Improved version of a binding energy formula for heavy and superheavy nuclei with $Z \geqslant 90$ and $N \geqslant 140$

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A local formula of binding energy for heavy and superheavy nuclei has very recently been proposed [Dong and Ren, Phys. Rev. C **72**, 064331 (2005)]. In this paper, the limit of the predictive ability of this local formula is investigated. It is found that the neutron-proton correlations should be considered when higher precision is required. On the one hand, we introduce a new term $|N - Z - 50|/A$, and on the other hand we consider the different strengths of proton-proton, neutron-neutron, and neutron-proton pairing correlations. For the first time, the standard deviation $\sqrt{\sigma^2}$ of the binding energies for 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ is reduced to 0.105 MeV. The α decay energies Q_α and half-lives T_α of nuclei with $Z = 102$–118 are reproduced quite well. The proton drip line of superheavy elements from Md ($Z = 101$) to Ds ($Z = 110$) are predicted.

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I. INTRODUCTION

To measure and calculate accurately the ground-state nuclear binding energies (or masses) are important goals of nuclear physicists [1–22]. At present, the synthesis of superheavy nuclei is a hot point in nuclear physics [23–35]. How to calculate accurately and predict reliably the binding energies of the known and unknown superheavy nuclei are important problems. Before the experiments are performed it is necessary to estimate the product cross section of the unknown nucleus by use of nuclear reaction models, where the binding energies are important input parameters. The binding energies also play important roles in the identification of newly synthesized nuclei. It is because up to now most newly synthesized superheavy nuclei are α emitters, only few nuclei mainly decay by spontaneous fission [23–35]. For example, among the 34 superheavy nuclei discovered at Dubna [29], 23 nuclei only decay by α emitting, five nuclei only decay by spontaneous fission. In this mass region the α decay half-life is very sensitive to the α decay energy which is nothing but the difference between the binding energies of the parent, daughter, and α particle. Therefore, an accurate formula of binding energy for superheavy nuclei is very useful. To meet this requirement we proposed a local formula of binding energy for heavy and superheavy nuclei with $Z \geqslant 90$ and $N \geqslant 140$ in a recent paper [36]. In this local formula the shell effects near $N = 152$ were described by analytical expressions. The experimental binding energies of 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ [36,37] were reproduced with the average and standard deviations 0.118 and 0.150 MeV, respectively. It is strongly hoped that the experimental binding energies can be calculated without deviation, namely the average and standard deviations are both reduced to zero. But it is impossible because of the complexity of the nuclear many-body problem. Though the zero deviation can not be reached at present, it is also interesting to investigate the limit of the predictive ability of this local binding energy formula. In other words, our purpose is to see whether there are still spaces to improve this formula.

II. NUMERICAL ANALYSIS AND DISCUSSIONS

In order to improve the local formula of binding energy for heavy and superheavy nuclei, it is natural to see whether there are systematic deviations between theoretical binding energies by the local formula proposed in Ref. [36] and the experimental ones [37]. After detailed analysis we find two characters of the deviations between experimental and theoretical binding energies: (i) the odd-even staggering exists in the deviations for many isotopic chains, especially for odd-Z ones [see Fig. 1(a)]; and (ii) a peak at $N - Z = 50$ is found when we plot the deviations against the neutron excess $N - Z$ [see Fig.2(a)].

Since there are systematic deviations between experimental and theoretical binding energies, it is possible to improve the local formula by removing these systematic deviations. The differences between theory and experiment often indicate new phenomenon in physics. So it is helpful to analyze the possible reasons of these systematic deviations before detailed calculations are performed. For this purpose, let us see the formula previously proposed in Ref. [36]:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2} + a_6 |A - 252| / A - a_7 |N - 152| / N. \quad (1)$$

In this formula the average pairing energy is described by $a_p \delta A^{-1/2}$, $\delta = 1$ for even-Z and even-N (even-even) nuclei, $\delta = -1$ for odd-Z and odd-N nuclei (odd-odd), 0 for odd-A nuclei. The coefficient zero for both even-odd (even-Z and odd-N) and odd-even (odd-Z and even-N) nuclei means that

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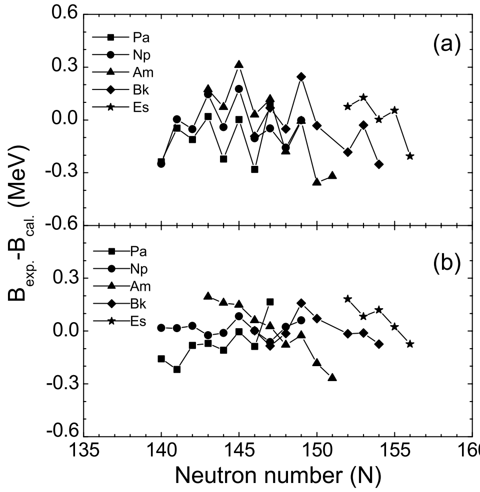

FIG. 1. The deviations between experimental and theoretical binding energies for odd-Z isotopic chains ranging from Pa ($Z = 91$) to Es ($Z = 99$) as the function of neutron number N. The upper part (a) shows the results from the previous local binding energy formula, and the lower part (b) shows the ones from the improved formula.

the average strengths of the neutron-neutron (nn) and proton-proton (pp) pairing are considered to be equal, and that the correlations between protons and neutrons (np) and perhaps other few-body correlations (e.g., α correlation) are ignored. Actually, the neutron-proton correlations play important roles in nuclear structure, and have been investigated extensively [38–42]. The α decay of medium and heavy nuclei and $(d, {}^6\text{Li})$ α transfer reactions [43,44] can be regarded as direct evidences of α correlation [45]. Particularly, up to now it has been found that α decay is the main decay mode in the superheavy region with only few exceptions [23–35]. So it is worthwhile to treat these correlations more carefully. In Ref. [45] the α correlation was discussed by introducing an additional term $\delta(\alpha)$ into the pairing energy for even-even nuclei. In this work we extend this method by considering the different strengths of neutron-neutron, proton-proton, and neutron-proton pairing correlations.

