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Exit doorway model for nuclear elastic breakup of weakly bound projectiles

M. S. Hussein[1,2] and R. Lichtenthäler[2]
[1]*Max-Planck-Institüt für Physik komplexer Systeme, Nöthnitzer Str. 38, D-01187, Dresden, Germay*
[2]*Instituto de Fisica, Universidade de São Paulo, C. P. 55318, 05315-970, São Paulo, SP, Brazil*
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We develop the exit doorway model for elastic breakup of loosely bound projectiles. We argue that this model could, in principle, supply an alternative simplified version of the continuum discretized coupled-channels (CDCC) model. We show that the cross section for elastic breakup can be generally written as a product of the cross section for inelastic excitation times a factor containing the excitation energy and width of the exit doorway and is generally bombarding energy dependent. The excitation energy of the exit doorway is identified with the Q value of the breakup channel. The width of the exit doorway is a measure of the energy range of the continuum that is discretized. We apply the theory to derive closed expressions for the nuclear breakup cross sections in the adiabatic limit using the Austern-Blair theory. We demonstrate the approximate validity of the scaling law that dictates that the nuclear breakup cross section scales linearly with the radius of the target. We also compare our results for the nuclear breakup cross section of ^{11}Be, ^{8}B, and ^{7}Be on several targets with recent CDCC calculation.

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I. INTRODUCTION

The breakup of nuclei is a common occurrence when the bombarding energy is high enough and/or the binding energies are sufficiently low. In the case of weakly bound nuclei the threshold for breakup is small and more so for bound unstable nuclei. The mechanism of breakup is assumed to consist of elongating the projectile, through the action of the interaction, which eventually leads to the production of two or more fragments. This interaction is composed of a short-range nuclear piece and a longer-range electromagnetic (EM) one.

Two distinct processes occur in the breakup reaction. The first is what is known as elastic breakup, where the target nucleus is left in its ground state. The second, commonly called inelastic breakup, the target nucleus may be excited and/or one of the fragments is captured by the target. This latter process is referred to as incomplete fusion. Several articles have been written on the this decomposition of the breakup cross section [1–3]. In the present article we concentrate our discussion on the elastic breakup of loosely bound nuclei. A lot of effort has been devoted in the past to the elastic breakup of the deuteron (see, e.g., Ref. [4]). The elastic breakup process is quite important as it supplies a mean to extract important structure information about the fragmenting nucleus. In particular, the knowledge of the electromagnetic component of the elastic breakup cross section can be used to obtain the $B(E\lambda)$ values of the exotic nucleus. To isolate the EM elastic breakup, also called electromagnetic dissociation cross section, one has to subtract from the data on elastic breakup the nuclear contribution.

A debate has been going on in the literature concerning the way the nuclear part of the elastic breakup cross section depends on the mass of the target nucleus that supplies the interaction. In most references [5–7], it is assumed that the dependence goes as the cubic root of the mass number. In Ref. [8], however, it is claimed that this dependence is more like linear! In a recent article [9], through a careful continuum discretized coupled-channels (CDCC) calculation, the former dependence ($A^{1/3}$) has been established, which corroborates the contention that the nuclear breakup cross section should follow the prediction of the Serber model [10].

It is interesting to compare the numerical CDCC calculation alluded to above with those of simpler analytical models. Specifically, the Austern-Blair adiabatic theory for inelastic scattering comes to mind. If one assumes that the breakup proceeds through a so-called exit doorway [11–13], then the process can be treated as an inelastic excitation. The idea of exit doorway has been used in the case of the influence of breakup on fusion [11,12] and in the excitation of giant resonances [13] with success. In a very recent article [14] a comparison of a preliminary CDCC calculation for the breakup cross section of the system ^{6}He+^{27}Al at low bombarding energies with a simple formula derived by us using the Austern-Blair model showed that such an idea is quite reasonable and encouraged us to pursue the matter further. We do this in the present article, where we fully develop the Austern-Blair model for the nuclear elastic breakup reaction cross section assumed to proceed through the excitation of an exit doorway [11–13].

