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Giant dipole resonance as a quantitative constraint on the symmetry energy

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The possible constraints on the poorly determined symmetry part of the effective nuclear Hamiltonians or effective energy functionals, i.e., the so-called symmetry energy $S(\rho)$, are very much under debate. In the present work, we show that the value of the symmetry energy associated with Skyrme functionals, at densities ρ around $0.1\ \mathrm{fm}^{-3}$, is strongly correlated with the value of the centroid of the Giant Dipole Resonance (GDR) in spherical nuclei. Consequently, the experimental value of the GDR in, e.g., ^{208}Pb can be used as a constraint on the symmetry energy, leading to $23.3\ \mathrm{MeV} < S(\rho = 0.1\ \mathrm{fm}^{-3}) < 24.9\ \mathrm{MeV}$.

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Introduction. The nuclear structure community is currently striving to determine a nuclear energy functional as universal and as accurate as possible. Extraction of this functional from a more fundamental theory like QCD is of course desirable, and there has been progress along this line. At present, however, it is still unavoidable to work with functionals that depend on free parameters that must be determined by some fitting procedure.

Existing functionals include those based on a covariant formulation [1] as well as those based on nonrelativistic formulations. Restricting ourselves to the latter case, we note that a nuclear energy functional can be defined in a general way, without being derived from an underlying Hamiltonian in conjunction with a reference state. However, most of the existing functionals to date are derived from an effective Hamiltonian H_{eff} that includes the kinetic energy plus a two-body interaction. In this case, the total energy is the expectation value of H_{eff} over the most general Slater determinant $|\Phi\rangle$. Both zero-range interactions like the one proposed by Skyrme at the end of the fifties, and systematically parametrized for Hartree-Fock (HF) calculations since the seventies [2,3], or finite-range interactions like the Gogny force [4] lead to satisfactory descriptions of many nuclear properties.

Our tool of choice in the present work is the zero-range Skyrme force, from which one can derive a functional $\mathcal{E}[\rho]$ which is a function of *local* densities only. For a system that is not symmetric in neutrons and protons, the total energy depends on both neutron and proton density:

$$E[\rho] = \int d_{3r}\mathcal{E}(\rho_n(\vec{r}), \rho_p(\vec{r})). \tag{1}$$

For the sake of simplicity, we have not indicated that, in general, the energy depends not only on the spatial densities but also on gradients $\nabla \rho_q$, on the kinetic energy densities τ_q, and on the spin-orbit densities J_q (where q labels n, p) [5,6].

In infinite matter, one has a simple expression in terms of the spatial densities only. Instead of ρ_n and ρ_p, one can use the total density ρ and the *local* neutron-proton asymmetry,

$$\delta \equiv \frac{\rho_n - \rho_p}{\rho}. \tag{2}$$

This quantity should not be confused with the *global* asymmetry $(N - Z)/A$. In asymmetric matter, we can make a further simplification on $\mathcal{E}(\rho, \delta)$ by making a Taylor expansion in δ and retaining only the quadratic term,

$$\begin{aligned}\mathcal{E}(\rho, \delta) &\approx \mathcal{E}_0(\rho, \delta = 0) + \mathcal{E}_{\mathrm{sym}}(\rho)\delta^2 \\ &= \mathcal{E}_0(\rho, \delta = 0) + \rho S(\rho)\delta^2.\end{aligned} \tag{3}$$

The first term on the r.h.s. is the energy density of symmetric nuclear matter $\mathcal{E}_{\mathrm{snm}}$, while the second term defines the main object of the present study, namely, the *symmetry energy* $S(\rho)$. The symmetry energy at saturation $S(\rho_0)$ is denoted by different symbols in the literature: J, a_τ, or a_4. We stress that Eq. (3) is not really a simplification: the coefficient of the term in δ^4 that should follow, for the Skyrme parameter sets employed in this work, is negligible at densities of the order of ρ_0. We remind the reader that the pressure can be written in a uniform system as

$$P = -\left.\frac{\partial E}{\partial V}\right|_A = \rho^2 \frac{\partial}{\partial \rho}\left.\frac{\mathcal{E}}{\rho}\right|_A. \tag{4}$$

This quantity is evidently related to the density dependence of the energy functional and of the associated symmetry part defined above.