In order to describe the additional binding energies of nuclei near the line $N - Z = 50$, we introduce a new term $|N - Z - 50|/A$ into the local formula. This term is similar to the Wigner term which was introduced into the mass formula by Myers and Swiatecki [1]. They found that for lighter nuclei with $N = Z$ there were additional binding energies, which can be expressed by $E_W = V_W \exp[-\lambda|N - Z|/A]$. Very recently, this term was also included into the Hartree-Fock mass formulas [5,6], but the parameters were different from the ones suggested by Myers and Swiatecki [1]. Then Goriely *et al.* [7] proposed a new form of the Wigner term $E_W = V_W \exp[-\lambda(\frac{N-Z}{A})^2] + V'_W|N - Z|\exp[-(\frac{A}{A_0})^2]$. These forms of the Wigner term [5–7] are all highly localized and contribute mainly to lighter nuclei with $N \sim Z$. For example, for $N = Z$ nuclei the Wigner corrections are about $2 \sim 3$ MeV for all of these mass formulas, whereas for heavy nuclei, such as ^{250}Fm, these terms are all negligible. It is because the Wigner term comes from the np correlations [38]. For the nuclei with $N = Z$ the protons and neutrons fill the same orbits, and hence the overlaps of neutron and proton wave functions are large. Very recently, the neutron-proton correlations in a large number of nuclei spanning the whole nuclear chart have been investigated [40–42]. It has been found that the neutron-proton correlations are not negligible for heavy nuclei, such as Fm and No isotopes [40]. It can be expected that the neutron-proton correlations in the heavy and superheavy nuclei can be described empirically by a new form. The peak of the deviations between experimental and theoretical binding energies near $N - Z = 50$ implies additional binding energies for nuclei near the line $N - Z - 50 = 0$. Since the last neutron(s) and proton(s) occupy different shell model orbits, the overlaps of proton and neutron wave functions will be small. Consequently, the strengths of np correlations in heavy and superheavy nuclei will be weaker than the ones in light nuclei.

According to above discussions, the improved local formula of binding energy reads

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p A^{-1/2} + a_6|A - 252|/A - a_7|N - 152|/N + a_8|N - Z - 50|/A. \quad (2)$$

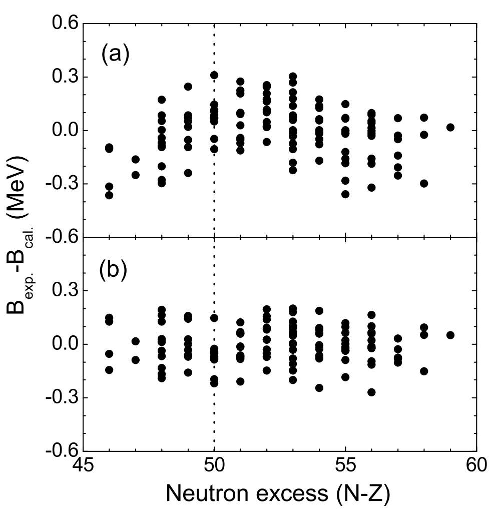

FIG. 2. The deviations between experimental and theoretical binding energies for 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ as the function of the neutron excess $N - Z$. The upper part (a) shows the results from the previous local binding energy formula, and the lower part (b) shows the ones from the improved formula.

Using the improved version of the binding energy formula we make the best fit to the 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ [37], which are the same as the ones used in the previous work [36]. The best fit parameters are

$$\begin{cases} a_v = 15.8032 \text{ MeV} \\ a_s = 17.8147 \text{ MeV} \\ a_c = 0.71478 \text{ MeV} \\ a_a = 97.6619 \text{ MeV} \\ a_6 = 5.33 \quad \text{MeV} \\ a_7 = 21.0 \quad \text{MeV} \\ a_8 = -15.25 \ \text{MeV} \end{cases}$$

The coefficients of the pairing energy are

$$a_p = \begin{cases} 12.66 \text{ MeV}, \text{ even-even nuclei} \\ 3.0 \quad \text{MeV}, \text{ even-odd nuclei} \\ 0 \quad \text{MeV}, \text{odd-even nuclei} \\ -8.0 \ \text{MeV}, \text{ odd-odd nuclei} \end{cases} \tag{3}$$

One can see that the pairing energy for even-odd nuclei is slightly larger than that for odd-even nuclei. It means that the average proton-proton pairing energy is larger than that of the neutron-neutron pairing. This agrees with the fact as has been pointed out by Bohr and Mottelson [46]. The larger magnitude of pairing energy for even-even nuclei than that of odd-odd ones indicates the existence of α correlation. The negative value of the eighth term means that the nuclei with the neutron excess $N - Z = 50$ have extra binding energies compared with the nuclei far away from this line.

The average deviation of the binding energies for the 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ calculated from the improved formula with above parameters is

$$\langle\sigma\rangle = \sum_{i=1}^{117} \left|B^i_{\text{exp.}} - B^i_{\text{cal.}}\right| \Big/ 117 = 0.086 \text{ MeV}, \tag{4}$$

and the standard deviation is

$$\sqrt{\sigma^2} = \left[\sum_{i=1}^{117} \left(B^i_{\text{exp.}} - B^i_{\text{cal.}}\right)^2 \Big/ 117\right]^{1/2} = 0.105 \text{ MeV}. \tag{5}$$

That is to say, the precision is improved by a factor of about one-third compared with the previous average and standard deviations 0.118 and 0.150 MeV [36]. In order to see clearly the improvement of the present results compared with the previous work we show the deviations between experimental and theoretical binding energies by these two formulas in Figs. 1 and 2. In these figures, the upper parts show the results from the previous formula, and the lower parts show the results from the improved version of the formula. One can see clearly that almost all deviations are less than 0.2 MeV. Especially, the odd-even effects and the peak at $N - Z = 50$ are both removed [see Figs. 1(b) and 2(b)].

