Density-induced suppression of the α-particle condensate in nuclear matter and the structure of α-cluster states in nuclei

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At low densities, with decreasing temperatures, in symmetric nuclear matter α particles are formed, which eventually give raise to a quantum condensate with four-nucleon α-like correlations (quartetting). Starting with a model of α matter, where undistorted α particles interact via an effective interaction such as the Ali-Bodmer potential, the suppression of the condensate fraction at zero temperature with increasing density is considered. Using a Jastrow-Feenberg approach, it is found that the condensate fraction vanishes near saturation density. Additionally, the modification of the internal state of the α particle due to medium effects will further reduce the condensate. In finite systems, an enhancement of the S-state wave function of the center-of-mass orbital of α-particle motion is considered as the correspondence to the condensate. Wave functions have been constructed for self-conjugate $4n$ nuclei that describe the condensate state but are fully antisymmetrized on the nucleonic level. These condensate-like cluster wave functions have been successfully applied to describe properties of low-density states near the $n\alpha$ threshold. Comparison with orthogonality condition model calculations in ^{12}C and ^{16}O shows strong enhancement of the occupation of the S-state center-of-mass orbital of the α particles. This enhancement is decreasing if the baryon density increases, similar to the density-induced suppression of the condensate fraction in α matter. The ground states of ^{12}C and ^{16}O show no enhancement at all, thus a quartetting condensate cannot be formed at saturation densities.

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I. INTRODUCTION

The properties of nuclear matter at very low densities and low temperatures are dominated by the formation of clusters, in particular α particles. As a well-known concept, α matter has been introduced where symmetric nuclear matter is described by a system of α particles, weakly interacting via effective α-α potentials fitted to the scattering phase shifts, such as the Ali-Bodmer interaction potential [1–3].

This concept becomes less valid with increasing density. First, at finite temperatures other correlations and also single nucleon states appear so we have a mixture of different constituents, described in chemical equilibrium by a mass action law. Second, at higher densities the internal fermionic structure of the α particles becomes relevant so the four-nucleon bound state will be modified by medium effects. A consistent approach can be given by quantum statistical methods [4]. Using thermodynamic Green functions, the effects of self-energy and Pauli blocking are included so the bound states are dissolved when the density exceeds a critical value. For α particles this critical density, which is also dependent on temperature, is about $\rho_0/5$, with $\rho_0 = 0.17\ \mathrm{fm}^{-3}$ as the saturation density [5].

An important phenomenon is the formation of a quantum condensate with strong four nucleon correlations at low temperatures [6]. At low densities where α particles are well-defined weakly interacting constituents of symmetric nuclear matter, we have Bose-Einstein condensation of α particles. With increasing density, quartetting occurs with medium-modified α particles and disappears at a density of about $\rho_0/3$. Note that quartet condensation has recently also been considered in the context of cold atom physics [7].

The Bose-Einstein condensation for ideal quantum gases is a well-known phenomenon. The occupation of single-particle states is given by the Bose distribution function. Below a critical temperature T_c, to obey normalization, the state of lowest energy is macroscopically occupied. This macroscopically enhanced coherent occupation of the lowest quantum state is denoted as quantum condensate. As well known, the fraction of bosons found in the condensate results for the ideal Bose gas as $n_{\mathrm{cond}}/n = 1 - (T/T_c)^{3/2}$.

However, this simple picture is no longer valid, if interaction is taken into account. For a recent determination of T_c in the interacting case, see Ref. [8]. Here, we want to concentrate on interaction effects at zero temperature. In general, the condensate fraction is given by the properties of the density matrix that contains a part that factorizes. According to Penrose and Onsager [9], the quantum condensate in a homogeneous interacting boson system at zero temperature is given by the off-diagonal long-range order in the density matrix. The nondiagonal density matrix in coordinate representation can be decomposed so in the limit $|\boldsymbol{r} - \boldsymbol{r}'| \to \infty$ follows

$$\lim_{|\boldsymbol{r}-\boldsymbol{r}'|\to\infty} \rho(\boldsymbol{r}, \boldsymbol{r}') = \psi_0^*(\boldsymbol{r})\psi_0(\boldsymbol{r}') + \gamma(\boldsymbol{r} - \boldsymbol{r}'). \tag{1}$$

The last contribution $\gamma(r)$ disappears at large distances, whereas the first contribution determines the condensate fraction

$$n_0 = \frac{\langle\Psi|a_0^\dagger a_0|\Psi\rangle}{\langle\Psi|\Psi\rangle} \tag{2}$$

in infinite matter. Exploratory calculation of the condensate fraction of α matter will be given in the following Sec. II. In contrast to Ref. [6,8] where the transition temperature T_c for quartetting was considered, we consider here the zero temperature case and analyze the ground-state wave function. It will be shown that due to the interaction, the condensate fraction is suppressed with increasing density.