II. EXIT DOORWAY MODEL OF ELASTIC BREAKUP

The exit doorway concept has been used in the development of reaction theories involving the the excitation of a doorway in the final state, in contrast to the conventional cases where such resonances are populated in the entrance channel [15]. In the breakup reactions of halo nuclei one may envisage that the process proceeds through the breakup exit doorway (dipole, quadrupole, etc.) into the continuum. The exit "doorway" here is not a resonance but rather a special threshold state. As such, the detailed description of the exclusive reaction, where the final channels are unspecified, will necessarily contain the full information about the exit doorway (its energy, width, etc.). This is the case that was encountered in the theory of the excitation of multiple giant resonances [13] and of the influence of the threshold "doorway" on the fusion of halo nuclei [11]. In the current article we will be content with the inclusive quantity of the integrated breakup cross section and the only reference to the exit doorway is made implicitly as a final state with complex excitation energy that has to be populated for breakup to occur. In an electromagnetic process, the exit doorway is defined simply as

$$|d_\mu\rangle = NO_\mu|0\rangle, \tag{1}$$

where the operator O_μ is the excitation operator (dipole, quadrupole, etc.) supplied by the EM field and $|0\rangle$ is the ground state of the halo nucleus. The factor N is a normalization constant. The above definition of the exit doorway is commonly used to describe, e.g., giant resonances. Here we generalize its use to describe the threshold excitation, which is coupled to the continuum. We also consider the operator O to represent a more general one containing both the EM and nuclear parts.

The full Hamiltonian that describes the colliding ions can be written as

$$H = H_0 + F, \tag{2}$$

where $H_0 = h_0 + K + V = h_0 + H^{(0)}$ is diagonal in open channel space, h_0 is the intrinsic part that describes the structure of the projectile and the target nuclei, K is the kinetic energy operator, and V is the optical potential that contains the complex nuclear plus the Coulomb parts. The operator F describes the coupling among the open channels.

The intrinsic Hamiltonian h_0, which for simplicity is taken here to represent the excitable projectile nucleus with the target considered structureless, is now written as

$$h_0 = |\phi_0\rangle E_0\langle\phi_0| + |d\rangle E_d\langle d| + \sum_i |i\rangle E_i\langle i| + \sum_i [|d\rangle\Delta_i\langle i| + |i\rangle\Delta_i^*\langle d|] + \sum_{ij}[|i\rangle\Omega_{ij}\langle j| + cc]. \tag{3}$$

The first three terms on the right-hand side above refer to the ground, exit doorway, and discretized continuum states, respectively. The fourth term couples the doorway to the discretized continuum states, and the last term represents the continuum-continuum coupling. If we remove the doorway from the above we get the CDCC intrinsic Hamiltonian

$$h_0 = |\phi_0\rangle E_0\langle\phi_0| + \sum_i |i\rangle E_i\langle i| + \sum_i [|\phi_0\rangle\Delta_i\langle i| + |i\rangle\Delta_i^*\langle\phi_0|] + \sum_{ij}[|i\rangle\Omega_{ij}\langle j| + cc]. \tag{4}$$

The exit doorway modulated CDCC Hamiltonian, Eq. (2), is our subject of study here. A full development of this new CDCC will be left for a future work. Here we concentrate our effort on understanding the consequence of reaching the breakup continuum from the entrance channel only through the exit doorway $|d\rangle$. For this purpose we ignore the last term in Eq. (2) and remind ourselves that, whereas $|\phi_0\rangle$ and $|i\rangle$ are eigenstates of h_0, $|d\rangle$ is not.

The full doorway-modulated CDCC equations can be obtained as follows. The full Schroedinger equation of the colliding system is

$$[E - (H_0 + F)]|\psi\rangle = 0, \tag{5}$$

which when projected onto the different channels gives

$$\left(E - E_0 - H_0^{(0)}\right)\psi_0^+ = \sum_i F_{0i}\psi_i^+ \tag{6}$$

$$\left(E - E_i - H_i^{(0)}\right)\psi_i^+ = F_{i0}\psi_0^+. \tag{7}$$

We now invoke the exit doorway hypothesis,

$$F_{0i} = F_{0d}\alpha_{di}^* \; F_{i0} = F_{d0}\alpha_{id}^* \tag{8}$$

The overlaps α_{di} and α_{id}^* and can be easily obtained from Eq. (2)(without the last term) [11,13],

$$|\alpha_{di}|^2 = (\Gamma^\downarrow/2\pi)\big/\left[(E_i - E_d)^2 + \left(\Gamma_d^\downarrow/2\right)^2\right] \tag{9}$$

where $\Gamma^\downarrow$, the exit doorway spreading width describing its average coupling to the continuum states of the projectile, is related to the Δ_i factors through

$$\Gamma^\downarrow = 2\pi\overline{|\Delta_i|^2}\rho, \tag{10}$$

where $\overline{|\Delta_i|^2}$ is an average value and ρ is the average density of discretized continuum states in the vicinity of d. Clearly the need to the continuum-continuum coupling terms would be very important if exclusive cross sections are to be calculated, because through them (and through the doorway) the elastic channels coupling to the breakup channel continuum can be fully acounted for. Including the c-c coupling term, would result in a more complicated expression for $|\alpha_{di}|^2$ than that of Eq. (9).

