PHYSICAL REVIEW C **77**, 058201 (2008)

Axial coupling from matching a constituent quark model to QCD

S. S. Afonin

V. A. Fock Department of Theoretical Physics, St. Petersburg State University, 1 ul. Ulyanovskaya, RU-198504 St. Petersburg, Russia

(Received 27 August 2007; revised manuscript received 26 March 2008; published 28 May 2008)

The axial-vector coupling g_A of a constituent quark is estimated from matching the constituent quark model to the operator product expansion in QCD in the limit of large number of colors under some assumptions. The obtained relation is $g_A \simeq \sqrt{7/11} \approx 0.80$, which is in agreement with the existing model estimates.

DOI: 10.1103/PhysRevC.77.058201 PACS number(s): 24.85.+p, 11.40.−q, 12.38.−t, 13.30.−a

The constituent quark model (CQM) of Manohar and Georgi [1] (often called the chiral quark model) has been used with great phenomenological success in the description of strong interactions at low energies, thus giving rise to various extensions and new applications (see, e.g., Ref. [2]). The underlying philosophy of this model is familiar from different branches of physics—the idea of rearrangement of physical degrees of freedom at certain energy scale. The proposed scenario assumes that in the energy region between the chiral symmetry breaking (CSB) scale, $\Lambda_{\rm CSB} \simeq 1$–$1.2$ GeV, and the confinement scale, $\Lambda_{\rm QCD} \simeq 100-300$ MeV, the almost massless strongly interacting ($\alpha_s(\Lambda_{\rm QCD}) > 1$) quarks entering the QCD Lagrangian are effectively rearranged into heavy weakly interacting ($\alpha_s(\Lambda_{\rm QCD}) \simeq 0.28$) constituent quarks with the effective mass $m \simeq 300-350$ MeV. Simultaneously, the interaction of fundamental quarks and gluons is rearranged below $\Lambda_{\rm CSB}$ into the interaction of the constituent quarks with the Goldstone bosons—the pions—associated with the spontaneous CSB and, possibly, with the low-energy gluons. For instance, the $SU(2)_L$ current in the effective CQM Lagrangian [1] is

$$j^{\rm CQM}_{\mu,L} = \bar{\psi}\gamma_\mu(1 - g_A\gamma_5)\vec{\tau}\psi + \text{terms involving } \pi, \tag{1}$$

while the same current in the QCD Lagrangian above $\Lambda_{\rm CSB}$ is

$$j^{\rm QCD}_{\mu,L} = \bar{\psi}\gamma_\mu(1 - \gamma_5)\vec{\tau}\psi. \tag{2}$$

It should be emphasized that, generally speaking, the quark and gluon fields in the CQM Lagrangian are not the same as the ones in the QCD Lagrangian. Neglecting the current quark masses, both fundamental and effective theories are chirally invariant, but the chiral symmetry is realized nonlinearly in the effective theory in contradistinction to the linear realization in the fundamental theory; thus the chiral symmetry undergoes a sort of "rearrangement" of its realization below $\Lambda_{\rm CSB}$ rather than breaking.

In this Brief Report we are concerned with the axial coupling g_A that is present in current (1). This coupling is of high importance in the phenomenology because the $SU(2)_L$ current (1) couples to the W boson, thus triggering some semileptonic decays. Making use of nonrelativistic quark model wave functions, g_A can be related with the axial constant G_A that parametrizes the amplitude of the nucleon β decay, the relation is [1]

$$g_A = \tfrac{3}{5}G_A. \tag{3}$$

Taking the modern experimental value for the axial constant [3], $G_A \approx 1.27$, relation (3) yields the estimate $g_A \approx 0.76$.

As was noted in Ref. [1], g_A should be calculable from QCD, although this is a hard nonperturbative calculation and there is still no idea how to perform it. At present there are only some estimates based on effective models and on the analog of the Adler-Weisberger sum rule for quark-pion scattering; see Ref. [4] for a brief review and also Refs. [5] and [6].