An important question is whether such properties of infinite nuclear matter are of relevance for finite nuclei. As well known, e.g., pairing obtained in nuclear matter within the BCS approach is also clearly seen in finite nuclei. Nuclei with densities near the saturation density are well described by the quasiparticle picture that leads to the shell model for finite nuclei. At low densities, a fully developed α cluster structure similar to α matter is expected. Cluster structures in finite nuclei have been well established. A density functional approach is able to include correlations and to bridge between infinite matter and finite nuclei.

An interesting aspect of finite nuclei is the enhancement of the occupation of single α-particle states similar to Bose-Einstein condensation in α-particle matter or condensation of bosonic atoms in traps. Recently, gaslike states have been investigated in self-conjugate $4n$ nuclei [10], where the α-particles in low-density excited nuclei move nearly freely in S states, contained only by the Coulomb barrier. The Tohsaki-Horiuchi-Schuck-Röpke (THSR) ansatz for the wave function given below in Sec. III, which is similar to the condensate state in infinite matter, has been shown to be appropriate in describing low-density isomers. In particular, ^{8}Be and the Hoyle state of ^{12}C are well described with this THSR wave function. Investigation of states near the four α threshold in ^{16}O is in progress [11,12]. Predictions for ^{20}Ne were given in Ref. [13].

In Sec. III, we will explain how the suppression of the condensate fraction, calculated for infinite nuclear matter, is also seen in the low-density isomers of self-conjugate $4n$ nuclei, in particular for $n = 3$ (^{12}C). First results for $n = 4$ (^{16}O) are also given. General conclusions are drawn in Sec. IV.

II. SUPPRESSION OF CONDENSATE FRACTION IN α MATTER AT ZERO TEMPERATURE

The theory of Penrose and Onsager [9] was first applied to a system with hard core repulsion. Depending on the filling factor, the suppression of the condensate was calculated. In particular, for liquid ^{4}He with a filling factor of 28% at normal conditions, the condensate fraction is reduced to $\approx$8%, in good agreement with experimental observations. To give an estimation for α matter, with an "excluded volume" of about 20 fm^3 [14], such a filling factor of 28% would arise at $\approx\rho_0/3$ so a substantial reduction of the condensate fraction already below saturation densities is expected for α matter.

Within a more systematic approach, we follow the work of Clark *et al.* [15,16]. We calculate the reduction of the condensate fraction as function of the baryon density within a variational calculation performed to lowest order in the density. A uniform Bose gas of α particles, interacting via the potential $V_\alpha(r)$, is considered. As is well known from thermodynamics, the constraint of a homogeneous density may be in conflict with a stable, inhomogeneous ground-state solution so the homogeneous solution may describe a metastable state. Furthermore, we disregard any change of the internal structure of the α particles at increasing density. In particular, the dissolution of the α particle as a four-nucleon bound state because of the Pauli blocking is not taken into account.

The simplest form of a trial wave function incorporating the strong spatial correlations implied by the interaction potential is the familiar Jastrow choice, $\psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_A) = \prod_{i<j} f(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$. Within our exploratory calculation we consider the lowest approximation with respect to the density to show the tendency of condensate suppression due to the interaction. The so-called normalization or unitarity condition, see Refs. [15,16], gives for the variational function the constraint

$$4\pi\rho_\alpha \int_0^\infty [f^2(r) - 1] r^2 dr = -1, \tag{3}$$

$\rho_\alpha = \rho/4$ being the density of α particles.

