

Consistent thermodynamic treatment for a quark-mass density-dependent model

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The ambiguities and inconsistencies in previous thermodynamic treatments for the quark-mass density-dependent model are addressed. A new treatment is suggested to obtain self-consistent results. A new independent variable of effective mass is introduced to make the traditional thermodynamic calculation with partial derivatives still practicable. The contribution from physical vacuum has been discussed. We find that the properties of strange quark matter given by the quark-mass density-dependent model are nearly the same as those obtained by the MIT bag model after considering the contribution of the physical vacuum. Besides the quark system, according to our argument, the new independent variable of effective mass should be taken seriously in any thermodynamically consistent treatment whenever the medium effect is represented in the mass term.

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I. INTRODUCTION

It is generally accepted that the effective masses of particles will change with density as a result of medium effects. Many theoretical considerations, including the finite-temperature quantum hadron dynamics (QHD) model [1], Brown-Rho scaling [2], and finite-temperature quark-meson coupling (QMC) model [3], have been suggested to investigate the effective masses of mesons and nucleons. Besides theoretical study, many experimental results that predict the changes of particle masses with density have been shown. In particular, the experiments of the TAGX Collaboration have shown directly that when the density of the nucleon medium equals $0.7n_0$, where n_0 is the saturation density, the effective mass of the neutral ρ -meson reduces to 610 MeV [4]. Both theoretical and experimental results confirm that the medium effects are important for studying the nuclear or quark systems.

To illustrate the medium effects more transparently in theory, instead of using a first principle calculation, many authors introduced different hypotheses to represent the medium contributions; for example, they supposed the density-dependent vacuum energy $B(\rho)$ to modify the QMC model [5,6] or suggested the density-dependent $NN\rho$ coupling to address liquid-gas phase transition [7]. Employing these hypotheses, authors have discussed many physical properties of nuclear matter, quark matter, the nucleon system, and the hyperon system. In particular, to simplify the calculations, many ideal quasiparticle models in which the effective mass depends on the density and/or the temperature have been suggested for studying the quark-gluon plasma [8,9], gluon plasma [10], and strange quark matter [11].

One such candidate is the quark-mass density-dependent (QMDD) model, which was first suggested by Fowler, Raha, and Weiner [11]. In this model, the masses of u , d quarks and

s quarks are given by

$$m_q^* = \frac{B}{3\rho_B}, \quad (q = u, \bar{u}, d, \bar{d}), \quad (1)$$

$$m_{s,\bar{s}}^* = m_{s0} + \frac{B}{3\rho_B}, \quad (2)$$

where B is a constant and m_{s0} is the current mass of the strange quark; and ρ_B , the baryon density, is defined as

$$\rho_B = \frac{1}{3}(\Delta\rho_u + \Delta\rho_d + \Delta\rho_s), \quad (3)$$

where

$$\Delta\rho_i = \rho_i - \rho_{\bar{i}} = \frac{g_i}{(2\pi)^3} \int d^3k \left(\frac{1}{\exp[\beta(\epsilon_i(k) - \mu_i)] + 1} - \frac{1}{\exp[\beta(\epsilon_i(k) + \mu_i)] + 1} \right), \quad (4)$$

where g_i is the degeneracy, and i ($i = u, s, d$) and \bar{i} correspond to the quarks and antiquarks, respectively. The *Ansätze* in Eqs. (1) and (2) correspond to a quark confinement mechanism, because if the quark goes to infinity, the volume of the system tends to infinity and the baryon number density goes to zero, then m_q^* approaches infinity according to Eqs. (1) and (2). The infinite mass prevents the quark from going to infinity.

Employing the QMDD model and considering the weak processes

$$u + d \leftrightarrow u + s, \quad s \rightarrow u + e^- + \bar{\nu}_e, \quad d \rightarrow u + e^- + \bar{\nu}_e, \\ u + e^- \rightarrow d + \nu_e, \quad (5)$$

and the condition of charge neutrality

$$2\Delta\rho_u = \Delta\rho_d + \Delta\rho_s + 3\Delta\rho_e, \quad (6)$$

many physical properties of strange quark matter [12–25] and strange quark star [26,27] have been studied; the results are in good agreement with those given by the MIT bag model [28].

Although the density-dependent quark masses Eqs. (1) and (2) can mimic the quark confinement mechanism, many difficulties will emerge when we discuss the thermodynamic behaviors of the system with such quarks. The dispersion

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relation for the quark,

$$\epsilon(k, \rho) = [k^2 + m^*(\rho)^2]^{1/2}, \quad (7)$$

will make many usual thermodynamic relations with partial derivatives no longer satisfactory. As is well known in thermodynamics, a proper choice of independent variables will have a suitable characteristic thermodynamic function, from which all the thermodynamic quantities can be obtained by partial derivatives without integration. For example, with the variables temperature T , volume V , and chemical potential μ , the characteristic function is the thermodynamic potential $\Omega = \Omega(T, V, \mu)$. From the differential relation for a reversible process

$$d\Omega = -SdT - pdV - \bar{N}d\mu, \quad (8)$$

we have

$$\begin{aligned} S &= -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu}, & p &= -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu}, \\ \bar{N} &= -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,V}, \end{aligned} \quad (9)$$

where S , p , and \bar{N} are entropy, pressure, and average particle number, respectively. Other thermodynamic quantities such as internal energy U , Helmholtz free energy F , enthalpy H , and Gibbs function G can be calculated by the combination of the quantities we obtained, based on their definitions or relations.