We consider a quite different way of addressing the problem. One can try to probe the vacuum by the vector (V) and axial-vector (A) currents of the fundamental and effective theories; the analysis and comparison of the corresponding responses might lead to definite conclusions. It is rather hard to garment such a general idea with precise calculations, a kind of guesswork is inescapable to advance. We propose an heuristic way for the analytical realization of this program, a way which, albeit qualitative, will result in a numerical estimate for g_A.

Our proposal is based on the expectation that the applicability of the CQM and that of the perturbative QCD should overlap in some energy region; i.e., both theories should give the same result in that region, thus they can be matched (similar ideas of matching were exploited in various effective models, see, e.g., Refs. [2] and [7]). First of all, let us estimate the matching region. The main difference between the CQM and QCD is induced by the pions as long as they are quark-antiquark bound states in QCD, while within the CQM the pions represent fundamental fields. It is expected (see, e.g., Ref. [6]) that the simple chiral quark model is only applicable in the low-energy region $\mu < m_\rho$, where m_ρ is the ρ-meson mass, $m_\rho = 775.5$ MeV [3], whereas in the intermediate energy region, $m_\rho < \mu < \Lambda_{\rm CSB}$, one should take into account the higher order derivative terms of the pion field and, probably, the ρ and σ mesons and long-range gluons as explicit degrees of freedom. Thus, it is reasonable to try to match the intermediate energy region to QCD. This point is partly supported by the success of the chiral perturbation theory [8], which basically is due to the existence of a natural small parameter $m_\pi^2/m_\rho^2 \simeq 0.03$ in the low-energy strong interactions. On the other hand, the operator product expansion (OPE) allows us to extend the applicability of the perturbative

QCD to the region below $\Lambda_{\rm CSB}$, up to the scale $\mu \simeq m_\rho$ [9]. We arrive thus at the conclusion that the matching region should be $m_\rho < \mu < \Lambda_{\rm CSB}$.

Let us for a while neglect the pion interactions in the CQM and consider only almost free constituent quarks sufficiently weakly interacting by means of low-energy gluons. The $SU(2)V$ and A quark currents can be then simply constructed,

$$j_{\mu,V}^{\rm CQM} = \bar{\psi}\gamma_\mu\vec{\tau}\psi, \qquad j_{\mu,A}^{\rm CQM} = \bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi. \tag{4}$$

Consider the two-point correlators of these currents,

$$\Pi_{\mu\nu,J}^{\rm CQM}(q^2) = \int d^4x\, e^{-iqx}\left\langle j_{\mu,J}^{\rm CQM}(x) j_{\nu,J}^{\rm CQM}(0)\right\rangle, \tag{5}$$

here $J = V, A$. As long as the effective coupling constant in the CQM is rather small, one may estimate the V and A correlators (5) by doing a standard one-loop perturbative calculation for the polarization function. Thus,

$$\Pi_{\mu\nu,V}^{\rm CQM}(q^2) \sim \int d^4p\,{\rm tr}\frac{\gamma_\mu}{\frac{\not q}{2}+\not p - m_{\rm con}}\frac{\gamma_\nu}{\frac{\not q}{2}-\not p - m_{\rm con}}, \tag{6}$$

$$\Pi_{\mu\nu,A}^{\rm CQM}(q^2) \sim \int d^4p\,{\rm tr}\frac{\gamma_\mu\gamma_5}{\frac{\not q}{2}+\not p - m_{\rm con}}\frac{\gamma_\nu\gamma_5}{\frac{\not q}{2}-\not p - m_{\rm con}}, \tag{7}$$

where $m_{\rm con}$ is the constituent quark mass. Taking the trace and neglecting the irrelevant for us terms quadratic in cutoff, we obtain

$$\Pi_{\mu\nu,V}^{\rm CQM}(q^2) \sim \{(-\delta_{\mu\nu}q^2 + q_\mu q_\nu)F_- + q_\mu q_\nu F_+\}I(q^2), \tag{8}$$

$$\Pi_{\mu\nu,A}^{\rm CQM}(q^2) \sim \{(-\delta_{\mu\nu}q^2 + q_\mu q_\nu)F_+ + q_\mu q_\nu F_-\}I(q^2), \tag{9}$$

where

$$F_\pm = 1 \pm \frac{4m_{\rm con}^2}{q^2}, \tag{10}$$

$$I(q^2) = \int d^4p \frac{1}{\left(\frac{q}{2}+p\right)^2 - m_{\rm con}^2}\frac{1}{\left(\frac{q}{2}-p\right)^2 - m_{\rm con}^2}. \tag{11}$$

