$K^- pp$ bound states from Skyrmions

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(Received 4 October 2007; published 5 May 2008)

The bound kaon approach to the strangeness in the Skyrme model is applied to investigating the possibility of deeply bound $K^- pp$ states. We describe the $K^- pp$ system as two Skyrmion, around which a kaon field fluctuates. Each Skyrmion is rotated in the space of SU(2) collective coordinate. The rotational motions are quantized to be projected onto the spin-singlet proton-proton state. We derive the equation of motion for the kaon in the background field of two Skyrmions at fixed positions. From the numerical solution of the equation of motion, it is found that the energy of K^- can be considerably small and that the distribution of K^- shows molecular nature of the $K^- pp$ system. For this deep binding, the Wess-Zumino-Witten term plays an important role. The total energy of the $K^- pp$ system is estimated in the Born-Oppenheimer approximation. The binding energy of the $K^- pp$ state is B.E. $\simeq$ 126 MeV. The mean square radius of the pp subsystem is $\sqrt{\langle r_{pp}^2 \rangle} \simeq 1.6$ fm.

DOI: 10.1103/PhysRevC.77.055202 PACS number(s): 24.85.+p, 12.39.Dc, 13.75.Jz, 14.20.Pt

I. INTRODUCTION

In recent years, lots of theoretical or experimental efforts to explore the possibility of nuclear $\bar{K}$-bound states [1] have been made. Although no firm evidence to show their existence is known, there is one result reported by the FINUDA collaboration [2] that *may* suggest the existence of the lightest nuclear $\bar{K}$-bound state, $K^- pp$. In the experiment at DAΦNE, it was observed that Λ and p from K^- absorption on ^{6}Li, ^{7}Li, and ^{12}C at rest has a strong back-to-back correlation and that the invariant mass spectrum of Λ and p shows a peak. The collaboration advocates that the observation can be interpreted as a signal of the formation of deeply bound $K^- pp$ state, whose binding energy is 115 MeV and the width is 67 MeV.

This experiment is motivated by the idea proposed by Akaishi and Yamazaki (AY) [1] suggesting the existence of deeply bound $\bar{K}$ nuclei. It is based on the assumption that $\Lambda(1405)$ baryon is a $\bar{K}N$ bound state formed by the strong attraction in the $I = 0$ $\bar{K}N$ channel. One may then expect that when a K^- is injected in a nucleus it may attract surrounding p's to form a shrunken nucleus. The K^- is bound deeply so an absorption reaction, $K^- p \to \pi\Sigma$, is energetically closed, and accordingly, it can have a long lifetime in a nucleus.

However, it has not yet been established that the peak observed by FINUDA really corresponds to the state proposed by AY. Magas *et al.* [3] claimed that the peak corresponds mostly to the process $K^- pp \to \Lambda p$ followed by final-state interactions of the produced particles with the daughter nucleus. Even if we suppose the peak to be a $K^- pp$ bound state, it is strange that it is much deeper than the original AY prediction [1]: the binding energy B.E. $= 48$ MeV and the width $\Gamma = 61$ MeV. Recently, two groups [4,5] performed $\bar{K}NN - \pi\Sigma N$ coupled-channels Faddeev calculations, based on the argument that $\bar{K}N$-$\pi\Sigma$ coupling is important [6] when one considers the $\Lambda(1405)$ or $K^- pp$ system. The authors in Ref. [4] obtained B.E. $= 55$–70 MeV and $\Gamma = 95$–110 MeV. The result in Ref. [5] are B.E. ~ 80 MeV and $\Gamma \sim 73$ MeV. They are at odds with both of the result by FINUDA and AY prediction. An attempt to describe the $\bar{K}$ nuclei as $\Lambda(1405)$ hypernuclei has also been made in Ref. [7].

A new experiment are planned to be performed at the Japan Proton Accelerator Research Complex (J-PARC) searching for the deeply bound $K^- pp$ state by the missing-mass spectrum of the ^{3}He(in-flight K^-, n) reaction, together with the invariant-mass spectra detecting all particles decaying from the $K^- pp$ bound state. It would be naively expected that a clear signal for formation of kaonic nuclei appears in this measurement, because a lighter nucleus is chosen as a target. Indeed, it has been suggested theoretically that a distinct peak of the $K^- pp$ bound state can be observed in the spectrum of the ^{3}He(in-flight K^-, n) reaction if some conditions for the $K^- pp$ optical potential are satisfied [8].

Our interests in this article are whether the deeply bound $K^- pp$ state can be realized in the context of the topological soliton model of baryons, the Skyrme model [9]. For this purpose, we employ the bound kaon approach to the strangeness in the Skyrme model [10], which describes hyperons as the bound states of an antikaon and a topological soliton of the pion field ("Skyrmion").

The Skyrme model is a low-energy effective theory of quantum chromodynamics (QCD) at large number of colors. In the limit of large number of colors, as was shown by t'Hooft, QCD reduces to a theory of weakly interacting mesons. The action of the Skyrme model is a chiral effective theory written in terms of the Nambu-Goldstone boson fields. Nucleons emerge as topological solitons of the $\mathrm{SU(2)_f}$ sector in the Skyrme Lagrangian [11].

One way to introduce the strangeness to the model is to assume a kaon field fluctuating around the SU(2) Skyrmion

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(bound kaon approach [12]). One finds the existence of bound states of the kaon and a Skyrmion, which can be identified to be hyperons. The lowest bound state has the quantum number $l = 1, t = 1/2$, where l is the orbital angular momentum of the kaon and t the combined angular momentum and isospin, $\boldsymbol{T} = \boldsymbol{L} + \boldsymbol{I}$, respectively. The parity of the $l = 1, t = 1/2$ state is in total positive, which is assigned to positive-parity hyperons. A notable feature is the presence of a bound state in the negative-parity state, $l = 0, t = 1/2$, which lies above the $l = 1, t = 1/2$ state. This state probably corresponds to $\Lambda(1405)$ baryon. Whereas the constituent quark models have difficulties in describing $\Lambda(1405)$, this approach predicts the static properties of $\Lambda(1405)$ [13] as well as octet and decouplet baryons in good agreement with the empirical values. In addition, an interaction originating from the WZW term acts on the $S = +1$ state, e.g., pentaquark, repulsively, and the state is pushed away into the continuum, whereas the interaction acts attractively on the $S = -1$ state, leading to the formation of the bound states.

Thus, the bound kaon approach is a theory naturally describing both of the positive-parity hyperons and the lowest negative-parity state, $\Lambda(1405)$, on the same ground. It is also worth mentioning that this approach has no parameter once we adjust F_π and e (for their definitions, see below) to fit the N and the Δ masses in $SU(2)_f$ sector. Therefore, the $\bar{K}N$ interaction, which is a key ingredient for the study of $\bar{K}$ nuclei, is unambiguously determined. In these respects, it is of great significance to investigate the issue of the exotic nuclei such as K^-pp, K^-ppn, K^-pnn, and so on in the context of the bound-kaon approach to the Skyrme model.

We describe the K^-pp system as two-Skyrmion around which a kaon field fluctuates. Each Skyrmion is rotated in the space of collective coordinate and its rotational motion is quantized to be projected onto a relevant two-nucleon state. We adiabatically treat the nucleon-nucleon radial motion and derive the kaon's equation of motion when the position of the Skyrmions are fixed first. Then we obtain the energy of kaon as a function of the relative distance between the two Skyrmions, which tells us whether the kaon can be deeply bound to the two-proton. For the existence of nuclear $\bar{K}$ bound states, it is necessary that $\bar{K}$ gains sufficiently large binding energy in nuclei. If such nuclei exist, the nuclear density distribution is rearranged under the influence of the strong attraction in $I = 0$ $\bar{K}N$. Then the nuclear part in $\bar{K}$ nuclei is excited relative to the original nuclear system. Therefore, the energy gained by $\bar{K}$ must be large enough to compensate the energy loss of nuclear component and to deeply bind the total system. If a strong binding of K^- is possible, it is also an interesting subject how the mechanism responsible for the strong binding is explained in the solitonic picture of baryons.

In our previous article [14], we have presented the derivation of the kaon's equation of motion and its numerical solution. It was shown that a strong binding of K^- to pp can occur. Needless to say, it cannot be taken as evidence for the actual deep binding of the K^-pp system until the two-proton radial motion is treated. In the present article, we give a detailed description of our approach and attempt to solve the dynamics of the radial motion of the protons under the strong attaractive force mediated by K^-, in addition to the ordinary nuclear force. Then we can estimate the binding energy of the total K^-pp system within the Born-Oppenheimer approximation. The possible structure of the K^-pp state is also discussed.

The organization of this article is as follows. In the second section, we derive the kaon's equation of motion and show its numerical solutions in the third section. We solve the radial motion of the two-proton in the fourth section. Discussion and summary are given in the fifth section. Full expression of the kaon Lagrangian and the useful formulas for collective coordinate quantization are gathered in the appendices.