But in the QMDD model, Ω not only is a function of T , V , and μ , but also depends explicitly on the quark density ρ because of Eq. (7), $\Omega = \Omega(T, V, \mu, m^*(\rho))$. How to tackle the thermodynamics self-consistently is still a problem. There have been many arguments in recent work [12–25]. The difficulty comes from the first and second laws of reversible process thermodynamics expressed by Eq. (8) and the partial derivatives by Eq. (9). Obviously, some extra terms involving the derivatives of m^* will emerge when partial derivatives are calculated following Eq. (9). Unfortunately, these extra terms for different treatments in different references contradict each other. For example, for an ideal quark gas system of quasiparticle with effective quark mass $m^* = m^*(\rho)$, the pressure and energy density ϵ were given as

$$p = -\tilde{\Omega} \equiv -\frac{\Omega}{V}, \quad (10)$$

$$\epsilon \equiv \frac{U}{V} = \tilde{\Omega} + \sum_i \mu_i \rho_i - T \frac{\partial \tilde{\Omega}}{\partial T}, \quad (11)$$

in Ref. [12–14]; as

$$p = -\left(\frac{\partial(\tilde{\Omega}/\rho)}{\partial(1/\rho)}\right)_{T,\{\mu_i\}} = -\tilde{\Omega} + \rho \left(\frac{\partial\tilde{\Omega}}{\partial\rho}\right)_{T,\{\mu_i\}} \quad (12)$$

$$\epsilon = \tilde{\Omega} - \rho \left(\frac{\partial\tilde{\Omega}}{\partial\rho}\right)_{T,\{\mu_i\}} + \sum_i \mu_i \rho_i - T \left(\frac{\partial\tilde{\Omega}}{\partial T}\right)_{\{\mu_i\},\rho}, \quad (13)$$

in Ref. [15,16]; and as

$$p = -\tilde{\Omega} + \rho \left(\frac{\partial\tilde{\Omega}}{\partial\rho}\right)_{T,\{\mu_i\}}, \quad (14)$$

$$\epsilon = \tilde{\Omega} + \sum_i \mu_i \rho_i - T \left(\frac{\partial\tilde{\Omega}}{\partial T}\right)_{\{\mu_i\},\rho}, \quad (15)$$

in Ref. [17].

For the quark-mass density- and temperature-dependent (QMDD) model [18–24] with the quark mass

$$m_q^* = \frac{B(T)}{3\rho_B}, \quad (q = u, \bar{u}, d, \bar{d}), \quad (16)$$

$$m_{s,\bar{s}}^* = m_{s0} + \frac{B(T)}{3\rho_B}, \quad (17)$$

where

$$B(T) = B_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right], \quad (18)$$

p and ϵ read

$$p = -\tilde{\Omega} - V \frac{\partial\tilde{\Omega}}{\partial V} + \rho_B \sum_i \frac{\partial\tilde{\Omega}}{\partial m_i^*} \frac{\partial m_i^*}{\partial\rho_B}, \quad (19)$$

$$\epsilon = \tilde{\Omega} - \sum_i \mu_i \frac{\partial\tilde{\Omega}}{\partial\mu_i} - T \frac{\partial\tilde{\Omega}}{\partial T} - T \sum_i \frac{\partial\tilde{\Omega}}{\partial m_i^*} \frac{\partial m_i^*}{\partial T}, \quad (20)$$

in Ref. [18], respectively. The ambiguity arises from the variable ρ in m^* , because it is not one of the characteristic variables of thermodynamic potential.

This paper evolves from an attempt to clear the above ambiguity and suggests a method for self-consistently calculating the thermodynamic quantities from partial derivatives. In the next section, we will address the traditional thermodynamic treatment with partial derivatives for the QMDD model transparently and emphasize that the difficulty cannot be overcome by the usual method. In fact, the thermodynamic inconsistency for an ideal quasiparticle system with effective mass $m^*(\rho, T)$ to describe the quark-gluon plasma or gluon plasma has been pointed out by many authors previously [8–10]. To avoid this difficulty, in Sec. III, we argue that we can calculate the thermodynamic quantities from an equilibrium state and show that if we choose the quasiparticle effective mass m^* as a new independent degree of freedom, we can calculate the thermodynamic quantities self-consistently along the reversible process by the usual partial derivatives. In Sec. IV, we apply this new treatment to studying the QMDD model. We show that we can eliminate previous difficulties with partial derivatives and take clear, reasonable, self-consistent results, which coincide with the results in the equilibrium state. In Sec. V, we consider the vacuum correction on the pressure and compare our result with that given by the MIT bag model. We show that even though our results agree with those given by Refs. [15,16], the mechanism and the physical reasons are completely different. In Refs. [15,16], the negative pressure comes from the partial derivative terms of m^* . But in our treatment, it comes from the physical vacuum. Our mechanism is just the same as that of the MIT bag model. The last section includes a summary and discussion.