Since the binding energies can be calculated very accurately, one can expect that the experimental single- and two nucleon- separation energies and α decay energies can be reproduced well, and that the values of unknown nuclei in this region can be predicted reliably. At first, we will show the α decay energies of superheavy nuclei. It is because up to now α decay is found to be the main decay mode in the superheavy region with only few exceptions [29]. The detailed results for even-Z and odd-Z isotopic chains are shown in Figs. 3 and 4, respectively. From these figures one can see

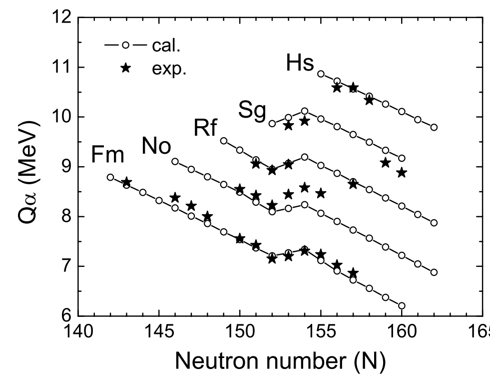

FIG. 3. The α decay energies of even-Z isotopic chains ranging from Fm ($Z = 100$) to Hs ($Z = 108$) obtained from the improved local binding energy formula.

that the α decay energies can be reproduced very well with only few exceptions. For the $N = 155$ isotones ^{256}Md, ^{257}No, and ^{258}Lr the deviations between experimental and theoretical values are a little larger. But for other three $N = 155$ isotones ^{255}Fm, ^{260}Db, and ^{262}Bh theoretical results agree well with experimental ones. There are two possible interpretations of the larger deviations for $N = 155$ isotones ^{256}Md, ^{257}No, and ^{258}Lr. One is the Z-dependence of the shell gap at $N = 152$, and the other is the different ground-state configuration of ^{256}Md, ^{257}No, and ^{258}Lr from that of lighter $N = 155$ isotones ^{253}Cf and ^{255}Fm. Very recently, Makii *et al.* [47] have reported the in-beam γ-ray spectroscopy of ^{245,246}Pu. They found that the $N = 152$ shell gap was reduced considerably for Pu isotopes [47]. From Fig. 3 one can see that the α decay energies for No ($Z = 102$) isotopes near $N = 152$ change more sharply than both Fm ($Z = 100$) and Rf ($Z = 104$) isotopes. But the α decay energies for Fm and Rf isotopes are reproduced very well. So we can not draw firm conclusion about the Z-dependence of the $N = 152$ shell gap in these isotopic chains. On the other hand, the discontinuity of the deviations along a isotopic chain may indicate the sharp change of the single-particle levels. Very recently, Asai *et al.* [48] have established the single-particle states of ^{257}No, and found that its ground-state configuration is different from that of lighter even-Z $N = 155$ isotones ^{253}Cf and ^{255}Fm. In the shell model, the single-particle energy is a function of the quantum numbers of its orbit. The sharp change of the level density

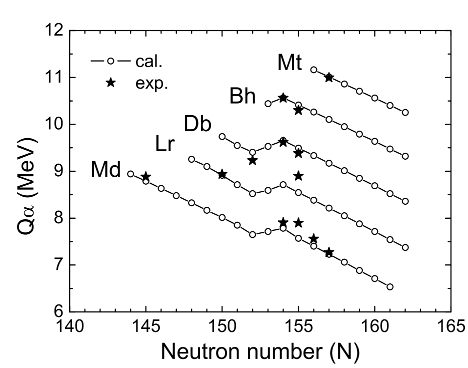

FIG. 4. The same as Fig. 3 but for odd-Z isotopic chains ranging from Md ($Z = 101$) to Mt ($Z = 109$).

near the Fermi surface will bring up discontinue change of the binding energy. However, due to the lack of experimental data of the single-particle configurations in this region, we can not give conclusion about the large deviations for the $N = 155$ isotones yet. The discrepancy between experiment and theory motivates more detailed studies in this region from both experimental and theoretical points of view in the near future. We believe that this problem can be clarified with the accumulation of experimental data in the future.

Besides α decay energies, we are also interested in the single-proton separation energies S_p of the proton-rich superheavy nuclei. The purpose is to predict the proton drip line of the elements with relatively smaller proton number in the superheavy region (e.g., $Z = 101$–110). It is because for these isotopic chains the proton drip line can be reached more easily before long. Since the odd-even staggering in binding energies have been removed by considering the different strengths of pairing correlations, it is expected that the proton drip line of the elements from Md ($Z = 101$) to Ds ($Z = 110$) can be predicted reliably. It can provide useful references for experimental physicists in planning experiments. The single-proton separation energies of isotopic chains ranging from Md to Ds are shown in Fig. 5. The zero values represent the proton drip line of these isotopic chains. From this figure one can see that the proton drip line nuclei are, respectively, ^{240}Md$_{139}$, ^{237}No$_{135}$, ^{246}Lr$_{143}$, ^{242}Rf$_{138}$, ^{252}Db$_{147}$, ^{248}Sg$_{142}$, ^{258}Bh$_{151}$, ^{254}Hs$_{146}$, ^{264}Mt$_{155}$, and ^{260}Ds$_{150}$. According to our prediction, the newly synthesized isotope ^{260}Bh [32] is very close to the proton drip line.