Equations (5) and (6) can be recast into the following, after setting $E_0 = 0$ and $F_{ij} = 0$,

$$\left[E - H_0^{(0)}\right]\psi_0^{(+)} = \alpha_{di}F_{0d}\psi_i^{(+)} \tag{11}$$

$$\left[E - E_i - H_i^{(0)}\right]\psi_i^{(+)} = \alpha_{id}^*F_{d0}\psi_0^{(+)}. \tag{12}$$

The breakup cross section, within the exit doorway model then becomes

$$\sigma_{\text{bup}} = \sum_i \frac{k_i}{k_0}|\alpha_{di}|^2|\langle\psi_i^{(-)}|F_{d0}|\psi_0^{(+)}\rangle|^2 \tag{13}$$

$$\approx \frac{|k_d|}{k_0}|\langle\psi_d^{(-)}|F_{d0}|\psi_0^{(+)}\rangle|^2, \tag{14}$$

where the sum over i has been performed by appropriate contour integration over E_i. Note that the Q value in $\psi_d^{(-)}$ is complex owing to the nonzero width of the exit dooway whose energy is $E_d - i\Gamma_d^\downarrow/2$. A simple way to see how the complex Q value arises is to eliminate $\psi_i^{(+)}$ in Eq. (5) in favor of $\psi_0^{(+)}$ by employing Eq. (6), which gives $\psi_i^{(+)} = [1/E - E_i - H_i^{(0)} + i\epsilon]F_{i0}\psi_0^{(+)}$. With this Eq. (5) becomes $\{E - E_0 - H_0^{(0)} - \sum_i F_{0i}\,[1\,/\,E - E_i - H_i^{(0)} + i\epsilon]F_{i0}\}\,\psi_0^{(+)} = 0$. With the exit doorway hypothesis, the polarization potential contribution, $\sum_i F_{0i}[1/E - E_i - H_i^{(0)} + i\epsilon]F_{i0}$ becomes

$$\sum_i F_{0d}\frac{\Gamma^{\downarrow}/2\pi}{(E_i-E_d)^2+(\Gamma_d^{\downarrow}/2)^2}\frac{1}{E-E_i-H_i^{(0)}+i\epsilon}F_{d0}$$
$$\approx F_{0d}\frac{1}{E-(E_d-i\Gamma_d^{\downarrow}/2)-H_d^{(0)}+i\epsilon}F_{d0}.$$

This suggests defining the exit doorway scattering-wave function by setting $H_i^{(0)} = H_d^{(0)}$ such that Eqs. (10) and (11) become

$$\left(E-H_0^{(0)}\right)\psi_0^{(+)} = F_{0d}\psi_d^{(+)} \qquad (15)$$

$$\left[E-(E_d-i\Gamma_d^{\downarrow}/2)-H_d^{(0)}\right]\psi_d^{(+)} = F_{d0}\psi_0^{(+)}. \qquad (16)$$

The "inelastic" cross-section is thus given by Eq. (13) above with the aforementioned proviso that the Q value of the excited state is complex. The width of this Q value is a measure of the continuum contribution to the coupling.

At this point we comment on the inclusion of the continuum-continuum coupling, namely the last term in Eq. (3). In this situation the amplitudes α_{di} are obtained by matrix diagonalization and, among other things, the resulting overlap probability $|\alpha_{di}|^2$ deviates from the Breit-Wigner form of Eq. (8). A possible form that may incorporate some of the c-c effects is a Lorentzian:

$$|\alpha_{\rm di}|^2 = \frac{2}{\pi}\frac{\Gamma_d^{\downarrow}E_i^2}{\left(E_i^2-E_d^2\right)^2+\Gamma_d^{\downarrow 2}E_i^2}$$

The above form results in an equation for $\psi_d^{(+)}$ with a modified form factor that depends on the position and width of the exit doorway $\tilde{V}_{d0} \approx f(E_d,\Gamma_d^{\downarrow})V_{d0}$, where $f(E_d,\Gamma_d^{\downarrow})$ is generally complex. Accordingly the cross section would be $\sigma^{\rm bup} = |f(E_d,\Gamma_d^{\downarrow})|^2\sigma_{\rm DWBA}$. In the limiting case of $\Gamma_d \ll E_d$, the factor $f(E_d,\Gamma_d^{\downarrow})$ is approximately given by $(1-\frac{i\Gamma_d^{\downarrow}}{2E_d})^{1/2}$, resulting a cross section given by $\sigma \approx \sqrt{1+(\frac{\Gamma_d^{\downarrow}}{2E_d})^2}\cdot\sigma_{\rm DWBA}$. In the case of coupling to the breakup continuum considered here, the other limit, $\Gamma_d^{\downarrow} \gg E_d$ is more appropriate, as E_d is roughly given by the Q value of the breakup ($\leqslant$1 MeV), whereas $\Gamma_d^{\downarrow}$ measures the extent in continuum excitation the discretization is performed ($\approx$10 MeV).