II. DERIVATION OF THE KAON'S EQUATION OF MOTION

Let us begin with showing how the K^- coupled to pp is described in the bound kaon approach to the Skyrme model. We consider two Skyrmions fixed at positions with the relative distance, R, and assume the presence of the kaon field fluctuating around the Skyrmions. The equation of motion for the kaon in the background field of the Skyrmions is then derived, from which we know the behavior of the K^- coupled to pp.

The action of the Skyrme model is given by

$$\Gamma = \int d^4x \left[\frac{F_\pi^2}{16} \mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \mathrm{tr}(\partial_\mu U U^\dagger, \partial_\nu U U^\dagger)^2 \right] + \Gamma_{\mathrm{SB}} + \Gamma_{\mathrm{WZW}}, \tag{1}$$

where U is the chiral SU(3) field built out of the eight Nambu-Goldstone bosons. Γ_{SB} is the symmetry breaking term [13] given by

$$\Gamma_{\mathrm{SB}} = \int d^4x \left\{ \frac{F_\pi^2 m_\pi^2 + 2F_K^2 m_K^2}{48} \mathrm{tr}(U + U^\dagger - 2) + \frac{F_\pi^2 m_\pi^2 - F_K^2 m_K^2}{24} \mathrm{tr}\left[\sqrt{2}\lambda_8(U + U^\dagger)\right] - \frac{F_\pi^2 - F_K^2}{48} \mathrm{tr} \times \left[\left(1 - \sqrt{3}\lambda_8\right)(U \partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger)\right] \right\}, \tag{2}$$

where $m_{\pi(K)}$ and $F_{\pi(K)}$ are the mass and the decay constant of the pion (kaon), respectively. The last term in Eq. (2) has a role to renormalize the kinetic energy term for the kaon, whereas the first two terms renormalize the mass term. Γ_{WZW} is the Wess-Zumino-Witten anomaly action [15]:

$$\Gamma_{\mathrm{WZW}} = -\frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\mu\nu\alpha\beta\gamma} \mathrm{tr} \times (U^\dagger \partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \partial_\gamma U), \tag{3}$$

where N_c denotes the number of colors.

We assume the following "product" ansatz for the chiral field representing KNN system,

$$U = U(1) U_K U(2), \tag{4}$$

where $U(1)$ and $U(2)$ are the fields of the baryon number $B = 1$ SU(2) Skyrmions located at $\boldsymbol{r}(1) = \boldsymbol{r} - \boldsymbol{R}/2$ and $\boldsymbol{r}(2) = \boldsymbol{r} + \boldsymbol{R}/2$, respectively. Their explicit expressions are as follows,

$$U(i) = \begin{pmatrix} u(i) & \boldsymbol{0} \\ \boldsymbol{0} & 1 \end{pmatrix}, \quad u(i) = e^{iF[r(i)]\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}(i)}, \quad (i = 1, 2), \tag{5}$$

where $r(i)=|\boldsymbol{r}(i)|$, $\hat{\boldsymbol{r}}(i)=\boldsymbol{r}(i)/r(i)$, $F(r)$ is the profile function of an isolated Skyrmion [11] and $\boldsymbol{\tau}$ the Pauli matrices. U_K is the field carrying strangeness. Its form is

$$U_K=\exp\left[i\frac{2\sqrt{2}}{F_K}\begin{pmatrix}\mathbf{0} & K\\ K^\dagger & \mathbf{0}\end{pmatrix}\right], \quad (6)$$

where K is the usual kaon isodoublet,

$$K=\begin{pmatrix}K^+\\ K^0\end{pmatrix}. \quad (7)$$

Each Skyrmion is rotated in the space of SU(2) collective coordinate, A_1 or A_2, as

$$u(1)\to A_1u(1)A_1^\dagger,\quad u(2)\to A_2u(2)A_2^\dagger. \quad (8)$$

By substituting the ansatz, Eq. (4), with the replacement Eq. (8) into the action, Eq. (1), we obtain the Lagrangian for the kaon field in the presence of the background $B=2$ Skyrmion. After expanding the Lagrangian up to the seceond order in K, we obtain

$$\begin{aligned}\mathcal{L}=&(D_\mu K)^\dagger D^\mu K-m_K^2K^\dagger K-\frac{1}{8}K^\dagger K\bigg[\mathrm{tr}(\partial_\mu U_{BB}^\dagger\partial^\mu U_{BB})\\ &+\frac{1}{e^2F_K^2}\mathrm{tr}(\partial_\mu U_{BB}U_{BB}^\dagger,\partial_\nu U_{BB}U_{BB}^\dagger)^2\bigg]\\ &-\frac{1}{e^2F_K^2}\bigg\{2(D_\mu K)^\dagger D_\nu K\,\mathrm{tr}(A^\mu A^\nu)\\ &+\frac{1}{2}(D_\mu K)^\dagger D^\mu K\,\mathrm{tr}(\partial_\nu U_{BB}^\dagger\partial^\nu U_{BB})\\ &-6(D_\mu K)^\dagger[A^\nu,A^\mu]D_\nu K\bigg\}\\ &-\frac{iN_c}{F_K^2}B^\mu[K^\dagger D_\mu K-(D_\mu K)^\dagger K],\end{aligned} \quad (9)$$

where U_{BB} represents the product of rotating solitons,

$$U_{BB}=A_1u(1)A_1^\dagger A_2u(2)A_2^\dagger. \quad (10)$$

In Eq. (9), the "covariant derivative" D_μ is defined by

$$D_\mu K=\partial_\mu K+V_\mu K \quad (11)$$

and

$$V_\mu=[L_\mu(1)+R_\mu(2)]/2,\quad A_\mu=[L_\mu(1)-R_\mu(2)]/2, \quad (12)$$

where

$$\begin{aligned}L_\mu(1)&=A_1u^\dagger(1)A_1^\dagger\partial_\mu[A_1u(1)A_1^\dagger],\\ R_\mu(2)&=A_2u(2)A_2^\dagger\partial_\mu[A_2u^\dagger(2)A_2^\dagger].\end{aligned} \quad (13)$$

The last term in Eq. (9) comes from the WZW term and B^μ is the baryon number current given by

$$B^\mu=\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{tr}(U_{BB}^\dagger\partial_\nu U_{BB}U_{BB}^\dagger\partial_\alpha U_{BB}U_{BB}^\dagger\partial_\beta U_{BB}). \quad (14)$$

Here we note that the Lagrangian, Eq. (9), has the same form as that for $B=1$ Skyrmion [10] except that the background Skyrmion is not the single $B=1$ Skyrmion but the product of $B=1$ Skyrmions, Eq. (10). It should be also noted that the *KNN* interaction is unambiguously determined as in Eq. (9) once the ansatz for U is given.

We neglect the terms suppressed by $1/N_c$ in Eq. (9). Because the time derivative of the collective coordinate is $\dot{A}_{1,2}\sim\mathcal{O}(N_c^{-1})$, we see A_0, V_0, and B_j are $\mathcal{O}(N_c^{-1})$ from their definitions, Eqs. (12) and (14),

$$A_0,V_0\sim\mathcal{O}(N_c^{-1}),\quad B_j\sim\mathcal{O}(N_c^{-1}). \quad (15)$$

Then the Lagrangian for the kaon field up to $O(N_c^0)$ terms reads as follows,

$$\begin{aligned}\mathcal{L}=&(\partial_0K)^\dagger\partial_0K\left[1+\frac{1}{2e^2F_K^2}\mathrm{tr}(\partial_jU_{BB}^\dagger\partial_jU_{BB})\right]\\ &+K^\dagger D_jD_jK-m_K^2K^\dagger K\\ &-\frac{1}{8}K^\dagger K\bigg[-\mathrm{tr}(\partial_jU_{BB}^\dagger\partial_jU_{BB})\\ &+\frac{1}{e^2F_K^2}\mathrm{tr}(\partial_jU_{BB}U_{BB}^\dagger,\partial_iU_{BB}U_{BB}^\dagger)^2\bigg]\\ &+\frac{1}{e^2F_K^2}\bigg\{2K^\dagger D_j[D_iK\,\mathrm{tr}(A_jA_i)]\\ &+\frac{1}{2}K^\dagger D_j[D_jK\,\mathrm{tr}(\partial_iU_{BB}^\dagger\partial_iU_{BB})]\\ &-6K^\dagger D_j([A_i,A_j]D_iK)\bigg\}\\ &-\frac{iN_c}{F_K^2}[K^\dagger\partial_0K-(\partial_0K)^\dagger K]B^0+O(N_c^{-1}).\end{aligned} \quad (16)$$