In the low-density limit, the binding energy per α particle is given by

$$E[f] = 2\pi\rho_\alpha \int_0^\infty \left\{ \frac{\hbar^2}{4M}\left[\frac{\partial f(r)}{\partial r}\right]^2 + V_\alpha(r) f^2(r) \right\} r^2 dr, \tag{4}$$

with M being the nucleon mass. The condensate fraction is calculated according to

$$n_0 = \exp\left\{-4\pi\rho_\alpha \int_0^\infty [f(r) - 1]^2 r^2 dr\right\}. \tag{5}$$

Note that these approximations [15] only hold in the low-density limit. At higher densities, the pair correlation function has to be evaluated. A more advanced approach based on a hypernetted-chain (HNC) calculation has been given by Clark, Ristig, and others; see Refs. [15–17].

For the evaluation of the condensate fraction (5) we use the Ali-Bodmer α-α interaction potential [2]

$$V_\alpha(r) = 457\mathrm{e}^{-(0.7r/\mathrm{fm})^2}\mathrm{MeV} - 130\mathrm{e}^{-(0.475r/\mathrm{fm})^2}\mathrm{MeV}. \tag{6}$$

According to Johnson and Clark [15] we choose the variational function as

$$f(r) = (1 - \mathrm{e}^{-ar})(1 + b\mathrm{e}^{-ar} + c\mathrm{e}^{-2ar}). \tag{7}$$

After determining the parameters a, b, c from the minimum of energy [18], the condensate fraction can be evaluated; see Fig. 1. Note that this variational function (7) is a simple choice. Alternative choices for the Jastrow correlation function $f(r)$ such as the Pandharipande-Bethe choice have been discussed in Ref. [15], which give a more adequate description of

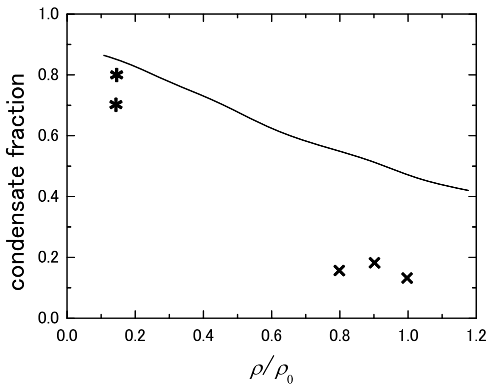

FIG. 1. Reduction of condensate fraction in α matter with increasing baryon density (ρ_0 denotes the saturation density). Full line shows variational calculation performed to lowest order in the density; crosses show HNC calculations by Johnson and Clark [15]; stars show the Hoyle state (see Sec. III).

correlations. It is intended to perform those more complicated calculations in the future. We, however, expect that the general trend will not be qualitatively changed.

In Fig. 1, the full line represents the result for the condensate fraction as function of the baryonic density according to the variational calculation performed to lowest order in the density. In the zero-density limit this fraction is expected to go to 1. Due to the chosen Jastrow wave function, we have a reduction of the condensate fraction with increasing density, but not the disapparence of the condensate at any finite density. This, however, is an artifact of the boson approximation for the α particles. In reality the α particles dissolve at high density. Calculations performed by Johnson and Clark [15] using a HNC calculation for the pair distribution function are given by crosses, showing a stronger suppression of the condensate fraction near the saturation density.

As found from the calculation of the critical temperature for the formation of a quartetting condensate [6], we expect that near the saturation density the condensate fraction will disappear. For this, we have not only to take into account the HNC-type improvement of the pair variational wave function (7) but also the Pauli blocking effects that modify the internal structure of the α particle so that the use of the Ali-Bodmer interaction potential is no longer justified. Improved versions of the α-α interaction have been proposed [19]. Another issue not detailed here is the consequence of α clustering and condensate formation for the thermodynamic stability of the equation of state of homogeneous nuclear matter with respect to phase separation, as discussed above. We mention only that the use of the Ali-Bodmer interaction would lead to a region of thermodynamic instability that is too large and an improved effective interaction between the α particles, including three-α forces, is expected to give a spinodal point for the instability of α matter that is positioned below saturation density [20]. Thus, the repulsive part of the α-α interaction (which also is a consequence of the Pauli blocking with respect to the internal nucleonic structure) is only a part of the suppression of the condensate, which is described here.