II. INCONSISTENCY OF TRADITIONAL THERMODYNAMIC TREATMENTS WITH PARTIAL DERIVATIVE

We first repeat the traditional treatment based on Eqs. (1) and (2). It has been established systematically in Refs. [15–17]. In Ref. [17], they derived

$$\begin{aligned} p &= -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu} = -\left(\frac{\partial(V\tilde{\Omega})}{\partial V}\right)_{T,\mu} = -\left(\frac{\partial(V\tilde{\Omega}/\bar{N})}{\partial(V/\bar{N})}\right)_{T,\mu} \\ &= -\left(\frac{\partial(\tilde{\Omega}/\rho)}{\partial(1/\rho)}\right)_{T,\mu} = -\tilde{\Omega} + \rho\left(\frac{\partial\tilde{\Omega}}{\partial\rho}\right)_{T,\mu}. \end{aligned} \quad (21)$$

Without losing generality, from here on we will display formulas with only one component for simplicity and clarity. Equation (21) is different from that of the normal expression $p = -\Omega/V = -\tilde{\Omega}$ by the last additional term because of the density dependence of effective mass. But Eq. (21) seems not as solid if we pay attention to the invariables. Noting that the chemical potential μ is determined by the particle density constraint $\rho = \bar{N}/V$, and the partial derivative of Eq. (21) is with respect to V but fixed T and μ , or equivalently fixed T and ρ , in this process \bar{N} must be changed. It is not a constant and so the third equal mark in Eq. (21) cannot hold. The result in Eq. (21) is error.

To avoid this ambiguity, in Ref. [18], instead of the thermodynamic potential Ω , the authors introduced the Helmholtz free energy F to change the characteristic variable μ to the average particle number \bar{N} , that is,

$$dF = -SdT - pdV + \mu d\bar{N}. \quad (22)$$

By using $F = \Omega + \mu\bar{N}$, they obtained

$$p = -\left(\frac{\partial F}{\partial V}\right)_{T,\bar{N}} = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\bar{N}} - \bar{N}\frac{\partial\mu}{\partial V}, \quad (23)$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,\bar{N}} = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\bar{N}} - \bar{N}\frac{\partial\mu}{\partial T}. \quad (24)$$

After some calculations, they gave

$$p = -\frac{\partial\Omega}{\partial V} - \left(\bar{N} + \frac{\partial\Omega}{\partial\mu}\right)\frac{\partial\mu}{\partial V} + \frac{\rho}{V}\frac{\partial\Omega}{\partial m^*}\frac{\partial m^*}{\partial\rho}, \quad (25)$$

$$S = -\frac{\partial\Omega}{\partial T} - \left(\bar{N} + \frac{\partial\Omega}{\partial\mu}\right)\frac{\partial\mu}{\partial T} - \frac{\partial\Omega}{\partial m^*}\frac{\partial m^*}{\partial T}. \quad (26)$$

For the internal energy, they required

$$\bar{N} = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{V,m^*,T}, \quad (27)$$

and found

$$U = \Omega + \mu\bar{N} + TS = \Omega + \mu\bar{N} - T\frac{\partial\Omega}{\partial T} - T\frac{\partial\Omega}{\partial m^*}\frac{\partial m^*}{\partial T}. \quad (28)$$

But in their formulas, the invariable quantities for the partial derivatives were not written explicitly. To show the errors of their calculation, we write these fixed quantities in the

following by using the standard mathematical chain rule of partial derivative with composition function:

$$\begin{aligned} p &= -\left(\frac{\partial F}{\partial V}\right)_{T,\bar{N}} = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\bar{N}} - \bar{N}\left(\frac{\partial\mu}{\partial V}\right)_{T,\bar{N}} \\ &= -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\bar{N},\mu,m^*} - \left(\frac{\partial\Omega}{\partial\mu}\right)_{T,\bar{N},m^*,V}\left(\frac{\partial\mu}{\partial V}\right)_{T,\bar{N}} \\ &\quad - \left(\frac{\partial\Omega}{\partial m^*}\right)_{T,\bar{N},\mu,V}\left(\frac{\partial m^*}{\partial V}\right)_{T,\bar{N}} - \bar{N}\left(\frac{\partial\mu}{\partial V}\right)_{T,\bar{N}} \\ &= -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\bar{N},\mu,m^*} - \left[\bar{N} + \left(\frac{\partial\Omega}{\partial\mu}\right)_{T,\bar{N},m^*,V}\right]\left(\frac{\partial\mu}{\partial V}\right)_{T,\bar{N}} \\ &\quad + \frac{\rho}{V}\left(\frac{\partial\Omega}{\partial m^*}\right)_{T,\bar{N},\mu,V}\left(\frac{\partial m^*}{\partial\rho}\right)_{T,\bar{N}}, \end{aligned} \quad (29)$$

$$\begin{aligned} S &= -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\bar{N},\mu,m^*} - \left[\bar{N} + \left(\frac{\partial\Omega}{\partial\mu}\right)_{V,\bar{N},m^*,T}\right]\left(\frac{\partial\mu}{\partial T}\right)_{V,\bar{N}} \\ &\quad - \left(\frac{\partial\Omega}{\partial m^*}\right)_{V,\bar{N},\mu,T}\left(\frac{\partial m^*}{\partial T}\right)_{V,\bar{N}}. \end{aligned} \quad (30)$$