The function $f(E_d,\Gamma_d^{\downarrow})$ can be calculated in such a situation, and the result is to add to the factor $\frac{i\Gamma_d^{\downarrow}}{2E_d}$ a contribution arising from the integration along the imaginary excitation energy axis of the exit doorway Green function weighted by the Lorentzian (see Ref. [12], Eq. A10). The existence of this integral in $|f(E_d,\Gamma_d^{\downarrow})|^2$ renders this factor energy dependent. For simplicity we leave this energy-dependent factor out in this article. The important point we are making here is that a DWBA calculation with complex excitation energy in the final state, and with a form factor of the type $f(E_d,\Gamma_d^{\downarrow})V_{d0}$, should be an adequate candidate to treat the elastic breakup process. Such exit doorway treatment of the elastic breakup of the deuteron has been reported earlier in Refs. [16,17], following a hitherto not-so-well-known treatment of the photodistintegration of the deuteron advanced by Schwinger [18].

The halo nucleus ^{11}Be is in fact quite similar to the deuteron insofar as the dipole response is concerned; the existence of a threshold peak of a predominantly dipole nature in the dissociation spectrum. This peak is then taken to be the exit doorway [16]. Of course, in contrast to the deuteron, ^{11}Be also exhibits the usual giant dipole resonance at about 20 MeV of excitation enegy. It was argued [16,19] that in such one-neutron halo nuclei, one may speak about the coherent production of these "soft" and "hard" dipole modes in the context of what might be called a nuclear instance of Schödinger cat state in halo nuclei [16].

In the following we take the exit doorway to be excited states of different multipolarities, defined as in Eq. (1), and with complex energies and use the Austern-Blair sudden/adiabatic theory [20,21]. We employ the distorted-wave Born approximation (DWBA) for $\psi_0^{(+)}$ and $\psi_d^{(-)}$. Within the adiabatic approximation, the complex Q value in the final channel distorted wave is ignored, and the only reference to the exit doorway in what follows resides in the important factor

$$|f(E_d,\Gamma_d^{\downarrow})|^2 = \sqrt{1+\left(\frac{\Gamma_d^{\downarrow}}{2E_d}\right)^2} \qquad (17)$$

that multiplies the cross section.

III. THE AUSTERN-BLAIR THEORY

The elastic breakup cross section and its dependence on the target masss can be analyzed within the DWBA. In Ref. [4], the DWBA was used to calculate the proton spectrum in the deuteron elastic breakup in combined EM and nuclear field of a gold target. In Ref. [16] the exit doorway model was used to study some features of the deuteron photodissociation cross section (see discussion in the previous section). Here we use the DWBA to obtain the angle- and energy-integrated total nuclear elastic breakup cross section for several loosely bound nuclei. If we treat the elastic breakup as an inelastic multipole process, the amplitude $T_{\rm LM} = \langle\psi_d^{(-)}|F_{d0}|\psi_0^{(+)}\rangle$ would look like

$$T_{\rm LM} = \sum_{l_f}(2l_f+1)^{1/2}(i)^{l_i-l_f}\langle l_f L;00|l_i 0\rangle$$
$$\langle l_f L;-MM|l_i 0\rangle R^L_{l_f,l_i}(k_f,k_i)$$
$$e^{i\sigma_{l_f}(k_f)+\sigma_{l_i}(k_i)}Y_{l_f,-M}(\theta,0). \qquad (18)$$

The unpolarized cross section of the dipole transition is then obtained from the expression

$$d\sigma_L/d\Omega = \frac{\mu}{(2\pi\hbar^2)^2}\frac{k_f}{k_i}\sum_{M=-L}^{M=L}|T_{\rm LM}|^2. \qquad (19)$$