A comment is in order here. It should be noted that in the Lagrangian, Eq. (16) U_{BB} must be the product not of the static solitons but of the rotating solitons, Eq. (10). In the case of $B=1$, the effect of rotation is suppressed in the limit of large N_c because the collective coordinate enters the Lagrangian only through its time derivative, which is $\mathcal{O}(N_c^{-1})$. As an example, lets us consider

$$\mathrm{tr}[\partial_\mu U\partial^\mu U^\dagger], \quad (17)$$

which appears in the coefficient of $K^\dagger K$ in Eq. (9). For $B=1$, under the replacement

$$u\to AuA^\dagger \quad (18)$$

with A being a collective coordinate, Eq. (17) reads

$$\begin{aligned}\mathrm{tr}(\partial_\mu u\partial^\mu u^\dagger)&\to\mathrm{tr}(A\partial_iuA^\dagger A\partial^iu^\dagger A^\dagger)+\mathcal{O}(N_c^{-1})\\ &=\mathrm{tr}(\partial_iu\partial^iu^\dagger)+\mathcal{O}(N_c^{-1}).\end{aligned} \quad (19)$$

Therefore, for $B=1$, the soliton may be regarded as a static one at $N_c\to\infty$. However, in the present $B=2$ case, because each Skyrmion is rotated independently,

$$u\to A_1u(1)A_1^\dagger A_2u(2)A_2^\dagger. \quad (20)$$

Eq. (17) reads as follows,

$$\begin{aligned}&\mathrm{tr}(\partial_\mu u\partial^\mu u^\dagger)\\ &\to\mathrm{tr}[\partial_i(A_1u(1)A_1^\dagger A_2u(2)A_2^\dagger)\partial^i(A_2u^\dagger(2)A_2^\dagger A_1u^\dagger(1)A_1^\dagger)]\\ &\quad+\mathcal{O}(N_c^{-1})\\ &=\mathrm{tr}[A_1\partial_iu(1)A_1^\dagger A_2u(2)A_2^\dagger+A_1u(1)A_1^\dagger A_2\partial_iu(2)A_2^\dagger\\ &\quad\times A_2\partial^iu^\dagger(2)A_2^\dagger A_1u^\dagger(1)A_1^\dagger+A_2u^\dagger(2)A_2^\dagger A_1\partial^iu^\dagger(1)A_1^\dagger]\end{aligned}$$

$$= \mathrm{tr}[\partial_i u(1) A_1^\dagger A_2 u(2) \partial^i u^\dagger(2) A_2^\dagger A_1 u^\dagger(1) + \partial_i u(2)\partial^i u^\dagger(2) + \partial_i u(1)\partial^i u^\dagger(1) + u(1)A_1^\dagger A_2 \partial_i u(2) u^\dagger(2) A_2^\dagger A_1 \partial^i u^\dagger(1)] + \mathcal{O}(N_c^{-1}). \quad (21)$$

Thus the collective coordinates themselves, which are not suppressed by $1/N_c$, enters the Lagrangian. As long as assuming that two solitons rotate independently, Eq. (4), the kaon inevitably couples not to static soliton but to rotating solitons in the limit of $N_c \to \infty$. Accordingly, we consider the kaon under the background of the two-Skyrmion projected onto a spin-isospin eigenstate of two-nucleon.

Our next task is to perform the collective coordinate quantization and project the rotation of each Skyrmion onto the relevant spin-isospin state. This procedure is done as follows. First, we rewrite the Lagrangian, Eq. (16), in terms of the adjoint matrix defined by

$$D_{ij}(A) = \mathrm{tr}(\tau_i A \tau_j A^\dagger)/2, \quad (22)$$

with A being a collective coordinate. The result, which is quite lengthy, is displayed in Appendix A. The matrix $D_{ij}(A)$ is known to be represented by the rotation matrix of rank 1. The wave function of the nucleon in the space of collective coordinate is also expressed by rotation matrix [11]:

$$\langle A|N_{I_3,J_3}\rangle = \frac{1}{2\pi}(-1)^{I_3+1/2} D^{1/2}_{-I_3 J_3}(\Omega), \quad (23)$$

where I_3 and J_3 denote the third component of the isospin and that of the spin, respectively, and Ω the Euler angles. Then, the projection of the Skyrmions onto physical two nucleon states is performed by sandwiching the Lagrangian, Eq. (16), with two nucleon states and integrating the Euler angles,

$$\int d\Omega_1 \int d\Omega_2 \langle N(1)N(2)|\mathcal{L}|N(1)N(2)\rangle, \quad (24)$$

where $N(i)$ denotes the ith nucleon. We assume that the proton-proton in the K^-pp system is in spin-singlet and project the rotational motion of the Skyrmions onto the spin-singlet proton-proton state. Thus we consider

$$\mathcal{L}_{ppK} \equiv \int d\Omega_1 \int d\Omega_2 \langle (pp)_{s=0}|\mathcal{L}|(pp)_{s=0}\rangle, \quad (25)$$

where $|(pp)_{s=0}\rangle$ is the wave function corresponding to the spin-singlet proton-proton state and is given by

$$|(pp)_{s=0}\rangle = \frac{1}{\sqrt{\mathcal{N}}}(|N_{\frac{1}{2},\frac{1}{2}}(1)N_{\frac{1}{2},\frac{-1}{2}}(2)\rangle - |N(1)_{\frac{1}{2},\frac{-1}{2}}N_{\frac{1}{2},\frac{1}{2}}(2)\rangle), \quad (26)$$

with $\mathcal{N}$ being the normalization constant. Equation (25) is the Lagrangian for the kaon coupled to two protons. Detailed description of the projection are shown in Appendix B.

Now, we derive the equation of motion for the kaon from the Lagrangian, Eq. (25). First, we average the direction of the line joining the two Skyrmions. To do that, we put $\boldsymbol{R}/2 = [(R/2)\sin\alpha\cos\beta, (R/2)\sin\alpha\sin\beta, (R/2)\cos\alpha]$ in the Lagrangian Eq. (25) and integrate the angles α and β,

$$\bar{\mathcal{L}}_{ppK} = \frac{1}{4\pi}\int_0^\pi d\alpha \int_0^{2\pi} d\beta \sin\alpha \mathcal{L}_{ppK}. \quad (27)$$

This corresponds to assuming that the proton-proton system is in S wave. Then the background field becomes spherical, which allows us to set the kaon field as

$$K(\boldsymbol{r}, t) = k(r, t) Y_{lm}(\theta, \phi), \quad (28)$$

with $Y_{lm}(\theta, \phi)$ the spherical harmonics. This ansatz, Eq. (28), is substituted into the Lagrangian, Eq. (27). We perform the θ and ϕ integrations before taking the variation with respect to $k(r, t)$. Up to this step, quite long and involved calculations are needed. Then the Euler-Lagrange equation for $k(r, t)$ yields

$$\left[-\bar{f}(r;R)\frac{d^2}{dt^2} - 2i\bar{\lambda}(r;R)\frac{d}{dt} - m_K^2 - \bar{V}_{\mathrm{eff}}(r;R,l) + \hat{\mathcal{O}}\right] \times k(r,t) = 0, \quad (29)$$

where the operator $\hat{\mathcal{O}}$ is defined as

$$\hat{\mathcal{O}} = c_1(r;R)\frac{\partial}{\partial r} + c_2(r;R)\frac{\partial^2}{\partial r^2}. \quad (30)$$

In Eqs. (29) and (30), the coefficients, $\bar{f}(r;R)$ and $\bar{\lambda}(r;R)$ are expressed as follows:

$$\bar{f}(r;R) = \frac{1}{4\pi}\int_0^\pi d\alpha \int_0^{2\pi} d\beta \sin\alpha \times \left\langle 1 + \frac{1}{2e^2F_K^2}\mathrm{tr}(\partial_j U_{BB}^\dagger \partial_j U_{BB})\right\rangle, \quad (31)$$

$$\bar{\lambda}(r;R) = \frac{1}{4\pi}\int_0^\pi d\alpha \int_0^{2\pi} d\beta \sin\alpha \frac{-N_c}{F_K^2}\langle B^0\rangle. \quad (32)$$

Here $\langle\cdots\rangle$ means taking an expectation value with respect to the spin-singlet proton-proton state as in Eq. (25). $-\bar{V}_{\mathrm{eff}}(r;R,l)$ and $\hat{\mathcal{O}}$ correspond to the terms with and without spatial derivative in the following equation, respectively,

$$\begin{aligned} -\bar{V}_{\mathrm{eff}}(r;R,l) + \hat{\mathcal{O}} &= \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta Y_{lm}(\theta,\phi)\cdot \frac{1}{4\pi}\int_0^\pi d\alpha \int_0^{2\pi} d\beta \sin\alpha \\ &\times \Big\langle D_j D_j - \frac{1}{8}\Big[-\mathrm{tr}(\partial_j U_{BB}^\dagger \partial_j U_{BB}) + \frac{1}{e^2F_K^2}\mathrm{tr}(\partial_j U_{BB}U_{BB}^\dagger, \partial_i U_{BB}U_{BB}^\dagger)^2\Big] \\ &+ \frac{1}{e^2F_K^2}\Big\{2D_j[D_i \mathrm{tr}(A_j A_i)] + \frac{1}{2}D_j[D_j \mathrm{tr}(\partial_i U_{BB}^\dagger \partial_i U_{BB})] - 6D_j([A_i, A_j]D_i)\Big\}\Big\rangle \\ &\times Y_{lm}(\theta,\phi). \end{aligned} \quad (33)$$

c_1 and c_2 in $\hat{\mathcal{O}}$ and $\bar{V}_{\mathrm{eff}}(r;R,l)$ are read off from this equation. Their explicit expressions are quite lengthy and are not particularly instructive. Therefore we do not display them here.