Equations (29) and (30) are just Eqs. (25) and (26) with the unchanged variable explicitly written out. Note that many partial derivatives are with four fixed variables. The term $\left(\frac{\partial\Omega}{\partial m^*}\right)_{V,\bar{N},\mu,T}$ must vanish, because $m^* = m^*(T, \rho)$, $\rho = \bar{N}/V$; to fix V, \bar{N} , and T is essentially to fix m^* . This means that all derivatives involving m^* in Eqs. (29) and (30) should be zero, therefore the extra terms $\left(\frac{\partial\Omega}{\partial m^*}\right)_{T,\bar{N},\mu,V}\left(\frac{\partial m^*}{\partial\rho}\right)_{T,\bar{N}}$ and $\left(\frac{\partial\Omega}{\partial m^*}\right)_{V,\bar{N},\mu,T}\left(\frac{\partial m^*}{\partial T}\right)_{V,\bar{N}}$ cannot survive. In fact, according to the definition of Gibbs function $G = \mu\bar{N} = U - Ts + pV$, we can directly find $\Omega = -pV$. The extra terms involving the derivative of m^* will surely destroy this relation.

The inconsistency appears not only in the expression of p and S , but also in \bar{N} . If we calculate the particle number \bar{N} from Eqs. (8) and (9), we obtain

$$\begin{aligned} \bar{N} &= \left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial\mu}\right)_{T,V,m^*} - \left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial m^*}\right)_{T,V,\mu} \\ &\quad \times \left(\frac{\partial m^*(T, \rho)}{\partial\mu}\right)_{T,V} \\ &= \sum_i \frac{g_i}{e^{\beta(\sqrt{m^{*2}+k_i^2}-\mu)}+1} - \left(\frac{\partial\Omega(T, V, \mu, m^*)}{\partial m^*}\right)_{T,V,\mu} \\ &\quad \times \left(\frac{\partial m^*(T, \rho)}{\partial\mu}\right)_{T,V}, \end{aligned} \quad (31)$$

where the summation is over all the quantum states with degeneracy g_i . Since $\rho = \bar{N}/V$ is usually taken as a constraint to deciding the value of μ , $\left(\frac{\partial m^*(T, \rho)}{\partial\mu}\right)_{T,V}$ does not vanish if m^* depends on ρ explicitly; so the extra term in Eq. (31) modifies the particle number of the system. For an ideal quasiparticle system, the number of quasiparticles $\bar{N} = \sum_i g_i n_i$, which is just the first term of Eq. (31); then we come to the conclusion that the result of Eq. (31) is incorrect. Besides, this result in Eq. (31) also conflicts with the requirement in Eq. (27),

so the inconsistency of the traditional treatment with partial derivatives is transparent.

In fact, the inconsistency of the usual thermodynamic treatment for the ideal quasiparticle system has been shown by many authors [8–10,25], so our presentation above is not a new result. Our aim was to demonstrate the inconsistency of thermodynamic treatment with partial derivatives along a reversible process explicitly.

III. A THERMODYNAMICALLY CONSISTENT TREATMENT

In Sec. II, we showed the inconsistency of traditional thermodynamic treatment with partial derivatives along the reversible process. In this section, we will suggest a consistent thermodynamic treatment for a quasiparticle system with density- and/or temperature-dependent particle mass $m^*(\rho, T)$. We want to point out that our method is universal, and the QMDD model is just a special example for using this method. Our method can also be applied to discussing the thermodynamic behavior of the QMDTD model [18–24] or other ideal quasiparticle system.

Noticing that at a fixed instant of the reversible process, the system is at an equilibrium state. We can denote the temperature and density of this system as T_0 and ρ_0 , respectively, then the effective mass of the quasiparticle becomes constant $m^*(T_0, \rho_0) = m_0$. The system reduces to the usual ideal gas system with constant mass m_0 quasiparticles. For this equilibrium state, the corresponding thermodynamic quantities can be directly obtained. For example, for the case of a one-component Fermi system:

$$\bar{N} \equiv \rho V = \sum_i g_i n_i = \sum_i \frac{g_i}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (32)$$

$$\Omega = - \sum_i g_i k T \ln \left(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)} \right), \quad (33)$$

$$U = \sum_i g_i n_i \epsilon_i = \sum_i \frac{g_i \sqrt{m_0^2 + k_i^2}}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (34)$$

$$G = \bar{N} \mu = \sum_i g_i n_i \mu = \sum_i \frac{g_i \mu}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \quad (35)$$

$$S = \frac{U - \Omega - G}{T}, \quad (36)$$

$$p = - \frac{\Omega}{V}, \quad (37)$$

where n_i is the particle number of the i th state and g_i is the corresponding degeneracy.