The radial integrals $R^L_{l_f,l_i}(k_f,k_i)$ for pure nuclear excitation are given by

$$R^L_{l_f,l_i}(k_f,k_i) = (4\pi/k_fk_i)\int dr f_{l_f}(k_f,r)F_L(r)f_{l_i}(k_i,r), \qquad (20)$$

where the form factors $F_L(r)$ are given by the following expressions for the monopole $L=0$, dipole $L=1$, and quadrupole excitations $L=2$ [22,23],

$$F_0(r) = -\delta_0^{(N)}\left[3V(r) + r\frac{dV(r)}{dr}\right] \tag{21}$$

$$F_1(r) = -\delta_1^{(N)}\left(\frac{3}{2}\right)\left(\frac{\Delta R_P}{R_P}\right)\left[\frac{dV(r)}{dr} + \left(\frac{R}{3}\right)\frac{d^2V(r)}{dr^2}\right] \tag{22}$$

$$F_2(r) = -\delta_2^{(N)}\frac{dV(r)}{dr}, \tag{23}$$

with $\Delta R_P = Rn - Rp$ being the difference between the rms radii of the neutron and proton distributions of the projectile and $V(r)$ the elastic-scattering channel optical potential. The quantities Rn and Rp can be extracted from the analysis of Refs. [24–26]. In Ref. [26] a power expansion in ΔR_P was employed in the analysis of α-inelastic scattering from neutron skin nuclei.

In the adiabatic limit, $k_i = k_f = k$, and for large orbital angular momenta, $l_f = l_i = l$, the radial integral can then be evaluated in closed form following the procedure of Austern and Blair [20,21]. For the dipole and quadrupole cases we have

$$R_{l,l}^{(1)}(k) = -i\delta_1^{(N)}(\pi\hbar^2/\mu)\left(\frac{3}{2}\right)\left(\frac{\Delta R_P}{R_P}\right) \times \left[\frac{dS_l^{(N)}(k)}{dl} + \left(\frac{R}{3}\right)\frac{d^2S_l^{(N)}(k)}{dl^2}\right] \tag{24}$$

$$R_{l,l}^{(2)}(k) = -i\delta_2^{(N)}(\pi\hbar^2/\mu)\frac{dS^{(N)}(k)}{dl}, \tag{25}$$

where $\delta_L^{(N)}$ is the nuclear deformation length given by $\delta_L^{(N)} = \beta_L^{(N)}R_P$ with $\beta_L^{(N)}$ being the nuclear deformation parameter and R_P is the radius of the excited projectile.

The above expression for the radial integrals can be associated with the nuclear elastic breakup radial integral. Thus we can obtain analytical expression for the integrated nuclear breakup cross section by simply integrating the cross-section formula, Eq. (2). In performing this calculation the angular momentum coupling coefficients are evaluated exactly and the sum over l_i can be performed by putting the Coulomb phase shifts both as functions of $l_f \equiv l$. The amplitude of Eq. (13) is given now by

$$T_{\mathrm{LM}} = i\sqrt{2}\sum_{l=0}^{\infty}(2l+1)^{1/2}R_{l,l}^{L}(k)e^{2i\sigma_l(k)}Y_{l,-M}(\theta,0). \tag{26}$$

with the condition that $T_{\mathrm{LM}} = 0$ if $L+M$ is odd. The integrated pure nuclear breakup cross section containing dipole and quadrupole contributions then becomes the following:

$$\sigma^{\mathrm{bup}} = N_{\mathrm{DW}}|f(E_d,\Gamma_d^{\downarrow})|^2\left\{[\delta_1^{(N)}]^2\left(\frac{3}{2}\right)^2\left(\frac{\Delta R_P}{R_P}\right)^2 + [\delta_2^{(N)}]^2\right\} \times \sum_{l=0}^{\infty}(2l+1)\left|\frac{dS^{(N)}(k)}{dl}\right|^2, \tag{27}$$

where $\delta_L^{(N)}$ is the nuclear deformation length given by $\delta_L^{(N)} = \beta_L^{(N)}R_P$ with $\beta_L^{(N)}$ being the nuclear deformation parameter and R_P is the radius of the excited projectile and where terms proportional to the second derivative of $S_l^{(N)}(k)$ have been dropped and the usual, energy-dependent, DWBA normalization coefficient, N_{DW}, is indicated.

A simple estimate of the above formula can be made by approximating the sum in l by an integral in $\lambda = l + 1/2$:

$$\sum_{l=0}^{\infty}(2l+1)\left|\frac{dS^{(N)}(k)}{dl}\right|^2 \rightarrow \int_0^{\infty}2\lambda\left|\frac{dS^{(N)}(k)}{d\lambda}\right|^2 d\lambda = I. \tag{28}$$