Let us expand the field $k(r, t)$ in terms of its eigenmodes:

$$k(r,t) = \sum_n [k_n(r)e^{i\omega_n t}a_n^\dagger + \tilde{k}_n(r)e^{-i\tilde{\omega}_n t}b_n], \quad (\omega_n,\ \tilde{\omega}_n > 0), \quad (34)$$

where a_n and b_n are the annihilation operators for the strangeness $S = \mp 1$ states, respectively. Substituting Eq. (34) into Eq. (29), we find the eigenmodes satisfy

$$\left[\bar{f}(r;R)\omega_n^2 - m_K^2 - V_K^{(-)}(r;\omega_n,R,l) + \hat{\mathcal{O}}\right]k_n(r) = 0, \quad (35)$$

$$\left[\bar{f}(r;R)\tilde{\omega}_n^2 - m_K^2 - V_K^{(+)}(r;\tilde{\omega}_n,R,l) + \hat{\mathcal{O}}\right]\tilde{k}_n(r) = 0. \quad (36)$$

Eqs. (35) and (36) are the equation of motions for $S = -1$ and $S = +1$ states, respectively. In Eqs. (35) and (36), $V_K^{(\mp)}(r;\omega,R,l)$ plays a role of potential term and can be separated into two terms,

$$V_K^{(\mp)}(r;\omega,R,l) = V_{\rm WZW}^{(\mp)}(r;\omega,R) + \bar{V}_{\rm eff}(r;R,l), \quad (37)$$

where $V_{\rm WZW}^{(\mp)}(r;\omega,R)$ originates from the WZW term and $\bar{V}_{\rm eff}(r;R,l)$ from remaining terms in the Skyrme Lagrangian. $V_{\rm WZW}^{(\mp)}(r;\omega,R)$ is given by

$$V_{\rm WZW}^{(\mp)}(r;\omega,R) = \mp 2\bar{\lambda}(r;R)\omega. \quad (38)$$

Thus the WZW term acts on the negative (positive) strangeness states in attarctive (repulsive) way. $k_n(r)$ and $\tilde{k}_n(r)$ obey the following normalization conditions,

$$\int 4\pi r^2 dr[\bar{f}(r;R)(\omega_n + \omega_{n'}) + 2\bar{\lambda}(r;R)]k_n(r)k_{n'}(r) = \delta_{nn'}, \quad (39)$$

$$\int 4\pi r^2 dr[\bar{f}(r;R)(\tilde{\omega}_n + \tilde{\omega}_{n'}) - 2\bar{\lambda}(r;R)]\tilde{k}_n(r)\tilde{k}_{n'}(r) = \delta_{nn'}. \quad (40)$$

III. NUMERICAL SOLUTION OF THE KAON EQUATION OF MOTION

We solved numerically the equation of motion, Eq. (35). Figure 1 shows the obtained energy eigenvalue of K^-, ω, as a function of the Skyrmion-Skyrmion relative distance, R. m_π, F_π, and e were taken to be $m_\pi = 0$, $F_\pi = 129$ MeV, and $e = 5.45$, which were adjusted to fit the masses of N and Δ [11]. The kaon mass was taken to be $m_K = 495$ MeV. For the ratio, F_K/F_π, we have examined two choices: $F_K/F_\pi = 1.00$ and the empirical value, $F_K/F_\pi = 1.23$. (In our previous work [14], we adopted $F_K/F_\pi = 1.00$.)

TABLE I. Energy eigenvalue of S-wave K^- ($\omega_{l=0}$) and that of P wave ($\omega_{l=1}$) for five cases of the proton-proton relative distance, R. F_K/F_π is taken to be the empirical value, $F_K/F_\pi = 1.23$.

R (fm)	$\omega_{l=0}$ (MeV)	$\omega_{l=1}$ (MeV)
1.0	121	369
1.5	262	402
2.0	356	435
2.5	411	460
3.0	442	475

We find that the lowest-lying mode is the S wave and that the P wave lies above the S wave. Note that this order is natural but different from the case of $B = 1$, where the lowest-lying mode is P wave, as mentioned in the introduction. We see that it is important to take into account the difference between F_K and F_π. As was shown in Ref. [13], by setting F_K/F_π equal to the empirical vaue, $F_K/F_\pi = 1.23$, hyperon masses are well reproduced, while when we set $F_K/F_\pi = 1$ they are overbound. The binding of K^- to pp is also weaker when taking the empirical value of F_K/F_π.

In Table I, the K^- energy eigenvalues for several values of R are displayed. Looking at the S-wave channel, the binding of the kaon is extremely strong for smaller distance, i.e., $R \lesssim 1.0$ fm. In this region, the repulsive nucleon-nucleon interaction dominates over the attractive interaction between K^- and pp. Nuclear matter in which average NN distance is $R \simeq 1.0$ fm is expected to be realized in the core region of compact stars. Our result shows that K^- in such high density nuclear matter can be lighter than the pion. This might be somehow related with the kaon condensation [16] that is considered to occur in compact stars. As R is increased, the binding becomes looser. However, at $R = 2.0$ fm, for instance, which is close to the average inter NN distance in normal nuclei, the binding is still deep: the binding energy is about 140 MeV (see Table I)

Next, we consider the R dependence of the K^- distribution. In Fig. 2, we plot the distribution of K^- in the S wave [kaon wave function $k(r)$ normalized by the condition, Eq. (39)]

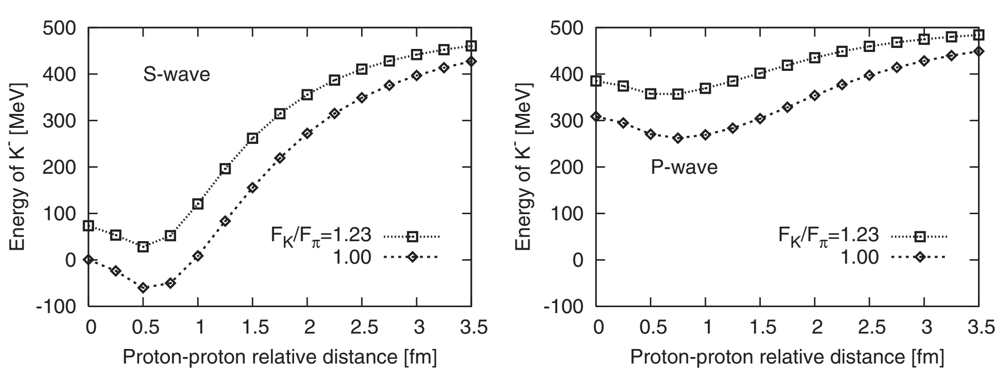

FIG. 1. The energy eigenvalues of S- and P-wave K^-, ω, as functions of the proton-proton relative distance, R. Two choices of the ratio F_K/F_π are examined: $F_K/F_\pi = 1.00$ and the experimental value, $F_K/F_\pi = 1.23$.