Another effect is the medium modification of the internal structure of the α particle as well as of the interaction that can be elaborated within a cluster mean field approximation [4]. The dissolution of α-like bound states due to Pauli blocking has been evaluated for an uncorrelated medium solving the Faddeev-Yakubowsky equation [5]. It has been shown [6] that the four-particle correlations in the condensate disappear due to Pauli blocking at around $\rho_0/3$ within a variational approach, approximating the four-nucleon wave function by the solution of the two-particle problem and describing the relative center-of-mass motion by a Gaussian wave function. Therefore, a medium-dependent α-α interaction of the Ali-Bodmer type may be expected to account for the features of this effect in an exploratory way. In principle, an *ab initio* calculation based on interacting nucleons should be performed, with Green functions, variational, or antisymmetrized molecular dynamics (AMD) techniques.

III. ENHANCEMENT OF CLUSTER CENTER-OF-MASS *S* ORBITAL OCCUPATION IN 4*n* NUCLEI

Signatures akin to Bose-Einstein condensation should arise already in finite nuclei. Low-density states of self-conjugate 4*n* nuclei clearly show an α-cluster structure, in particular, for the Hoyle state ($n = 3$) being the 0_2^+ state of ^{12}C. This state is well known as the key state for the synthesis of ^{12}C in stars and also as one of the typical mysterious 0^+ states in light nuclei that are very difficult to understand from the point of view of the shell model [21]. The counterpart of a condensate in infinite α matter, where the occupation of the ground state is enhanced and becomes of the same order as the total particle number, will be the enhancement of the occupation number of a single-α orbital of the α clusters in a low-density state of the nucleus.

The α-clustering nature of the nucleus ^{12}C has been studied by many authors using various approaches [22]. Among these studies, solving the fully microscopic three-body problem of α clusters gives us the most important and reliable theoretical information of α clustering in ^{12}C. First solutions of the microscopic 3α problem where the antisymmetrization of nucleons is exactly treated, have been given by Uegaki *et al.* [23] and by Kamimura *et al.* [24]. Their calculations reproduced reasonably well the observed binding energy and rms radius of the ground 0_1^+ state that is the state with normal density, whereas they both predicted a very large rms radius for the second 0_2^+ state that is larger than the rms radius of the ground 0_1^+ state by about 1 fm, i.e., by over 30%. The observed 0_2^+ state lies slightly above the 3α breakup threshold. The energies of the calculated 0_2^+ state reproduced reasonably well the observed value, together with the electron scattering form factors with respect to the 0_2^+ state [23,24]. The dilute character of the 0_2^+ state can be described by a gaslike structure of 3α particles that interact weakly among one another, predominantly in relative *S* waves. This *S*-wave dominancy in the 0_2^+ state had been already suggested by Horiuchi on the basis of the 3α orthogonality condition model (OCM) calculation [25].

Recently, based on the investigations of the possibility of α-particle condensation in low-density nuclear matter [6], the present authors proposed a conjecture that near the $n\alpha$ threshold in self-conjugate $4n$ nuclei there exist excited states of dilute density that are composed of a weekly interacting gas of self-bound α particles and that can be considered as an $n\alpha$ condensed state [10]. This conjecture was backed by examining the structure of ^{12}C and ^{16}O using a new α-cluster wave function of the α-cluster condensate type.

The new α-cluster wave function [10], which has been denoted above as THSR wave function, represents a condensation of α clusters. This is clearly seen by the following expression

$$|\Psi\rangle = \mathcal{P}(C_\alpha^\dagger)^n|\text{vac}\rangle, \tag{8}$$

with

$$\langle 1234|C_\alpha^\dagger|\text{vac}\rangle = \Phi(\boldsymbol{P})\delta_{\boldsymbol{P},\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3+\boldsymbol{p}_4} \times \phi_\alpha(1234)a_1^\dagger a_2^\dagger a_3^\dagger a_4^\dagger, \tag{9}$$

with $\Phi(\boldsymbol{P})$ describing the center-of-mass motion of the α cluster and ϕ_α the internal wave function of the four-nucleon cluster. The operator $\mathcal{P}$ is projecting out the total center-of-mass motion of the $4n$ nucleus. In the limit of infinite nuclear matter, the Φ orbitals are plane waves, and the projection operator $\mathcal{P}$ can be neglected. In the case considered here, the use of a product ansatz with Gaussians for ϕ_α allows the explicit separation of the center-of-mass motion of the four-nucleon cluster as well as of the whole $4n$ nucleus. It should also be noted that Eq. (8) contains two limits exactly: the one of a pure Slater determinant relevant at higher densities and the one of a 100% ideal α-particle condensate in the dilute limit [10]. All intermediate scenarios are also correctly covered.