Assuming a real nuclear S matrix that depends on λ through $[1+\exp(\lambda-\Lambda)/\Delta]^{-1}$ then the derivative of S would peak around the grazing angular momentum Λ with a width given by Δ. The integral (28) is then obtained as $I = \frac{\Lambda}{3\Delta}$ for $\Lambda/\Delta \gg 1$. Using $\Lambda = kR$ $\Delta = ka$, with $R = r_{0p}A_p^{1/3} + r_{0t}A_t^{1/3}$ and a being the diffuseness of the optical potential we find the simple formula for σ:

$$\sigma^{\mathrm{bup}} = N_{\mathrm{DW}}|f(E_d,\Gamma_d^{\downarrow})|^2 \times \left\{[\delta_1^{(N)}]^2\left(\frac{3}{2}\right)^2\left(\frac{\Delta R_P}{R_P}\right)^2 + [\delta_2^{(N)}]^2\right\}\frac{R}{3a}. \tag{29}$$

The quantity $|f(E_d,\Gamma_d^{\downarrow})|^2$ in the above two equations is a constant normalization factor that depends on the exit doorway nature of the excited state and can be calculated following the work of Ref. [12] (see discussion above). In the application to follow, we introduce the following notation for the cross section:

$$\sigma^{\mathrm{bup}} = c\left\{[\delta_1^{(N)}]^2\left(\frac{3}{2}\right)^2\left(\frac{\Delta R_P}{R_P}\right)^2 + [\delta_2^{(N)}]^2\right\}\frac{R}{3a}, \tag{30}$$

where we have introduced the overall normalization c,

$$c = N_{\mathrm{DW}}|f(E_d,\Gamma_d^{\downarrow})|^2. \tag{31}$$

It is clear that σ^{bup} depends linearly on the radius of the target and, more importantly, on the square of the nuclear dipole and quadrupole deformation lengths. Thus, the $A_T^{1/3}$ dependence is established.

Several models have been developed for the calculation of the low-lying multipole strength. Semianalytical, albeit realistic, models based on the use of the asymptotic normalization coefficient of the bound-state wave function of the loosely bound nucleus, in conjunction with the separation energy of the removed nucleon and on its scattering length, have been shown to account well for $\frac{dB(E\lambda)}{dE}$ distributions for several one-nucleon halo nuclei [27,28]. In the calculation to follow, however, we use the cluster model to calculate deformation lengths for the different multipolarities [29–31], which gives a bit larger values of $B(E\lambda)$, and correspondingly larger deformation lengths, than those of Refs. [27,28]. The cluster model assumes that the projectile is composed of two clusters, a core of mass and charge A_c and Z_c and a "valence" particle with A_b and Z_b. The separation energy is denoted by Q, the Q value of the breakup. Calling the spectroscopic factor of finding the cluster configuration in the ground state of the projectile, S one obtains the following expression for the distribution of $B(E\lambda)$ in the excitation energy E_x [29,31].

$$\frac{dB(E\lambda)}{dE_x} = SN_0^2\frac{2^{\lambda-1}}{\pi^2}(\lambda!)^2(2\lambda+1)\left(\frac{\hbar^2}{\mu_{cb}}\right)^{\lambda} \times \frac{Q^{1/2}(E_x-Q)^{\lambda+1/2}}{E_x^{2\lambda+2}} \times \left[\frac{Z_bA_c^{\lambda}+(-1)^{\lambda}Z_cA_b^{\lambda}}{A_p^{\lambda}}\right]^2 e^2, \tag{32}$$

where N_0 is normalization factor that takes into account the finite range, r_0 of the $c+b$ potential. The latter is assumed to be such as to give a Yukawa type wave function at large distances, $\psi_{bc}(r) = N_0\sqrt{K/(2\pi)}\frac{e^{-Kr}}{r}$ with $K=\sqrt{2\mu_{bc}Q/\hbar^2}$ and $N_0 = \frac{e^{Kr_0}}{\sqrt{1+Kr_0}}$. It is easy to obtain $B(E\lambda)$ by simply integrating of Eq. (32) and employing the expression:

$$\int_0^{\infty}\frac{y^{\lambda+\frac{1}{2}}}{(y+1)^{2\lambda+2}}dy = \frac{(-)^{2\lambda+3}\pi}{(2\lambda+1)!\sin\left[\left(\lambda+\frac{3}{2}\right)\pi\right]} \times \prod_{k=1}^{2\lambda+1}\left(\lambda+\frac{3}{2}-k\right), \tag{33}$$

We get for the cluster-model deformation lengths δ_1^2 and δ_2^2 the following:

$$\left[\delta_1^{(N)}\right]^2 = \left(\frac{2\pi}{3}\frac{A_P}{Z_PN_P}\right)^2\frac{B(E1)}{e^2} = N_0^2S\left(\frac{2\pi A_P}{3Z_PN_P}\right)^2 \times \frac{3}{16\pi}\frac{\hbar^2}{\mu_{bc}}\left(\frac{A_cZ_b-A_bZ_c}{A_P}\right)^2\frac{1}{Q}, \tag{34}$$

$$\left[\delta_2^{(N)}\right]^2 = \left(\frac{4\pi}{3Z_PR_P}\right)^2\frac{B(E2)}{e^2} = N_0^2S\left(\frac{4\pi}{3Z_PR_P}\right)^2 \times \frac{5}{32\pi}\left(\frac{\hbar^2}{\mu_{bc}}\right)^2\left(\frac{A_c^2Z_b+A_b^2Z_c}{A_P^2}\right)^2\frac{1}{Q^2}, \tag{35}$$

where $p(=b+c)$ refers to the projectile.