The magnitude and the density dependence of the symmetry energy $S(\rho)$ are not yet well understood [7]. In brief, there exist at present three main research lines aimed at constraining the behavior of the symmetry energy, by using nuclear structure data, observables related to heavy-ion collisions, or evidences from the study of neutron stars.

Within the realm of nuclear structure, the symmetry energy affects, of course, all properties of nuclei having neutron excess, including basic ones like masses and radii. In particular, much attention has been focused on radii since Typel and Brown [8,9] have noted that the neutron skin thickness $\delta R \equiv \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ is correlated with the pressure in neutron matter $P_{\mathrm{nm}}(\rho = 0.1\ \mathrm{fm}^{-3})$ (see also Ref. [10]). The issue is also investigated in Ref. [11], where the correlations between the neutron skin thickness and other quantities are discussed (see also Ref. [12]). The experimental accuracy is not sufficient (so far) to limit the acceptable range of the neutron skin thickness so that this can constrain a given equation of state; the Parity Radius Experiment (PREX) at JLAB promises to achieve this task [13]. Another interesting way to fix the value of the neutron skin thickness, and extract information on the symmetry energy, is to go through the study of the isovector Spin-Dipole Resonance (SDR) sum rule; also in this case, the experimental difficulties hinder a too definite conclusion (see Ref. [14]).

In this article we shall instead concentrate on the correlation between the symmetry energy and the energy of the Giant Dipole Resonance (GDR). The idea is not new, but in the present work we develop it based on a fully microscopic approach, namely, within a self-consistent Random Phase Approximation (RPA) scheme to calculate the GDR properties. In the past [15], as well as in recent works [16], the connection with the symmetry energy has been discussed starting from a macroscopic, hydrodynamical description of the GDR (in particular, using the Steinwedel-Jensen ansatz, which is known to be not fully reliable). Consequently, we believe that our results are more relevant from a quantitative point of view.

If one tries to constrain the symmetry energy by means of the study of heavy-ion collisions, or by neutron star observables, it is likely that somewhat different physics is needed. In general, the behavior of the symmetry energy on a *broader* range of densities is involved. In heavy-ion collisions maximum densities up to ∼4–5 ρ_0 can be attained [17]. On the other hand, data at lower incident energies are believed to be able to constrain the nuclear EOS below ρ_0 [18]. The study of neutron stars, as well, brings in the physics of both low-density and high-density neutron matter (for a comprehensive review, cf. Ref. [19]). A recent study [20], which is close in spirit to ours, has examined a large set of Skyrme forces, trying to determine those that have a satisfactory behavior in reproducing the neutron-star observables. One should remark, however, that there are many caveats in the literature against the use of functionals in a density regime far from that in which the functionals are fitted and usually employed. In particular, we mention here that Monte Carlo calculations of neutron matter at low density [21] show that in this regime E/A is about one half of the Fermi energy of the noninteracting neutron gas, and this behavior is not reproduced by any effective mean field functional.

Consequently, we do not discuss in detail in the present work the possibility of an overall constraint on the symmetry energy extracted by different kinds of studies. We briefly discuss in the conclusions to what extent our results can be compared with a few others in the literature.

The correlation between the GDR and the symmetry energy. Our starting point will be the hydrodynamical model of giant resonances, proposed by Lipparini and Stringari [22]. They assume an energy functional that is simplified yet sufficiently realistic, solve the macroscopic equations for the densities and currents, and extract expressions for the moments m_1 and m_{-1} associated with an external operator F ($m_k \equiv \int dE S(E) E^k$, where S is the strength function associated with F). The expression for m_1 is proportional to $(1+\kappa)$, where κ is the well-known "enhancement factor," which in the case of Skyrme forces is associated with their velocity dependence. The expression for m_{-1}, in the case of an isovector external operator, includes integrals involving $\mathcal{E}_{\rm sym}$ and F. They can be evaluated in a simple way if one assumes the validity of the leptodermous expansion. We write the volume and surface coefficients of the expansion of $\mathcal{E}_{\rm sym}$ as $b_{\rm vol}$ and $b_{\rm surf}$, respectively. By specializing F to the isovector dipole case, the following expression is obtained (for details, cf. Ref. [22]),

$$E_{-1} \equiv \sqrt{\frac{m_1}{m_{-1}}} = \sqrt{\frac{3\hbar^2}{m\langle r^2\rangle}\frac{b_{\rm vol}}{\left[1+\frac{5}{3}\frac{b_{\rm surf}}{b_{\rm vol}}A^{-\frac{1}{3}}\right]}(1+\kappa)}. \quad (5)$$

