γ rays are distinguishable in the figure. These facts lead to the conclusion that the 9735 keV level in ⁵⁶Ni decays mainly by proton emission into the ground state of ⁵⁵Co.

In other observed prompt proton decays in the mass region, the proton is typically emitted as an $\ell = 4$, $1g_{9/2}$ proton, i.e., reducing the number of protons in this shape driving orbital. [26,27]. Supposing the same for the present case, the SD2 band should have negative parity.

IV. CRANKED NILSSON-STRUTINSKY INTERPRETATION

Previously, the near spherical states in ⁵⁶Ni were investigated using spherical shell-model calculations. The agreement

FIG. 4. Sum of spectra in coincidence with the 1573, 1946, 2318, and 2748 keV γ -ray transitions in a $\gamma\gamma$ matrix consisting of data from the 2α and the $2\alpha 1p$ channel and if at least one additional γ ray, with the energies mentioned above, was detected. The 2938 keV transition has been added on top of the rotational band. The tentative γ rays of 845 and 2083 keV are also labeled. The resolution of the spectrum is 4 keV per channel.



FIG. 5. (Color online) Proton center-ofmass energy spectrum created by the summed coincidence spectra of the transitions in SD2 (including the 1200 keV γ ray) in ⁵⁶Ni (black solid curve) and the ground state transition in ⁵⁵Co (red dashed curve). The resolution of the spectrum is 50 keV per channel. The proton peak has an energy of 2540(30) keV with a FWHM of 250 keV. See text for more details.

between experimental results and theoretical predictions is very good, with the exception of the 8_1^+ and the 8_2^+ states [1,2]. The rotational bands have been studied in mean field calculations with the Skyrme interaction [3] and also in the cranked Nilsson-Strutinsky (CNS) [6] approach, using the formalism in Refs. [28,29]. Here we will carry out some additional CNS calculations similar to those in Ref. [6] but using the new features presented in Ref. [30]. CNS calculations have successfully been performed for an ample amount of nuclei all over the nuclear chart, and they have been particularly helpful in classifying collective structures in the $A \sim 60$ region, such as ⁵⁹Cu [31],⁶⁰Ni [32],⁶¹Cu [33], and ⁶²Zn [34]. With recent developments of the CNS formalism [30], the total nuclear energy is calculated relative to a rotating liquid drop energy, which makes it straightforward to compare calculated and observed energies on an absolute scale. The rotating liquid drop energy is obtained from the Lublin-Strasbourg drop (LSD) [35] with a diffuse surface when calculating the rigid moment of inertia.

A description of the nucleus can be obtained from both the spherical shell model and in mean field calculations such as the cranked Nilsson-Strutinsky approach. A significant difference between the two is the model space and the nuclear interaction. In the shell-model calculations, it is necessary to restrict the model space and create an effective interaction suitable for the respective model space. In the CNS approach, the model space is practically unlimited, but only some aspects of the nuclear interaction are taken into account. The CNS calculations have been performed using the modified oscillator potential and a standard set of single-particle parameters [28].



FIG. 6. γ -ray energy spectrum obtained in coincidence with the proton peak at 2.54 MeV and any γ -ray transition in SD2 in an E_p - E_{γ} matrix for the $2\alpha 1 p$ channel. The labeled transitions all belong to ⁵⁶Ni, and no transitions belonging to ⁵⁵Co are present in the spectrum. The resolution of the spectrum is 4 keV per channel.

For a given configuration, the total energy is minimized with respect to two quadrupole parameters, ϵ_2 (elongation) and γ (nonaxiality), and one hexadecapole parameter, ϵ_4 . The effects of pairing interactions are disregarded, which implies that the results from the CNS calculations should agree better with experimental results at high spin than at lower spins. The configuration of the rotational bands is labeled in the usual short-hand notation for the $A \sim 60$ region. The high-*i* particles and holes are denoted $[p_1p_2, n_1n_2]$, where $p_1(n_1)$ is the number of proton (neutron) holes in the $1f_{7/2}$ subshell. The number of proton (neutron) particles in the $1g_{9/2}$ orbital is given by $p_2(n_2)$. Any other valence nucleons occupying the other fp shells $(2p_{3/2}, 1f_{5/2}, \text{ or } 2p_{1/2})$ are unlabeled. It should be noted that the *j* shells are not pure, i.e., the labeling refers to those *j* shells where the respective orbitals have their dominating amplitudes. In the numerical calculation, the major part of the mixing of the *j* shells in the mean field caused by deformation and rotation is taken care of [28]. It is common to classify high spin rotational bands with the quantum number signature α . It reveals how the system behaves when rotating it by the angle π . For nuclei with an even number of nucleons,

and

$$= 0 \quad \text{for} \quad I = 0, 2, 4, \dots$$

α

$$\alpha = 1$$
 for $I = 1, 3, 5, \dots$

The results of the present CNS calculation can be seen in Fig. 7. Figure 7(a) shows the energy of the experimentally observed bands, SD1 and SD2, with the rotating liquid drop energy removed [30], as a function of spin. In Fig. 7(b), selected predicted bands are shown. The calculated band, which is associated with the experimental SD1 band, has a



FIG. 7. (Color online) (a) Experimental rotational bands, SD1 and SD2. (b) Bands predicted by the CNS model. (c) Difference between the experimental and calculated band. See text for details.