For our three nuclei discussed here, we have ^{11}Be $=$ ^{10}Be $+ n$, ^{8}B $=$ ^{7}Be $+ p$ and ^{7}Be $=$ ^{4}He $+$ ^{3}He, which define their cluster character, with the corresponding breakup Q values, 0.504, 0.137, and 1.587 MeV. The factor N_0^2S could be related to the asymptotic normalization coefficient (ANC) of the bound state wave function and is taken as a parameter to be adjusted so as to account for the experimentally known $B(E\lambda)$. We have used for the matter radii the values (in fm) $R_P = 2.90\pm0.05, 2.50\pm0.04,$ and 2.31 ± 0.02 for ^{11}Be, ^{8}B, and ^{7}Be, respectively. These values were collected from Refs. [32,33].

Simple estimate of $\frac{\Delta R_P}{R_P}$ can be obtained from Refs. [22,23], who gave $\frac{\Delta R_P}{R_P} \approx \frac{|N_P^{1/3}-Z_P^{1/3}|}{A_P^{1/3}}$. In Table I we present the results of the deformation lengths obtained using the cluster model of formulas (34) and (35). The value of δ_2^N for ^{8}B obtained from Eq. (35) is way too high (21.58 fm). This is due to the neglect, in the cluster model (originally developed for neutron-halo nuclei), of the Coulomb repulsion effects between the core and the halo proton. We have therefore opted to employ an upper value for this deformation length in this nucleus to conform to the accepted wisdom that it has to be within the confines of the value of the radius. The value we cite in the table is 2.5 fm. For ^{11}Be, we used $B(E1) = 1.05\pm0.06\ e^2$fm^2 [5] and we get $[\delta_1^{(N)}]^2 = 0.71\pm0.04$ fm^2. Further, $\delta_2^{(N)} = 1.27\pm0.25$ fm from the same reference. The factor $c = N|f(E_d,\Gamma_d^{\downarrow})|^2$ in Eq. (30) normalizes the cross section.

TABLE I. Deformation lengths for the ^{7}Be, ^{8}B, and ^{11}Be projectiles. The deformation lengths for ^{7}Be and ^{8}B have been calculated using formulas (34) and (35) using $N_0^2S = 1$. For the ^{11}Be the $\delta_1^{(N)}$ and $\delta_2^{(N)}$ (in units of fm) are the values from Ref. [5].

Projectile	$\delta_1^{(N)}$	$\delta_2^{(N)}$	$\left(\frac{\Delta R}{R}\right)^2$	c
^{7}Be	0.33	1.58	0.00576	0.61
^{8}B	1.91	2.5	0.0179	0.55
^{11}Be	0.84(2)	1.27(25)	0.0214	2.01

In Fig. 1 we compare our results of Eq. (30) using $a = 0.65$ fm and $r_{0t} = 1.2$ fm with the CDCC calculation of Ref. [9] at $E_{\text{lab}} = 200A$ MeV for ^{11}Be, $44A$ MeV for ^{8}B, and $100A$ MeV for the weakly bound nucleus ^{7}Be. Clearly we underestimate the CDCC calculation. The reason resides in the neglect, in our model, of the higher-order channel-coupling terms alluded to above. We also show in Fig. 1 the comparison with the CDCC calculation for ^{8}B and ^{7}Be. Clearly the scaling law is better obeyed in the "normal" nucleus ^{7}Be as has already been discussed in Ref. [9]. The value of the factor c is of the order of the unity for the "normal" nuclei ^{7}Be and for the ^{8}B. For the ^{11}Be breakup a higher normalization is required, which is due in part to higher-order effects that are not accounted for