To compare these expressions with the OPE in QCD we have to perform the Wick rotation and consider the transverse part $\Pi_{J\perp}^{\rm CQM}(q^2)$ only,

$$\Pi_{V\perp}^{\rm CQM}(Q^2) \sim \left(1 - \frac{4m_{\rm con}^2}{Q^2}\right)I(Q^2), \tag{12}$$

$$\Pi_{A\perp}^{\rm CQM}(Q^2) \sim \left(1 + \frac{4m_{\rm con}^2}{Q^2}\right)I(Q^2). \tag{13}$$

Is is seen that the V and A correlators are equal in the limit of exact chiral symmetry, $m_{\rm con} \to 0$, and in the limit of asymptotic chiral symmetry, $Q^2 \to \infty$. In the limit of vanishing euclidean momentum, $Q^2 \to 0$, they have opposite signs, but equal absolute value. This sign flip could be regarded as a signal of change of chiral symmetry realization at low energies. The second term in Eqs. (12) and (13) emerges because of the chiral symmetry breaking. It is different for the V and A channels; let us denote it $\Pi_{{\rm CSB},J}^{\rm CQM}(Q^2)$. Of interest for us is the fraction

$$\frac{\Pi_{{\rm CSB},V}^{\rm CQM}(Q^2)}{\Pi_{{\rm CSB},A}^{\rm CQM}(Q^2)} = -1. \tag{14}$$

The inclusion of pion interactions should correct the simple picture above because the derivative of the pion field enters the axial-vector current; moreover, in the matching region, $m_\rho < \mu < \Lambda_{\rm CSB}$, the higher order derivative terms of the pion field may become significant. However, such derivative terms can affect strongly the longitudinal parts of the correlators while they should not couple to the transverse parts in the chiral limit. Because we are working with the transverse parts only, it looks reasonable to neglect the terms involving π in Eq. (1). Moreover, we will match the CQM to QCD in the large-N_c limit [10]; hence, the same limit has to be taken from the CQM side, this provides a suppression of possible multiparticle contributions to current (1). Thus, the residual effect of the strong interactions reduces to the renormalization of the axial-vector current [factor g_A in Eq. (1)]. This constitutes our first assumption.

Our second assumption concerns a concrete realization of this renormalization. We propose an alternative interpretation of the origin of the axial coupling g_A in Eq. (1). In QCD, one constructs the V and A currents from the same quark spinors, but in the effective theory it is not evident that we are allowed to do this. Actually, the axial-vector sector is affected strongly by the pion interactions. This may lead to the fact that we should use another quark spinors in the A channel; let us denote this circumstance by a prime. But phenomenological success of the CQM suggests that, neglecting the direct pion contributions, the action of the A' current may be simulated as the action of the A current (constructed from the same quark spinors as the V current) if we accept the following renormalization prescription: $A = g_A A'$. The $V - A$ current in QCD, Eq. (2), turns into the $V - g_A A'$ current below $\Lambda_{\rm CSB}$. The identical notation for the quark spinors in the vector and axial-vector parts of current (1) should be then understood symbolically only.

We would provide the following qualitative support in favor of this hypothesis. If the constituent quarks are almost free, the corresponding left nucleon current is expected to experience the same renormalization. The nucleon analog $j_{\mu,L}^N$ of the left quark current (1) enters the amplitude of the nucleon β decay and it can be written in the form

$$j_{\mu,L}^N = \bar{\psi}_p\gamma_\mu(1 - G_A\gamma_5)\psi_n. \tag{15}$$

This current is constructed following the Fermi V-A theory of weak interactions. Hence, initially one has the V-A' current but then in calculations one uses the same $\psi_{p,n}$ for the vector and axial-vector parts, i.e., one works with the V-$g_A^{-1}A$ hadron current. Consequently, we expect $g_A \approx G_A^{-1} \approx 0.79$, which is reasonable (notice that from relation (3) we would formally obtain $g_A = \sqrt{0.6} \approx 0.77$, which is also not bad). If our hypothesis is right this numerology is not accidental.