[20,20] configuration, i.e., two proton and two neutron holes in the $1 f_{7/2}$ subshell, and four nucleons are excited into any of the upper fp orbitals. The configuration has a positive parity and signature, like the experimental SD1 band, consistent with Refs. [2-4,6]. The experimental SD2 band is best described by the calculated band with a [21,20] configuration, i.e., two proton and two neutron holes in the $1 f_{7/2}$ subshell, with three nucleons excited into the upper *fp* shell and finally one proton in the $1g_{9/2}$ orbital. This is noteworthy because observed prompt proton decays typically proceed from the $1g_{9/2}$ orbital (cf. III B). A [20,21] band with similar properties is calculated at a slightly higher energy. The observed band is built as a mixture of the [21,20] and [20,21] bands, but because no such mixing is included in the CNS approach, we will from now on only consider the [21,20] band. It has an negative parity and the signature $\alpha = 1$. Experimentally, the SD2 band is determined to have $\alpha = 1$, but the parity is undetermined. However, based on the excellent agreement with the CNS results, a negative parity is highly likely. This parity is also is in agreement with results from Refs. [3,6,7].

In Fig. 7(c), the difference between the experimental and theoretical bands is shown. If perfect agreement between the theoretical and experimental bands existed, then the curves should be situated at $\Delta E = 0$ for all spins. The agreement improves with increasing spin. This is due to the neglected pairing forces in the interaction. Even so, it appears that pairing is of minor importance in the observed bands of ⁵⁶Ni, because they are very well described by the calculation in their full spin range. Band [20,20] illustrates that if many particles occupy low-*j* shells, the energy cost to build high spin states is high, as the black solid curve in Fig. 7(b) is increasing considerably for increasing spin. On the other hand, band [21,20] with one proton in the high-j orbital $1g_{9/2}$ has a lower energy cost when creating angular momentum. The dashed red curve at first decreases for increasing spin, but then increases again. Note that the same number of valence nucleons are active in both bands. It is only their distribution that is different.

The shape trajectories in the (ϵ_2, γ) plane are plotted in Fig. 8. This shows how the deformation changes as a function of spin. Band [20,20] has a near prolate shape (collective rotation at $\gamma \sim 0^{\circ}$) at low spins. As the spin increases, the deformation decreases but remains essentially prolate. The [20,20] configuration terminates in a noncollective state at $I_{\text{max}} = 20^+$, where I_{max} is the maximum spin that can be built in the pure *j*-shell configuration. In the present case, $I_{\text{max}} = 2 \times 6 + 2 \times 4$, where 6 and 4 are the maximal spin contributions from two $f_{7/2}$ holes and two fp particles, respectively. In the [21,20] configuration, one proton is lifted from the fp orbitals to the lowest $g_{9/2}$ orbital, which means that the maximum spin is $I_{\text{max}} = 20 - 1.5 + 4.5 = 23$. However, the shape trajectory in Fig. 8 shows that this does not really terminate at this spin value. Instead, the calculated band can be followed to higher spins. In general, it is expected that rotational bands will not become noncollective when reaching I_{max} if the coupling between the N shells is too strong so that an appreciable amount of angular momentum can be built from this coupling. Experimental evidence for such a behavior was recently presented in the case of ⁷⁴Kr [36]. In the present case, it is, however, rather the direct coupling between one



FIG. 8. (Color online) Deformation trajectories calculated in the CNS model. The solid black curve shows how the deformation changes as a function of angular momentum for the configuration having no particles in the N = 4 shell (i.e., four valence protons and four valence neutrons of each signature in the N = 3 shell). The dashed red curve is drawn for the configuration with one $\alpha = -1/2$ proton excited from N = 3 to the favored $\alpha = 1/2$ orbital of the N = 4 shell. In the latter case, the lowest minimum at noncollective shape ($\gamma = 60$) for I = 23 is also indicated. This minimum is less than 100 keV above the absolute I = 23 minimum. At low spin values, the minimum corresponding to collective rotation at $\gamma \approx 0$ is chosen, and this minimum is then followed to termination. The solid line is equivalent to the trajectory for the [20,20] configuration, while it is only at spin values up to I = 21 that the trajectory drawn by the dashed line corresponds to the [21,20] trajectory. See text for more details.