Another important quantity in the identification of newly synthesized superheavy nuclei is α decay half-life. Very recently, α decay half-lives have been investigated by various methods (see for example Refs. [49–53]). A simple but widely used formula of α decay half-life is the Viola-Seaborg formula [54,55]. The Viola-Seaborg formula plays an important role in the identification of newly synthesized elements [29]. Here we use the Viola-Seaborg formula with the parameters proposed recently [55] to calculate the α decay half-lives using the theoretical α decay energies. The calculated and experimental α decay energies (Q_α) and partial half-lives (T_α) of even-Z isotopes with neutron number near $N = 152$ are listed in Table I. In this table, the experimental half-lives of ^{255}Rf, ^{257}Rf, and ^{266}Hs are taken from Refs. [56,57], and [25], respectively. From this table one can see again that most of the theoretical α decay energies are very close to experimental ones. Seven of these 15 experimental α decay energies are reproduced within 0.1 MeV. The largest deviation is 0.4 MeV for the $N = 155$ isotone ^{257}No. The possible reasons of the large deviation has been discussed in the above. With these theoretical α decay energies the α decay half-lives are reproduced very well by the Viola-Seaborg formula [55]. 17 of the 21 experimental half-lives are reproduced within a factor of 1–4. Only for three nuclei ^{251}No, ^{256}No, and ^{259}Sg, the ratios between experimental and theoretical half-lives are slightly larger than 10. The α decay half-life of ^{253}Rf is reproduced with a factor of 7.7. The largest value of $T_{\rm cal.}/T_{\rm exp.}$ is 16.7 for ^{251}No. The calculated and experimental α decay half-lives for odd-Z nuclei from ^{253}Lr to ^{267}Mt are listed in

TABLE I. Theoretical and experimental α decay energies (in MeV) and half-lives (in seconds) for even-Z nuclei ($Z = 102$–108) with the neutron number near $N = 152$. Theoretical α decay energies are used to calculate the half-lives [T_α(Cal)] by the Viola-Seaborg formula [55].

Nucl.	Q_α(Cal)	Q_α(Exp)	T_α(Cal)	T_α(Exp)	Ref.
^{250}No	8.801		0.636		
^{251}No	8.643		15.3	0.916	[37]
^{252}No	8.493	8.550	5.84	$\approx$3.64	[37]
^{253}No	8.290	8.421	0.21×10^3		
^{254}No	8.096	8.226	0.122×10^3	0.567×10^2	[37]
^{255}No	8.164	8.442	0.557×10^3	0.305×10^3	[37]
^{256}No	8.238	8.581	40.1	$\approx$2.91	[37]
^{257}No	8.066	8.466	0.121×10^4		
^{253}Rf	9.522		0.202	$\approx 0.26 \times 10^{-1}$	[37]
^{254}Rf	9.336		0.855×10^{-1}		
^{255}Rf	9.140	9.058	2.47	3.154	[56]
^{256}Rf	8.952	8.930	1.14	2.016	[37]
^{257}Rf	9.071	9.044	3.95	5.34/4.43	[57]
^{258}Rf	9.195		0.219	0.923×10^{-1}	[37]
^{259}Rf	9.028		5.30	3.04	[37]
^{260}Rf	8.870		2.04		
^{261}Rf	8.702	8.650	53.2		
^{258}Sg	9.867		0.139×10^{-1}	$>0.165 \times 10^{-1}$	[37]
^{259}Sg	9.991	9.830	0.509×10^{-1}	0.644	[37]
^{260}Sg	10.121	9.920	0.301×10^{-2}	0.95×10^{-2}	[37]
^{261}Sg	9.961		0.611×10^{-1}	$\approx$0.23	[37]
^{262}Sg	9.810		0.197×10^{-1}	$>0.364 \times 10^{-1}$	[37]
^{263}Sg	9.648		0.428	$1.0 \sim 1.43$	[37]
^{264}Hs	10.721	10.591	0.411×10^{-3}	$\approx 1.08 \times 10^{-3}$	[37]
^{265}Hs	10.566	10.590	0.769×10^{-2}	$\approx 0.21 \times 10^{-2}$	[37]
^{266}Hs	10.418	10.336	0.228×10^{-2}	0.23×10^{-2}	[25]
^{267}Hs	10.260		0.451×10^{-1}	0.32×10^{-1}	[37]

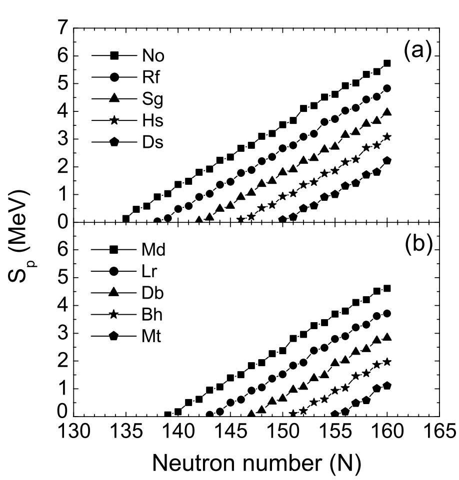

FIG. 5. The single-proton separation energies S_p for isotopic chains ranging from Md ($Z = 101$) to Ds ($Z = 110$) calculated from the improved local binding energy formula. The zero points of S_p show the proton drip line of these isotopic chains.

TABLE II. The same as Table I but for odd-Z nuclei ($Z=$ 103–109).