This THSR wave function was applied to study the structure of ^{12}C and ^{16}O and actually succeeded to place a level of dilute density (about one-third of saturation density) in each system of ^{12}C and ^{16}O in the vicinity of the 3α and 4α breakup thresholds, respectively, without using any adjustable parameter. In the case of ^{12}C, this success of the new α-cluster wave function may seem rather natural, because the microscopic 3α cluster models had predicted a gaslike structure of 3α particles for the 0_2^+ state, as mentioned above.

The detailed structure analyses of ^{12}C [26] showed that the 0_2^+ wave function of ^{12}C that was obtained in past by solving the full three-body problem of the microscopic 3α cluster model is almost completely equivalent to the wave function of the 3α THSR state. This result gives us strong support to our opinion that the 0_2^+ state of ^{12}C has a gaslike structure of 3α clusters with "Bose condensation." The rms radius for this THSR state was calculated as $R(0_2^+)_{\text{THSR}} = 4.3$ fm, which fits well with experimental data for the form factor of the Hoyle state; see Ref. [27]. It confirms the assumption of low density as a prerequisite for the formation of an α-cluster structure for which the Bose-like enhancement of the occupation of the S orbit is possible.

A very interesting analysis of the applicability of the THSR wave function can be performed by comparing with stochastic variational calculations [28] and OCM calculations [29]. The α-particle density matrix $\rho(\boldsymbol{r},\boldsymbol{r}')$, defined by integrating out of the total-density matrix all intrinsic α-particle coordinates, is diagonalized to study the single-α orbits and occupation probabilities in ^{12}C states. Figure 2 shows the occupation probabilities of the L orbits with S, D, and G waves belonging to the k-th largest occupation number (denoted by Lk) for the ground and Hoyle state of ^{12}C obtained by diagonalizing the density matrix $\rho(\boldsymbol{r},\boldsymbol{r}')$. We found that in the Hoyle state the α-particle S orbit with zero node ($S1$ in Fig. 2) is occupied to more than 70% by the three α particles (see also Ref. [28] and Fig. 1). Taking into account the finite size of the nucleus, a reduction of the condensate fraction from 100% to about 70% is not surprising, and the remaining fraction (about 30%) is due to higher orbits originating from antisymmetrization among nucleons. This huge percentage means that an almost ideal α-particle condensate is realized in the Hoyle state. One should remember that superfluid ^{4}He has only 8% of the particles in the condensate, what represents a macroscopic amount of particles nonetheless. Please also note that the S-wave occupancy of the Hoyle state is larger than the occupancy of any other state by at least a factor of 10 (Fig. 2). Independent of the absolute occupancy of the S-wave state, this is a clear signature of quantum coherence, i.e., of condensation (see also Ref. [30] for a more detailed discussion of this point).

However, in the ground state of ^{12}C, the α-particle occupations are equally shared among $S1$, $D1$, and $G1$ orbits, where they have two, one, and zero nodes, respectively, reflecting the SU(3)$(\lambda\mu) = (04)$ character of the ground state [29]. This fact thus invalidates a condensate picture for the ground state.

To get a more extended analysis, OCM calculations have been performed [29] for studying the density dependence of the S-orbit occupancy in the 0^+ state of ^{12}C on the different densities $\rho/\rho_0 \sim [R(0_1^+)_{\text{exp}}/R]^3$, in which the rms radius (R) of ^{12}C is taken as a parameter and $R(0_1^+)_{\text{exp}} = 2.56$ fm. A Pauli-principle respected OCM basis $\Psi_{0^+}^{\text{OCM}}(\nu)$ with a size parameter ν is used, in which the value of ν is chosen to reproduce a given rms radius R of ^{12}C, and the α-particle density matrix $\rho(\boldsymbol{r},\boldsymbol{r}')$ with respect to $\Psi_{0^+}^{\text{OCM}}(\nu)$ is diagonalized to obtain the S-orbit occupancy in the 0^+ wave function. The results are shown in Fig. 3. The S-orbit occupancy is $70 \sim 80\%$ around $\rho/\rho_0 \sim [R(0_1^+)_{\text{exp}}/R(0_2^+)_{\text{THSR}}]^3 = 0.21$, whereas it decreases with increasing ρ/ρ_0 and amounts to about $30 \sim 40\%$ in the saturation density region. A smooth transition of the S-orbit is observed from the zero-node S-wave nature ($\rho/\rho_0 \simeq 0.2$) to a