This equation yields values of the centroid energy that are in rather good agreement with those of microscopic RPA calculations. It turned out to be useful in a previous study [23], to constrain directly the parameters of the isovector part of the Skyrme interaction. Here we use it as a guideline and try instead to find a quantitative connection between the energy of the GDR and the symmetry energy.

The ratio $\frac{b_{\rm surf}}{b_{\rm vol}}$ can be evaluated through the calculation of a semi-infinite nuclear slab. This has been done, e.g., in Ref. [24] (cf. their Sec. 3.2.3). We do not discuss here the approximations made in the derivation, but we use the fact that the mentioned ratio can be written in terms of the symmetry energy and its derivatives. If we insert this result into Eq. (5) we obtain

$$E_{-1} = \sqrt{\frac{6\hbar^2}{m\langle r^2\rangle}g_A(\rho_0)(1+\kappa)}, \quad (6)$$

where

$$g_A(\rho) = \frac{S(\rho)}{1+\frac{5}{S(\rho)}\left[\rho\frac{dS}{d\rho}-\frac{\rho^2}{4}\frac{d^2S}{d\rho^2}\right]A^{-\frac{1}{3}}}. \quad (7)$$

For a given heavy nucleus we can safely consider the first of the three factors under the square root as a constant, because different Skyrme forces do not vary widely in their predictions for $\langle r^2\rangle$. We have evaluated the term $g_A(\rho)$ at $\rho=\rho_0$ for $A=40, 124, 208$ and for a number of Skyrme forces. We have found that it is strongly correlated with the value of $S(\rho)$ in the range $\rho = 0.08$–0.12 fm^{-3}. The specific case $A=208$ is displayed in Fig. 1 in the case $\rho = 0.1$ fm^{-3}, for which the correlation coefficient is maximum.

Although we have not been able to deduce this correlation in an analytic way from the expression of the Skyrme functional, this result, together with Eq. (6), motivates us to look for a direct correlation between the centroid of the GDR and the

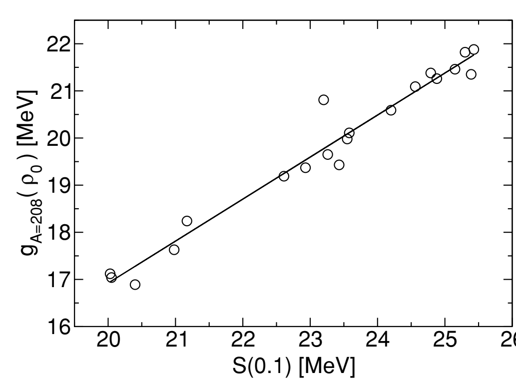

FIG. 1. Correlation between the quantity $g_{A=208}(\rho_0)$ and the symmetry energy $S(\rho)$ for the Skyrme forces listed in Table I, at $\rho = 0.1$ fm^{-3}. The value of the correlation coefficient is $r = 0.981$.

symmetry energy, through the quantity

$$f(\rho) \equiv \sqrt{S(\rho)(1+\kappa)}, \quad (8)$$

for $\rho \sim 0.1$ fm^{-3}. In the next section, we discuss this correlation for $\rho = 0.1$ fm^{-3} in the case of ^{208}Pb. We also discuss in some detail the choice of the Skyrme forces that we have employed.

Results. We have obtained results for the GDR in ^{208}Pb by using a series of microscopic Hartree-Fock (HF) plus Random Phase Approximation (RPA) calculations. Skyrme-RPA theory has been well known for many years, especially in its matrix formulation. Recently, we have developed a scheme that is fully self-consistent and is discussed in Ref. [25]. There is no approximation in the residual interaction, in that all its terms are taken into account (including the two-body spin-orbit and Coulomb interactions). The occupied states are determined by solving the HF equations in a radial mesh extending up to 24 fm. The continuum is discretized by using box boundary conditions (the box size is 24 fm). Particle-hole (p-h) configurations, which constitute the basis for the RPA matrix equations, are included up to typically 60 MeV so that the value of m_1 is at least 98% of the well-known value obtained from the double commutator. In a few cases, due to the instability of RPA, we had to resort to Tamm-Dancoff Approximation (TDA) calculations but this does not affect significantly our results.