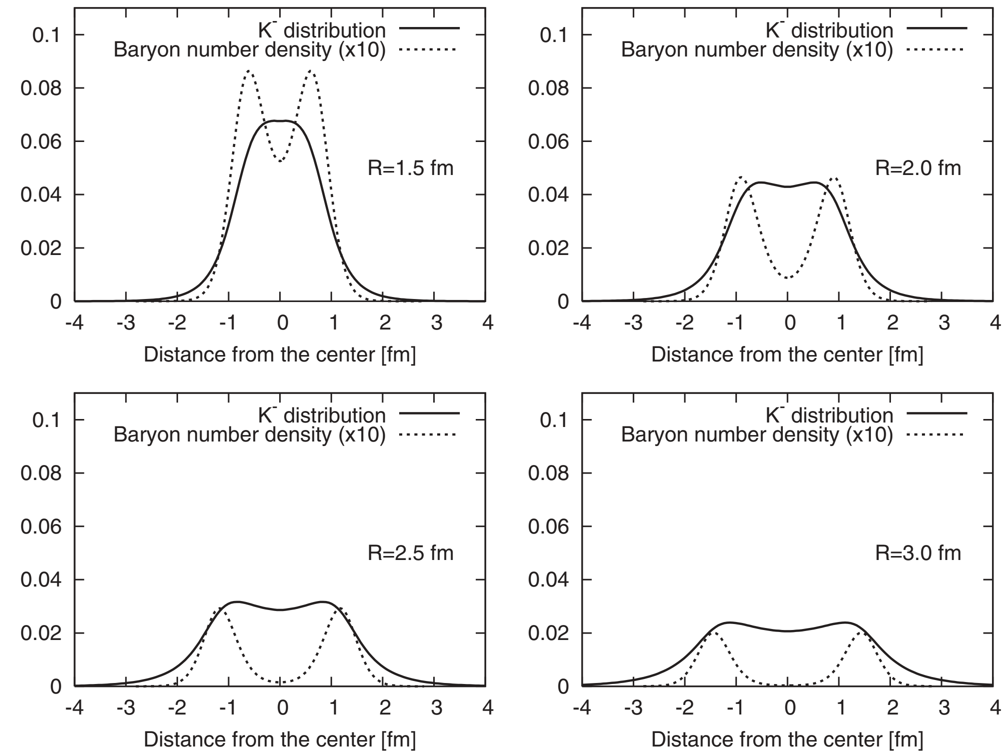

FIG. 2. S-wave K^- distribution [normalized wave function $k(r)$] and baryon number density [Eq. (41)] for the relative distance of the two Skyrmions, $R = 1.5$, 2.0, 2.5, and 3.0 fm. Both are in units of eF_π. The horizontal axis is the distance from the origin. $F_K/F_\pi = 1.23$ was chosen. The baryon number density is multiplied by a factor 10.

for several cases of R, $R = 1.5$, 2.0, 2.5, and 3.0 fm. For comparison, the baryon number denisity given by

$$\langle B^0 \rangle \equiv \int d\Omega_1 \int d\Omega_2 \langle (pp)_{s=0} | B^0 | (pp)_{s=0} \rangle, \tag{41}$$

is also plotted. At $R = 1.5$ fm, it can be seen that K^- is localized in the narrow region between the two protons. At relatively larger separation, $R \gtrsim 2.0$ fm, K^- has large probability to stay near the proton's respective positions, which is characteristic to molecular orbital states [14,17].

IV. PROTON-PROTON RADIAL MOTION

Now, we solve the dynamical problem of proton-proton radial motion under the strong attaractive potential mediated by K^-, in addition to the ordinary nucleon-nucleon potential. We assume that the radial motion of the two-proton is governed by the following Hamiltonian,

$$H = T_{NN}(R) + V_{NN}(R) + V_{KNN}(R), \tag{42}$$

where T_{NN} is the NN kinetic energy term,

$$T_{NN} = -\frac{1}{M_N}\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right). \tag{43}$$

Here, the nucleon is regarded as not a soliton with finite size but a pointlike particle with the mass $M_N = 939$ MeV and its motion is assumed to be nonrelativistic. $V_{NN}(R)$ is the state-independent central part of the NN potential obtained from the product of $B = 1$ Skyrmion [18]. We have neglected the contribution from the spin and the isospin-dependent part because they give a minor contribution compared with the state-independent part. The last term is the energy generated by the bound kaon in S wave,

$$V_{KNN}(R) = \omega_{l=0}(R) - m_K, \tag{44}$$

where $\omega_{l=0}(R)$ is the S-wave kaon's energy obtained in the previous section. V_{KNN} corresponds to the K^--pp "potential." In Fig. 3, we show the behavior of $V_{NN}(R)$, $V_{KNN}(R)$, and

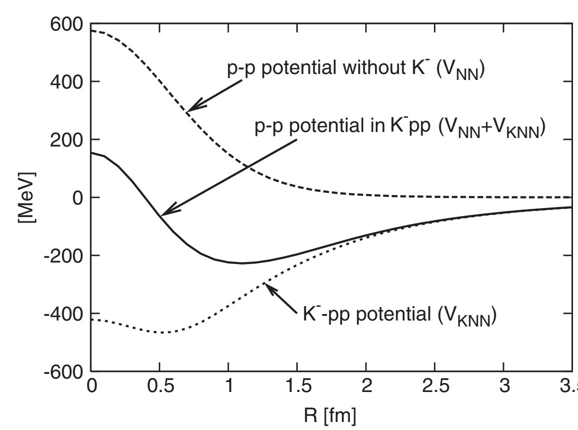

FIG. 3. Potential terms in Eq. (42) as functions of the proton-proton relative distance, R. The upper curve is the proton-proton potential in the absence of K^-, $V_{NN}(R)$. The lower one represents the K^--pp potential, $V_{KNN}(R)$. The middle one corresponds to their sum, $V_{NN}(R) + V_{KNN}(R)$, the effective proton-proton potential in the $K^- pp$ system.

TABLE II. Calculated total energy of the K^-pp bound state relative to $2M_N + m_K$ and its decomposition. The root mean square radius of NN subsystem, $\sqrt{\langle r_{NN}^2\rangle}$, is also shown. For the definition of each component, see the text.

F_K/F_π	$\langle T_{NN}\rangle$ (MeV)	$\langle V_{NN}\rangle$ (MeV)	$\langle V_{KNN}\rangle$ (MeV)	Total (MeV)	$\sqrt{\langle r_{NN}^2\rangle}$ (fm)
1.00	50.4	97.3	−380.5	−232.7	1.46
1.23	42.0	74.5	−239.2	−125.5	1.63

their sum. $V_{NN}(R)$ has repulsive core at short distances and one-pion exchange tail at long distances. Medium-range attraction cannot be produced within the product ansatz of $B = 1$ Skyrmions [19]. However, the attractive force generated by bound kaon, $V_{KNN}(R)$, is so strong that it overcomes the strongly repulsive $V_{NN}(R)$. As a result, the effective NN potential in the K^-pp system, $V_{KNN}(R) + V_{NN}(R)$, is strongly attractive in the medium range.

The energy of the K^-pp state relative to $2M_N + m_K$, E, is obtained by solving the Schrodinger equation,

$$H\Psi_N = E\Psi_N. \tag{45}$$

In Table II, we show the binding energy of K^-pp measured from $2M_N + m_K$ (threshold) and its decomposition into the NN kinetic energy, $\langle T_{NN}\rangle$, the NN potential energy, $\langle V_{NN}\rangle$, and the kaon's energy, $\langle V_{KNN}\rangle$, obtained by solving Eq. (45). The expectation value, $\langle \mathcal{O}\rangle$, is defined by

$$\langle \mathcal{O}\rangle \equiv \int_0^\infty 4\pi R^2 dR\,\Psi_N(R)^*\mathcal{O}\Psi_N(R) \Big/ \times \int_0^\infty 4\pi R^2 dR\,\Psi_N(R)^*\Psi_N(R). \tag{46}$$

The root-mean-square radius for NN subsystem, $\sqrt{\langle r_{NN}^2\rangle}$, is also shown.

When the F_K/F_π is taken to be the experimental value, $F_K/F_\pi = 1.23$, the binding energy of the K^-pp bound state is estimated to be

$$B_{K^-pp} \simeq 126\,\text{MeV}. \tag{47}$$

Another fact worth noting is the smallness of the NN kinetic energy, $\langle T_{NN}\rangle \sim 40$ MeV. As long as looking at this fact, the Born-Oppenheimer approximation seems to be not so poor. The root-mean-square radius of the two-nucleon subsystem is

$$\sqrt{\langle r_{NN}^2\rangle} \simeq 1.6\,\text{fm}, \tag{48}$$

which is smaller than the average inter-NN distance in normal nuclei.

V. DISCUSSION AND SUMMARY

Lets us consider the mechanism responsible for the strong binding of K^- to pp. This is turned out to be attributed to the WZW term. It is known that in $B = 1$ sector the existence of the WZW term leads to various important results. The WZW term is included in the action from the requirement that the effective theory written in terms of the Nambu-Goldstone bosons should reproduce the anomaly the fundamental theory possesses. The Skyrme Lagrangian without the term has a fictitious symmetry: an invariance under $U \leftrightarrow U^\dagger$. This symmetry forbids processes changing even-oddness of meson number, e.g., $K^+K^- \to \pi^+\pi^-\pi^0$. The WZW term breaks this extra symmetry [15]. In addition, for odd N_c, the WZW term ensures that the quantized Skyrmion has a half-odd spin and thus behaves as a fermion. The effect of the WZW term goes beyond these rules. In the bound kaon approach, the interaction Lagrangian of the kaon and the nucleon that comes from the WZW term has the form of

$$\mathcal{L}_{\text{WZW}} = -\frac{iN_c}{F_K^2}[K^\dagger\partial_0 K - (\partial_0 K)^\dagger K]B^0. \tag{49}$$