Nucl.	Q_α(Cal)	Q_α(Exp)	T_α (Cal)	T_α(Exp)	Ref.
^{253}Lr	8.912	8.937	2.50	0.644	[37]
^{254}Lr	8.713		23.7	17.105	[37]
^{255}Lr	8.521		41.4	25.882	[56]
^{256}Lr	8.595		55.5	31.765	[37]
^{257}Lr	8.714		10.1	≈0.646	[37]
^{258}Lr	8.545	8.900	80.3	$4.1 \sim 4.32$	[37]
^{259}Lr	8.383		1.160×10^{2}	7.949	[37]
^{256}Db	9.550		0.408	2.5	[58]
^{257}Db	9.407	9.230	0.443	$1.53 \sim 1.63$	[37]
^{258}Db	9.529		0.465	7.03	[37]
^{259}Db	9.655	9.620	0.903×10^{-1}	0.51	[37]
^{260}Db	9.494	9.380	0.585	$1.52 \sim 1.68$	[37]
^{261}Db	9.337		0.699	$1.8 \sim 2.20$	[37]
^{258}Bh	10.446		0.819×10^{-2}		
^{259}Bh	10.309		0.777×10^{-2}		
^{260}Bh	10.437	10.364	0.863×10^{-2}	0.35×10^{-1}	[32]
^{261}Bh	10.568	10.560	0.177×10^{-2}	0.137×10^{-1}	[37]
^{262}Bh	10.412	10.300	0.994×10^{-2}		
^{263}Bh	10.263		0.101×10^{-1}		
^{264}Mt	11.306		0.315×10^{-3}		
^{265}Mt	11.162		0.286×10^{-3}		
^{266}Mt	11.010	10.996	0.149×10^{-2}		
^{267}Mt	10.864		0.141×10^{-2}		

Table II. In this table, the experimental half-lives of ^{255}Lr and ^{256}Db are taken from Refs. [56] and [58], respectively. The experimental α decay energy and half-life of the newly synthesized nucleus ^{260}Bh are taken from Ref. [32]. The α decay energy is deduced from the kinetic energy of α particle E_α by using the standard relation [59]: $Q_\alpha = [A_p/(A_p - 4)]E_\alpha + (65.3\, Z_p^{7/5} - 80.0\, Z_p^{2/5}) \times 10^{-6}$ MeV, where Z_p and A_p are the proton and mass numbers of the α emitter, respectively. From the measured α particle energy $E_\alpha =$ 10.16 MeV [32], we obtain its α decay energy $Q_\alpha =$ 10.364 MeV, which is very close to the theoretical value 10.437 MeV (see Table II). Our result is also close to the value 10.470 MeV estimated by Audi *et al.* [37]. Except for the $N = 155$ isotone ^{258}Lr, the α decay energies are reproduced within 0.2 MeV. Five of these nine experimental α decay energies are reproduced within 0.1 MeV. The largest deviation is 0.355 MeV for the $N = 155$ isotone ^{258}Lr. The α decay half-lives of these odd-Z nuclei are also reproduced well. For ^{257,258,259}Lr and ^{258}Db, the ratios between experimental and theoretical half-lives are slightly larger than 10. On the whole, the α decay half-lives of nuclei from No ($Z = 102$) to Mt ($Z = 109$) isotopes can be predicted quite accurately by combining the improved binding energy formula with the Viola-Seaborg formula [55].

Through the above discussions one can see that the α decay energies and half-lives for nuclei with $Z = 102$–109 can be reproduced very well. In the next we will test the validity of the improved local binding energy formula in a larger range of mass number. Very recently, 34 new superheavy nuclei with $Z = 104$–118 have been discovered at Dubna [26–29]. These

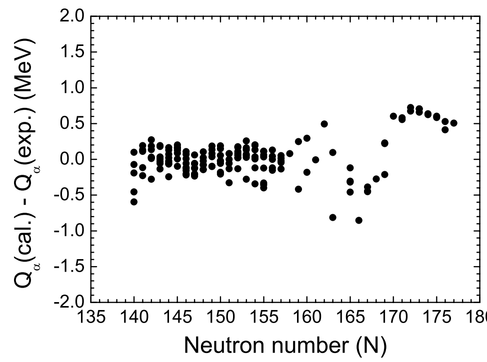

FIG. 6. Deviations between theoretical and experimental α decay energies for 162 nuclei with $Z \geqslant 90$ and $N \geqslant 140$. Experimental data are taken from Refs. [29,37].

nuclei provide good groundwork to test the validity of the improved binding energy formula. The deviations between theoretical and experimental α decay energies for 162 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ are shown in Fig. 6. Among these 162 experimental α decay energies, 27 data are taken from Ref. [29]. From this figure one can see that for most nuclei the α decay energies can be reproduced within 0.2 MeV. Especially, one can see clearly that there is a valley near $N =$ 164 in the deviations between theoretical and experimental α decay energies. The systematic deviations mean that $N = 162$ is a magic number which has been discussed in many works (see for example Refs. [30,50]). From the above discussions one can see that α decay energies can be used to calculate α decay half-lives by combining with the Viola-Seaborg formula. Now we will take α decay half-lives for newly discovered superheavy nuclei ranging from $Z = 110$ to $Z =$ 118 [29] as examples to test the predictive ability of our improved binding energy formula. The numerical results are shown in Table III. From this table one can see that the α decay half-lives can be reproduced quite well. Seventeen of the twenty experimental half-lives are reproduced within a factor of 10. Especially, for ^{279}Rg, ^{291}116, and ^{293}116 theoretical α decay half-lives are almost equal to experimental ones. For only three nuclei ^{278}Rg, ^{286}114, and ^{288}114 the deviations between experimental and theoretical results are a little larger. The largest value of the ratio between experimental and theoretical half-lives $T_{\text{exp.}}/T_{\text{cal.}}$ is about 50 for ^{286}114. It is well known that the production cross sections for nuclei heavier than Ds ($Z = 110$) are very small. For some nuclei only one or two events have been observed [29]. Consequently the experimental error bars of α decay half-lives are a little larger. Therefore, the average deviation by a factor of about 10 is satisfactory. It means that the improved version of the local binding energy formula is also valid for nuclei heavier than Ds ($Z = 110$).