The calculations have been performed for a set of 20 Skyrme interactions. It is well known that more than 100 Skyrme parametrizations have been proposed in the literature, including some that have been employed only in limited and specific cases. It is hard to define in a clear-cut way a "standard" subset to be analyzed; however, we have decided, in the present context, not to consider parametrizations that (a) have an associated K_∞ outside of the range 210–270 MeV, in keeping with the conclusions reached in Ref. [26] by studying the giant monopole resonance (GMR) in ^{208}Pb, and (b) do not reproduce the experimental value of the GDR in ^{208}Pb (13.46 MeV [27]) within ±2 MeV.

We have determined our set including forces proposed by different groups and at different times. In this sense the set can be considered representative enough. In the cases in which several forces have been proposed in the same reference, we have not included more than two forces to avoid a too strong bias.[1] The forces are listed in Table I; we also provide the references from which the parameter sets have been taken, the value for $E_{-1}(\mathrm{RPA}) \equiv \sqrt{\frac{m_1}{m_{-1}}}$ obtained from our RPA calculation, and the values of the quantities $f(0.1)$, $S(0.1)$, and κ [cf. Eq. (8)].

We find a strong linear correlation between the values of $E_{-1}(\mathrm{RPA})$ and $f(0.1)$, which are shown in Fig. 2 together with the interpolating straight line $f(0.1) = a + bE_{-1}(\mathrm{RPA})$.

[1]We have nonetheless checked that, expanding the set shown in Table I, that is, including the forces SkT1, SkT2, SkT3, SkT5, SkT7, SkT8, SkT9, Es, Zs, SkI3, SkI5, MsK1, MSk2, MSk3, MSk4, MSk5, MSk6, v105, v100, v090, v080, BSk1 (whose references can be found in Ref. [20]), the final result for the constraint on $S(0.1)$ reported in Eq. (13) does not vary within 200 keV.

TABLE I. For the Skyrme parameter sets considered in this work, we provide the values of $E_{-1}(\mathrm{RPA})$, $f(0.1)$, $S(0.1)$, and κ. All these quantities are defined in the text.

	Ref.	E_{-1} (MeV)	$f(0.1)$ (MeV$^{1/2}$)	$S(0.1)$ (MeV)	κ
SkA	[28]	15.14	6.27	23.43	0.68
SkM	[15]	13.94	5.65	23.26	0.37
SGI	[29]	14.16	5.62	20.40	0.55
SGII	[29]	13.56	5.34	20.98	0.36
SkM*	[30]	13.89	5.63	22.93	0.38
RATP	[31]	15.17	6.06	23.55	0.56
SkT4	[32]	11.47	4.86	23.58	0.00
SkT6	[32]	12.17	4.92	24.20	0.00
Rs	[33]	12.76	5.20	20.05	0.35
Gs	[33]	12.62	5.20	20.03	0.35
SkI2	[34]	12.29	5.00	21.17	0.18
SLy230a	[35]	12.49	5.04	25.43	∼0
SLy4	[5]	13.40	5.45	25.15	0.18
SLy5	[5]	13.28	5.42	24.88	0.18
SkO'	[36]	13.85	5.00	22.61	0.11
MSk7	[37]	12.10	4.86	24.56	−0.04
v110	[38]	12.13	4.80	24.79	−0.07
v075	[38]	13.97	5.62	25.30	0.25
SK255	[39]	13.98	5.94	25.39	0.39
LNS	[40]	13.95	5.43	23.20	0.27

The value of the correlation coefficient is $r = 0.909$. Before discussing the extraction of the value of the symmetry energy, we should stress that we have not been able to correlate the GDR simply with $S(\rho_0)$; this may be possible (cf., e.g., Ref. [41]) at the price of restricting oneself to a small set of Skyrme forces.