It can be clearly seen that these formulas are the same as those of the standard ideal gas. In fact, for example, if we derive the energy from the very beginning of the statistical definition in the grand canonical distribution,

we have

$$\begin{aligned} U = \bar{E} &= \frac{1}{\Xi} \sum_N E_N e^{-\beta(E_N - \mu N)} \\ &= \frac{1}{\Xi} \sum_N \left[\left(- \frac{\partial}{\partial \beta} \right)_{\mu, m^*, V} + \mu N \right] e^{-\beta(E_N - \mu N)} \\ &= - \left(\frac{\partial \ln \Xi}{\partial \beta} \right)_{\mu, m^*, V} + \mu \bar{N}, \end{aligned} \quad (38)$$

where the invariant μ is clearly seen, while m^* and V are fixed, since E_N is a function of m^* and V for each N . Take the quasiparticle ideal gas thermodynamic potential, Eq. (33), which means the grant partition function satisfies

$$\ln \Xi = \sum_i g_i \ln \left(1 + e^{-\beta(\sqrt{m^{*2} + k_i^2} - \mu)} \right), \quad (39)$$

it is easy to find

$$\begin{aligned} U &= - \left(\frac{\partial \ln \Xi}{\partial \beta} \right)_{\mu, m^*, V} + \mu \bar{N} \\ &= \sum_i g_i n_i (\sqrt{m^{*2} + k_i^2} - \mu) + \mu \bar{N} \\ &= \sum_i g_i n_i \epsilon(m^*, k_i), \end{aligned} \quad (40)$$

where $n_i = (e^{\beta[\epsilon(m^*, k_i) - \mu]} + 1)^{-1}$ is the average particle number of the i th state. This result is consistent with the interaction-free quasiparticle picture of the QMDD model.

We see from Eqs. (32)–(37) that the contribution of medium effect is included in the effective value of mass and appears in the exponential of the Fermi distribution. A remarkable property of these formulas is that the extra terms related to the partial derivative of m^* do not appear. This is reasonable because these thermodynamic quantities are functions of an equilibrium state, they do not depend on the change of the quasiparticle mass, since at this state the quasiparticle mass $m^*(T, \rho)$ becomes constant $m_0(T_0, \rho_0)$.

To compare the above treatment with those given in Sec. II, as an example, we calculate the entropy of the ideal quasiparticle system. Denoting the entropy calculated by equilibrium state as S_{sta} , we have, from Eqs. (33)–(36),

$$\begin{aligned} S_{\text{sta}} &= \frac{U - G - \Omega}{T} = \sum_i g_i \left[\frac{n_i \sqrt{m_0^2 + k_i^2} - n_i \mu}{T} \right. \\ &\quad \left. + k \ln \left(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)} \right) \right] \\ &= k \sum_i g_i \left[n_i \beta (\sqrt{m_0^2 + k_i^2} - \mu) \right. \\ &\quad \left. + \ln \left(\frac{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)}} \right) \right] \end{aligned}$$

$$\begin{aligned}
 &= k \sum_i g_i \left[n_i \ln \left(\frac{1}{n_i} - 1 \right) + \ln \left(\frac{1}{1 - n_i} \right) \right] \\
 &= -k \sum_i g_i [n_i \ln(n_i) + (1 - n_i) \ln(1 - n_i)]. \quad (41)
 \end{aligned}$$

This is a familiar formula for an equilibrium state whose physical meaning is transparent. Denoting the entropy calculated by the partial derivative of Ω following Eqs. (8) and (9) as S_{der} , we have

$$\begin{aligned}
 S_{\text{der}} &= - \left(\frac{\partial \Omega(T, V, \mu, m^*(T, \rho))}{\partial T} \right)_{V, \mu} \\
 &= - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial T} \right)_{V, \mu, m^*} \\
 &\quad - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial m^*} \right)_{T, V, \mu} \left(\frac{\partial m^*(T, \rho)}{\partial T} \right)_{V, \mu}, \quad (42)
 \end{aligned}$$

and for the equilibrium state with $m^*(T_0, \rho_0) = m_0$,

$$\begin{aligned}
 &- \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial T} \right)_{V, \mu, m^*=m_0} \\
 &= k \sum_i g_i \ln \left(1 + e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT} \right) \\
 &\quad + kT \sum_i g_i \frac{e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT} \left(\frac{\sqrt{m_0^2 + k_i^2} - \mu}{kT^2} \right)}{1 + e^{(\mu - \sqrt{m_0^2 + k_i^2})/kT}} \\
 &= k \sum_i g_i \ln \left(\frac{1}{1 - n_i} \right) + k \sum_i g_i n_i \ln \left(\frac{1}{n_i} - 1 \right) \\
 &= -k \sum_i g_i [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] \\
 &= S_{\text{sta}}. \quad (43)
 \end{aligned}$$

The first term of Eq. (42) is just the result given by the equilibrium state. The difference between these two treatments is significant. They cannot be accorded together. Noticing that the contribution of the medium effect at the equilibrium state is included within the value of m^* in the distribution, and the entropy describing the disorder of quasiparticles in a system does not depend on the intrinsic quantity such as the effective mass of the quasiparticle, the correctness of S_{sta} is obvious. The thermodynamic consistency of the new treatment is transparent, because it is based on an equilibrium state and $m_0(T_0, \rho_0)$ is constant.