III. SUMMARY

In summary, we improve the local formula of binding energy for heavy and superheavy nuclei. After detailed analysis we find two characters in the deviations between

TABLE III. Experimental and theoretical α decay energies (in MeV) and half-lives for superheavy nuclei with $Z = 110$–118. Experimental data are taken from Ref. [29]. The theoretical half-lives are calculated by the Viola-Seaborg formula [55].

Nucl.	Q_α(Cal)	Q_α(Exp)	T_α (Cal)	T_α(Exp)
^{279}Ds	9.627	9.84 ± 0.06	10.6 s	$2.0^{+0.5}_{-0.4}$ s
^{278}Rg	10.399	10.85 ± 0.08	0.195 s	$4.2^{+7.5}_{-1.7}$ ms
^{279}Rg	10.248	10.52 ± 0.16	0.208 s	170^{+810}_{-80} ms
^{280}Rg	10.090	9.87 ± 0.06	1.28 s	$3.6^{+4.3}_{-1.3}$ s
^{283}112	10.250	9.67 ± 0.06	0.894 s	$3.8^{+1.2}_{-0.7}$ s
^{285}112	9.941	9.28 ± 0.05	6.25 s	29^{+13}_{-7} s
^{282}113	11.008	10.78 ± 0.04	23.9 ms	73^{+134}_{-29} ms
^{283}113	10.861	10.26 ± 0.09	23.5 ms	100^{+490}_{-45} ms
^{284}113	10.706	10.15 ± 0.06	0.133 s	$0.48^{+0.58}_{-0.17}$ s
^{286}114	11.020	10.33 ± 0.06	5.14 ms	$0.26^{+0.08}_{-0.04}$ s
^{287}114	10.866	10.16 ± 0.06	96.7 ms	$0.48^{+0.16}_{-0.09}$ s
^{288}114	10.719	10.08 ± 0.06	28.9 ms	$0.8^{+0.27}_{-0.16}$ s
^{289}114	10.563	9.96 ± 0.05	0.569 s	$2.6^{+1.2}_{-0.7}$ s
^{287}115	11.467	10.74 ± 0.09	3.28 ms	32^{+155}_{-14} ms
^{288}115	11.315	10.61 ± 0.06	17.2 ms	87^{+105}_{-30} ms
^{290}116	11.625	11.00 ± 0.08	0.735 ms	$7.1^{+3.2}_{-1.7}$ ms
^{291}116	11.474	10.89 ± 0.07	12.9 ms	18^{+22}_{-6} ms
^{292}116	11.330	10.80 ± 0.07	3.61 ms	18^{+16}_{-6} ms
^{293}116	11.177	10.69 ± 0.06	65.9 ms	61^{+57}_{-20} ms
^{294}118	12.223	11.81 ± 0.06	0.125 ms	$0.89^{+1.07}_{-0.31}$ ms

experimental and theoretical binding energies. One is the odd-even staggering in many isotopic chains, and the other is the extra binding energies for the nuclei near the line $N - Z = 50$. The underlying physics of these two characters are both related to the neutron-proton correlations. Correspondingly, we consider the different strengths of pairing correlations, such as proton-proton, neutron-neutron, neutron-proton correlations, and introduce a new term $|N - Z - 50|/A$ into the local formula. Through the best fit we obtain a new set of parameters. By use of this improved local formula of binding energy the standard deviation for the 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$ can be reduced to 0.105 MeV. The precision is improved by a factor of about 1/3 compared with the previous formula [36]. The odd-even staggering and the peak near $N - Z = 50$ in the deviations between experimental and theoretical binding energies are both removed. Most of the experimental α decay energies for superheavy nuclei ranging from ^{243}Fm to ^{266}Mt ($Z = 100$–109) are reproduced accurately by this new formula. The possible reasons of the slightly larger deviations for the $N = 155$ isotones ^{256}Md, ^{257}No, and ^{258}Lr are discussed. The discrepancy between theory and experiment shows the necessity of detailed studies for these nuclei. The proton drip line of superheavy elements ranging from Md ($Z = 101$) to Ds ($Z = 110$) are predicted. By combining this local binding energy formula with the Viola-Seaborg formula, we calculate the α decay half-lives of the nuclei ranging from ^{250}No to ^{267}Mt ($Z = 102$–109). In general, the half-lives of these nuclei can be reproduced quite well. For few nuclei the ratios between theoretical and experimental half-lives are slightly larger than 10. Particularly, the α decay energy and half-life of the newly synthesized nucleus ^{260}Bh are reproduced very well. It means that the improved formula has a strong predictive ability for the nuclei with the neutron number near $N = 152$. The proton drip line of elements ranging from Md ($Z = 101$) to Ds ($Z = 110$) are predicted. In addition, the α decay energies and half-lives for superheavy nuclei with $110 \leqslant Z \leqslant 118$ are calculated and compared with experimental data. The systematic deviations between theoretical and experimental α decay energies show that $N = 162$ is a magic number. On the whole, the half-lives of these nuclei can be reproduced within a factor of 10. It means that the improved binding energy formula is also valid for superheavy nuclei beyond Ds ($Z = 110$).