This is nothing but the so-called Tomozawa-Weinberg term. This term gives an effective attractive contribution to negative strangeness states that is crucial for obtaining the correct values of the masses of ground state hyperons. In particular, without the term, the S-wave bound state of a kaon and a Skyrmion, which corresponds to $\Lambda(1405)$, does not exist. Also in the present $B = 2$ case, the role of the WZW term is revealed to be important. In the equation of motion, Eq. (35), there exist two terms, $V_{\text{WZW}}^{(-)}$ and $\bar{V}_{\text{eff}}$ in Eq. (37), which effectively play a role of the potential acting on the kaon. Among them, it is $V_{\text{WZW}}^{(-)}$ that originates from the WZW term. To see the effects of the WZW term, we switched off $V_{\text{WZW}}^{(-)}$ and calculated the kaon's energy. The result for the S-wave K^- is shown in Fig. 4. One observes that the WZW term additionally gives a substantial attractive contribution to the binding of the kaon. Here, we should note that $V_{\text{WZW}}^{(-)}$ is stronger than that for $B = 1$ because the interaction Eq. (49) is proportional to the baryon number density. It is therefore quite a natural result that K^-

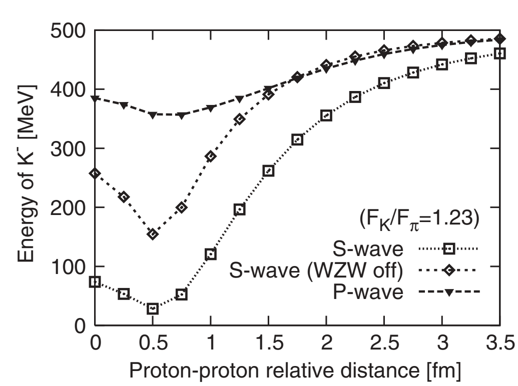

FIG. 4. The energy eigenvalue of K^- as a function of the proton-proton relative distance, R, compared with the case when the Wess-Zumino-Witten term is switched off. The ratio of the decay constant is taken to be $F_K/F_\pi = 1.23$. The bottom and upper curves are the energies of K^- in S and P waves, respectively. The middle one represents the energy of S-wave K^- when the Wess-Zumino-Witten term is switched off. P-wave K^- is unbound when the Wess-Zumino-Witten term is switched off.

is bound to a two-proton more deeply than to a one proton. However, this does not necessarily mean that the larger the baryon number B becomes the deeper the binding of the kaon is, because $V_{\rm WZW}^{(-)}$ does not necessarily become stronger with the increasing B: $V_{\rm WZW}^{(-)}$ is proportional to the kaon's energy [see Eq. (38)].

The K^- distribution shown in Fig. 2 suggests that the K^-pp state is a molecular orbital state. This observation is quite natural in the following sense. The two protons in the K^-pp system should keep some distance so as to avoid the repulsive core of the nuclear potential. For the two-proton with finite separation, it can be shown that the potential acting on the kaon, $V_K^{(-)}(r;\omega,R,l)$ in Eq. (37), is a double-well potential that is most attractive at the proton's respective position. It is the molecular orbit that the kaon occupies under such a double-well potential. If the K^-pp is really a molecular orbital state, it is plausible that the binding of K^- to two-proton is stronger than to one proton because K^- experiences the strong attraction from the two protons without increase of the kinetic energy [17].

The system considered in our article is the bound state of K^- and two-Skyrmion projected onto pp state. More realistic treatment is to rotate not only the solitons but also the kaon field in the collective coordinate space and perform the quantization of the rotations and the projection of the system onto spin-isospin eigenstates. If this task is achieved, the components other than K^-pp may be included[1]. This elaborate task, however, is left for our future work.

In summary, we have applied the Skyrme model to a study of the lightest $\bar{K}$-nuclear bound state, K^-pp. We have derived the equation of motion for the kaon coupled to a two-proton at a fixed position. The two-proton is expressed by a two-Skyrmion whose rotational motion in the space of collective coordinate is quantized and projected onto spin-singlet proton-proton state. Numerical solution of the equation of motion shows that K^- can be strongly bound even for relatively large interproton-to-proton distances. Next, we have solved the two-proton radial motion by assuming that the protons move under the strong attractive potential generated by K^- in addition to the ordinary NN potential. Then we have found that K^-pp state can be realized as a very deeply bound and compact state, whose binding energy is $B_{K^-pp}\simeq 126$ MeV and the mean inter-NN distance is $\sqrt{\langle r_{NN}^2\rangle}\simeq 1.6$ fm. The obtained value of the binding energy is surprisingly close to the experimental result obtained by FINUDA collaboraton, $B_{K^-pp}^{\rm exp.}=115$ MeV. However, considering the crudeness of our treatment, we are not allowed to satisfy this agreement.

ACKNOWLEDGMENTS

This work was supported in part by the 21st Century COE Program at Tokyo Institute of Technology "Nanometer-Scale Quantum Physics" from the Ministry of Education, Culture, Sports, Science and Technology, Japan.

[1]It should be noted that this model does not necessarily means the hyperons to be $\bar{K}N$ bound states but quantized states of the bound system of a SU(2)$_{\rm f}$ soliton and kaon fluctuating around the soliton.

APPENDIX A: FULL EXPRESSION OF THE KAON LAGRANGIAN

In this appendix, we rewrite the Lgrangian, Eq. (16), in terms of the adjoint matrix, Eq. (22). The adjoint matrix is included in U_{BB}, V_j, and A_j. V_j and A_j can be rewritten using the adjoint matrix as follows,

$$V_j = i\tau_m\tfrac{1}{2}\left[D_{mi}(A_1)L^i_j(1)+D_{mi}(A_2)R^i_j(2)\right], \quad \text{(A1)}$$

$$A_j = i\tau_m\tfrac{1}{2}\left[D_{mi}(A_1)L^i_j(1)-D_{mi}(A_2)R^i_j(2)\right]. \quad \text{(A2)}$$

Here $L^i_j(1)$ and $R^i_j(2)$ are defined as

$$L^i_j(1) = \hat{\delta}(1)_{ji}\frac{1}{r(1)}C(1)S(1)+\hat{r}(1)_j\hat{r}(1)_iF'(1) + \epsilon_{jik}\hat{r}(1)_k\frac{1}{r(1)}S(1)^2, \quad \text{(A3)}$$

$$R^i_j(2) = -\hat{\delta}(2)_{ji}\frac{1}{r(2)}C(2)S(2)-\hat{r}(2)_j\hat{r}(2)_iF'(2) + \epsilon_{jik}\hat{r}(2)_k\frac{1}{r(2)}S(2)^2, \quad \text{(A4)}$$

where

$$\hat{\delta}(\alpha)_{ji} = \delta_{ji}-\hat{r}(\alpha)_j\hat{r}(\alpha)_i, \quad \text{(A5)}$$

$$F(\alpha) = F[r(\alpha)], \qquad F'(\alpha)=\frac{dF(\alpha)}{dr(\alpha)}, \quad \text{(A6)}$$

$$C(\alpha) = \cos F(\alpha), \qquad S(\alpha)=\sin F(\alpha), \qquad (\alpha=1,2). \quad \text{(A7)}$$

Now we use these equations to rewrite the Lagrangian in terms of the adjoint matrix and Skyrmion profile function. Let us show the result for each term in Eq. (16).

A. $(\partial_0 K)^\dagger\partial_0 K$ term

$\mathrm{tr}(\partial_j U^\dagger_{BB}\partial_j U_{BB})$ in the first term of Eq. (16) is expressed as follows,

$$\mathrm{tr}(\partial_j U^\dagger_{BB}\partial_j U_{BB}) = 2\left[L^i_j(1)L^i_j(1)+R^i_j(2)R^i_j(2)-D_{ik}(A_1^\dagger A_2)2L^i_j(1)R^k_j(2)\right]. \quad \text{(A8)}$$

B. $K^\dagger K$ term

The first term in the square bracket of the fourth term is given in Eq. (A8). The second term in the square bracket is rewritten as follows,

$$\frac{1}{e^2F_\pi^2}\mathrm{tr}(\partial_\mu U_{BB}U^\dagger_{BB},\partial_\nu U_{BB}U^\dagger_{BB})^2 = \frac{4}{e^2F_\pi^2}\Big(\left[L^k_j(1)L^k_i(1)+R^k_j(2)R^k_i(2)\right]\left[L^l_j(1)L^l_i(1) + R^l_j(2)R^l_i(2)\right]-\left[L^i_j(1)L^i_j(1)+R^i_j(2)R^i_j(2)\right]^2 + D_{kn}(A_1^\dagger A_2)\Big\{-2\left[L^k_j(1)R^n_i(2)+L^k_i(1)R^n_j(2)\right] \times\left[L^l_i(1)L^l_j(1)+R^l_i(2)R^l_j(2)\right]+4L^k_j(1)R^n_j(2) \times\left[L^l_m(1)L^l_m(1)+R^l_m(2)R^l_m(2)\right]\Big\}$$

$$+ D_{kn}(A_1^\dagger A_2) D_{lm}(A_1^\dagger A_2)\{[L_j^k(1)R_i^n(2) + L_i^k(1)R_j^n(2)] \times [L_i^l(1)R_j^m(2) + L_j^l(1)R_i^m(2)] - 4L_j^k(1)R_j^n(2)L_i^l(1)R_i^m(2)\}). \quad (A9)$$

C. Terms containing D_jK

In this subsection, we display the results for the terms containing spatial "covariant derivative," D_jK. The second term in Eq. (16) is rewritten as

$$K^\dagger D_j D_j K = K^\dagger\{\partial_j^2 - \tfrac{1}{4}[L_j^i(1)L_j^i(1) + R_j^i(2)R_j^i(2)] + i\tau_m D_{mi}(A_1)\tfrac{1}{2}[\partial_j L_j^i(1) + L_j^i(1)\partial_j] + i\tau_m D_{mi}(A_2)\tfrac{1}{2}[\partial_j R_j^i(2) + R_j^i(2)\partial_j] + D_{ik}(A_1^\dagger A_2)[-\tfrac{1}{2}L_j^i(1)R_j^k(2)]\}K. \quad (A10)$$

The three terms in the curly bracket of the fifth term have very lengthy expressions.