Thus, the assumptions above result in the following conclusion: In ratio (14) we had different quark spinors in the numerator and in the denominator. If we want to compare the correlators calculated with the same spinors, say with the ones entering the vector current (let us refer to them as bare spinors), we should renormalize the A correlator in the following way,

$$\Pi^{\mathrm{CQM}}_{\mathrm{CSB},A}(Q^2) \to g_A^{-2}\Pi^{\mathrm{CQM}}_{\mathrm{CSB},A}(Q^2). \tag{16}$$

This renormalization effectively takes into account the contribution of pion interactions.

Now we formulate our matching condition between the CQM and the QCD in the region $m_\rho < \mu < \Lambda_{\mathrm{CSB}}$, where both are expected to describe the same physics related to CSB. We require that the *bare* quark spinors in the CQM Lagrangian can be replaced by those of the QCD Lagrangian at $m_\rho < \mu < \Lambda_{\mathrm{CSB}}$ with ensuing equality of operators of quark currents,

$$\bar{\psi}\gamma_\mu\vec{\tau}\psi|^{\mathrm{bare}}_{\mathrm{CQM}} \simeq \bar{\psi}\gamma_\mu\vec{\tau}\psi|_{\mathrm{QCD}}, \tag{17}$$

$$\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi|^{\mathrm{bare}}_{\mathrm{CQM}} \simeq \bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi|_{\mathrm{QCD}}. \tag{18}$$

We require also that the same identification is valid between the fundamental gluon fields in the QCD Lagrangian and the long-range gluons in the CQM. Then relations (14) and (16) lead to

$$\frac{\Pi^{\mathrm{QCD}}_{\mathrm{CSB},V}(Q^2)}{\Pi^{\mathrm{QCD}}_{\mathrm{CSB},A}(Q^2)} \simeq -g_A^2. \tag{19}$$

The problem now is to find $\Pi^{\mathrm{QCD}}_{\mathrm{CSB},J}(Q^2)$, i.e., the parts of the corresponding QCD correlators that appear due to the CSB. A solution of such a task is known in the euclidean region due to the OPE method [9]. Accepting the chiral and large-N_c [10] limits, the OPE for $\Pi^{\mathrm{QCD}}_{\mu\nu,J}(q^2)$ in the one-loop approximation reads as follows at large euclidean momentum Q,

$$\Pi^{\mathrm{QCD}}_{J\perp}(Q^2) = \frac{N_c}{12\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{Q^2} + \frac{\alpha_s}{12\pi}\frac{\langle G^2\rangle}{Q^4} + \frac{4\pi\xi_J\alpha_s}{9}\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right), \tag{20}$$

where

$$\xi_V = -7, \qquad \xi_A = 11, \tag{21}$$

and we have defined

$$\Pi^{\mathrm{QCD}}_{\mu\nu,J}(Q^2) = (-\delta_{\mu\nu}Q^2 + Q_\mu Q_\nu)\Pi^{\mathrm{QCD}}_{J\perp}(Q^2). \tag{22}$$

The symbols $\langle G^2\rangle$ and $\langle\bar{q}q\rangle$ denote the gluon and quark condensate, respectively. The power-like expansion (20) shows explicitly that the CSB effects set in since the $\mathcal{O}(1/Q^6)$ terms. This agrees with our naive model calculation above—expanding Eqs. (12) and (13) at large Q^2 we obtain the same qualitative behavior for the CSB contribution. The part of $\Pi^{\mathrm{QCD}}_{J\perp}(Q^2)$ that absorbs the leading contributions related to the CSB (more precisely, the contributions that are different for the V and A channels), $\Pi^{\mathrm{QCD}}_{\mathrm{CSB},J}(Q^2)$, is evident from Eq. (20),

$$\Pi^{\mathrm{QCD}}_{\mathrm{CSB},J}(Q^2) = \frac{4\pi\xi_J\alpha_s}{9}\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right). \tag{23}$$

The relation above gives

$$\frac{\Pi^{\mathrm{QCD}}_{\mathrm{CSB},V}(Q^2)}{\Pi^{\mathrm{QCD}}_{\mathrm{CSB},A}(Q^2)} = \frac{\xi_V}{\xi_A} + \mathcal{O}\left(\frac{1}{Q^2}\right). \tag{24}$$

Collecting Eqs. (19), (24), and (21), we get our final result,

$$g_A^2 \simeq \tfrac{7}{11}, \tag{25}$$

which yields $g_A \approx 0.80$ in a good agreement with the existing phenomenological estimates.