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[1] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966).
[2] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[3] J. Duflo and A. P. Zuker, Phys. Rev. C **52**, R23 (1995).
[4] H. Koura, M. Uno, T. Tachibana, and M. Yamada, Nucl. Phys. **A674**, 47 (2000).
[5] S. Goriely, F. Tondeur, and J. M. Pearson, At. Data Nucl. Data Tables **77**, 311 (2001).
[6] M. Samyn, S. Goriely, P.-H. Heenen, J. M. Pearson, and F. Tondeur, Nucl. Phys. **A700**, 142 (2002).
[7] S. Goriely, M. Samyn, P.-H. Heenen, J. M. Pearson, and F. Tondeur, Phys. Rev. C **66**, 024326 (2002).
[8] Zhongzhou Ren, Phys. Rev. C **65**, 051304(R) (2002).
[9] Zhongzhou Ren, Fei Tai, and Ding-Han Chen, Phys. Rev. C **66**, 064306 (2002).
[10] J. Barea, A. Frank, J. G. Hirsch, and P. Van Isacker, Phys. Rev. Lett. **94**, 102501 (2005).
[11] S. Liran, A. Marinov, and N. Zeldes, Phys. Rev. C **62**, 047301 (2000).
[12] F. Sarazin, H. Savajols, W. Mittig, F. Nowacki, N. A. Orr, Z. Ren, P. Roussel-Chomaz, G. Auger, D. Baiborodin, A. V. Belozyorov, C. Borcea, E. Caurier, Z. Dlouhý, A. Gillibert, A. S. Lalleman, M. Lewitowicz, S. M. Lukyanov, F. de Oliveira, Y. E. Penionzhkevich, D. Ridikas, H. Sakuraï, O. Tarasov, and A. de Vismes, Phys. Rev. Lett. **84**, 5062 (2000).
[13] W. Mittig, A. Lépine-Szily, and N. A. Orr, Annu. Rev. Nucl. Part. Sci. **47**, 27 (1997).