The first term in the bracket:

$$\begin{aligned}
2K^\dagger D_j[D_iK\,\mathrm{tr}(A_jA_i)] &= 2K^\dagger\big(-\tfrac{1}{2}\partial_j[L_j^l(1)L_i^l(1) + R_j^l(2)R_i^l(2)]\partial_i \\
&+ \tfrac{1}{4}[L_j^p(1)L_i^p(1) + R_j^p(2)R_i^p(2)][L_j^l(1)L_i^l(1) \\
&+ R_j^l(2)R_i^l(2)] + D_{lk}(A_1^\dagger A_2)\tfrac{1}{2}\partial_j[L_j^l(1)R_i^k(2) \\
&+ L_i^l(1)R_j^k(2)]\partial_i - D_{pm}(A_1^\dagger A_2)D_{lk}(A_1^\dagger A_2) \\
&\times \tfrac{1}{4}[L_j^p(1)R_i^m(2) + L_i^p(1)R_j^m(2)][L_j^l(1)R_i^k(2) \\
&+ L_i^l(1)R_j^k(2)] - i\tau_n D_{nq}(A_1)\tfrac{1}{4}\{\partial_j L_i^q(1)[L_j^l(1)L_i^l(1) \\
&+ R_j^l(2)R_i^l(2)] + [L_j^l(1)L_i^l(1) + R_j^l(2)R_i^l(2)]L_j^q(1)\partial_i\} \\
&- i\tau_n D_{nq}(A_2)\tfrac{1}{4}\{\partial_j R_i^q(2)[L_j^l(1)L_i^l(1) + R_j^l(2)R_i^l(2)] \\
&+ [L_j^l(1)L_i^l(1) + R_j^l(2)R_i^l(2)]R_j^q(2)\partial_i\} \\
&+ i\tau_n D_{nq}(A_1)D_{lk}(A_1^\dagger A_2)\tfrac{1}{4}\{\partial_j L_i^q(1)[L_j^l(1)R_i^k(2) \\
&+ L_i^l(1)R_j^k(2)] + [L_j^l(1)R_i^k(2) + L_i^l(1)R_j^k(2)]L_j^q(1)\partial_i\} \\
&+ i\tau_n D_{nq}(A_2)D_{lk}(A_1^\dagger A_2)\tfrac{1}{4}\{\partial_j R_i^q(2)[L_j^l(1)R_i^k(2) \\
&+ L_i^l(1)R_j^k(2)] + [L_j^l(1)R_i^k(2) \\
&+ L_i^l(1)R_j^k(2)]R_j^q(2)\partial_i\}\big)K. \quad (A11)
\end{aligned}$$

The second term:

$$\begin{aligned}
\tfrac{1}{2}K^\dagger D_j[D_jK\,\mathrm{tr}(\partial_i U_{BB}^\dagger \partial_i U_{BB})] &= K^\dagger\big(\partial_j[L_m^n(1)L_m^n(1) + R_m^n(2)R_m^n(2)]\partial_j \\
&- \tfrac{1}{4}[L_j^i(1)L_j^i(1) + R_j^i(2)R_j^i(2)]^2] \\
&- D_{nk}(A_1^\dagger A_2)2\partial_j L_m^n(1)R_m^k(2)\partial_j \\
&+ D_{il}(A_1^\dagger A_2)D_{nk}(A_1^\dagger A_2)L_j^i(1)R_j^l(2)L_m^n(1)R_m^k(2) \\
&+ i\tau_l D_{li}(A_1)\tfrac{1}{2}\{L_j^i(1)[L_m^n(1)L_m^n(1) + R_m^n(2)R_m^n(2)]\partial_j \\
&+ \partial_j[L_m^n(1)L_m^n(1) + R_m^n(2)R_m^n(2)]L_j^i(1)\} \\
&+ i\tau_l D_{li}(A_2)\tfrac{1}{2}\{R_j^i(2)[L_m^n(1)L_m^n(1) + R_m^n(2)R_m^n(2)]\partial_j \\
&+ \partial_j[L_m^n(1)L_m^n(1) + R_m^n(2)R_m^n(2)]R_j^i(2)\} \\
&+ i\tau_l D_{li}(A_1)D_{nk}(A_1^\dagger A_2)2[-L_j^i(1)L_m^n(1)R_m^k(2)\partial_j \\
&- \partial_j L_m^n(1)R_m^k(2)L_j^i(1)] + i\tau_l D_{li}(A_2)D_{nk}(A_1^\dagger A_2) \\
&\times 2[-R_j^i(2)L_m^n(1)R_m^k(2)\partial_j - \partial_j L_m^n(1)R_m^k(2)R_j^i(2)]\big)K. \quad (A12)
\end{aligned}$$

The third term:

$$\begin{aligned}
-6K^\dagger D_j[A_i, A_j]D_iK &= -3K^\dagger[-\tfrac{1}{4}\{[L_j^l(1)L_j^l(1) - R_j^l(2)R_j^l(2)][L_i^k(1)L_i^k(1) \\
&- R_i^k(2)R_i^k(2)] - [L_j^k(1)L_i^k(1) - R_j^k(2)R_i^k(2)] \\
&\times [L_j^l(1)L_i^l(1) - R_j^l(2)R_i^l(2)]\} \\
&- i\tau_m D_{mk}(A_1)(\epsilon_{knl}\partial_j L_i^n(1)L_j^l(1)\partial_i \\
&- \tfrac{1}{2}\{L_i^k(1)[L_j^l(1)L_j^l(1) - R_j^l(2)R_j^l(2)] \\
&- L_j^k(1)[L_i^l(1)L_j^l(1) - R_i^l(2)R_j^l(2)]\}\partial_i \\
&+ \tfrac{1}{2}\partial_j\{L_i^k(1)[L_i^l(1)L_j^l(1) - R_i^l(2)R_j^l(2)] \\
&- L_j^k(1)[L_i^l(1)L_i^l(1) - R_i^l(2)R_i^l(2)]\}) \\
&- i\tau_m D_{mk}(A_2)(\epsilon_{knl}\partial_j R_i^n(2)R_j^l(2)\partial_i \\
&+ \tfrac{1}{2}\{R_i^k(2)[L_j^l(1)L_j^l(1) - R_j^l(2)R_j^l(2)] \\
&- R_j^k(2)[L_i^l(1)L_j^l(1) - R_i^l(2)R_j^l(2)]\}\partial_i \\
&- \tfrac{1}{2}\partial_j\{R_i^k(2)[L_i^l(1)L_j^l(1) - R_i^l(2)R_j^l(2)] \\
&- R_j^k(2)[L_i^l(1)L_i^l(1) - R_i^l(2)R_i^l(2)]\}) \\
&+ i\tau_m D_{mk}(A_1)D_{lr}(A_1^\dagger A_2)\{\epsilon_{kln}\partial_j L_j^n(1)R_i^r(2)\partial_i \\
&- \tfrac{1}{2}L_j^k(1)[L_i^l(1)R_j^r(2) - L_j^l(1)R_i^r(2)]\partial_i \\
&- \tfrac{1}{2}\partial_j L_i^k(1)[L_j^l(1)R_i^r(2) - L_i^l(1)R_j^r(2)]\} \\
&+ i\tau_m D_{mk}(A_2)D_{lr}(A_1^\dagger A_2)\{\epsilon_{krn}\partial_j L_i^l(1)R_j^n(2)\partial_i \\
&+ \tfrac{1}{2}R_j^k(2)[L_i^l(1)R_j^r(2) - L_j^l(1)R_i^r(2)]\partial_i \\
&+ \tfrac{1}{2}\partial_j R_i^k(2)[L_j^l(1)R_i^r(2) - L_i^l(1)R_j^r(2)]\} \\
&+ D_{pr}(A_1^\dagger A_2)[\epsilon_{klp}L_i^k(1)L_j^l(1)R_j^r(2)\partial_i \\
&+ \epsilon_{klr}L_j^p(1)R_i^k(2)R_j^l(2)\partial_i + \epsilon_{klp}\partial_j L_i^k(1)L_j^l(1)R_i^r(2) \\
&+ \epsilon_{klr}\partial_j L_i^p(1)R_i^k(2)R_j^l(2)] + D_{kr}(A_1^\dagger A_2)D_{lt}(A_1^\dagger A_2) \\
&\times \tfrac{1}{4}[L_i^k(1)R_j^r(2) - L_j^k(1)R_i^r(2)] \\
&\times [L_j^l(1)R_i^t(2) - L_i^l(1)R_j^t(2)]]K. \quad (A13)
\end{aligned}$$