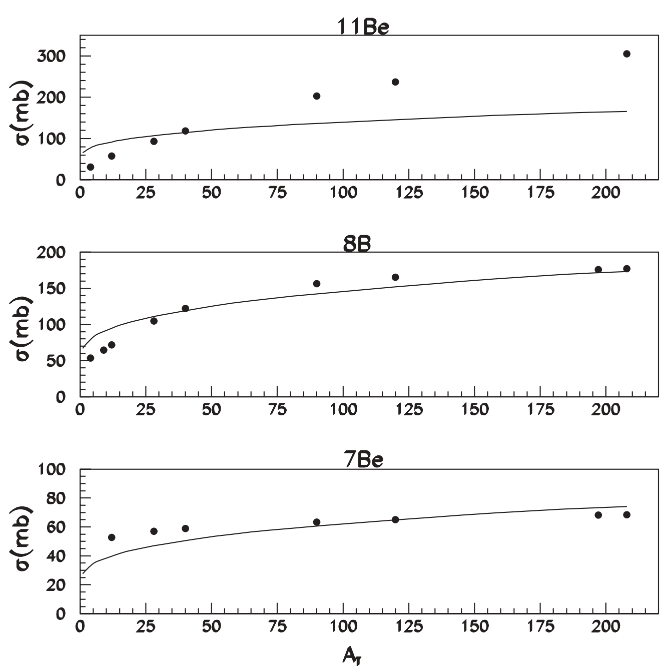

FIG. 1. CDCC calculations for the nuclear breakup (dots) as a function of the target mass, compared to the results of Eq. (30). See text for details.

by our DWBA description and because of the larger-than-unity value of the exit doorway factor $|f(E_d,\Gamma_d^{\downarrow})|^2$ [12].

Equation (30) can be rewritten for the ^{11}Be as

$$\sigma^{\rm bup} \approx 17(R_P + R_T)(\rm mb). \tag{36}$$

The above expression for $\sigma^{\rm bup}$ is independent of the bombarding energy. The CDCC calculation shows a clear trend of a decreasing $\sigma^{\rm bup}$ with the bombarding energy. We trace this to the energy-dependent factor referred to above, c [12], that contains the exit doorway factor $|f(E_d,\Gamma_d^{\downarrow})|^2$ and to higher-order effects not accounted for by the DWBA (partly contained in the energy-dependent factor $N_{\rm DW}$). For the purpose of comparison with the CDCC results from Ref. [9], we rewrite Eq. (36) as:

$$\sigma^{\rm bup} = A(E) + B(E)A_T^{1/3}(\rm mb), \tag{37}$$

where the coefficients $A(E)$ and $B(E)$ are found to be system dependent. For halo nuclei, $A(E)$ was found to be negative, whereas it is positive for the normal nucleus ^{7}Be. Further, both coefficients are slowly decreasing functions of the bombarding energy. Our DWBA-based formula applied for the ^{11}Be, Eq. (36), has $A = 17R_P$ and $B = 17r_{0t}$. Neither coefficients are dependent on energy. In fact in the CDCC calculation of Ref. [9] the values of the coefficients $A(E)$ and $B(E)$ for the three cases studied above, namely for ^{11}Be ($E_{\rm lab} = 200A$ MeV), ^{8}B ($E_{\rm lab} = 44A$ MeV) and ^{7}Be ($E_{\rm lab} = 100A$ MeV) are, respectively (in units of mb), (−114, 70.6), (8.17, 30.7), and (43.8, 4.23), to be contrasted with our values from Eq. (30), (49.3, 20.4), (45.1, 21.65), and (18.04, 9.37).

IV. CONCLUSIONS

In this article we developed the exit doorway model of elastic breakup. The proper treatment of the continuum can be made by including several exit doorways that are coupled among themselves. This would take into account the continuum-continuum coupling effects. The obtained elastic breakup cross section within the DWBA theory has the general form $\sigma^{\rm bup} = |f(E_d,\Gamma_d^{\downarrow})|^2\sigma_{\rm DWBA}$ where the exit doorway factor $|f(E_d,\Gamma_d^{\downarrow})|^2$ is generally bombarding energy dependent. We have applied the model to elastic breakup of weakly bound projectiles. The obtained elastic breakup cross-section $\sigma^{\rm bup}$ exhibits the scaling law, albeit approximately especially in ^{11}Be as is the case in the CDCC calculation [9] and should serve to supply a simple mean for an estimate of the nuclear breakup contribution. The geometric dependence R/a in the expression, Eq. (30), is quite similar to the one obtained using the so-called free dissociation (FD) model [34,35]. This latter model gives $\sigma_{FD} = \frac{\pi}{12}(4\ln 2 - 1)\frac{R_T}{\kappa}$, where $\kappa = \frac{\sqrt{(2\mu Q)}}{\hbar}$ [35]. As such, the range of validity of our model is similar to that of the FD model. Our model has the advantage of relating the nuclear elastic breakup cross section to the deformation lengths of the fragmenting projectile.

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