 $f_{7/2}$ orbital and one fp proton orbital that becomes too strong at the high frequencies; i.e., it becomes essentially impossible to distinguish between the [21,20] and [31,20] configurations, which come very close in energy in the I = 17-21 range. Therefore, the shape trajectory in Fig. 8 has been drawn only with the constraint that there is one $g_{9/2}$ proton, and then the collective minimum starting at $\epsilon_2 \approx 0.3$, $\gamma \approx 0$ at low spin is followed. At I = 23, the minimum is found in the collective plane. Indeed, this minimum is very soft and extends to the noncollective axis where the energy is calculated less than 100 keV above the absolute minimum. The energy balance favoring a termination and the more collective behavior, respectively, will of course depend on parameters, so it is not really possible to predict the outcome if it becomes possible to observe this band to I = 23. Going beyond I = 23 at even higher rotational frequencies, it appears that the mixing between the $f_{7/2}$ and fp orbitals becomes even stronger so that more holes are effectively created in the $f_{7/2}$ orbitals leading to higher I_{max} values and a final termination at $I = 29^{-}$. Similar results were obtained in Ref. [6], and as shown in a diagram there, the terminating 29⁻ state is calculated high above yrast and therefore of no experimental interest.

It can also be seen in Fig. 8 that the [21,20] band is generally somewhat more deformed than the [20,20] band, which is expected because of one of the protons being placed in the deformation driving $1g_{9/2}$ orbital. In Ref. [31], an empirical formula for describing the average quadrupole deformation for

rotational bands in ⁵⁹Cu was derived, namely,

$$\epsilon_2 \approx 0.09 + 0.04q,$$

 $q = p_1 + p_2 + n_1 + n_2.$
(1)

A similar formula would be expected for ⁵⁶Ni, suggesting that band [20,20] ([21,20]) would have $\epsilon_2 \sim 0.25$ (0.29). Indeed, comparing these numbers to the results in Fig. 8, a good average agreement for the deformation exists for the experimentally observed spin range.

It is apparent, when comparing these results with those obtained for some midshell $A \sim 60$ nuclei studied with CNS [31,33], that their shape evolution is different from the CNS prediction for especially the [20,20] band in ⁵⁶Ni. Typically, the nuclei change from a prolate shape and gradually become noncollective oblate shaped when reaching termination. This is not the case for bands SD1 and SD2 in ⁵⁶Ni. As particles and holes have different shape driving properties, a state consisting of both holes and particles has counteracting shape driving components, which makes this state less favored energetically. A high-*j* particle (hole) with its full angular momentum will basically rotate around the equator of the nucleus in a circular orbit. This leads to an approximately oblate (prolate) mass distribution at termination. The fact that both bands in ⁵⁶Ni essentially remain prolate throughout all spins and terminate at close to spherical shape is a result of the counteracting shape driving forces of the equal number of holes and particles.



Similar shape driving properties but with slightly negative γ values instead have been calculated for the ground band of the midshell nucleus ⁴⁸Cr [37].

In Fig. 9, the energy of some additional calculated bands is displayed as a function of spin. Their CNS configurations are given as well. The solid (dashed) lines correspond to positive (negative) parity states and filled (open) symbols correspond to a signature of $\alpha = 0(1)$. The ⁵⁶Ni nuclei are produced in the experiment at spins $I \ge 20$. Included in Fig. 9 are ten bands which all have a rather low excitation energy for $I \simeq 20$. Although this is only a selection of all the bands that can be formed in the yrast region when combining possible proton and neutron configurations, it is still evident from the figure that it is possible to populate a number of different bands around spin 20. It is interesting to note that band SD1, which is best described by the [20,20] configuration, is yrast for spins smaller than 14. Experimentally it is observed up to spin 12^+ . Band SD2, which is best described by [21,20] is yrast up to spin 17, which is also the maximum spin identified from the experiment. One could note that the [21,20] configuration has a signature partner calculated at similar energies for I = 10 - 18 and clearly at lower energies for $I \ge 18$. Thus, it appears likely that it should be possible to observe this signature partner in some future experiment. For $I \approx 20$, many bands compete, which means that the intensity is fragmented over several states. The abundant number of possible bands can be the reason for the nonobservation of FIG. 9. (Color online) Theoretical bands calculated by the CNS model, which approach the yrast line for $I \approx 20$. As discussed in the text, it is sometimes difficult to distinguish between $f_{7/2}$ and fp particles. However, for spin values up to I = 23 covered in this figure, such difficulties might lead to uncertainties of 100–200 keV in some energies, but they will not influence the general features. The configurations assigned the SD1 band (black solid curve) and the SD2 band (red dashed curve) are yrast up to $I \approx 12$ and $I \approx 17$, respectively. These are the maximum spin values observed in these two bands. See text for more details.

more experimental rotational bands in ⁵⁶Ni, i.e., the γ -ray transitions from each band will in general have a very low intensity, until all the different bands decay into the respective yrast bands. A possible explanation for the 2938 keV γ ray on top of SD2 is an $18^+ \rightarrow 17^-$ decay proceeding from the [21,21] structure into SD2 ([21,20]).

V. SUMMARY

Through the present analysis, the level scheme of ⁵⁶Ni has been extended up to spin I = 18 and excitation energy $E_x \sim 22.5$ MeV. The two experimental rotational bands have been matched to theoretical predictions from cranked Nilsson-Strutinsky calculations. A prompt proton decay from one of the rotational bands in ⁵⁶Ni into the ground state of ⁵⁵Co has been firmly established.

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