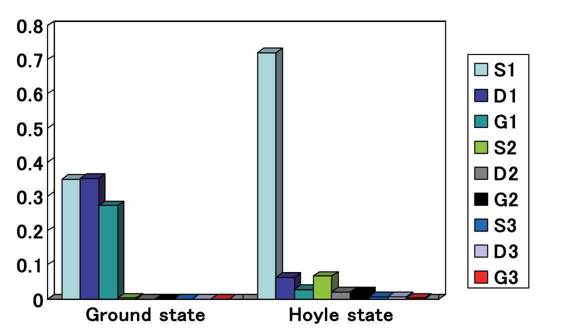

FIG. 2. (Color online) Occupation of the single-α orbitals of the ground state of ^{12}C compared with the Hoyle state [29]. For explanation see the text.

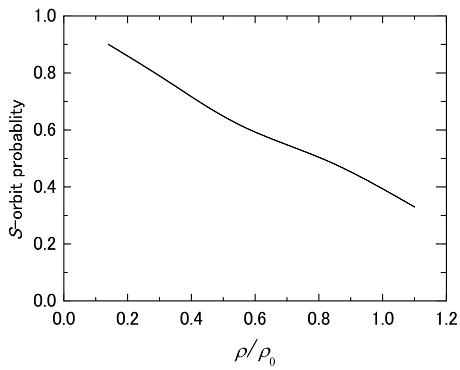

FIG. 3. Occupation of the $S1$ orbital as function of density using the 3α OCM [29].

two-node S-wave one ($\rho/\rho_0 \sim 1$) with increasing ρ/ρ_0 [29]. The feature of the decrease of the enhanced occupation of the S orbit is in striking correspondence with the density dependence of the condensate fraction calculated for nuclear matter (see Fig. 1).

An interesting item is whether there exist other nuclei showing the Bose condensate-like enhancement of the S-orbit occupation number. Then, the suppression of the condensate with increasing density is also of relevance for those nuclei. After we discussed the case of ^{12}C corresponding to $n = 3$ we will now shortly discuss the situation in the next nucleus ^{16}O corresponding to $n = 4$, where great efforts are being performed to investigate low-density excitations in the 0^+ spectrum in theory as well as in experiments.

In analogy to the aforementioned OCM calculation for ^{12}C [29], we recently performed a quite complete OCM calculation also for ^{16}O, including many cluster configurations (a full account is given in a separate publication [12]). We were able to reproduce the full spectrum of 0^+ states with 0_2^+ at 6.4 MeV, 0_3^+ at 9.4 MeV, 0_4^+ at 12.6 MeV, 0_5^+ at 14.1 MeV, and 0_6^+ at 16.5 MeV. Also the rms radii are obtained. The largest values are found as $R(0_6^+)_{\rm OCM} = 5.6$ fm, followed by $R(0_4^+)_{\rm OCM} = 4.0$ fm. We tentatively make a one to one correspondence of those states with the six lowest 0^+ states of the experimental spectrum. In view of the complexity of the situation, the agreement can be considered as very satisfactory. The analysis of the diagonalization of the α-particle density matrix $\rho(\boldsymbol{r}, \boldsymbol{r}')$ (as was done in Ref. [29]) showed that the newly discovered 0^+ state at 13.6 MeV [31], as well as the well known 0^+ state at 14.01 MeV, corresponding to our states at 12.6 and 14.1 MeV, respectively, have, contrary to what we assumed previously [32], very little condensate occupancy of the zero-node S-orbit (about 20%). However, the sixth 0^+ state at 16.5 MeV calculated energy, to be identified with the experimental state at 15.1 MeV, has 61% of the α particles being in the zero-node S orbit.