The obtained value calls for a comment concerning the large-N_c behavior of g_A. In the literature there is a discrepancy with regard to the question of whether or not $g_A = 1$ in the large-N_c limit; see Refs. [4–6] for discussions. Our estimate has been performed taking this limit from the outset since it was used in OPE (20)—the factorized form of the numerator in $\mathcal{O}(1/Q^6)$ term takes place by virtue of the vacuum saturation hypothesis, which is justified in the large-N_c limit only [9]. We conclude thus that the deviation of g_A from unity is not an artifact of the $\mathcal{O}(1/N_c)$ corrections, at least within the presented approach.

Let us summarize our scheme. We have considered the transverse parts of vector and axial-vector two-point correlators and extracted the leading contributions coming from the spontaneous chiral symmetry breaking, which are different for the vector and axial-vector channels. In the constituent quark model, the ratio of these contributions in the vector and axial-vector channels is -1 whereas from the QCD side in the large-N_c limit it is equal to $-7/11$. We assumed that the difference appears mainly from the fact that the QCD vector and axial-vector currents, j^V_μ and j^A_μ, are built from the same quark spinors while in the constituent quark model this is not the case, the effect can be effectively described as the renormalization of the current j^A_μ, $j^A_\mu|_{\mathrm{ren}} = g_A^{-1}j^A_\mu|_{\mathrm{bare}}$, and, in fact, by default one uses the renormalized current in the two-point correlator $\langle j^A_\mu j^A_\nu\rangle$. The correct matching with QCD correlators, however, should be achieved with the unrenormalized currents if we want to escape the double counting of nonperturbative effects. Thus, the axial-vector correlator of the constituent quark model should be multiplied by the factor g_A^{-2} when doing matching to QCD. This gives immediately the relation for the axial coupling, $g_A^2 \simeq 7/11$.

Finally, realizing that the undertaken reasoning is expected, at best, to give an order-by-magnitude estimate, it looks quite spectacular that the obtained value for g_A is so close to the accepted phenomenological estimate. Perhaps, this somewhat justifies *a posteriori* the assumptions made.

A discussion of the work with A. A. Andrianov and D. Espriu is gratefully acknowledged. The work was supported by the Ministry of Education of Russian Federation, Grant RNP.2.1.1.1112.

[1] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[2] A. A. Andrianov and D. Espriu, J. High Energy Phys. 10 (1999) 022.
[3] W.-M. Yao *et al*., J. Phys. G **33**, 1 (2006).
[4] S. Peris and E. de Rafael, Phys. Lett. **B309**, 389 (1993).
[5] W. Broniowski, M. Lutz, and A. Steiner, Phys. Rev. Lett. **71**, 1787 (1993).
[6] M. Jacob and H. Zheng, Phys. Lett. **B321**, 125 (1994).
[7] A. A. Andrianov and V. A. Andrianov, Zap. Nauchn. Semin. **245**, 5 (1996), arXiv:hep-ph/9911383; A. A. Andrianov, D. Espriu, and R. Tarrach, Nucl. Phys. **B533**, 429 (1998); A. A. Andrianov, V. A. Andrianov, and S. S. Afonin, arXiv:hep-ph/0101245, arXiv:hep-ph/0209260; V. A. Andrianov and S. S. Afonin, Eur. Phys. J. A **17**, 111 (2003); J. Math. Phys. Sci. **125**, 99 (2005); Zap. Nauchn. Semin. **291**, 5 (2002).
[8] J. Gasser and H. Leutwyler, Ann. Phys. (NY) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
[9] M. A. Shifman, A. I. Vainstein, and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
[10] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); E. Witten, Nucl. Phys. **B160**, 57 (1979).