We can now avail ourselves of the experimental values of the GDR centroid $E_{-1}(\exp)$ and of the enhancement factor κ to deduce the best value of the symmetry energy. While the value of $E_{-1}(\exp)$ in ^{208}Pb has been rather well determined from photoabsorbtion measurements, $E_{-1}(\exp) = 13.46$ MeV [27],

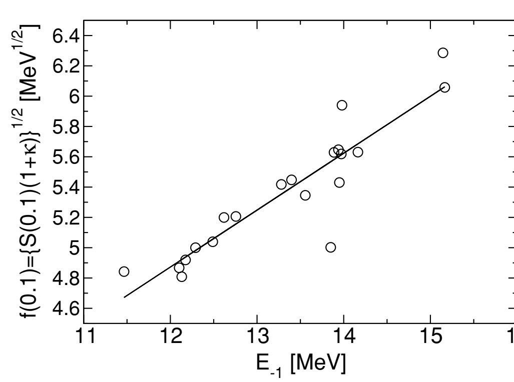

FIG. 2. Correlation between the energy of the GDR and the quantity $f(0.1)$. The definition of these quantities, and the related discussion, can be found in the text.

the value of κ is less precise. Because

$$\int_0^{\infty} \sigma(E)dE = 60\left(\frac{NZ}{A}\right)(1+\kappa)\text{ MeV mb}, \tag{9}$$

κ would be determined if the integrated photoabsorbtion cross section $\sigma(E)$ had been measured up to large energies (essentially, up to the pion production threshold). For obvious experimental difficulties, the photoabsorbtion cross section has been measured up to much lower energies (25 MeV in the case at hand, namely, ^{208}Pb [42]). The experimental value with the associated error can be divided by the well-known Thomas-Reiche-Kuhn (TRK) sum rule and provides an "effective" κ that we can call $\kappa_{25} = 0.17 \pm 0.08$. Furthermore, we have noticed that there is a very strong linear correlation between κ_{25} and κ for all the Skyrme forces listed in Table I, leading to $\kappa = 0.22 \pm 0.04$. This is in agreement with the estimate reported in Ref. [22], where it has been stated that κ should lie, approximately, between 0.2 and 0.3.

At this point, the best value for $S(0.1)$ is found as

$$S(0.1) = \frac{\left(a + bE_{-1}^{\text{exp}}\right)^2}{(1+\kappa)}, \tag{10}$$

where a, b come from the fit. The error is obtained from

$$\sigma_{\sqrt{S(0.1)(1+\kappa)}} = \sigma_b \sqrt{\sigma_{E_{-1}}^2 + (\bar{E}_{-1} - E_{-1}^{\text{exp}})^2}, \tag{11}$$

where σ_b comes from the fit and the variance $\sigma_{E_{-1}}^2$ is calculated with respect to the interpolating straight line. Having determined the $\pm 1\sigma$ interval around the mean value for the quantity $\sqrt{S(0.1)(1+\kappa)}$, we obtain

$$\frac{5.419}{\sqrt{1+\kappa_{\max}}} < \sqrt{S(0.1)} < \frac{5.422}{\sqrt{1+\kappa_{\min}}}. \tag{12}$$

This can be considered one of the main results of the present investigation. By introducing the values of $\kappa_{\min}$ and $\kappa_{\max}$ discussed above, that is, 0.18 and 0.26, one obtains a further (more direct) constraint, that is,

$$23.3 \text{ MeV} < S(0.1) < 24.9 \text{ MeV}. \tag{13}$$

Conclusions. Using a representative set of Skyrme effective functionals we have found a clear correlation between the energy of the GDR in ^{208}Pb and a simple function of the symmetry energy at density $\rho \sim 0.1$ fm^{-3} and of the enhancement factor κ associated with the velocity dependence (and with the effective mass) of the various functionals. Using the well-established experimental value of the GDR we have extracted a range of acceptable values for $S(0.1)$ [cf. Eq. (13)].

It will be important to test whether other classes of effective functionals lead to a similar result. More generally, it will be essential to study the interplay between constraints coming from the different kinds of works mentioned in the Introduction, which deal with different energy and density regimes. In fact, a better knowledge of the symmetry part of the nuclear effective functionals, and in particular of its density dependence, would be highly instrumental for the study of systems ranging from exotic nuclei to pure neutron matter.

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