These results confirm our statement that the α-particle condensate in nuclear matter is suppressed with increasing density and, consequently, a well-developed condensate state in nuclei can be expected only at very low densities. For ^{16}O, the relative densities ρ/ρ_0 are estimated as $[R(0_1^+)_{\rm exp}/R(0_4^+)_{\rm OCM}]^3 = 0.32$ and $[R(0_1^+)_{\rm exp}/R(0_6^+)_{\rm OCM}]^3 = 0.12$. Therefore we expect a significant enhancement of the S-orbit occupation number only for the 0_6^+ state, in full agreement with the OCM calculation cited above. The very large radius of that state is again a clear indication of an α-particle gas- (Hoyle) like state, and the THSR wave function is expected to describe this state in a sufficient approximation. Work to determine the complete spectrum of THSR states in ^{16}O showing the relevance of a Bose-condensate like state is in progress [11].

IV. CONCLUSIONS

Multiple successful theoretical investigations concerning the Hoyle state in ^{12}C have established, beyond any doubt, that it is a dilute gaslike state of three α particles held together only by the Coulomb barrier and describable to first approximation by a wave function of the form $(C_\alpha^\dagger)^3|\mathrm{vac}\rangle$, where the three bosons $(C_\alpha^\dagger)$ are condensed into the S orbital. There is no objective reason why in ^{16}O,^{20}Ne,$\cdots$ there should not exist similar "Hoyle"-like states. At least the calculations with THSR and OCM approaches show this to be the case, systematically. In this work, we give preliminary results of a complete OCM calculation that reproduces the six first 0^+ states of ^{16}O to rather good accuracy. In that calculation the 0_6^+ state at 16.5 MeV, which might be identified with the experimental 0^+ state at 15.1 MeV, shows the characteristics typical for a Hoyle-like state, that is high α-particle S-wave occupancy combined with an unusually large radius.

Therefore, the main quantity for the formation of an α cluster state is the density that should be low. Then, the occurrence of a THSR state where all α particles occupy the same orbit with respect to the center-of-mass motion is an interesting effect that corresponds to the formation of an α-particle condensate in symmetric nuclear matter. The condensate fraction decreases with increasing density because of correlations, as is known from interacting Bose systems. In addition, the internal structure of the four-nucleon cluster is changed due to Pauli blocking if density is increasing.

The α particles may be considered as independent bosons moving relatively free like quasiparticles only in the very low density limit. A mean field approach of the interaction that is assumed to be weak would give a Gross-Pitaevskii equation [33]. Then we can apply the approach of a noninteracting Bose gas where the α particles may occupy the same center-of-mass orbital. The enhanced occupation of the ground state (plane wave) in infinite matter is the standard description of Bose-Einstein condensation. This corresponds, in finite nuclei, to the enhanced occupation of the same orbital for the center-of-mass motion so that the THSR state will be a good approximation for the many-nucleon wave function. We stress the similarity to two-particle pairing where the concept of a BCS state was successfully applied to finite nuclei. The question of finite number of Cooper pairs in the nuclear BCS state is also to be considered in analogy with the finite number of α particles in the THSR state.

With increasing contribution of the interaction, e.g., with increasing density, the condensate state becomes more complex. Calculations in infinite matter ($T = 0$) show that the condensate state becomes increasingly nonideal (the condensate fraction is smaller than one). The same is also observed in OCM calculations for finite nuclei where with increasing density the condensate state becomes gradually depleted. We conclude that there are similarities between the structure of the ground-state wave function of α matter and the α gaslike states in finite nuclei.

In addition to the effect of interaction, mixing higher states of center-of-mass orbits to the ground-state wave function, there is also the dissolution of the internal wave function of the α particle due to medium effects. The transition from the cluster picture with well-defined α states to a shell model where nucleons move independently in a mean field is also reproduced in harmonic oscillator approximation but needs a first principle approach to calculate the many-nucleon wave function.

These results are also of relevance for other phenomena that arise if the local density approach is used. Low-density matter arises in the halo of heavy nuclei so that preformation of α clusters is an interesting issue there, but also in heavy-ion reactions or during supernova explosions. Cluster condensation very likely will soon also become an important subject in cold atom physics. Theoretical investigations already have appeared [7]. So far nuclear physics is at the forefront of this subject.

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