D. WZW term

The last term in Eq. (16), which is proportional to the baryon number density, B^0, comes from the WZW term. B^0 can be written in terms of L and R as

$$\begin{aligned}
B^0 = -\frac{1}{12\pi^2}\{&\epsilon^{ijk}\epsilon_{pqr}[L_i^p(1)L_j^q(1)L_k^r(1) \\
&- R_i^p(2)R_j^q(2)R_k^r(2)] + D_{ps}(A_1^\dagger A_2)3\epsilon^{ijk} \\
&\times [\epsilon_{qrs}L_i^p(1)R_j^q(2)R_k^r(2) - \epsilon_{qrp}R_i^s(2)L_j^q(1)L_k^r(1)]\}. \quad (A14)
\end{aligned}$$

APPENDIX B: SPIN-ISOSPIN PROJECTION

In this appendix, we show the procedure of spin-isospin projection in detail. The first step is to replace the adjoint matrix in the Lagrangian with Wigner D function. For example,

$$D_{mi}(A_1) \to D^1_{MM'_i}(\Omega_1), \tag{B1}$$

$$D_{ij}(A_1^\dagger A_2) = D_{mi}(A_1)D_{mj}(A_2) \to D^1_{MM'_i}(\Omega_1)D^1_{MM'_j}(\Omega_2). \tag{B2}$$

Next, we sandwich them between the relevant nucleon wave function, Eq. (23), and integrate the Euler angle. We demonstrate the procedure by taking two examples below. For later use, we display two basic formulas for D functions,

$$D^{J_1}_{M_1M'_1}(\Omega)D^{J_2}_{M_2M'_2}(\Omega) = \sum_{J=|J_1-J_2|}^{J_1+J_2} \langle J_1J_2M_1M_2|JM\rangle\langle J_1J_2M'_1M'_2|JM'\rangle D^J_{MM'}(\Omega), \tag{B3}$$

$$\int d\Omega D^{J_1*}_{M_1M'_1}(\Omega)D^{J_2}_{M_2M'_2}(\Omega) = \frac{8\pi^2}{2J_1+1}\delta(J_1,J_2)\delta(M_1,M_2)\delta(M'_1,M'_2). \tag{B4}$$

A. Example 1

In this subsection, we calculate the matrix element $\langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)|I_3J_3\rangle$ as an example. Here and hereafter, we denote the nucleon state with the third component of the isospin I_3 and that of the spin J_3 simply by $|I_3J_3\rangle$. This matrix element is given by the following integral,

$$\langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)|I_3J_3\rangle \equiv \frac{1}{(2\pi)^2}(-1)^{2I_3+1}\int d\Omega D^{1/2}_{-I_3-J_3}(\Omega)^* D^1_{M_1M'_1}(\Omega)D^{1/2}_{-I_3J_3}(\Omega). \tag{B5}$$

Here, from Eq. (B3), we note that

$$\begin{aligned} D^1_{M_1M'_1}(\Omega)D^{1/2}_{-I_3J_3}(\Omega) &= \sum_{J=1/2}^{3/2}\left\langle 1\,\frac{1}{2}M_1 - I_3\right|\left. JM\right\rangle\left\langle 1\,\frac{1}{2}M'_1J_3\right|\left. JM'\right\rangle D^J_{MM'}(\Omega)\\ &= \left\langle 1\,\frac{1}{2}M_1 - I_3\right|\left.\frac{1}{2}M\right\rangle\left\langle 1\,\frac{1}{2}M'_1J_3\right|\left.\frac{1}{2}M'\right\rangle D^{1/2}_{MM'}(\Omega)\\ &\quad + \left\langle 1\,\frac{1}{2}M_1 - I_3\right|\left.\frac{3}{2}M\right\rangle\left\langle 1\,\frac{1}{2}M'_1J_3\right|\left.\frac{3}{2}M'\right\rangle D^{3/2}_{MM'}(\Omega). \end{aligned} \tag{B6}$$

We substitute this equation into Eq. (B5) and integrate the Euler angle using Eq. (B4). The result reads as follows,

$$\begin{aligned} \langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)|I_3J_3\rangle &= \left\langle 1\,\frac{1}{2}M_1 - I_3\right|\left.\frac{1}{2}M\right\rangle\left\langle 1\,\frac{1}{2}M'_1J_3\right|\left.\frac{1}{2}M'\right\rangle\\ &\quad\times\delta(-I_3,M)\delta(-J_3,M')\\ &= (-1)^{1/2-I_3}\frac{\sqrt{2}}{3}\delta(0,M_1)\left[-\delta(-1,M'_1)\delta\left(\frac{1}{2},J_3\right)\right.\\ &\quad\left.+\delta(1,M'_1)\delta\left(-\frac{1}{2},J_3\right)\right]. \end{aligned} \tag{B7}$$

B. Example 2

The second example is the matrix element of two D functions: $\langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)D^1_{M_2M'_2}(\Omega)|I_3J_3\rangle$ given by

$$\begin{aligned} \langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)D^1_{M_2M'_2}(\Omega)|I_3J_3\rangle &= \frac{1}{(2\pi)^2}(-1)^{2I_3+1}\int d\Omega D^{1/2}_{-I_3-J_3}(\Omega)^* D^1_{M_1M'_1}\\ &\quad\times(\Omega)D^1_{M_2M'_2}(\Omega)D^{1/2}_{-I_3J_3}(\Omega). \end{aligned} \tag{B8}$$

Using Eq. (B3) two times, we obtain

$$\begin{aligned} D^1_{M_1M'_1}(\Omega)D^1_{M_2M'_2}(\Omega)D^{1/2}_{-I_3J_3}(\Omega) &= \langle 11M_1M_2|00\rangle\langle 11M'_1M'_2|00'\rangle D^{1/2}_{-I_3J_3}(\Omega)\\ &\quad+\langle 11M_1M_2|1M_3\rangle\langle 11M'_1M'_2|1M'_3\rangle\big(\big\langle 1\tfrac{1}{2}M_3 - I_3\big|\tfrac{1}{2}M\big\rangle\\ &\quad\times\big\langle 1\tfrac{1}{2}M'_3J_3\big|\tfrac{1}{2}M'\big\rangle D^{1/2}_{MM'}(\Omega)+\big\langle 1\tfrac{1}{2}M_3 - I_3\big|\tfrac{3}{2}M\big\rangle\\ &\quad\times\big\langle 1\tfrac{1}{2}M'_3J_3\big|\tfrac{3}{2}M'\big\rangle D^{3/2}_{MM'}(\Omega)\big)+\langle 11M_1M_2|2M_3\rangle\\ &\quad\times\langle 11M'_1M'_2|2M'_3\rangle D^2_{M_3M'_3}(\Omega)D^{1/2}_{-I_3J_3}(\Omega). \end{aligned} \tag{B9}$$

We substitute this equation into Eq. (B8) and carry out the integration of the Euler angle using Eq. (B4) to obtain

$$\begin{aligned} \langle I_3 - J_3|D^1_{M_1M'_1}(\Omega)D^1_{M_2M'_2}(\Omega)|I_3J_3\rangle &= \langle 11M_1M_2|1M_3\rangle\langle 11M'_1M'_2|1M'_3\rangle\left\langle 1\,\frac{1}{2}M_3 - I_3\right|\left.\frac{1}{2}M\right\rangle\\ &\quad\times\left\langle 1\,\frac{1}{2}M'_3J_3\right|\left.\frac{1}{2}M'\right\rangle\delta(-I_3,M)\delta(-J_3,M')\\ &= (-1)^{1/2-I_3}\frac{\sqrt{2}}{3}\frac{1}{2}[\delta(M_1,1)\delta(M_2,-1)\\ &\quad-\delta(M_1,-1)\delta(M_2,1)]\left\{[\delta(M'_1,1)\delta(M'_2,0)\right.\\ &\quad-\delta(M'_1,0)\delta(M'_2,1)]\delta\left(-\frac{1}{2},J_3\right)\\ &\quad-[-\delta(M'_1,-1)\delta(M'_2,0)+\delta(M'_1,0)\delta(M'_2,-1)]\\ &\quad\left.\times\ \delta\left(\frac{1}{2},J_3\right)\right\}. \end{aligned} \tag{B10}$$

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