If one hopes to extend this treatment to a reversible process, the accordance between the results from the calculation along a reversible process and the results obtained at a fixed equilibrium state must hold. So to get a self-consistent calculation between the equilibrium state and the reversible process in thermodynamics, we are compelled by the above discussion to introduce an intrinsic degree of freedom m^* for the quasiparticle in the thermodynamic system to describe the medium effect. We should rewrite Eq. (8) as

$$d\Omega = -SdT - pdV - \bar{N}d\mu + Xdm^*, \quad (44)$$

then Eq. (9) becomes

$$\begin{aligned}
 S &= - \left(\frac{\partial \Omega}{\partial T} \right)_{V, \mu, m^*}, \quad p = - \left(\frac{\partial \Omega}{\partial V} \right)_{T, \mu, m^*}, \\
 \bar{N} &= - \left(\frac{\partial \Omega}{\partial \mu} \right)_{T, V, m^*}, \quad X = \left(\frac{\partial \Omega}{\partial m^*} \right)_{T, V, \mu}, \quad (45)
 \end{aligned}$$

where X is an extensive quantity corresponding to the intensive variable m^* . In Eq. (44), the intrinsic degree of freedom m^* for the quasiparticle has been added as an independent variable in the thermodynamic system. The quantities S , p , and \bar{N} shown in Eq. (45) agree with the results obtained by the formulas of an equilibrium state in Eqs. (32)–(37), because we fixed m^* as an unchanged parameter in the partial derivative calculations. For example, the pressure given by Eq. (9) is

$$\begin{aligned}
 p &= - \left(\frac{\partial \Omega(T, V, \mu, m^*(\rho_B))}{\partial V} \right)_{T, \mu} \\
 &= - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial V} \right)_{T, \mu, m^*} - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial m^*} \right)_{T, V, \mu} \\
 &\quad \times \left(\frac{\partial m^*(\rho_B)}{\partial V} \right)_{T, \mu} \\
 &= - \frac{\Omega}{V} - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial m^*} \right)_{T, V, \mu} \left(\frac{\partial m^*(\rho_B)}{\partial V} \right)_{T, \mu}, \quad (46)
 \end{aligned}$$

whereas it is given by Eq. (45) as

$$p = - \left(\frac{\partial \Omega(T, V, \mu, m^*)}{\partial V} \right)_{T, \mu, m^*} = - \frac{\Omega}{V}. \quad (47)$$

In Eq. (46), the first term is just the result of the equilibrium state, but the second term, according to the treatment in Sec. II, will lead to inconsistency in thermodynamics.

Usually, the thermodynamic parameters such as S , p , and T depend on the whole system. They are independent of the intrinsic property of the particle or subsystem, no matter whether the subsystem is a simple point particle or a quasiparticle with an inner structure and different intrinsic properties. Ordinary thermodynamic variables depend on the collection of the subsystem only. Similarly, mass is an intrinsic quantity of a particle; it does not affect the collective thermodynamic properties of the whole system. When we consider the medium effect or the confinement mechanism and summarize this effect into the effective mass $m^*(T, \rho)$ under a quasiparticle approximation, the dynamic interaction can be concentrated on the effective mass m^* of the quasiparticle by using the finite-temperature quantum field calculation [29] or directly by confinement *Ansätze*. But the macroscopic thermodynamic variables cannot describe these microscopic dynamic interactions. We must choose new variables to represent these dynamic interactions or the medium effect. Obviously, the effective mass m^* appears as the suitable independent variable. Furthermore, we want to emphasize that the application of this new independent variable m^* to rewriting Eqs. (8) and (9) is limited to a quasiparticle approximation only. In a quasiparticle approximation, the interaction between particles, the medium effect, or the confinement mechanism are summarized in the effective mass m^* . Then m^* becomes an independent variable to represent all those physical effects;

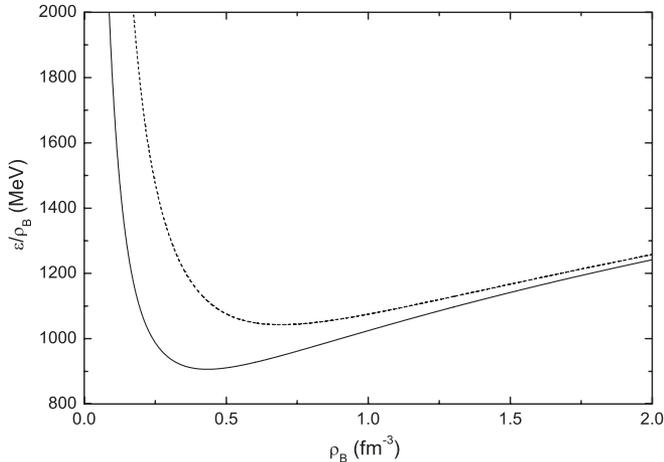


FIG. 1. Energy per baryon as a function of the baryon density ρ_B at $T = 0$ for different treatments of the QMDD model. The solid curve represents our treatment and the dashed line refers to the treatment in Ref. [15].

its dependence on density and temperature is due to the equilibrium condition of the whole system. Introducing m^* in a quasiparticle physical picture to represent the medium effect and taking it as a variable are twins in thermodynamics of quasiparticle system. If we employ finite-temperature quantum field theory to calculate the thermodynamic potential beyond the quasiparticle approximation [30–32], we can use Eq. (2) to calculate the thermodynamic quantities and investigate the interaction between particles by different orders of Feynman diagrams. Then we will not need to introduce the quasiparticle and use the corresponding Eqs. (44) and (45).