[14] D. Lunney, J. M. Pearson, and C. Thibault, Rev. Mod. Phys. **75**, 1021 (2003).
[15] U. Hager, V.-V. Elomaa, T. Eronen, J. Hakala, A. Jokinen, A. Kankainen, S. Rahaman, S. Rinta-Antila, A. Saastamoinen, T. Sonoda, and J. Äystö, Phys. Rev. C **75**, 064302 (2007).
[16] U. Hager, T. Eronen, J. Hakala, A. Jokinen, V. S. Kolhinen, S. Kopecky, I. Moore, A. Nieminen, M. Oinonen, S. Rinta-Antila, J. Szerypo, and J. Äystö, Phys. Rev. Lett. **96**, 042504 (2006).
[17] B. Fogelberg, K. A. Mezilev, V. I. Isakov, K. I. Erokhina, H. Mach, E. Ramström, and H. Gausemel, Phys. Rev. C **75**, 054308 (2007).
[18] C. Guénaut, G. Audi, D. Beck, K. Blaum, G. Bollen, P. Delahaye, F. Herfurth, A. Kellerbauer, H.-J. Kluge, J. Libert, D. Lunney, S. Schwarz, L. Schweikhard, and C. Yazidjian, Phys. Rev. C **75**, 044303 (2007).
[19] A. M. García-García, J. G. Hirsch, and A. Frank, Phys. Rev. C **74**, 024324 (2006).
[20] G. Royer and C. Gautier, Phys. Rev. C **73**, 067302 (2006).
[21] H. Nakamura and T. Fukahori, Phys. Rev. C **72**, 064329 (2005).
[22] H. Olofsson, S. Åberg, O. Bohigas, and P. Leboeuf, Phys. Rev. Lett. **96**, 042502 (2006).
[23] S. Hofmann, Rep. Prog. Phys. **61**, 639 (1998).
[24] S. Hofmann and G. Münzenberg, Rev. Mod. Phys. **72**, 733 (2000).
[25] S. Hofmann, F. P. Heßberger, D. Ackermann, S. Antalic, P. Cagarda, S. Ćwiok, B. Kindler, J. Kojouharova, B. Lommel, R. Mann, G. Münzenberg, A. G. Popeko, S. Saro, H. J. Schött, and A. V. Yeremin, Eur. Phys. J. A **10**, 5 (2001).
[26] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, I. V. Shirokovsky, Yu. S. Tsyganov, G. G. Gulbekian, S. L. Bogomolov, A. N. Mezentsev, S. Iliev, V. G. Subbotin, A. M. Sukhov, A. A. Voinov, G. V. Buklanov, K. Subotic, V. I. Zagrebaev, M. G. Itkis, J. B. Patin, K. J. Moody, J. F. Wild, M. A. Stoyer, N. J. Stoyer, D. A. Shaughnessy, J. M. Kenneally, and R. W. Lougheed, Phys. Rev. C **69**, 021601(R) (2004).
[27] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, I. V. Shirokovsky, Yu. S. Tsyganov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, S. Iliev, V. G. Subbotin, A. M. Sukhov, A. A. Voinov, G. V. Buklanov, K. Subotic, V. I. Zagrebaev, M. G. Itkis, J. B. Patin, K. J. Moody, J. F. Wild, M. A. Stoyer, N. J. Stoyer, D. A. Shaughnessy, J. M. Kenneally, P. A. Wilk, R. W. Lougheed, R. I. Il'kaev, and S. P. Vesnovskii, Phys. Rev. C **70**, 064609 (2004).
[28] Yu. Ts. Oganessian, V. K. Utyonkov, Yu. V. Lobanov, F. Sh. Abdullin, A. N. Polyakov, R. N. Sagaidak, I. V. Shirokovsky, Yu. S. Tsyganov, A. A. Voinov, G. G. Gulbekian, S. L. Bogomolov, B. N. Gikal, A. N. Mezentsev, V. G. Subbotin, A. M. Sukhov, K. Subotic, V. I. Zagrebaev, G. K. Vostokin, M. G. Itkis, R. A. Henderson, J. M. Kenneally, J. H. Landrum, K. J. Moody, D. A. Shaughnessy, M. A. Stoyer, N. J. Stoyer, and P. A. Wilk, Phys. Rev. C **76**, 011601(R) (2007).
[29] Yu. Ts. Oganessian, J. Phys. G: Nucl. Part. Phys. **34**, R165 (2007).
[30] J. Dvorak, W. Brüchle, M. Chelnokov, R. Dressler, Ch. E. Düllmann, K. Eberhardt, V. Gorshkov, E. Jäger, R. Krücken, A. Kuznetsov, Y. Nagame, F. Nebel, Z. Novackova, Z. Qin, M. Schädel, B. Schausten, E. Schimpf, A. Semchenkov, P. Thörle, A. Türler, M. Wegrzecki, B. Wierczinski, A. Yakushev, and A. Yeremin, Phys. Rev. Lett. **97**, 242501 (2006).
[31] P. A. Wilk, K. E. Gregorich, A. Türler, C. A. Laue, R. Eichler, V. Ninov, J. L. Adams, U. W. Kirbach, M. R. Lane, D. M. Lee, J. B. Patin, D. A. Shaughnessy, D. A. Strellis, H. Nitsche, and D. C. Hoffman, Phys. Rev. Lett. **85**, 2697 (2000).
[32] S. L. Nelson, K. E. Gregorich, I. Dragojević, M. A. Garcia, J. M. Gates, R. Sudowe, and H. Nitsche, Phys. Rev. Lett. **100**, 022501 (2008).
[33] K. Morita, K. Morimoto, D. Kaji, T. Akiyama, Sin-ichi Goto, H. Haba, E. Ideguchi, R. Kanungo, K. Katori, H. Koura, H. Kudo, T. Ohnishi, A. Ozawa, T. Suda, K. Sueki, H. Xu, T. Yamaguchi, A. Yoneda, A. Yoshida, and Y. Zhao, J. Phys. Soc. Jpn. **73**, 2593 (2004).
[34] K. Morita, K. Morimoto, D. Kaji, T. Akiyama, Sin-ichi Goto, H. Haba, E. Ideguchi, K. Katori, H. Koura, H. Kikunaga, H. Kudo, T. Ohnishi, A. Ozawa, N. Sato, T. Suda, K. Sueki, F. Tokanai, T. Yamaguchi, A. Yoneda, and A. Yoshida, J. Phys. Soc. Jpn. **76**, 045001 (2007).
[35] Z. G. Gan, J. S. Guo, X. L. Wu, Z. Qin, H. M. Fan, X. G. Lei, H. Y. Liu, B. Guo, H. G. Xu, R. F. Chen, C. F. Dong, F. M. Zhang, H. L. Wang, C. Y. Xie, Z. Q. Feng, Y. Zhen, L. T. Song, P. Luo, H. S. Xu, X. H. Zhou, G. M. Jin, and Zhongzhou Ren, Eur. Phys. J. A **20**, 385 (2004).
[36] Tiekuang Dong and Zhongzhou Ren, Phys. Rev. C **72**, 064331 (2005).
[37] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. **A729**, 337 (2003).
[38] W. Satuła, D. J. Dean, J. Gary, S. Mizutori, and W. Nazarewicz, Phys. Lett. **B407**, 103 (1997).
[39] F. Šimkovic, Ch. C. Moustakidis, L. Pacearescu, and A. Faessler, Phys. Rev. C **68**, 054319 (2003).
[40] R. B. Cakirli, D. S. Brenner, R. F. Casten, and E. A. Millman, Phys. Rev. Lett. **94**, 092501 (2005).
[41] R. B. Cakirli and R. F. Casten, Phys. Rev. Lett. **96**, 132501 (2006).
[42] M. Stoitsov, R. B. Cakirli, R. F. Casten, W. Nazarewicz, and W. Satuła, Phys. Rev. Lett. **98**, 132502 (2007).
[43] F. D. Becchetti, L. T. Chua, J. Jänecke, and A. M. VanderMolen, Phys. Rev. Lett. **34**, 225 (1975).
[44] F. D. Becchetti and J. Jänecke, Phys. Rev. Lett. **35**, 268 (1975).
[45] Z. Ren and G. O. Xu, Phys. Rev. C **38**, 1078 (1988).
[46] A. Bohr and B. R. Mottelson, *Nuclear Structure* (W. A. Benjamin, New York, 1975), Vol. 1.
[47] H. Makii, T. Ishii, M. Asai, K. Tsukada, A. Toyoshima, M. Matsuda, A. Makishima, J. Kaneko, H. Toume, S. Ichikawa, S. Shigematsu, T. Kohno, and M. Ogawa, Phys. Rev. C **76**, 061301(R) (2007).
[48] M. Asai, K. Tsukada, M. Sakama, S. Ichikawa, T. Ishii, Y. Nagame, I. Nishinaka, K. Akiyama, A. Osa, Y. Oura, K. Sueki, and M. Shibata, Phys. Rev. Lett. **95**, 102502 (2005).
[49] D. N. Poenaru, I. H. Plonski, R. A. Gherghescu, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **32**, 1223 (2006).
[50] D. N. Poenaru, I. H. Plonski, and W. Greiner, Phys. Rev. C **74**, 014312 (2006).
[51] P. Roy Chowdhury, C. Samanta, and D. N. Basu, Phys. Rev. C **73**, 014612 (2006).
[52] C. Samanta, P. Roy Chowdhury, and D. N. Basu, Nucl. Phys. **A789**, 142 (2007).
[53] A. Parkhomenko and A. Sobiczewski, Acta Phys. Pol. B **36**, 3095 (2005).

[54] V. E. Viola and G. T. Seaborg, J. Inorg. Nucl. Chem. **28**, 741 (1966).
[55] Tiekuang Dong and Zhongzhou Ren, Eur. Phys. J. A **26**, 69 (2005).
[56] J. K. Tuli, S. Singh, and A. K. Jain, Nucl. Data Sheets **107**, 1347 (2006).
[57] A. K. Jain, S. Singh, and J. K. Tuli, Nucl. Data Sheets **107**, 2103 (2006).
[58] N. Nica, Nucl. Data Sheets **106**, 813 (2005).
[59] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **45**, 2247 (1992).