IV. QMDD MODEL

To illustrate the basic difference between our treatment based on Eqs. (44) and (45) and those of traditional treatment based on Eqs. (8) and (9), we employ the QMDD model as an example to calculate the thermodynamic property of strange quark matter. We fix the parameters in Eqs. (1) and (2) $B = 170 \text{ MeV fm}^{-3}$, $m_{s0} = 150 \text{ MeV}$, as in Ref. [21]. The temperature is set at $T = 0$, as that of Refs. [12, 15, 17], for convenience of comparison. Our results are shown in Figs. 1 and 2. In Fig. 1, we draw the curves of energy per baryon ε/ρ_B vs baryon number density ρ_B , comparing the results of our treatment (solid curve) with those in Ref. [15] (dashed curve). We see that the saturation point of the solid curve is $\varepsilon/\rho_B = 906.3 \text{ MeV}$ at $\rho_B = 0.433 \text{ fm}^{-3}$, which locates in a reasonable range. The saturation point of the dashed curve is $\varepsilon/\rho_B = 1042.8 \text{ MeV}$ at $\rho_B = 0.692 \text{ fm}^{-3}$. It is higher than our result because it has an additional term $-(\partial\tilde{\Omega}/\partial\rho_B)_{T,\{\mu_i\}}$.

The equations of state for different treatments are shown in Fig. 2, where the solid curve refers to our treatment, and the dotted and dashed curves refers to those given by Refs. [15] and [17], respectively. We see from Fig. 2 that there are two remarkable differences between our treatment and the others. First, the pressure is always positive in our treatment. This is of course reasonable, because our Hamiltonian $\hat{H} = \sum_{k,s} \varepsilon_{ks} \hat{a}_{ks}^\dagger \hat{a}_{ks}$ is an ideal gas Hamiltonian, so in principle it

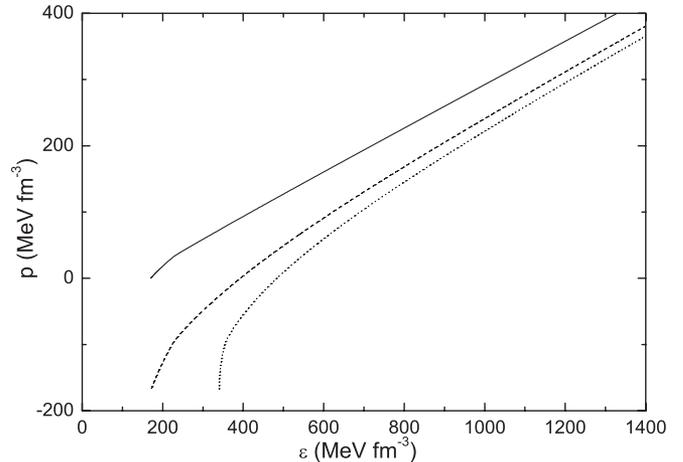


FIG. 2. Pressure P as a function of the energy density $\varepsilon = U/V$ for different treatments of the QMDD model. The solid, dotted, and dashed curves represent our treatment, the treatment in Ref. [15], and that in Ref. [17], respectively. The tendencies of these curves are similar at the large energy density region, but at the small energy density region, different treatments have quite different behaviors. In our treatment, the pressure never goes to negative.

cannot give negative pressure. But for the other treatments, the pressure becomes negative in the small energy density regions because of the incorrect modification term

$$\rho_B \left(\frac{\partial \tilde{\Omega}}{\partial \rho_B} \right)_{T=0, \{\mu_i\}} = - \sum_i \frac{g_i}{48\pi^2} \frac{4m_i^* B}{\rho_B} \left[\mu_i \sqrt{\mu_i^2 - m_i^{*2}} - m_i^{*2} \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^{*2}}}{m_i^*} \right) \right]. \quad (48)$$

Second, although the tendencies of all the curves are similar in the large energy density region, in the small energy density region, their behaviors are very different. In particular, the end points at $\rho_B \rightarrow 0$ for these three curves are different. They are $\{\varepsilon = 170 \text{ MeV fm}^{-3}, p = 0\}$ for the solid curve, $\{\varepsilon = 340 \text{ MeV fm}^{-3}, p = -170 \text{ MeV fm}^{-3}\}$ for the dashed curve, and $\{\varepsilon = 170 \text{ MeV fm}^{-3}, p = -170 \text{ MeV fm}^{-3}\}$ for the dotted curve.

Can we obtain a negative pressure in the QMDD model, as occurs in the MIT bag model? We will answer this question in next section.

V. THE CONTRIBUTION OF PHYSICAL VACUUM

To answer this question, let us recall that in the MIT bag model the negative pressure comes from the physical vacuum. Let us extend the quasiparticle Hamiltonian to

$$\hat{H} = \sum_{k,s} \varepsilon_{ks} \hat{a}_{ks}^\dagger \hat{a}_{ks} + H_0, \quad (49)$$

where H_0 is the system energy in the absence of quasiparticle excitations. We then note that in a quasiparticle system, the physical vacuum energy H_0 is possibly a function of T and ρ , since ε_{ks} depends on T and ρ via the effective mass m^* . Corresponding to H_0 , the additional term of thermodynamic

potential is Ω_0 , and the pressure becomes

$$p' = -\frac{\Omega + \Omega_0}{V} = p - B_0, \quad (50)$$

the energy density becomes $\varepsilon + B_0$, but the entropy density $s = (\varepsilon + p - \sum_i \mu_i \rho_i)/T$ is unchanged.

Next we determine Ω_0 in the QMDD model. It was first suggested by Gorenstein and Yang [10] that the thermodynamic consistency of the quasiparticle system be guaranteed by means of a vacuum correction, for which they gave a set of constraints. While in Ref. [25], Ω_0 is chosen to satisfy

$$\left(\frac{\partial(\tilde{\Omega} + \tilde{\Omega}_0)}{\partial \rho_B} \right)_{\{\mu_i\}} = 0, \quad (51)$$

where $\tilde{\Omega}_0 = \Omega_0/V$ is virtually B_0 in Eq. (50). At zero temperature, Eq. (51) reduces to

$$d\tilde{\Omega}_0(\rho_B) = - \left(\frac{\partial \tilde{\Omega}}{\partial m^*} \right)_{T=0, V, \mu} \frac{dm^*}{d\rho_B} d\rho_B, \quad (52)$$

because m^* depends on ρ_B only, $\tilde{\Omega}_0$ can be obtained by integration. Based on Eqs. (1), (2), and (33), we obtain

$$\begin{aligned} \frac{d\tilde{\Omega}_0(\rho_B)}{d\rho_B} &= - \left(\frac{\partial \tilde{\Omega}}{\partial \rho_B} \right)_{T=0, \{\mu_i\}} \\ &= \sum_i \frac{g_i B m_i^*}{12\pi^2 \rho_B^2} \left[\mu_i \sqrt{\mu_i^2 - m_i^{*2}} \right. \\ &\quad \left. - m_i^{*2} \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^{*2}}}{m_i^*} \right) \right], \quad (53) \end{aligned}$$

therefore

$$\begin{aligned} \tilde{\Omega}_0(\rho_B) &= \int_{\rho_0}^{\rho_B} \sum_i \frac{g_i B m_i^*}{12\pi^2 \rho_B^2} \left[\mu_i \sqrt{\mu_i^2 - m_i^{*2}} - m_i^{*2} \right. \\ &\quad \left. \times \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^{*2}}}{m_i^*} \right) \right] d\rho_B + \tilde{\Omega}_0(\rho_0). \quad (54) \end{aligned}$$

Choosing $\tilde{\Omega}_0(\infty) = B_\infty$, as that of Ref. [25], we get

$$\begin{aligned} \tilde{\Omega}_0(\rho_B) &= - \int_{\rho_B}^{\infty} \sum_i \frac{g_i B m_i^*}{12\pi^2 \rho_B^2} \left[\mu_i \sqrt{\mu_i^2 - m_i^{*2}} - m_i^{*2} \right. \\ &\quad \left. \times \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^{*2}}}{m_i^*} \right) \right] d\rho_B + B_\infty. \quad (55) \end{aligned}$$

Equations (53)–(55) are just Eqs. (7), (8), and (14) in Ref. [25], then the stability and the negative pressure have been shown there. The negative pressure and the stable point do exist, and we do not recalculate the numerical results here.

Of course, the constraint of Eq. (51) is one of the possible ways to get $\tilde{\Omega}_0$ only. The aim of this choice is to demonstrate that we can get negative pressure and a stable strange quark matter. We must emphasize here again that the negative pressure as well as the stability of the strange quark matter come from the physical vacuum. It is completely different from that of Refs. [15–17], in which the negative pressure comes from the extra partial derivative term $\rho_B \frac{\partial \Omega}{\partial \rho_B}$. As has been shown in Sec. II, this term cannot exist since it destroys the thermodynamic consistency. Our mechanism is the same as that of the MIT bag model. The conclusion [15] that the properties of strange quark matter in the QMDD model are nearly the same as those obtained in the MIT bag model is still valid, but their argument was incorrect. The correct argument is shown in Secs. III–V.

VI. SUMMARY AND DISCUSSION

In summary, we have shown the shortcomings of the previous treatments for the QMDD model, which are based on the partial derivative of thermodynamic functions along a reversible process. A new method is suggested. For the reversible process, we have introduced a new intrinsic degree of freedom m^* for the quasiparticle system. We have proved that the thermodynamic quantities calculated by the partial derivatives concerning this independent variable are in agreement with those obtained for the equilibrium state. The difficulties and controversies in previous references are removed. We also find that the properties of strange quark matter in the QMDD model are nearly the same as those obtained in the MIT bag model [23], if we take the vacuum contribution into account. Our method is applicable if the quasiparticle with effective mass is introduced to represent the medium effect. Since such quasiparticle models are quite commonly used in the study of nuclear matter, quark matter, and quark-gluon plasma, we hope our discussion helps the theoretical study in those fields.

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